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# Comparison of aerodynamic planform optimization of non-planar rotors using blade element momentum method and a vortex cylinder model

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**Abstract.** The present work compares non-planar rotors designed using the blade element momentum (BEM) method and a vortex cylinder model. In a previous work, it is shown that blade element theory coupled with the superposition of the vortex cylinder model (BEVC) is able to model the loads of non-planar rotors. The result predicted by the BEVC model is in significantly improved agreement with higher-fidelity models than the loads as predicted using the BEM method. In this work, the BEM method and the BEVC method are integrated into a gradient-based optimization framework for aerodynamic planform optimization, in which the analytical gradients are obtained using the algorithmic differentiation (AD) method. In the present study, the rotor is assumed to be stiff for all cases such that the pure aerodynamic effects are highlighted. Loads of the optimized non-planar rotors with different geometries under different constraints designed from both methods are calculated using the BEM method, the BEVC method and also the higher-fidelity lifting-line (LL) method. Within the constraints of the present work it was found that the advantage of the BEVC method is not significant when comparing the integrated aerodynamic loads: the non-planar rotor designed using the BEM method gives similar total thrust and power as the rotor designed using the BEVC method when the designs are evaluated with the higher-fidelity LL method. However, the results confirmed that the distributed aerodynamic loads of the non-planar rotors predicted by the BEVC method are in improved agreement with the LL method compared to the BEM method.

## 1. Introduction

The blade element momentum (BEM) method [1] is the main working horse for the aeroelastic load calculation and design optimization of horizontal-axis wind turbines due to its low computational cost. In the BEM method, the calculation of the force and velocity can take into account the blade out-of-plane and in-plane geometries. However, when calculating the inductions from the momentum theory, it is implicitly assumed the blade is straight and stays in the rotor plane [2]. The sectional forces of each blade section are projected with respect to an imaginary planar actuator disc to get the axial and tangential loads. The axial and tangential induced velocities are obtained from the momentum theory, which is shown by Branlard and Gaunaa [3] to implicitly correspond to a planar rotor. Once the induced velocities are obtained they are then projected back to each local airfoil section. With the sectional flow velocity, the 2-D lift and drag forces are calculated from the angle of attack and the relative velocity. The



calculation is iterated until the results of the induction and loads are converged for all blade sections. For a non-planar rotor, the starting position of the trailed vortices is shifted in the axial direction compared to a planar rotor, which is not modelled in the momentum part of the BEM method. This means that the impact of the nonplanarity on the induction is then not modelled. As a result, the BEM method is not able to correctly model the aerodynamic effects due to blade dihedral.

In a previous work [2], it is shown that using the blade element theory coupled with the superposition of the vortex cylinders (BEVC) is able to model the non-planar effects. In the BEVC method, the starting position of the vortex cylinders are following the curved bound vortex surface swept by the dihedral blades such that the starting position of the concentric vortex tubes are displaced in the axial direction compared to the planar rotor case. The impact of non-planar blade geometry on both axial and radial induced velocities is modelled in the BEVC method. As shown in [2], the relations for the tangential induction is independent of the axial displacement of the annular elements. In that previous work, it was shown that the radial distribution of the aerodynamic loads of non-planar rotors predicted by the BEVC model is in significantly improved agreement with higher-fidelity models compared to the loads as predicted using the BEM method. Therefore, the BEVC method is expected to have an advantage over the BEM method for the aerodynamic planform optimization of non-planar rotors. However, it should be noted that from previous experience, the BEM method performs surprisingly well when predicting the aerodynamic performance in the overall rotor level, such as integrated rotor power and thrust.

The study in the present work is to show whether there is any advantage of the BEVC method over the BEM method when used for aerodynamic planform optimization for rigid non-planar rotors. Non-planar rotors with different dihedral geometries are optimized using the BEM method and the BEVC method. The aerodynamic performance of these optimized rotors are subsequently predicted using BEM, BEVC and the higher-fidelity lifting-line (LL) model to assess the sensitivity of the aerodynamic model used in the design process on the rotor performance.

## 2. Methodology

There has been previous work by Ning [4] that uses the algorithmic differentiation (AD) method [5] to obtain the analytical gradients of the BEM method for use in gradient-based optimization. In the present work, the analytical gradients of the BEM method and the BEVC method are also obtained using the AD method in forward and reverse modes. Since the BEVC method no longer has radial independence, the total derivative has to be computed over the entire model. Then, a gradient-based optimization framework is used for the aerodynamic planform optimization in this study. The advantage of having analytical gradients over the finite differencing method is that the gradients are calculated with high accuracy and relatively low computational effort. The disadvantage is the added complexity when implementing the model. The analytical gradients are validated against the finite differencing method. In the present study, the rotor is assumed to be stiff for all cases such that the isolated effects due to only the aerodynamics are highlighted.

### 2.1. Models for comparison

The BEM method and the BEVC method [2] are implemented in a gradient-based optimization framework using OpenMDAO[6]. For the optimization, the chord and twist of the blade are represented by Akima splines with 21 control points along the span with uniform spacing. The implementation of the BEM method is similar to the implementation in the HAWC2 code [7]. However, the induced velocities are computed without the drag force included. The BEVC method is implemented following the descriptions in the previous work [2]. For the special case of uniform inflow applied perpendicular to the rotor plane, the steady-state results are tested

to be identical to the results from the same implementation of BEM and BEVC methods in the HAWC2 code. For both methods, the blade is discretized into 80 sections for the calculation. The pitch rate drag correction is included in both methods and the wake rotation effect [3] is included in the BEVC method.

The lifting-line module in the MIRAS code [8] is the higher-fidelity model used for the comparison. It is implemented in a time-marching free vortex wake fashion. For the setup in this study, each blade is discretized into 50 sections with cosine spacing. Each simulation is calculated for 20 thousand time steps and each step corresponds to  $1.5^\circ$  of azimuthal rotation, resulting in a total of 83.3 rotor revolutions. The vortex core size is 0.1% of the local chord length. The curved bound vortex influence is included in the model [9]. The implementation uses the flow information at both the 3/4 chord point and the 1/4 chord point. The flow information at the 3/4 chord point is used to determine the magnitude of the lift and the flow information at the 1/4 chord point is used to determine the direction of the lift and drag. In a previous work [2], the results from this implementation of the LL method are shown to be in good agreement with fully-resolved RANS results from EllipSys3D [10] for rotors with different dihedral blades and at different operating conditions.

### 3. Results

The blades used for the test cases in the present study are modified based on the blade of the IEA-10.0-198 10 MW reference wind turbine (RWT) [11]. The airfoil data used for all methods is from 2-D fully turbulent Reynolds-averaged Navier-Stokes (RANS) computations [11]. Firstly, a straight blade is modified based on the RWT to have zero prebend and sweep by aligning the main-axis of the half-chord line into a straight line. The blades have zero coning, no tilt or pitch so that the blades stay in the rotor plane.

#### 3.1. Planar rotor with straight blades, no constraints

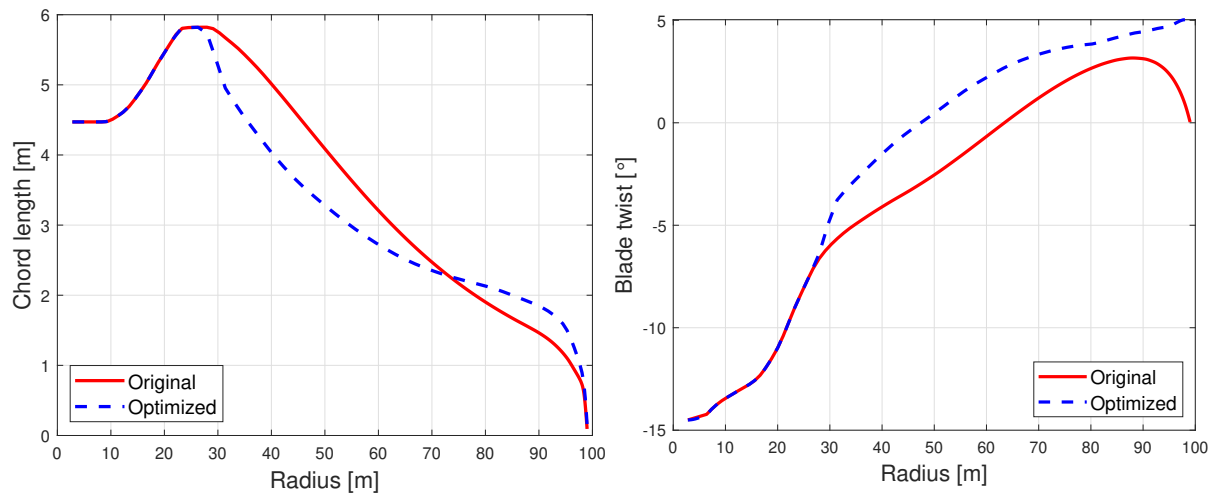
The chord and twist distribution of the straight blade from 25% span (radius of 28 m) until the blade tip is further optimized to achieve maximum aerodynamic power at the operating condition of uniform wind speed of 8 m/s with the tip-speed ratio of 9.0 and zero pitch angle. The radial distribution of airfoil relative thickness is kept unchanged. No load constraints are applied in this optimization case.

For such a planar rotor, the BEM and BEVC methods predict identical induction and loads when the wake rotation effect is excluded, which is as expected [2]. This optimized straight blade is the baseline of this study. The thrust coefficient is 0.89 as predicted by the BEM method. The chord and twist distribution of the optimized straight blade is plotted together with the original straight blade in Figure 1. Comparing to the original blade, the chord of the optimized blade is decreased from radius of 28 m to 74 m and is increased from 74 m until the tip. The twist of the optimized straight blade is increased from the radius of 28 m until the blade tip. The results are expected to be smoother if fewer design points are used for the splines.

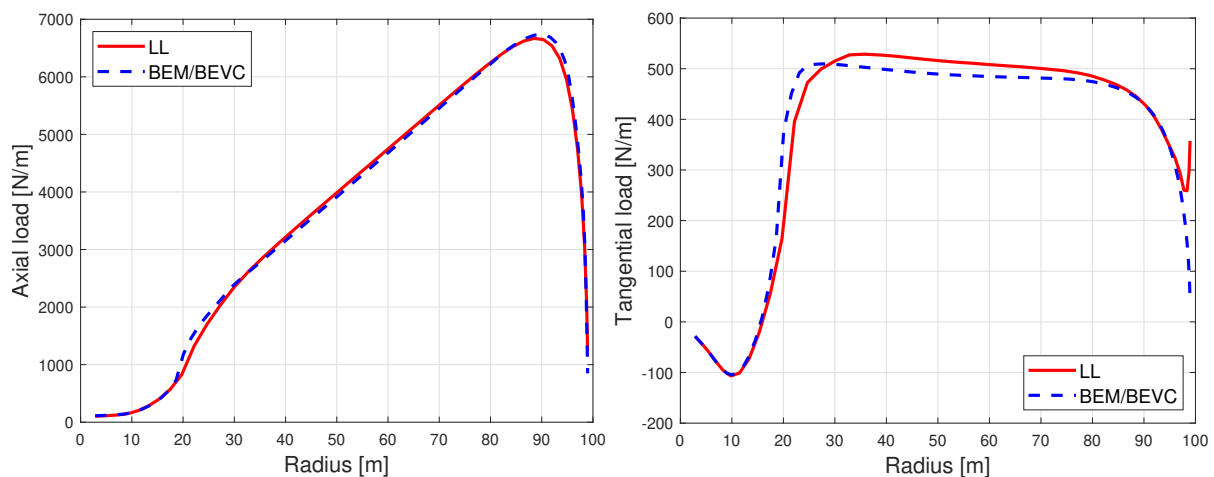
The axial and tangential loads of the optimized straight blade without cone calculated from the BEM/BEVC method are compared with the loads from the LL method in Figure 2. It can be seen from the figure that the results from the BEM/BEVC method are in relatively good agreement with the LL method.

#### 3.2. Non-planar rotor with dihedral blades, no constraints

The non-planar rotors with the dihedral blades are optimized at the same operating condition as the straight blade. The upwind dihedral blade geometry is identical with the blade W-1 in the previous work [2]. The blade has the same radius as the straight blade when not coned. It is bending upwind from 50% of radius until the blade tip, the tip dihedral magnitude is 10% and the tip dihedral angle is  $20^\circ$ . The downwind dihedral blade D-1 is modified based on W-1 by



**Figure 1.** The chord (left) and twist (right) distribution of the optimized straight blades and the original straight blade.

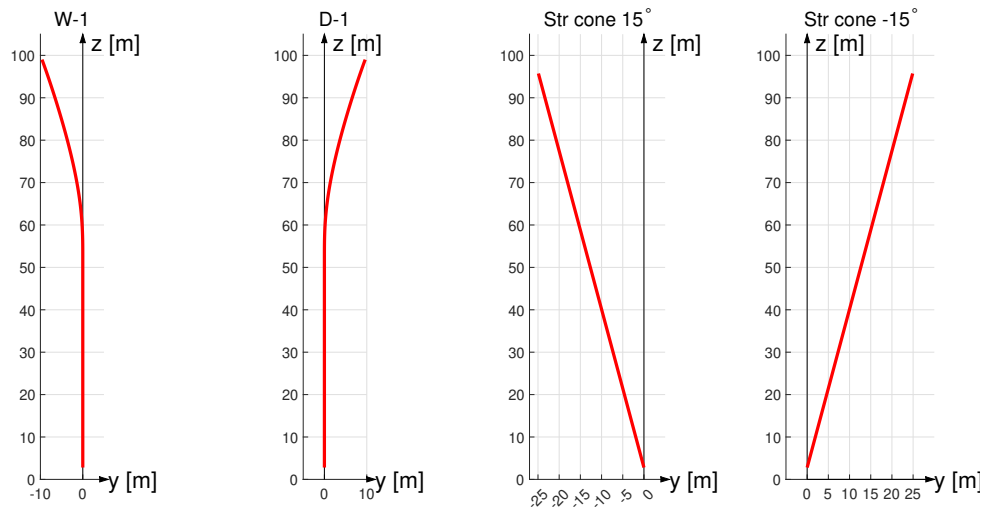


**Figure 2.** The axial load (left) and tangential load (right) of the optimized straight blades calculated from LL and BEM/BEVC.

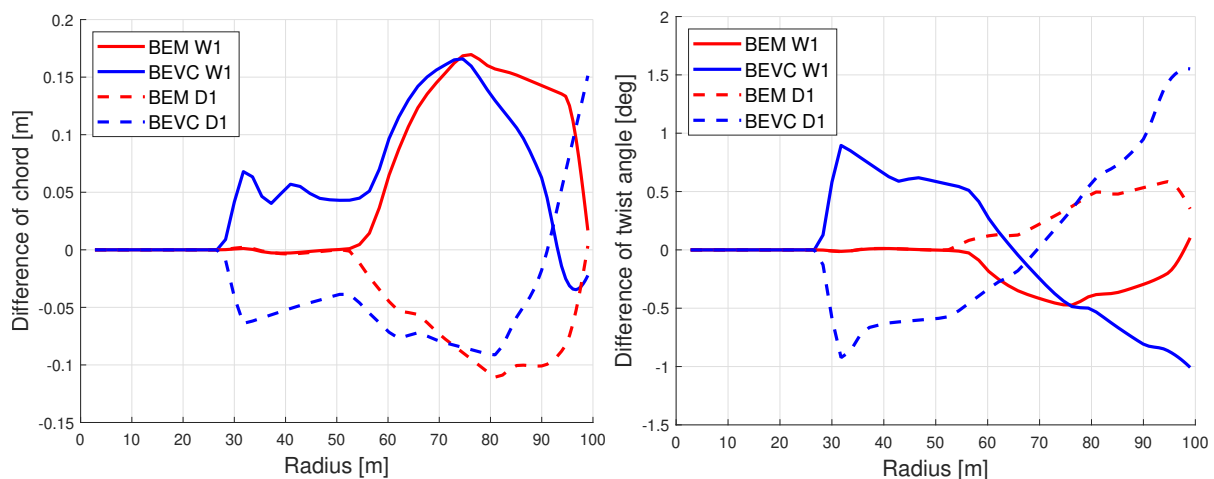
changing the dihedral direction. The side view of the main-axes of different dihedral geometries are shown in Figure 3.

The chord and twist of the dihedral blades from a radius of 28 m until the blade tip are optimized to obtain maximum aerodynamic power without load constraints. The radial distribution of airfoil relative thickness is kept unchanged. To clearly illustrate the optimized planform, the difference of the chord and twist of the optimized dihedral blades compared to the optimized baseline straight blade are shown in Figure 4.

Both optimized upwind and downwind dihedral blades from the BEM method have identical chord and twist for the straight part of the blade that has radius less than 50 m. This is as expected since the BEM method assumes radially independent blade sections, so the influence of the outer dihedral part of the blade on the inboard straight part can not be felt by the inner straight part. On the other hand, the BEVC method is able to model the influence of the changed axial position of the outer sections felt at the inner sections, and vice versa. Thus, the chord and twist distribution of the straight part of the dihedral blades optimized by the BEVC



**Figure 3.** Side-view of the main-axes of the blades used for the comparison.



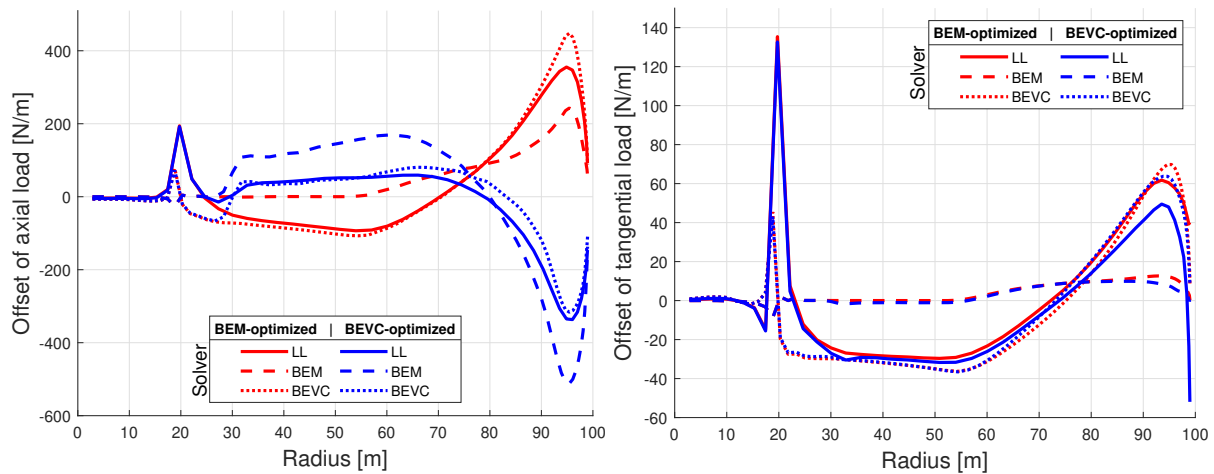
**Figure 4.** The difference in chord length (left) and blade twist (right) of the optimized dihedral blades compared to the optimized straight blade.

method are different compared to the optimized straight blade.

For both upwind and downwind dihedral cases, the chord and twist distribution of the two optimized designs from BEM and BEVC are different, but they do show similar trends. On the other hand, when comparing the designs for the upwind and downwind dihedral cases from each method, the change of chord and twist are approximately symmetric to the zero-value line.

To evaluate the performance of the different designs, the distributed loads of the different designs are calculated from BEM, BEVC and also the higher-fidelity LL method. To better show the difference between different designs, the difference of the loads of the optimized dihedral blades compared to the optimized straight blade are shown in Figure 5 and 6 for upwind and downwind dihedral respectively. The loads correspond to aerodynamic force per unit radius.

For the upwind dihedral case in Figure 5, the axial loads of the two designs are different. For the BEM design, the axial load is decreased compared to the optimized straight blade for the part of the blade with a radius of less than 70 m and is increased for the tip part of the blade. Instead, the axial load of the BEVC design is firstly increased for the inboard part of the



**Figure 5.** The difference of the axial load (left) and tangential load (right) of the two different optimized upwind dihedral blades based on W-1 calculated from the LL, BEM and BEVC.

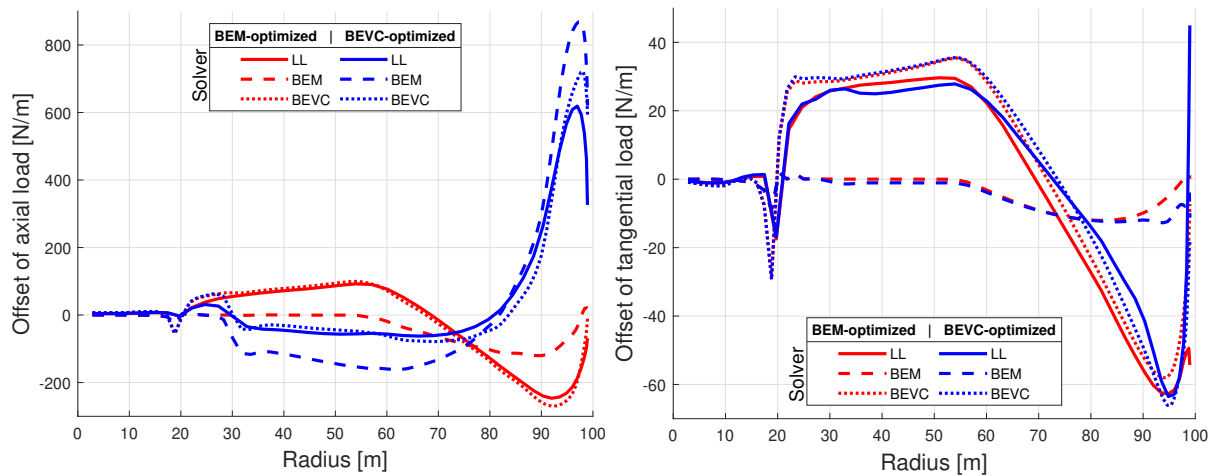
blade and is then decreased for the tip part. There is good agreement between the axial loads predicted by the BEVC method and the LL method for both designs. The prediction by the BEM method generally has an offset but can still predict some of the trends. The tangential loads of the two different designs are fairly similar. The LL method predicts that the tangential loads are firstly decreased for the inboard part of the blade that has radius less than 75 m and is increased for the tip part of the blade. The BEVC method predicts very similar results as the LL method. However, the BEM method predicts a quite different behavior than the high-fidelity method: a small increase of the tangential load throughout the span, for both designs. As the lifting-line method is based on vortex elements with inherently singular nature, sudden changes in circulation have a large effect on local induced velocities. This explains the peak seen at  $r=20$  m, because this location is at the edge between the root section, where the original root section is kept, and the part of the blade where the optimizer can change the loading, causing a high local gradient in the load, and thereby also the bound circulation.

For the downwind dihedral blades, the conclusions are similar to the upwind dihedral cases, but with opposite values.

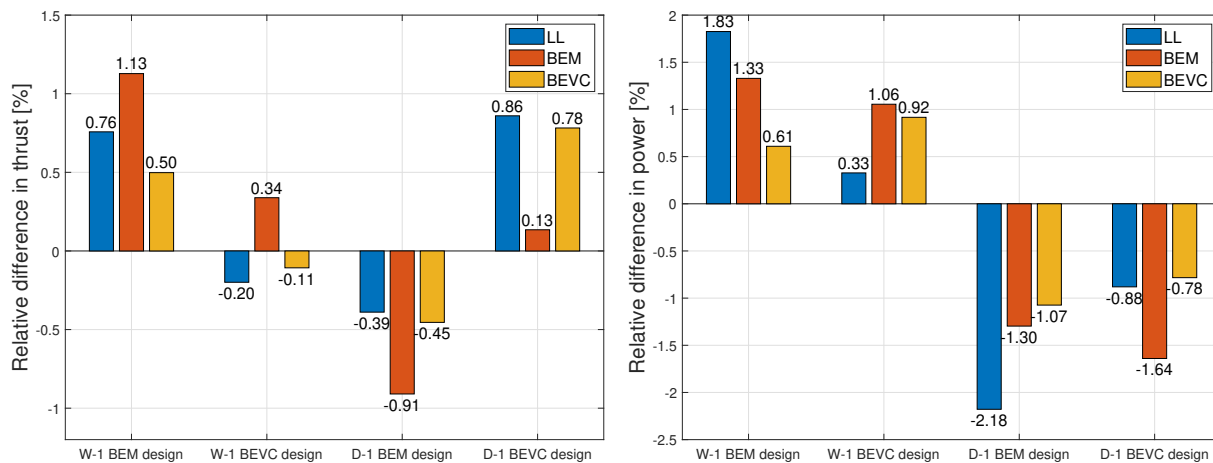
To show the rotor-integrated aerodynamic loads of different designs, the relative difference of the thrust and power of the upwind and downwind dihedral rotors designed using BEM and BEVC method compared to the planar rotor with optimized straight blades predicted by BEM, BEVC and LL method are summarized in the bar plots in Figure 7.

Firstly, the abilities of the two methods to predict the integrated thrust and power of different dihedral rotors are compared. For the aerodynamic thrust, the predictions from the BEVC method have better agreement with the LL method compared to the BEM method for all four designs. However, it is not possible to conclude which method has better performance when predicting the aerodynamic power. The BEM method predicts better agreement with LL for the blades designed using BEM while the BEVC method is in better agreement with LL for the blades designed using BEVC. Secondly, it can be concluded from the results that the optimized upwind dihedral blades have increased power compared to the straight blade while the optimized downwind dihedral blades have decreased power. All simulation methods show this conclusion. Thirdly, the performance of the two methods on the aerodynamic design are compared. Unfortunately, it is not possible to conclude which method designs better dihedral blades. For the upwind dihedral case, the BEM design has higher power while for the downwind dihedral case, the BEVC design has higher power. However, it should be emphasized that the





**Figure 6.** The difference of the axial load (left) and tangential load (right) of the two different optimized downwind dihedral blades based on D-1 calculated from different aerodynamic methods.



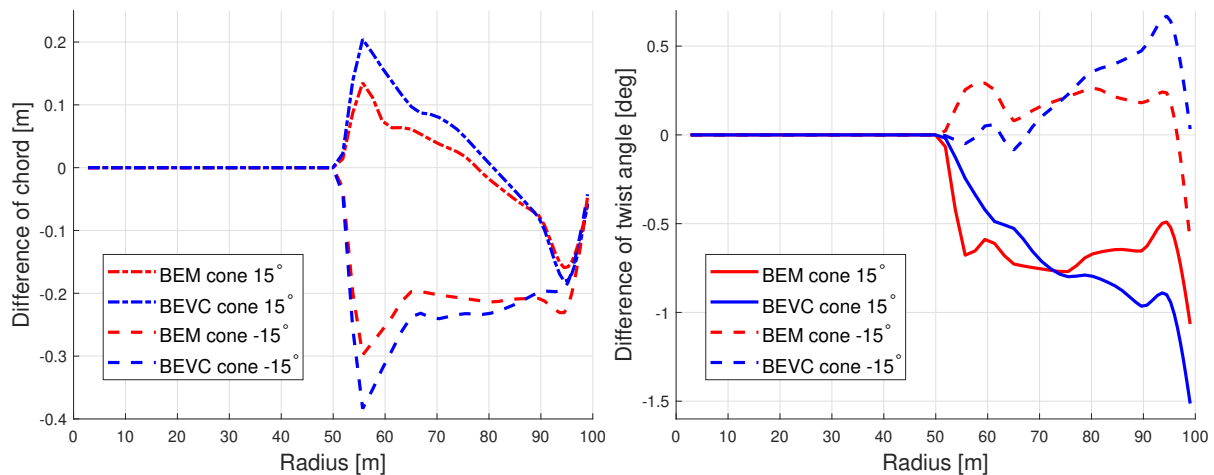
**Figure 7.** The relative difference in aerodynamic thrust (left) and power (right) of the upwind dihedral blades W-1 and downwind dihedral blade D-1 designed using BEM and BEVC method with respect to the planar rotor with optimized straight blades.

performance of the BEVC method on the aerodynamic optimization is not fully represented by the resulting integrated aerodynamic power of the optimized rotor. The advantage of the BEVC method over the BEM method when predicting distributed loads is significant, so the design from the BEVC method is believed to be closer to the actual optimal design.

### 3.3. Coned rotor with straight blades, with constraints

The chord and twist of the coned rotors with straight blades that have the same radius as the baseline rotor without coning but with increased total blade length are optimized for the same operational condition using BEM and BEVC methods. The optimization is performed to achieve maximum aerodynamic power, under the constraints that the total thrust force and the root out-of-plane aerodynamic moment should not increase compared to the un-coned baseline rotor. The coned rotors with both 15° of upwind coning and with 15° of downwind coning are optimized. To avoid large increase of the chord closer to the root region that will distort the

results, only the outboard part of the blade from 50% span to the blade tip is optimized. The focus of this optimization study is the mid-span to tip region of the blade. For each of the optimized designs, the constraint on the root out-of-plane aerodynamic moment is reached and the constraint on the total thrust force is not reached. The difference of the chord and twist distribution of the coned blades compared to the optimized baseline straight blade are shown in Figure 8.



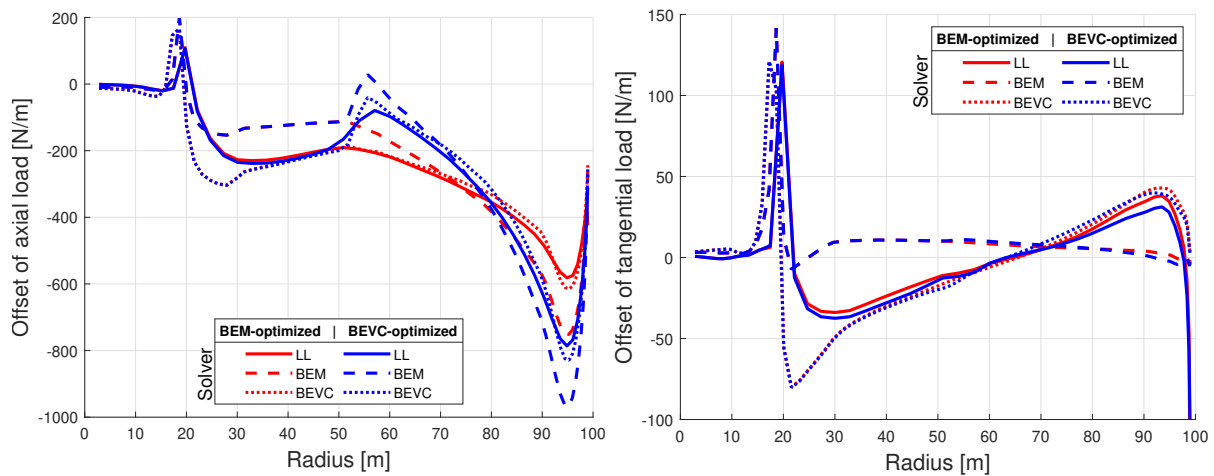
**Figure 8.** The difference of the chord (left) and twist (right) distribution of the straight blades with  $15^\circ$  of coning upwind and  $15^\circ$  of coning downwind (labelled as cone  $-15^\circ$ ) optimized using the BEM method and the BEVC method compared to the baseline straight blade.

For either upwind or downwind coning case, the chord and twist distribution of the two optimized designs from BEM and BEVC methods have some differences, but the overall trends and shapes are still similar. When comparing the designs for the upwind coning and downwind coning cases, the chord and twist have relatively large differences, for the designs from each method. The relative trend is predicted similarly for both methods.

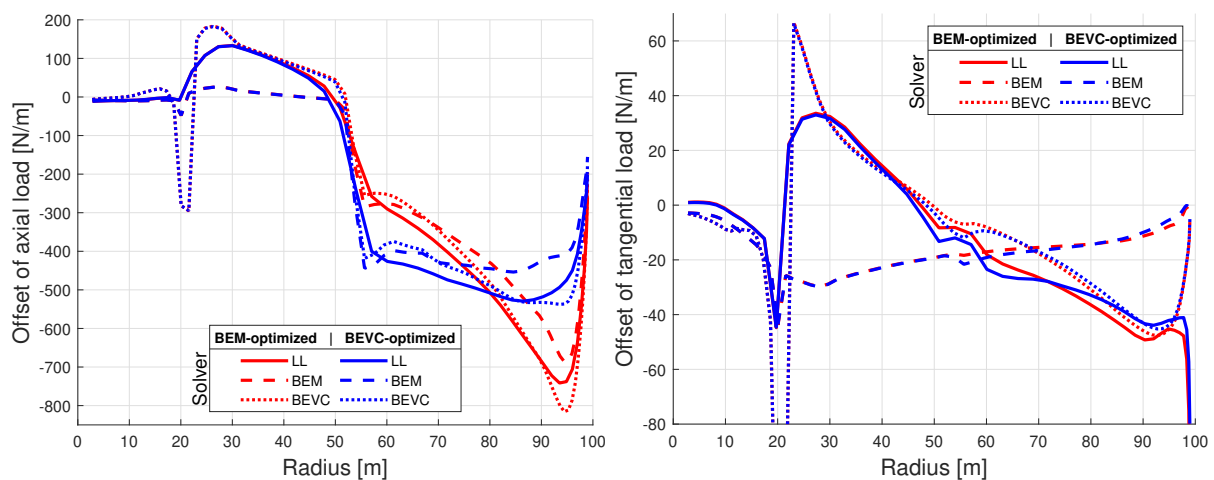
The aerodynamic loads of different optimized coned blades are calculated from LL, BEM and BEVC. To better compare the performance of different designs, the difference of the loads of the optimized coned blades compared to the baseline straight blade are shown in Figure 9 and 10. As previously, the distributed loads are aerodynamic force per unit radius.

For the upwind coning case in Figure 9, the two different designs have different axial loads. The BEVC design has higher axial loads compared to the BEM design from radius of 50 m to 80 m and has lower axial loads from radius of 80 m until the blade tip. A similar difference is also observed for the case of upwind dihedral blades optimized without constraints as shown in Figure 5. The axial loads predicted by the BEVC and LL methods are in good agreement for both designs. The BEM method does predict the trends, but the results are slightly further away from the LL results than the BEVC results. The tangential loads of the two designs are similar, which are decreased from radius of 20 m until 65 m and then increased from radius of 65 m until the blade tip. The BEVC method is able to correctly predict this trend but the BEM method predicts a completely different trend: a small increase of the tangential load throughout the span. For the downwind coning case in Figure 10, the conclusions are similar to the upwind coning case but the changes are in opposite directions.

The integrated aerodynamic power and thrust of the coned rotor are calculated to further evaluate the performance of the designs and methods. To show the influence of the blade coning on the integrated aerodynamic loads of the whole rotor, the relative difference of the thrust and power of the upwind and downwind coned rotors designed using the BEM and BEVC methods



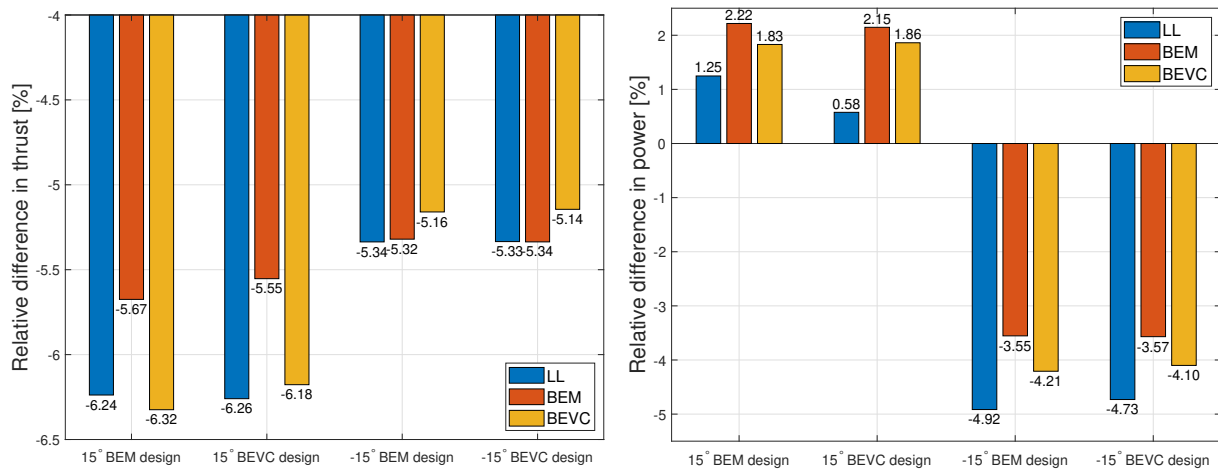
**Figure 9.** The difference of the axial load (left) and tangential load (right) of the two optimized  $15^\circ$  upwind coned blades calculated from the LL method, the BEM method and the BEVC method.



**Figure 10.** The difference of the axial load (left) and tangential load (right) of the two optimized  $15^\circ$  downwind coned blades calculated from the LL method, the BEM method and the BEVC method.

compared to the planar rotor with optimized straight blades predicted by BEM, BEVC and LL method are summarized in the bar plots in Figure 11.

The performance of the two methods on the aerodynamic planform optimization are compared first. For both upwind coning and downwind coning cases, the designs from BEM and BEVC have almost identical rotor thrust forces. For the aerodynamic power, the BEM design has slightly higher power for the upwind coning case but has slightly lower power for the downwind coning case. However, as has been argued for the dihedral blades optimized without constraints, it is believed the BEVC design is closer to the actual optimal design, because the local features predicted with the BEVC method agree much better with the high fidelity LL results than the BEM results. Secondly, the abilities of the two methods to predict the thrust and power of different coned rotors are compared. For the aerodynamic power, the predictions from the BEVC method are in better agreement with LL for all designs. For the aerodynamic thrust, BEM and BEVC have similar performance: BEVC predicts better for upwind coning cases while BEM



**Figure 11.** The relative difference in aerodynamic thrust (left) and power (right) of the 15° upwind coning and 15° downwind coning designed using BEM and BEVC method with respect to the planar rotor with optimized straight blades.

predicts better for downwind coning cases. Thirdly, it can be concluded from the LL result that the optimized upwind coned case has approximately 1% lower thrust compared to the downwind coned case but has approximately 6% higher power. This difference can be correctly predicted by both BEM and BEVC methods.

#### 4. Conclusions and future work

The present study involved aerodynamic planform optimization of rotors with dihedral blades and also coned straight blades under different constraints. The performance of the BEM and BEVC engineering models was benchmarked with a high-fidelity LL method. The blade element vortex cylinder (BEVC) model shows an advantage over the blade element momentum (BEM) model for predicting the distributed aerodynamic loads, which is in agreement with an earlier study on that matter. For the prediction of the integrated aerodynamic loads of the rotor, such as thrust and power, the BEM method again shows surprisingly good performance. There is only a small advantage of the BEVC model over the BEM method. Both the BEM and BEVC methods predict that the optimized upwind dihedral rotor has higher aerodynamic power compared to the optimized downwind dihedral rotor under the same constraints. For either the upwind or downwind dihedral cases, the aerodynamic power of the BEVC designs does not show significant improvement compared to the BEM designs. However, it is believed that the BEVC design is closer to the actual optimal design due to its significantly improved ability to predict distributed loads of non-planar rotors. It is expected that the BEVC method may show more advantages in the wind turbine design when the blade structure properties and elastic deformation are included in the design loop. In addition, the advantages are also expected when there are constraints locally at certain spanwise locations, such as limiting the flap-wise bending moment at a specific blade section. Future work is also favourable on comparing the aerodynamic design from the BEVC method with the design directly from a higher-fidelity model, such as the lifting-line method or blade geometry resolving Navier-Stokes simulations.

#### Acknowledgement

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