Multi-scale optimization of the design of offshore wind farms

Cazzaro, Davide; Trivella, Alessio; Corman, Francesco; Pisinger, David

Published in: Applied Energy

Link to article, DOI: 10.1016/j.apenergy.2022.118830

Publication date: 2022

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Multi-scale optimization of the design of offshore wind farms

Davide Cazzaro a,d,*, Alessio Trivella b, Francesco Corman c, David Pisinger a

a DTU Management, Technical University of Denmark, Akademivej 358, DK-2800 Kgs. Lyngby, Denmark
b Industrial Engineering and Business Information Systems, University of Twente, 7500 AE Enschede, The Netherlands
c IVT-Institute for Transport Planning and Systems, ETH Zurich, 8093 Zurich, Switzerland
d Vattenfall BA Wind, Jupitervej 6, 6000 Kolding, Denmark

A R T I C L E    I N F O
Keywords:
Offshore wind farms
Wind energy
Integrated design
Area selection
Shape optimization

A B S T R A C T
The traditional optimization of a wind farm layout consisted of arranging the wind turbines inside a designated area. In contrast, the 2021 tender from the UK government, Offshore Wind Leasing Round 4 (“UK Round-4”), and upcoming bids only specify large regions where the wind farm can be built. This leads to the new challenge of selecting the wind farm shape and area out of a larger region to maximize its profitability. We introduce this problem as the “wind farm area selection problem” and present a novel optimization framework to solve it efficiently. Specifically, our framework combines three scales of design: (i) on a macro-scale, choosing the approximate location of the wind farm out of larger regions, (ii) on a meso-scale, generating the optimal shape of the wind farm, and (iii) on a micro-scale, choosing the exact position of the turbines within the shape. In particular, we propose a new constructive heuristic to choose the best shape of a wind farm at the meso-scale, which is scarcely studied in the literature. Moreover, while macro and micro-scales have already been investigated, our framework is the first to integrate them. We perform a detailed computational analysis using real-life data and constraints from the recent UK Round-4 tender. Compared to the best rectangular-shaped wind farm at the same location, our results show that optimizing the shape increases profitability by 1.1% on average and up to 2.8%, corresponding to 46 and 109 million Euro respectively.

1. Introduction

The offshore wind energy market is developing at a rapid pace. To reduce greenhouse gas emissions in 2030 by at least 55% compared to 1990, the European Union plans to increase its offshore wind energy from the current 12 GW of installed capacity to 60 GW in 2030, and to 300 GW in 2050 [1]. The United States aims at increasing its offshore wind energy capacity to 30 GW by 2030, and in Asia the trend is the similar with Japan planning to add 10 GW by 2030, South Korea 12 GW by 2030, and Taiwan 5.7 GW by 2025 [2].

Wind farms typically undergo a tendering process, organized by governments, in which the companies that are interested in developing, constructing, and operating the wind farms submit bids for a project. Based on the evaluation criteria of the tender, the government then selects the winning proposal. In Europe, in 2021 alone, Poland is tendering 5.9 GW of wind energy, Denmark 2.2 GW, France 1 GW (plus 250MW of floating wind energy), Germany 1 GW, the Netherlands 1.5 GW, and the United Kingdom (UK) 6 GW [3].

Wind projects used to be subsidized by the governments, and thus company interest was to bid for a lower subsidy than competitors. Due to this pressure and thanks to advances in wind turbine technology, wind park optimization, and economy of scale factors, the level of subsidy needed for wind farm projects to become profitable has steadily decreased in the last decade [4]. A turning point was reached in 2018 when Vattenfall won the Hollandse Kust Zuid wind farm in the Netherlands, the first subsidy-free offshore wind farm.

In February 2021, the “Offshore Wind Leasing Round 4” tender in the UK (henceforth simply “UK Round-4”) changed again the market landscape of offshore wind energy. UK Round-4 is indeed the first tender to allow for a “negative bidding”, where companies pay the UK government to win the rights to build the wind farms. This development has been heavily criticized, because it is feared that the additional costs sustained by wind farm developers will ultimately result in higher energy prices for end consumers [5]. Therefore, in this setting, it is evermore important to drive down the energy costs of wind farms.

Traditionally, the specific area where to build the wind farm was assigned by the government while the companies’ only choice was to place the individual wind turbines within this area. Existing wind farms present various shapes with different degrees of regularity [6,7], where

* Corresponding author at: DTU Management, Technical University of Denmark, Akademivej 358, DK-2800 Kgs. Lyngby, Denmark.
E-mail addresses: dacaz@dtu.dk (D. Cazzaro), a.trivella@utwente.nl (A. Trivella), francesco.corman@ivt.baug.ethz.ch (F. Corman), dapi@dtu.dk (D. Pisinger).

https://doi.org/10.1016/j.apenergy.2022.118830
Received 29 October 2021; Received in revised form 31 January 2022; Accepted 22 February 2022
Available online 15 March 2022
0306-2619/© 2022 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).
Starting with the UK Round-4, the tender rules only specify larger regions that can be used for new wind farms, leaving it up to the companies to decide the exact location and shape of the wind farm. Fig. 1 shows these regions for the UK Round-4, with upcoming tenders already following similar procedures. Since these regions are much larger than the size of a wind farm, the developers are faced with the new problem to select the best shape and area for the wind farm itself. We call this new optimization problem the wind farm area selection problem (WFASP), which explores the questions: “How to choose the area for the wind farm out of a larger region?” and “What is the best shape for a wind farm?” The objective is to maximize the profitability of the wind farm while satisfying the tender constraints, which may include a maximum covered area, a maximum wind farm capacity, a minimum energy density, and a maximum perimeter-to-area ratio for the wind farm shape.

Tackling this problem is challenging because elements from both a macro-scale (i.e., choosing a smaller area out of the larger regions) and a micro-scale (i.e., deciding the location of each wind turbine) affect the wind farm power production and its costs. Moreover, these elements need to be combined with the new “meso-scale” problem to define the shape of the wind farm, for which no decision framework exists to the best of our knowledge. In addition, there is no clear understanding of what is the optimal shape of a wind farm that maximizes its power production and minimizes its costs. Because it has only recently become relevant, the WFASP has been scarcely studied in the extant literature but indications are that the shape of a wind farm has a major impact on its power production, as shown by studies on simplified shapes such as squares [8] or rectangles [9]. Moreover, we expect this impact to grow larger as the size of modern wind farms increases.

To investigate the WFASP, we consider the UK Round-4 as a real-life case study, accounting for all its constraints. We then propose a novel multi-scale optimization framework that integrates three components (macro, meso, and micro-scale), and generates optimized wind farms satisfying all tender rules.

The first macro-scale component performs an initial screening to find the most promising areas out of the larger regions. This optimization phase of our framework is necessary because the four bidding regions of the UK Round-4 span an area that is significantly larger than the maximum one allowed for the wind farm. Specifically, after discretizing the regions into smaller cells, we use a rectangular wind farm with turbines placed on a regular grid to estimate the maximum profitability that a wind farm can obtain in that cell. The rectangular wind farm is evaluated for different orientations and side ratios, similarly to [9], and the best combination is selected for each cell of the bidding regions.

The second component of our framework tackles the meso-scale optimization and is the core of the WFASP. Using the information from the first phase, the task is to find the shape (i.e., boundary) of the new wind farm that leads to the highest production and lowest costs possible, which is complex since the shape of the wind farm is subject to a number of constraints. For the UK Round-4, these constraints include the following: the wind farm area must be contained within one of the larger regions, obstacles and existing wind farms must be avoided, the wind farm has a maximum extension, a maximum capacity, a minimum energy density, and a maximum perimeter-to-area ratio. We account for all these constraints in a novel constructive heuristic designed to find the optimal shape for the wind farm. At a high level, the idea is to “grow” the wind farm starting from a central seed position determined during the screening phase, such that the shape maximizes the power production and minimizes costs while satisfying all the constraints. At the end of its execution, the heuristic generates a feasible shape for the wind farm. To the best of our knowledge, this is the first attempt to determine the shape of a wind farm without relying on a simplified/predefined geometrical shape or through a manual approach. In the industry, this step is typically performed manually by experts, even though it is not clear what an optimal shape looks like.

Once the shape is generated, the third and final component of our framework optimizes the location of the wind turbines in the selected area to further improve the profitability of the park. Plenty of papers have investigated this micro-scale problem of placing the turbines, also known as micro-siting or wind farm layout optimization problem. In our
We tested our framework in an extensive computational study using the UK Round-4 tender and real data on wind, bathymetry, and cost parameters. We found that the solutions of our initial screening roughly match the same areas in the larger regions of the real projects that won the UK Round-4, highlighting the soundness of our macro-scale optimization phase. Moreover, our results suggest that the profitability of a wind farm is significantly affected by its shape. Specifically, our screening phase shows that a rectangular wind farm has a higher net present value (NPV) when it is oriented perpendicular to the dominant wind direction and when the ratio between sides of the rectangle is as high as the constraints allow, which is consistent with the literature on simplified geometries [9]. Nevertheless, our constructive heuristic for the meso-scale shows that the wind farm will choose a non-regular shape when it is free to grow, for example by preferring shallower waters, and that the energy density and perimeter-to-area constraints play a major role in shaping the area of the wind farm. After the micro-siting optimization, the wind farm with optimized shape generated by our method exhibits a higher NPV compared to the best rectangular wind farm in the same location. In particular, the NPV increases by 1.14% on average across eight instances that we have tested, and up to 2.76% in highly constrained areas. This improvement corresponds to an average and maximum NPV increase of about 46 and 109 million Euro (MEuro), respectively. Given these results, the developed methodology appears very promising to submit more competitive bids for upcoming wind energy projects, especially since solving the WFSAP efficiently will be critical from now onwards in the design of modern wind farms.

The remainder of this paper is organized as follows. In Section 2, we review the related literature and summarize the contributions of our research. In Section 3, we formally define the WFSAP, including objective function and constraints. In Section 4, we describe in detail the three steps of our optimization framework for the WFSAP. In Section 5, we present the results of our numerical study and discuss our findings and implications for practice. We conclude in Section 6.

2. Related works and novelty

We start by reviewing the problems faced at macro and micro-scale, which have been studied previously in the literature, before moving to the meso-scale.

At the macro-scale, the objective is to find the most promising areas for new wind farm developments. Studies have been conducted for onshore wind farms in Iran [11], the New York State [12], and Nigeria [13], where terrain configurations and wind site characteristics are used to screen a large region (often the entire country) to select the wind farm areas with the highest power production potential. Similar investigations have been carried out for offshore wind farms in South Korea [14], in Hong Kong [15], on the U.S. East Coast [16], and in the Netherlands [17]. The latter study includes a comprehensive amount of factors that affect the business viability of wind energy projects in relation to the areas used by the government in future tenders, such as wind site conditions, water depths, seabed soil conditions, electrical infrastructures (i.e., inter-array cables and export cable), and operation and maintenance costs. The first phase of our optimization framework involves a detailed screening of the locations inside the larger regions using factors consistent with the literature. However, we measured the performance of each location using rectangular-shaped wind farms with varying orientation and side ratio [9].

At the micro-scale, the layout optimization problem concerns with the placement of wind turbines within a predefined boundary. At this scale, it is critical to consider the wakes generated among turbines (a shadowing effect that lowers the wind speed for downstream turbines), which is estimated to account for a 10 to 20% lower power production in large wind farms [18]. Thus, several models have been developed to describe the wake effect, such as the Jensen model [19], the Frandsen model [20], the Bastankhah and Porté-Agel model [21], and models that account for deep-array wake effects [22–24].

Since the wind farm site was traditionally predetermined in the tendering process, the layout optimization at the micro-scale has been explored extensively in the literature following both discrete and continuous approaches [10]. In the discrete approach, a number of locations where to place the turbines must be chosen out of a larger set of available candidates. To this purpose, researchers have developed meta-heuristics like genetic algorithms [25–27], simulated annealing [28,29], and variable neighborhood search [10,30], as well as exact methods based on mixed-integer programming [31–33] and constraint programming [34]. The discrete approach has the advantage to easily account for additional elements affecting profitability other than the wakes, such as the cost of foundations. Moreover it is straightforward to consider boundaries and obstacles in the optimization. In contrast, the continuous approach represents the coordinates of each wind turbine position by means of continuous variables varying on a plane. This variant has been tackled using meta-heuristics [35–37] and gradient-based methods [38–40]. In general, a continuous approach has the potential to reach higher power production levels than a discrete one due to the higher degree of freedom when choosing the turbine locations. In addition, gradient-based approaches can employ high-accuracy wake models based on computational fluid dynamics [41]. Nevertheless, continuous approaches do not consider foundation costs while obstacles and boundaries add complexity to the model. For these reasons, most continuous models simplify the boundaries and consider a rectangular or circular shape. Due to this limitation, and since the goal of this paper is to consider arbitrary boundary shapes in a flexible manner, we employ a discrete approach when solving the layout optimization problem.

The new meso-scale problem of choosing the shape of a wind farm sits somehow in-between the macro and micro-scale problems. Indeed, macro-scale investigations only determine larger areas where to construct a wind farm, neglecting its shape and the wind turbine locations, whereas the micro-scale literature assumes that a boundary for the wind farm is given and cannot be altered. In particular, the considered boundary shapes range from simple rectangles, especially in the continuous approach, to more complex shapes in the discrete approach that avoid obstacles and/or forbid some areas in the region. A first study on the wind farm shape is found in [8], where it is observed that the energy production of a wind farm increases when orienting a square-shaped area with a diagonal perpendicular to the main wind direction. Although limiting to squares is a rather simplified example, this study shows how wind farm power production can benefit from this type of optimization. A rectangular wind farm shape is studied in [9] allowing for both rotations of the rectangle and varying side ratios. In particular, a set of orientation and side ratio combinations is generated randomly, and the corresponding rectangular wind farms are then optimized by choosing the type of turbines and their locations using a mixed-discrete particle swarm optimization. The 25 MW wind farm at Baker, North Dakota, is used as a case study and includes 10 turbines. Their results show that the optimal energy production depends on the wind farm shape, and that the wind farm achieves higher energy production when using higher aspect ratios between the two sides of the rectangle and when the orientation is close to the dominant wind direction. Even though they describe an onshore wind farm, no constraints are considered other than the wind conditions. In contrast, our study includes practical constraints from the UK Round-4 tender, allows for generic shapes, and is tested on much larger wind farms.

Overall, our approach tries to bridge the gaps in the literature by allowing the wind farm shape to grow without geometric simplifications, by integrating micro, meso, and macro-scale design optimization, and by including real-life constraints. In particular, the main contributions of our work are the following:
• We formalize and study the WFASP, which is an emerging optimization problem that arises in the tendering process of new offshore wind energy projects.
• We propose an optimization framework to solve the WFASP, which integrates all the scales from macro region screening to turbine micro-siting. Within this framework, we design a novel constructive heuristic for the meso-scale problem to efficiently determine the best shape of the wind farm that maximizes profitability while accounting for all tender constraints.
• We perform an extensive computational study involving real data and constraints from the UK Round-4, which is a highly relevant case for industrial applications, and perform a sensitivity analysis on critical parameters.
• We provide managerial insights that can be useful to both governments organizing upcoming tenders and participating companies. In particular, our study shows that the NPV can increase significantly when optimizing the shape of a wind farm, as done by our constructive heuristic, and that some tender constraints such as the minimum energy density and the maximum perimeter-to-area ratio have a major impact in shaping the wind farm area and determining its profitability.

3. Problem statement

Given a larger region, the goal of the WFASP is to select a smaller area and its shape where to build a wind farm, and place the turbines in this area. Solving this problem requires combining a screening of the region, choosing the wind farm shape, and optimizing its layout. We aim to maximize the net present value of the park (Section 3.1) while fulfilling a set of constraints (Section 3.2).

3.1. Objective function

The objective of the wind farm area selection problem is to maximize the NPV of the wind farm, that is, making the project as profitable as possible for the company that will develop and operate it. Indeed the NPV is a widely used metric to assess the economic viability of an energy project, and is computed as the cumulative discounted cash flows accrued by the wind project over its lifetime [42]. For an offshore wind farm, the NPV can be defined as:

$$\text{NPV} = \sum_{t=0}^{T} \frac{\text{Revenues}_t - \text{Costs}_t}{(1 + r)^t}$$

$$= \sum_{t=0}^{T} \frac{\text{EP}_t \times \text{AEP}_t - (\text{CAPEX}_t + \text{OPEX}_t)}{(1 + r)^t},$$

where EP, is the electricity price in MEuro/MWh/y and AEP, is the annual energy production, in MWh/y, which is computed based on the wind statistics and by modeling the wake effects between turbines. Together, these two terms provide the expected revenues from the wind farm energy production in year t. CAPEX are CAPital EXPenditures, in MEuro, which for offshore wind farms include the supply and installation costs of turbines, of their foundations, of inter-array cables, of export cables, and of substations; costs due to commissioning, consenting, engineering, insurance, and decommissioning of the wind farm; and contingency costs. OPEX are OPerational EXPenditures, in MEuro, which include the costs due to operations and maintenance of turbines and of the balance of plant logistics, servicing, testing, repairing, monitoring, and project management [17]. These two terms jointly collect all costs of a wind farm project. Finally, T is the expected lifetime of the wind farm and r is the discount rate. In our study, the following factors have been included to estimate the NPV:

1. The annual energy production, which accounts for the wake effect among turbines not only of the same park but also of nearby wind farms.
2. The foundation costs, which depend on water depth and on soil conditions of the seabed.
3. The export cable cost, which mainly depends on the distance from shore.
4. The inter-array cable costs, depending on the distances between turbines.

The NPV is not the only metric used to assess the profitability of wind farm projects [43]. The levelized cost of energy (LCOE) is a popular alternative used especially to compare different energy projects. The LCOE is computed as the ratio between the total costs of the wind farm (CAPEX and OPEX) and the total energy production (AEP), discounted over the park lifetime. For a proper assessment of the LCOE, though, all the cost and revenue components of the park must be included, otherwise, the LCOE ratio can be skewed by the missing factors. The NPV is instead additive, hence the improvement gap for different configurations of the same park does not change depending on how many cost factors are included. Since our study does not include all cost factors, the NPV is a more robust metric and we consider it as our optimization objective.

3.2. Constraints

In real-life wind farm tendering processes, a variety of constraints must be satisfied in the WFASP. Below, we describe the constraints we account for in our model, which are defined based on the UK Round-4 tender.

1. A maximum total area for the wind farm, which can be set explicitly or can instead be implied by the next two constraints.
2. A maximum total installed capacity, which is an upper limit on the nominal energy production of the wind park.
3. A minimum energy density of the wind farm, which measures how much energy is produced per square kilometer.
4. A maximum ratio of the perimeter of the wind farm to the square root of its area. Practically, this implies that the shape cannot be too stretched.
5. The wind farm must be fully contained inside one of the larger bidding regions.
6. The wind farm must be connected, that is, it cannot be split into multiple discontinuous areas.
7. A minimum distance must be observed from the existing wind farms, from the areas reserved for already planned wind parks, and from other restricted areas such as natural reserves.
8. Obstacles must be avoided, such as existing pipelines, cables, infrastructure in the seabed, as well as fishing and aviation corridors.

4. Method

Our optimization framework is divided into three phases. The aim of the first phase is to select a rough location from the larger regions where to build the wind farm (Section 4.1). Starting from this location, the second phase finds the best shape and area for the wind farm (Section 4.2). Finally, the third phase finds the best arrangement of the turbines in the selected shape (Section 4.3).

4.1. Macro-scale: screening

The first phase in our framework involves a screening of the larger regions to identify the most promising locations for the wind farm. At a high level, our strategy consists in discretizing the regions using a fine grid and evaluating the profitability of the cells in the grid. More specifically, for each cell, we consider a rectangular-shaped wind farm with equally spaced turbines and with center coinciding with the center of the cell. Multiple orientations and side ratios of this rectangular
The tender.

We use a similar approach for the maximum area of the wind farm and heuristic, and add a penalty when this ratio exceeds its allowed limit. The ratio of the convex hull defined by the turbines placed so far by the main wind direction of that particular location and of its geometry. For a given cell and wind farm orientation and side ratio, the four different NPV components (see Section 3.1) are computed as follows:

- The annual energy production is estimated by taking into account the wind conditions for the specific cell, the wakes generated by the turbines within the park itself and by the turbines of nearby wind farms.
- The foundation costs are estimated based on the water depth in the location of each individual wind turbine, by following the model in [44].
- The export cable cost is estimated as a linear function of the length of the cable, which is the distance of the center of the wind farm to shore.
- The cost of inter-array cables is estimated as a linear function of the total cable length, which is computed by solving a minimum spanning tree between the turbines and a substation located in the center of the wind farm.

The process ends when all the cells are evaluated. A heatmap is generated as output of this phase, encoding the most promising locations for a wind farm that can be used as starting positions in the next phase.

4.2. Meso-scale: constructive heuristic

The goal of the second phase, which is the core of the WFASP, is to generate the shape of the wind farm that maximizes its NPV. For this task, we propose a constructive heuristic that iteratively takes greedy decisions when placing the turbines, and produces as output a wind farm fulfilling the constraints on shape.

We start by selecting an initial seed location based on the results from the previous screening phase, and place the first turbine exactly at this seed. Then, the constructive heuristic iteratively performs the following two steps:

1. A very fine grid of positions is created around the already placed turbines (initially, just the seed location), which represent the candidate positions for the heuristic to place the subsequent turbines. This grid is constructed by considering disks with a given radius around the already placed turbines, merging them, and discretizing the resulting area.
2. A point among the available candidates is selected to place the next turbine. To make this choice, for each candidate location, we evaluate the NPV of the partial wind farm consisting of all the already placed turbines plus a temporary turbine placed in that location. The candidate yielding the highest NPV is chosen for the actual placement of the turbine.

Since solutions are built around a fixed seed, note that the costs of export cable and inter-array cables vary only marginally across the constructed solutions (although these costs are critical to the screening phase). Thus, when evaluating a partial solution, we assume that these costs are constant in the NPV calculation. In other words, the meso-scale optimization is driven by minimizing wakes and foundation costs. In addition, we introduce penalties to the NPV when the constraints on shape are violated. Specifically, we compute the perimeter-to-area ratio of the convex hull defined by the turbines placed so far by the heuristic, and add a penalty when this ratio exceeds its allowed limit. We use a similar approach for the maximum area of the wind farm and its minimum density, depending on how this constraint is specified by the tender.

When turbines have been placed with cumulative capacity equal to the desired wind farm capacity, the constructive algorithm ends and the convex hull defined by the complete set of turbines is chosen as the final wind farm shape.

4.3. Micro-scale: layout

The final phase of our framework determines the best locations for the turbines within the shape defined by the constructive heuristic. In fact, goal of the previous phase was to define a wind farm shape subject to the tender constraint, while the placement of individual turbines within the shape was guided by greedy decisions. Thus, there is potential to further improve the NPV of the wind farm by running a state-of-the-art layout optimization algorithm to reposition the turbines within the chosen shape.

To this end, we employ the optimizer proposed in [10] based on a variable neighborhood search heuristic in a discrete setting. This approach is convenient to use since it accepts arbitrary wind farm shapes as input. We adapt it in this paper to our NPV objective. The solution obtained by the optimizer is then considered as our final design for the wind farm, which at this point should be in one of the most promising areas of the larger regions, with the shape optimized for its location while satisfying all the constraints, and with the highest possible NPV.

4.4. Wake model

Offshore wind parks generate wake losses and additional turbulence affecting nearby wind farms. Therefore, it is critical to include these effects in all three phases of our optimization framework previously described.

As mentioned in Section 2, different wake models exist in the literature. These include "bottom-up" approaches where the combined effect of several wakes at one point is calculated by overlaying single wakes. Most notably in this class is the Jensen model [19]. In contrast, "top-down" models capture deep-array wake effects that arise in large wind farms, i.e., when several rows of turbines may cause single turbine wakes to merge into a common farm wake. By accounting for atmospheric properties, such models have the potential for a more precise wake estimation [22–24]. Finally, full computational fluid dynamic simulations may also be used to estimate the wake effects.

In this paper, we employ the Jensen model to compute the energy production of wind farms. We do this consistently in all the optimization scales and account for losses that arise due to both the wakes within the park and from nearby wind farms. The motivation for our choice is that this model is frequently used in the industry, is typically robust to estimate the wakes, and is computationally efficient to evaluate even for large wind farms [45]. In fact, a computational fluid dynamic simulation would have been too computationally demanding for our large-scale optimization because wakes are computed and evaluated in hundreds of iterations over the various optimization phases. On the other hand, top-down models would require more data to capture interactions of atmospheric stability and temperature [46].

While not using more sophisticated wake models or simulations may be seen as a limitation of our approach, note that our framework only requires a function that takes a given candidate configuration of turbines as input and returns its associated annual energy production. Thus, the framework could be adapted (conceptually, at no change) to incorporate a different wake model (or annual energy production estimation) provided that sufficient computational resources are available as well as the relevant data.
5. Numerical study

In this section, we present a computational study based on the Offshore Wind Leasing Round 4 in the UK. We first describe the instances, including real-life constraints and input data, and the computational setup (Section 5.1). Then, we discuss the results from the screening phase (Section 5.2), from the micro-siting optimization (Section 5.3) and from the overall approach (Section 5.4). Finally, we perform a sensitivity analysis on critical parameters of the tender constraints and of the algorithm (Section 5.5).

5.1. Instances and computational setup

At the macro-scale, our screening covers the four large sea regions defined in the UK Round-4, which are called Dogger Bank, Eastern Regions, South East, and Northern Wales & Irish Sea (see Fig. 1). These regions have been discretized using a regular grid of 0.0625 degrees (roughly 4km at these latitudes), which is used to compute a heatmap of the most promising areas.

At the meso-scale, we consider all the constraints stated in the UK Round-4 (see Section 3.2) to ensure that the selected shape and area satisfy the tender requirements. These constraints are specified as follows:

1. The ratio of perimeter over square root of area, \( \frac{\text{perimeter}}{\sqrt{\text{area}}} \), must not exceed 5. Note that the minimum theoretical ratio is achieved when using a circle and is \( \frac{2\pi}{\sqrt{\pi}} \approx 3.54 \). The maximum ratio is obtained when the area of the shape tends to 0 and the ratio to infinity.
2. The minimum energy density for the wind farm is equal to 3 MW/km².
3. The nominal installed capacity of the wind farm must not exceed 1.5 GW.
4. The area of the wind farm must not exceed 500 km², which is implied by the constraints on minimum energy density and maximum capacity.
5. A minimum distance of 7 km must be observed from existing wind farms, from sites reserved for planned wind farms, and from other restricted areas.

Our framework benefits from data on weather and site conditions to obtain an accurate estimation of the wind farm production. In particular, wind statistics are fundamental to estimate the energy production of a wind farm. Due to the significant extension of the bidding regions, we use weather data from ERA5 [47], which is a European project providing the data collected by the Copernicus satellites since 1979. In particular, we extract statistics based on wind direction and intensity data from the last 5 years at an hourly time resolution. This data is available on a regular latitude-longitude grid with 0.25 by 0.25 degrees resolution (about 16 km at these latitudes), to characterize the wind inside the larger regions, covering an area of about 8 by 6 degrees overall.

For the wind farm to develop, we use the 15 MW offshore wind turbine generator by NREL [48], which has full thrust and power curves available thereby making the energy production calculation more accurate. We assume that the expected lifetime \( T \) of the wind farm is 25 years, which is standard [49]. We account for planned and existing wind farms, assuming that wakes from wind farms farther away than 60 kilometers are negligible [50].

The various cost components are crucial in computing the NPV and hence affect the design of the wind farm. In particular, the costs of turbine foundations depend on water depth and on the conditions of the seabed soil, among others. We use the bathymetry dataset published by GEBCO [51] to estimate the foundation costs of each turbine based on the water depth at each location. The geographic latitude and longitude resolution of this dataset is of 15 arc-seconds (about 0.27 km), which is very accurate for our purpose and more detailed than the wind data.

To estimate the inter-array cable cost, we assume that two types of electrical cables are available. The smaller cable has a cross-section of 400 mm² and a cost per meter of 240 Euro/m, while the larger one has a cross-section of 630 mm² and a cost per meter of 336 Euro/m [52]. These two types are used in the cable routing solution according to the capacity needed at each section. The cost of the export cable, which is larger, is set to 1500 Euro/m [53].

Finally, in the NPV calculation, we use the energy price of 39.59 Euro/MWh, which is the average Day-Ahead Nord Pool UK auction price for the year 2020 [54], and a discount rate \( r \) for a typical wind farm of 6% [55].

The algorithm was implemented using Python and run on a server with an Intel® Xeon® Processor E5-2673 v4 at 2.30 GHz and with 16 GB RAM available. The average running time to execute the initial screening and constructive heuristic was about 1680 and 30 min, respectively, while a time limit of 30 min was set when running the meta-heuristic at the micro-scale level.

5.2. Results of screening

Fig. 2 shows the results from the macro-scale optimization for all UK Round-4 regions and displays as well the six winning bids (in red). As can be seen, the best locations identified by our screening (i.e., those with a higher NPV) match quite closely with the areas selected by the winning bids. In addition, the existing wind farms are also located in zones where the NPV is relatively high, which explains why these areas were selected during previous tenders. Among the four bidding regions, the South East region appears the least promising according to our screening, mainly due to lower average wind speeds.

For the second UK Round-4 region known as “Eastern regions”, we provide a breakdown of the components that most contribute to the NPV in Fig. 3.

Fig. 3(a) shows the distribution of the annual energy production in the region. It can be seen that, in general, moving farther away from shore leads to higher mean wind speeds and therefore higher production levels. In the northern part of this region, especially, many wind parks have already been built or planned. Fig. 3(b) reports the estimated foundation costs of each location, which are correlated with the water depth. The north-western corner of the region is the most favorable in this respect. Fig. 3(c) shows the cost of the export cable connecting the substation of the wind farm to shore. As expected, the export cable is shorter for the locations that are closer to shore, and consequently cheaper, whereas locations farther away are associated with higher costs. Finally, Fig. 3(d) reports the full NPV for the region, which includes all previous factors. Notice that the locations with higher power production are typically those that are far from shore (i.e., requiring expensive cable connections) and with deeper waters (i.e., associated with expensive foundation costs). Thus, the key trade-off here is between placing the turbines in shallower waters closer to shore or in more expensive locations with higher energy production. Our NPV objective allows managing this trade-off by balancing the different factors, also by means of the energy price constant. Overall, our screening suggests that the north-west part of this region is the most promising for new wind farms, which is consistent with the location of the real UK Round-4 winning bid in this region.

5.3. Micro-siting optimization

Given a wind farm boundary, the value from optimizing the location of individual turbines within the boundary is well established in the literature. We performed a set of experiments to confirm that this is indeed the case also in our instances. Specifically, we compare on the same input shape the wind farm resulting from our meta-heuristic algorithm with two benchmarks that do not use optimization. These benchmarks arrange the turbines either on a regular grid (as we do in the screening) or on a staggered grid (see, e.g., [56]).
Fig. 2. Heatmap of the NPV (MEuro) for the four UK Round-4 bidding regions. The higher the NPV (i.e., lighter color), the more promising is the location for a potential wind farm.

Fig. 3. Heatmaps of main NPV components from the screening of the Easter Regions. For all graphs, lighter colors identify better locations with respect to the related NPV component.

Based on the results of the screening, we chose two locations in each region of UK Round-4, totaling eight seed points, and compared the three solutions on the rectangular shape centered in these points and with the best side ratio and orientation. Consistently with the literature, our results indicate that micro-siting optimization significantly improves the power production of a wind farm by reducing its wakes compared to a fixed grid-based positioning of turbines. Since these results are expected and studying micro-siting alone is not the
focus of this paper, we relegate more details on these experiments to Appendix A. Finally, when looking at the two grid-based parks, although staggering may help depending on the wakes of nearby wind farms and the site characteristics, there is no clear winner between regular and staggered configurations.

5.4. Results of multi-scale optimization

We now investigate the performance of our multi-scale approach. Recall that the meso-scale optimization decides the shape of the wind farm starting from a seed point. For each of the 8 seed points from the screening previously mentioned, our constructive heuristic grows the area and shape of the wind farm as described in Section 4.2, while the subsequent micro-scale phase optimizes the positions of the wind turbines inside the selected shape as discussed in Section 4.3.

We compare the final NPV of the wind farm constructed by following our multi-scale optimization approach with that of a rectangular-shaped wind farm placed in the same seed point. For the rectangular wind farm, note that the combination of orientation and side ratio leading to the highest value is chosen. Moreover, to have a fair comparison, the location of the turbines in the rectangular wind farm is optimized using the same meta-heuristic algorithm described in Section 4.3 and with the same time limit. Therefore, the only difference between the two wind farms is represented by the shape used as input to the final optimization, i.e., the shape resulting from our meso-scale constructive heuristic versus a fixed rectangular shape with the best side ratio and orientation.

Table 1 reports the details of this comparison, including the coordinates of the seed locations and their region, the final NPV obtained by our approach, its perimeter-to-area ratio and energy density, the final NPV of the rectangular wind farms, and their difference. These results show that the shape selection is an important step when optimizing the design of a wind farm and that the NPV improves in all 8 cases. In particular, seed \( C \) has the smallest objective function gap between optimized and rectangular shape, with a delta of only 3.5 MEuro. In contrast, the improvement is especially large for seeds \( D \), \( F \), \( G \), and \( H \) with a difference of 65.9 MEuro, 52.4 MEuro, 58.1 MEuro, and 108.6 MEuro, respectively. Across the 8 seeds, the improvement is 1.14% on average and up to 2.76%. These percentages correspond to an average and maximum improvement of 46 MEuro and 109 MEuro, respectively. Across the 8 seeds, the improvement is 1.14% on average and up to 2.76%. These percentages correspond to an average and maximum improvement of 46 MEuro and 109 MEuro, respectively. Across the 8 seeds, the improvement is 1.14% on average and up to 2.76%. These percentages correspond to an average and maximum improvement of 46 MEuro and 109 MEuro, respectively. Across the 8 seeds, the improvement is 1.14% on average and up to 2.76%. These percentages correspond to an average and maximum improvement of 46 MEuro and 109 MEuro, respectively.

Moreover, notice that the energy density of the final shapes is always close to its lower bound, and the perimeter-to-area ratio is close to its upper bound too. Thus, both constraints are binding.

Next, we analyze more in detail the results for the first seed point \( A \). Fig. 4 visualizes the wind farm with optimized and rectangular shape in panel (a) and (b), respectively. For this seed point, one would intuitively expect that a rectangular wind farm performs well. In fact, the boundaries of the region are too far away to influence the shape optimization, and the existing wind farms in the north-west direction are also not too relevant as they are located downstream compared to the main wind direction in that area. Indeed the two resulting shapes are quite similar. Nonetheless, the optimized shape improves the NPV by 28.1 MEuro because it can better exploit the shallow waters. As we
We then analyze in Fig. 5 the behavior of our algorithm with respect to the perimeter-to-area ratio (top) and the energy density (bottom) constraints, for seed location $A$, during the process of incrementally adding one turbine to the current solution of the constructive heuristic. The solid blue lines in this figure represent the evaluation of the perimeter-to-area ratio and the energy density for a partial wind farm solution with $n \in \{3, \ldots, 100\}$ turbines, while the dotted red lines define the limits on these values set by the tender. Initially, the constraints fluctuate quite heavily but eventually stabilize close their respective limits. Thus, both constraints are binding in the later stages, impacting on the final shape of the wind farm. This is intuitive since the energy density is related to the amount of area that the wind farm is allowed to use; hence, it is more beneficial to spread turbines apart as much as possible at the beginning to reduce the wake losses. Similarly, stretching the shape of the wind farm perpendicular to the main wind direction reduces the wakes internal to the wind farm, which drives the algorithm to stay close to the upper bound on the perimeter-to-area ratio.

5.5. Sensitivity analysis

We perform a sensitivity analysis on the most important parameters affecting the wind farm design. Specifically, we test instances at varying perimeter-to-area ratio and energy density limits. These parameters influence a wind farm significantly in terms of both profitability and allowed shape. Thus, our analysis can inform policy makers on how to set them. We also test a parameter of the algorithm that may affect its performance, namely the radius used in the constructive heuristic to identify the candidate locations to place the next turbine.

Fig. 6(a) shows the NPV of the wind farm for an allowed ratio of perimeter over square root of area (PtA) ranging from 3.7 to 14 (the limit in UK Round-4 is 5). Higher values of this threshold constrain less the problem, allowing for more flexibility to choose shapes with increased power production and decreased foundation costs. In particular, the wind farm NPV increases by roughly 100 M€uro (2%) when the limit on PtA is increased from 5 to 14. Fig. 6(b) shows the NPV of the wind farm when the minimum energy density is varied from 1 to 8 MW/km² (the value specified by the tender is 3 MW/km²). The lower the minimum density, the better the objective value because turbines can be spread farther apart thus reducing wake losses. The NPV is affected substantially by this parameter and changes by about 12% between the two extreme values tested.

In Fig. 7, we visualize the shape of the wind farms resulting from a PtA ratio of 3.7 and 14, respectively, in panel (a) and (b). A PtA ratio equal to 3.7 is close to the theoretical minimum and the shape chosen by the constructive heuristic is indeed close to a circle. Such a wind farm uses a compact area in the sea but is subject to very high wakes that negatively affect energy production. On the other extreme,
with an allowed PtA ratio of 14, the shape is very much unconstrained and the resulting wind farm is stretched as much as possible along the direction orthogonal to the dominant wind direction. This shape is convenient because it reduces the wake losses generated inside the wind farm. In fact, most of the turbines directly receive the main winds most of the time; hence, the energy production is maximized. However, such a shape would not lead to an efficient utilization of the nearby sea areas, which is one of the reasons why this constraint was introduced in the tender rules. The NPV difference between the two limiting cases amount to 154 MEuro.

In Fig. 8, we visualize the shape of two wind farms resulting from different energy density limits. In particular, in panel (a) of this figure the limit is $\frac{2}{\text{MW/km}^2}$ while in panel (b) the limit is $\frac{7}{\text{MW/km}^2}$, which are respectively smaller and larger than the limit specified in UK Round-4. It is clear that the lower-density wind farm uses a much larger area than the higher-density one, leading to lower wake losses and more choice on the turbine foundations. When the energy density is high, the wind farm is limited in the amount of sea area available and the resulting shape is more compact and subject to higher wake losses. In addition, there are less opportunities to place turbines in locations with shallow waters compared to lower-densities cases. The difference in NPV between these two cases is about 379 MEuro, which is almost 8% of the value.

Overall, our analysis shows that the constraints on perimeter-to-area ratio and energy density are critical in shaping the wind farm area and determining its profitability. The government organizing a tender has to carefully select these two limits to balance the trade-off between an efficient utilization of sea areas by means of compact shapes (which may be critical when many wind farms are planned in a given region) and the competitiveness of the individual wind farms, which increases substantially as the constraints on shape are loosened.

Finally, we analyze the effect of varying the radius used at the meso-scale optimization on the wind farm shape. In contrast to the former parameters examined, notice that this is an internal parameter of the algorithm used to select the pool of candidate locations to place the next turbine at each iteration of the constructive heuristic. A too small radius would include too few candidates thereby not being able to efficiently optimize the shape, while a very large radius would consider too many candidates and thus unnecessarily increasing runtime. We investigate this trade-off for values of the radius ranging from 2.5 to 15 km and report the results in Fig. 9. The left graph in this figure reveals that the runtime increases approximately linearly with increasing radius. This is mainly due to the need to compute the wake effects for each
candidate point to each other turbine already placed in the solution. The right graph in Fig. 9 reports the final NPV of the wind farm in relation to the chosen radius, showing that there is not a clear influence on solution quality as the NPV values are all quite close to each other. This suggests that employing a conservative radius may be sufficient to speed up computation without compromising quality.

6. Conclusion

In this paper, we have introduced and studied the “wind farm area selection problem”, an emerging problem in the design of offshore wind farms that involves choosing the best shape and area of a wind farm to maximize its value (NPV). To tackle this problem, we proposed an optimization framework comprised of three phases: a macro-scale phase that identifies the most promising areas out of larger bidding regions, a meso-scale phase that optimizes the shape of the wind farm, and a micro-scale phase that decides the final positions of the turbines within the shape. We put particular emphasis on selecting the optimal shape at the meso-scale, which is a new problem in the extant wind energy literature and which we solve using a novel constructive heuristic. Moreover, to the best of our knowledge, our work is the first to integrate all three scales of design.

We tested our optimization framework using the real-life case of the recent UK Round-4 tender. Our method is able to handle all the practical constraints relevant for the industry, e.g., obstacles and nearby wind farms, including newly introduced constraints specific to this tender such as a maximum perimeter-to-area ratio and a minimum energy density. Our results showed that optimizing the shape of the wind farm could improve the NPV significantly compared to the best rectangular shape, for all seed locations analyzed. The average improvement was 1.1% (corresponding to about 46 MEuro) and it was up to 2.8% under more challenging conditions regarding nearby boundaries and wind parks. Finally, our sensitivity analysis on the main constraints of the tender provided insights that may support policy makers in specifying the rules of future tenders.

Overall, our work shows that solving the wind farm area selection problem by integrating all three scales of design can bring major benefits to wind energy projects while respecting the newer tender rules in the wind industry. We hope this paper fosters future research on the multi-scale design of wind farms, e.g., by further improving the meso-scale shape optimization which is understudied albeit crucial according
to our results, or by embedding different wake models. Moreover, although this work focuses on offshore wind farms, onshore projects may benefit from a similar decision framework. At a higher level, we hope this work may contribute to improving the competitiveness of wind energy technologies and thus to accelerating the transition to renewable energy.

CRediT authorship contribution statement

**Davide Cazzaro:** Conceptualization, Methodology, Software, Validation, Data curation, Writing – original draft, Writing – review & editing, Visualization.  
**Alessio Trivella:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing, Supervision.  
**Francesco Corman:** Conceptualization, Writing – review & editing, Supervision.  
**David Pisinger:** Conceptualization, Methodology, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

We thank Jesper Runge Kristoffersen and David Franz Koza, from Vattenfall BA Wind, and Gabriele Bedon, for their expertise on wind farm design and the useful suggestions provided during the writing of this paper. We also thank the review team for useful feedback that helped us improving the manuscript. This research is partly funded by the Innovation Fund Denmark (IFD) under File No. 9065-00162B.

Appendix A. Comparison of rectangular wind farms

In this appendix we provide some more details on our comparison of rectangular wind farms, including: (i) an optimized micro-siting solution, (ii) a regular configuration with turbines placed in a regular grid within the rectangular area, and (ii) a staggered configuration where even rows of turbines are offset with respect to the odd ones. This “zigzag” pattern is placed perpendicular to the main wind direction to maximize power production. An example of rectangular and staggered solutions is shown in Fig. A.10 for seed point A. An illustration for the corresponding optimized solutions is instead reported in Fig. 4.

The different configurations are evaluated for the 8 seed locations selected from the screening (see Table 1 for the coordinates and other details about such locations). The results of the comparison are shown in Table A.2.

![Fig. A.10. Regular and staggered wind farms in seed point A.](image)

### Table A.2

<table>
<thead>
<tr>
<th>Seed</th>
<th>Regular NPV MEuro</th>
<th>Staggered NPV MEuro</th>
<th>Delta MEuro %</th>
<th>Delta best shape MEuro %</th>
<th>Delta rectangle MEuro %</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4671.87 4673.00</td>
<td>4714.00 4673.00</td>
<td>-61.0 -1.29</td>
<td>-32.9 -0.70</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>4637.72 4632.48</td>
<td>4631.45 4632.48</td>
<td>-64.6 -1.37</td>
<td>-37.4 -0.80</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>4480.78 4475.73</td>
<td>4480.78 4475.73</td>
<td>-24.1 -0.53</td>
<td>-20.6 -0.46</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>3898.81 3877.42</td>
<td>3898.81 3877.42</td>
<td>-122.8 -3.05</td>
<td>-56.9 -1.44</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>3968.58 3960.75</td>
<td>3968.58 3960.75</td>
<td>-76.4 -1.89</td>
<td>-51.1 -1.27</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>3849.91 3865.29</td>
<td>3849.91 3865.29</td>
<td>-122.5 -3.07</td>
<td>-64.4 -1.64</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>3939.82 3935.45</td>
<td>3939.82 3935.45</td>
<td>-104.5 -2.58</td>
<td>-52.1 -1.30</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>3872.13 3871.65</td>
<td>3872.13 3871.65</td>
<td>-168.5 -4.17</td>
<td>-59.9 -1.52</td>
<td></td>
</tr>
</tbody>
</table>
in Table A.2, which reports the NPV of regular and staggered configurations (columns 2–3), the difference between these two solutions (columns 4–5 titled “Delta”), and their average difference with respect to both the optimized shape (columns 6–7) and the optimized rectangular wind farm (columns 8–9). Note that the NPV of the latter two optimized solutions is shown in Table 1.

The regular configuration achieves a 0.08% higher NPV on average compared to the staggered one. In two cases, though, the staggered configuration performs better than the regular one. Overall, the difference in NPV between these two grid-based configurations is rather small in most instances and we cannot say that one approach dominates the other. Both regular and staggered configurations, however, perform substantially worse than the optimized solutions from our meta-heuristic approach, with an average decrease in NPV of 93.1 MEuro and 46.9 MEuro, respectively, compared to the best shape and the optimized rectangular wind farm. These results confirm the already well-established advantages of micro-siting optimization, which enables obtaining far more profitable wind parks than (non-optimized) grid-based layouts.

Appendix B. Illustration of resulting wind farms in all seed locations

See Figs. B.11–B.17.
Fig. B.13. Comparison of the wind farms for seed point $D$.

Fig. B.14. Comparison of the wind farms for seed point $E$. 
Fig. B.15. Comparison of the wind farms for seed point $F$.

Fig. B.16. Comparison of the wind farms for seed point $G$. 
References


[31] Fagerfjäll P. Optimizing wind farm layout: more bang for the buck using mixed integer linear programming. Chalmers University of Technology and Gothenburg University 2010;111.


[45] VanLuanvan DR. Investigation of observed and modeled wake effects at Horns Rev using WindPRO. Tech. rep., Technical University of Denmark, Department of Mechanical Engineering; 2006.


