Collaborative Pricing in a Power-Transportation Coupled Network: A Variational Inequality Approach

Xie, Shiwei; Wu, Qiuwei; Hatziargyriou, Nikos D.; Zhang, Menglin; Zhang, Yachao; Xu, Yinliang

Published in:
IEEE Transactions on Power Systems

Link to article, DOI:
10.1109/TPWRS.2022.3162861

Publication date:
2022

Document Version
Peer reviewed version

Citation (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
Collaborative Pricing in a Power-Transportation Coupled Network: A Variational Inequality Approach

Shiwei Xie, Member, IEEE, Qiuwei Wu*, Senior Member, IEEE, Nikos D. Hatzigiariou, Life Fellow, IEEE, Menglin Zhang, Yachao Zhang, Member, IEEE, Yinliang Xu, Senior Member

Abstract—This paper presents a collaborative pricing scheme for a power-transportation coupled network based on the variational inequality (VI) approach. In the proposed scheme, nodal electricity prices and congestion tolls on roads and at charging stations are considered to coordinate the coupled networks in order to minimize the operational cost of the whole system. The prices are determined by a second-order cone-based AC power flow model and a mixed user equilibrium model, respectively. A collaborative pricing model (CPM) is then built based on the two models and the interactions between them. In order to avoid the intractability of the developed non-convex model, the CPM is transformed into the VI formulation. With proven existence and uniqueness of solutions of the VI formulation, a new prediction-correction algorithm is proposed to accelerate the solution of the CPM problem, which is guaranteed to converge to the optimal solution. The proposed models and algorithm are verified using case studies on a real-world test system. The results show that the proposed pricing scheme can reduce the operational cost and the proposed algorithm shows improved convergence and higher computational efficiency compared with the existing algorithms.

Index Terms—collaborative pricing, power-transportation coupled network, prediction-correction algorithm, variational inequality.

I. INTRODUCTION

THE increasing deployment of electric vehicles (EVs) in urban cities is creating a complex relationship between two critical infrastructures: power distribution networks (PDNs) and transport networks (TNs) [1]. On one hand, variations in the spatial and temporal occurrence of traffic congestion affect the travel behaviors of EVs and thus the flow patterns in the TN. On the other hand, the traffic flow patterns of EVs will impact the spatial and temporal distribution of the power charging demands as well as the operation of the PDNs and the electricity price at charging stations (CSs). These factors in turn affect the decisions made by EV drivers. This interconnection of systems brings great challenges in optimizing the management strategy of power-transportation coupled networks (PTCNs) [2]-[4].

Recently, studies have been conducted to investigate the interdependence of systems in PTCNs. Studies in [5], [6] showed that the system-level interactions between the related networks can be designed to reach an equilibrium of the network. These studies determined both the user equilibrium (UE) in the TN and the optimal power flow conditions in the PDN. Other models representing this interaction are proposed in [8]-[10]. These models optimized the overall system cost by accounting for the relationship between power and traffic flows. Furthermore, the participation of EVs in the model was included to enhance the system flexibility. For instance, the authors in [11] proposed a model to schedule the spatio-temporal charging flexibility of EVs in PTCNs, while respecting the transit schedule and the constraints imposed by the EV battery. Also, in further research [12]-[13], the EV charging flexibility was considered an improvement for the day-ahead operation of power grids.

In practice, regulating traffic and power flow can realize the social optimum of PTCNs, which is usually achieved by price making [14]. Instead of considering prices as static [8]-[10], a number of studies treated the prices for the EV drivers as either the locational marginal price (LMP) [6], [15], [16] or the retail price [17], [18]. Furthermore, [19], [20] investigated the road pricing mechanism for managing road congestion and routing individual drivers to improve social welfare. By jointly considering the PDN and TN, several studies proposed integrated pricing strategies [2], [14], [21], in which the toll prices of the TNs and the electricity prices of PDNs were determined collaboratively.

Despite the great progress in the integrated pricing of the PTCNs, there are several research gaps.

1) Most existing research omitted the joint participation of EVs and gasoline vehicles (GVs) in such a pricing problem [2], [9], [14], [21]-[22]. However, this may be improper in the current case where multiple vehicles co-exist in urban cities to satisfy different traffic demands. The other studies [6], [7], [17] modeling such co-existence only concentrated on the charging prices faced by EVs at CSs, but ignored the critical role of road pricing in regulating users’ choices and alleviating traffic congestions. In addition, different charging requirements and driving range constraints for EVs were usually relaxed in literature, which becomes more vital when addressing electricity pricing design.

2) A high computational burden may arise from solving such
a pricing design problem. For instance, it is challenging to directly solve the traffic assignment problem (TAP) including integral and non-linear travel time variables in the objective function. Approximations of the objective function typically use piecewise linear (PWL) methods [8], [10], [22]. This introduces a large number of discontinuous integer variables, resulting in a high computational burden. On the other hand, conventional methods cannot directly obtain electricity prices that are implicitly determined by the dual variables [20]. The approach to deal with this difficulty is solving the problem in a distributed manner, such as using the dual decomposition algorithm [2] and the best-response decomposition algorithm (BRDA) [6]. Nevertheless, mixed-integer programming may be repeatedly called when using these algorithms to solve the TAP, leading to an excessive high computational burden.

3) Due to the above nonlinearities in such a model, the existing methods may fail to converge to the global optimum [2], [6]. Specifically, the distributed algorithms like BRDA highly depend on the initial value, which should be close enough to the solution. Otherwise, it may fail to converge, even an equilibrium exists [6]. In addition, these discontinuities and nonlinearities result in a non-compact set, which become a major hurdle in exploring the distributed optimization methods in terms of the existence and uniqueness of solutions.

To address the challenges mentioned above, this paper attempts to solve the collaborative pricing problem for PTCNs under a new scheme, which is supported by variational inequality (VI) theory. The VI has been used to model and solve the market equilibrium problems [23], [24], [25]. To the best knowledge of the authors, there is no research exploring the application of VI theory in the collaborative pricing problem of PTCNs. Different from the existing methods, using VI method can avoid the intractability of the non-convex pricing model and reduce the complexity of analyzing the existence and uniqueness of solutions. To solve the collaborative pricing problem, the proposed scheme also inspires a VI-based algorithm that provides guarantees on the convergence. The main contributions of this paper include:

- Propose a new collaborative pricing scheme for PTCNs. The distribution LMPs (DLMPs) and congestion tolls on traffic roads and at charging stations are jointly considered to manage the PTCNs to maximize the social welfare. Different from existing studies, an expanded network is used to describe the behaviors of traveling, queuing, and charging for EV drivers with multiple charging demands. Congestion pricing strategies guarantee that the different behaviors of EVs and GVs can be managed towards the optimal traffic flow pattern.

- Recast the non-convex collaborative pricing model (CPM) with SOC constraints into an equivalent VI formulation. With the proposed VI reformulation of the CPM, the DLMPs can be explicitly included as a part of variables and thus can avoid the intractability of the non-convex CPM and reduce the computational burden. The VI approach can theoretically prove the existence and uniqueness of solutions, as well as the convergence of the developed algorithm, which is hard for a discontinuous and non-convex problem.

- Propose an improved prediction-correction algorithm (IPCA) to solve the proposed CPM in the VI form. The proposed algorithm incorporates an adaptive stepsize and a relaxation technique, which accelerates the convergence speed compared to the basic method. With the VI form, the convergence of the IPCA is established both rigorously and conveniently.

II. PRELIMINARIES AND PROBLEM STATEMENT

In this study, the collaborative pricing strategies are investigated under a static setting to manage the integration of urban power and traffic networks to achieve a social optimum. To satisfy various traffic demands, EVs as well as GVs coexist in the TN [17]. All drivers are assumed to be rational and have access to traffic information. This includes the availability of CSs, electricity prices for charging, and road congestion conditions, upon which the drivers decide their paths to minimize their travel costs. This is a realistic assumption as it is natural to extend the popular navigation apps to include trip planning information [18]. Only EV drivers need to find a CS along the path to charge, and do so at the minimal charging cost, which is a function of location-based prices and congestion conditions at CSs. EVs, which do not currently need to be charged, are treated as GVs since they have the same routing criteria [7]. Note that EVs arriving at their destinations can receive a battery charge before the next journey [18]; however, this is out of the scope of this work. In the PDN, a number of DGs including renewable ones has been installed at certain buses. The PTCN is operated by a non-profit independent system operator (ISO) [10], [22], who has access to data of both networks, and aims at minimizing the cost of the whole system using pricing strategies, as shown in Fig. 1. Specifically, the ISO optimizes the operation of PTCN by 1) dispatching the PDN optimally according to the power load and EV charging demand, 2) determining the prices at the CSs to regulate EV charging choices, and 3) imposing congestion tolls at roads and CSs based on the flow patterns to manage users’ behaviors. Notably, the ISO cannot control drivers directly, but can decide the congestion tolls on the PTCN to guide their route choices towards the optimal flow pattern. Although the power and transportation systems are
managed by different entities currently, new entities will emerge with the progress of transportation electrification in the near future [9], [21].

III. TRANSPORTATION MODELING

Network representation: A directed graph \((V, A)\) is used to model a TN with a finite number of CSs, where \(V\) and \(A\) are the sets of traffic nodes and arcs, respectively. The CS node set is denoted as \(N_{CS} (N_{CS} \subseteq V)\). EV's (GV's) travel demands are \(Q^e_i (Q^g_i)\), and are between a set of Origin-Destination (O-D) pairs \(W^e (W^g)\). Each O-D pair \((i, j)\) is composed of an origin node \(i \in V\) and a destination node \(j \in V\), which are connected by a set of paths. We use \(P_i (P^g_i)\) to represent the feasible path set of EVs (GVs) between an O-D pair \((i, j)\) in \(W^e (W^g)\), and \(P\) as the set of all feasible paths in the TN. Each path \(p \in P\) is a concatenated collection of arcs \(a \in A\). Denote \(\delta_{ij}\) as the path-arc incidence, which equals 1 if arc \(a\) is on path \(p\) of an EV (GV) and 0 otherwise. Any path for EVs must pass by at least one CS, while no CSs are required along paths for GVVs. Given that the driving range of an EV is limited by its remaining energy, some paths between its O-D pair will be unusable.

To incorporate the driving range constraints into the model, the definition of energy-feasible paths is employed [26]: a path for EVs is defined as energy-feasible if an EV travelling upon it is able to reach at least one CS before running out of energy and can, upon leaving this CS, complete the journey under its own power without further need to charge its batteries. With this definition, the ISO can pre-determine all energy-feasible paths off-line (refer to [2], [26] for more discussions).

A. Expanded TN model with CSs

![Diagram of Expanded TN model with CSs](image)

For modeling the travel time that a user spends on a route, arcs in the expanded graph can be classified into four types:

1) Regular arc \((a \in A^e)\): Both EVs and GVVs can visit a regular arc and their travel time will depend on the traffic flow. The popular BPR function is adopted to compute the time needed to travel through a regular arc:

\[
\tau_a(x_a) = \tau^0_a \left[1 + 0.15 \left(\frac{x_a}{\xi_a}\right)^4\right], \quad \forall a \in A^e
\]

where \(x_a\) is the arc traffic flow, \(\tau^0_a\) is the free travel time, and \(\xi_a\) is the arc capacity.

2) Queuing arc \((a \in A^q)\): Due to the limited number of chargers as well as congestion at the CS, EVs may experience queuing time prior to plugging in for charging, which will occur in the queuing arc. Here, the Davidson function is employed to evaluate such queuing time:

\[
\tau_a(x_a) = R \cdot \frac{x_a}{\xi_a}, \quad \forall n \in N_{CS}, n \in a \in A^q
\]

where \(R\) is a shape factor and \(\xi_a\) is the CS capacity.

3) Charging arc \((a \in A^c, n \in N_{CS})\): The travel time spent on a charging arc \(a\) at CS \(n\) is equivalent to the charging time at the CS. If the average rated power of chargers is \(P_c\) and the amount of energy charged in a charging arc \(a\) is \(E_a\), this time is equal to,

\[
\tau_a(x_a) = E_a / P_c, \quad \forall a \in A^c, n \in N_{CS}
\]

In (3), the EV charging time is considered to be linear with respect to the charging power [2], [6], [7], which is reasonable for such a system-level study.

4) Bypass arc \((a \in A^b)\): GVVs do not need to be charged and skip the CS via a bypass arc with zero travel time:

\[
\tau_a(x_a) = 0, \quad \forall a \in A^b
\]

B. Travel cost function of individual drivers

All individual drivers decide what paths to take to minimize their travel costs. For GV drivers, their travel costs on path \(p\) (denoted by \(\Psi^g_p\)) can be evaluated by the sum of the time spent on all passing arcs:

\[
\Psi^g_p = M \cdot \sum_{a \in P^g_p} \tau_a(x_a) \delta_{ij}, \quad \forall p \in P^g, (i, j) \in W^g
\]

where \(M\) is the monetary cost of the travel time.

EV drivers must, additionally, look for CS locations to charge their EV batteries at the minimum charging cost based on the electricity price at each station. Thus, the travel cost of EVs on path \(p\) (denoted by \(\Psi^e_p\)) can be calculated by,

\[
\Psi^e_p = M \cdot \sum_{a \in P^e_p} \tau_a(x_a) \delta_{ij} + E_a \lambda^e_{ij} \chi^e_{ij}, \quad \forall p \in P^e, (i, j) \in W^e
\]

where \(\lambda^e_{ij}\) is the price of electricity at charging arc \(a\) that is connected to CS bus \(n \in N_{CS}\) in the PDN, and \(\chi^e_{ij}\) is node-path incidence coefficient (a 0-1 indicator variable).

C. Mixed traffic flow constraints

In the presence of both EVs and GVVs, the network flow pattern is constrained by the following equations.

\[
x_a = \sum_{i \in V^e \cap p^e} h^e_{iij} + \sum_{i \in V^g \cap p^g} h^g_{iij}, \quad \forall a \in A
\]

\[
h_p \in \mathbb{R}^+ : \sum_{p \in P^g} h_p = Q^g_{ij}, \quad \forall i, j \in W^g
\]
IEEE TRANSACTIONS ON POWER SYSTEMS

\[ h_p \in \mathbb{R}_+ : \sum_{p \in C_p} h_p = Q_{ij}^p, \forall i,j \in \mathcal{N}^p \] (9)

where \( h_p \) is the path flow and \( \mathbb{R}_+ \) is the set of non-negative real numbers. Eq. (7) defines the link-path flow relation, and (8)-(9) are flow conservation equations. For brevity, the matrix form of the flow constraints is,

\[ \Omega_T = \left\{ (h,x) : x = \Delta h, h \in \mathbb{R}_+^{[p]} \right\} \] (10)

where \( x = [x_a]_{a \in \mathcal{A}_C}, h = [h_p]_{p \in \mathcal{P}_G}, \Delta = [\Delta_E \Delta_G]^T, \Delta_E = [\delta_{pq}^e], \Delta_G = [\delta_{pq}^g], Q^e = [Q_{ij}^e]_{i,j \in \mathcal{V}^e}, Q^g = [Q_{ij}^g]_{i,j \in \mathcal{V}^g}, \Delta_E (\Delta_G) \) is the (EV (GV) OD-path incidence matrix.

D. Mixed user equilibrium

If each driver selects the path according to its own interest, the aggregate effect of individual decisions made by all drivers will lead to a user equilibrium (UE) state [27]. An equilibrium is such that no driver has any incentive to change paths and all utilized paths are minimum-cost paths. Given that our model involves two kinds of vehicles, a mixed user equilibrium (MUE) state can be similarly defined as follows: A flow pattern is in an MUE state if the travel costs of both GVs and EVs on all used paths between any given O-D pair are equal, and they are no larger than those of any unused paths.

Mathematically, the MUE definition can be interpreted as,

\[ h_p > 0, p \in \mathcal{P}_G^e \Rightarrow \psi_p^e = v_p^e, \forall (i,j) \in \mathcal{N}^e \] (11)

\[ h_p > 0, p \in \mathcal{P}_G^g \Rightarrow \psi_p^g = v_p^g, \forall (i,j) \in \mathcal{N}^g \] (12)

\[ \psi_p^e \geq v_p^e, p \in \mathcal{P}_G^e \Rightarrow h_p = 0, \forall (i,j) \in \mathcal{N}^e \] (13)

\[ \psi_p^g \geq v_p^g, p \in \mathcal{P}_G^g \Rightarrow h_p = 0, \forall (i,j) \in \mathcal{N}^g \] (14)

where \( v_p^e (v_p^g) \) is the infimum of cost levels achievable by EV (GV) drivers for the O-D pair \( (i,j) \).

E. Tolls that lead the system towards a social optimum

This subsection discusses how ISOs can transform an MUE flow pattern into a socially optimal flow pattern. First, the travel cost incurred by all vehicles traveling on each arc is defined as,

\[ \phi_a (x_a) = \begin{cases} M_{\tau_a} (x_a), & \forall a \in \mathcal{A}^e \cup \mathcal{A}^g \cup \mathcal{A}^b \\ \delta_{pq}^e \lambda_c, & \forall a \in \mathcal{A}^c, n \in \mathcal{N}_{CS} \end{cases} \] (15)

Note that the charging cost of EV drivers at the CS is included in the charging arc \((a \in \mathcal{A}^c, n \in \mathcal{N}_{CS})\). Accordingly, for a given \( \lambda_c \), the optimal travel cost is the solution of the following traffic system optimum problem:

\[ \min_{x} \mathcal{F}_{TN} = \sum_{a \in \mathcal{A}_C} x_a \cdot \phi_a (x_a) \]

s.t. \( \text{CnsTN} \equiv \{ x = \Delta h, Q^e = \Delta_E h, Q^g = \Delta_G h, h \in \mathbb{R}_+^{[p]} \} \] (16)

where the objective \( \mathcal{F}_{TN} (x) \) is the total travel cost in the TN. For brevity, the notation \( \Omega_T \) is used to replace the feasible set "CnsTN".

The MUE flow led by drivers’ self-interested behaviors is most unlikely consistent with the optimal traffic flow in (16). To guide both EV and GV drivers towards the socially optimal flow pattern, a congestion toll \( \xi = [\xi_a]_{a \in \mathcal{A}_C} \) at each arc and charging station is introduced. To identify such a toll vector, the KKT conditions of (16) are observed first, as follows:

\[ \nabla h_p \cdot \mathcal{F}_{TN} - \psi_p^e - \psi_p^g = 0, \forall p \in \mathcal{P}_G^e, (i,j) \in \mathcal{N}^e \]

\[ \nabla h_p \cdot \mathcal{F}_{TN} - \psi_p^e - \psi_p^g = 0, \forall p \in \mathcal{P}_G^g, (i,j) \in \mathcal{N}^g \]

\[ \psi_p^e h_p = 0, \forall p \in \mathcal{P}_G^e, (i,j) \in \mathcal{N}^e \]

\[ \psi_p^g \geq 0, \forall p \in \mathcal{P}_G^g, (i,j) \in \mathcal{N}^g \] (17)

where

\[ \nabla h_p \cdot \mathcal{F}_{TN} = \frac{\partial \mathcal{F}_{TN}}{\partial h_p} = \sum_{a \in \mathcal{A}_C} \frac{\partial h_p}{\partial x_a} = \sum_{a \in \mathcal{A}_C} \frac{\partial [c_a (x_a)]}{\partial x_a} = \sum_{a \in \mathcal{A}_C} \frac{[c_a (x_a) + \xi_a]}{\partial x_a} \]

(18)

Note that (18) refers to the definitions of marginal arc latency and marginal path latency, i.e.,

\[ MC_a = \phi_a (x_a) + x_a \frac{dc_a (x_a)}{dx_a}, \quad MC_p = \sum_{a \in \mathcal{A}_C} \delta_a \cdot MC_a. \]

It then follows from substituting (18) into (17) that

\[ h_p > 0, p \in \mathcal{P}_G^e \Rightarrow MC_p = \psi_p^e, \forall (i,j) \in \mathcal{N}^e \]

\[ h_p > 0, p \in \mathcal{P}_G^g \Rightarrow MC_p = \psi_p^g, \forall (i,j) \in \mathcal{N}^g \]

\[ MC_p \geq \psi_p^e, p \in \mathcal{P}_G^e \Rightarrow h_p = 0, \forall (i,j) \in \mathcal{N}^e \]

\[ MC_p \geq \psi_p^g, p \in \mathcal{P}_G^g \Rightarrow h_p = 0, \forall (i,j) \in \mathcal{N}^g \] (19)

The difference between (19) and MUE conditions in (11)-(14) is the link travel cost for users, i.e., \( \phi_a \) and \( MC_a \). This indicates that if a toll \( \xi_a \) is imposed at each link and CS, where

\[ \xi_a = x_a \frac{d \phi_a (x_a)}{dx_a}, \]

the solution of the MUE with these travel costs \((MC_a \downarrow \mathcal{A}_C)\) will produce an optimal flow pattern in (16), which is equivalent to the solution of a user equilibrium problem with imposed tolls \( \xi_a \). Thus, the congestion pricing scheme can be obtained by solving the optimization problem (16).

IV. POWER DISTRIBUTION NETWORK MODELING

A. Optimal dispatch problem in PDN

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>NOTATIONS USED IN THE SOC-BASED ACOPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision Variables</td>
<td>( p_i^o, q_i^o ) Active / Reactive power generation at bus ( n ).</td>
</tr>
<tr>
<td>( p_i^c, q_i^c ) Active / Reactive power demand at bus ( n ).</td>
<td></td>
</tr>
<tr>
<td>( p_{min}, q_{min} ) Active / Reactive power flow in line ( f ), ( (m,n) \in \mathcal{L} ).</td>
<td></td>
</tr>
<tr>
<td>( \Delta p_{max}, \Delta q_{max} ) Active / Reactive power load shedding at bus ( n ).</td>
<td></td>
</tr>
<tr>
<td>( U_{min}, I_{min} ) Square magnitude of voltage / current.</td>
<td></td>
</tr>
<tr>
<td>Parameters</td>
<td>( p_i^c ) Rated power of DG generation at bus ( n ).</td>
</tr>
<tr>
<td>( \omega ) The maximum curtailment rate of DGs at bus ( n ).</td>
<td></td>
</tr>
<tr>
<td>( U_{max}, I_{max} ) Lower / Upper bound of voltage magnitude.</td>
<td></td>
</tr>
<tr>
<td>( p_{max}, q_{max} ) Fixed active / reactive power demand at bus ( n ).</td>
<td></td>
</tr>
<tr>
<td>( \Delta p_{max}, \Delta q_{max} ) Upper bound of active / reactive power flow / current.</td>
<td></td>
</tr>
<tr>
<td>( \alpha, h_n, h_x ) Cost coefficients in the objective function.</td>
<td></td>
</tr>
<tr>
<td>( R_{min}, X_{min}, Z_{min} ) Resistance / Reactance / Impedance of line ( f ).</td>
<td></td>
</tr>
</tbody>
</table>
IEEE TRANSACTIONS ON POWER SYSTEMS

In this paper, a PDN with a radial topology represented by \((\mathcal{N}, \mathcal{L})\) is considered, where \(\mathcal{L}\) denotes the set of lines and \(\mathcal{N} = \{0\} \cup \mathcal{N}_e\) denotes the set of buses that is the union set of the slack bus 0 and the rest in \(\mathcal{N}_e = \{1, \ldots, |\mathcal{N}_e|\}\). The outputs of DGs can be curtailed within a certain range due to line congestion. To serve the charging load of EVs, CSs are connected to certain distribution buses of the PDN. The charging prices for the EV drivers at the CSs are provided by the market clearing prices of electricity, i.e., DLMPs. To dispatch the PDN in a socially optimal fashion and derive the DLMPs at the specific nodes, operators can solve the following branch-flow-based ACOPF model [28], in which the notations used are summarized in Table I.

\[
\min \sum_{m, n} \left[ a_m (p_m^p)^2 + b_m p_m^p + b_m p_m^{pq} + h_n (\Delta p_n^p + \Delta q_n^p) \right] \tag{20}
\]

s.t. \(p_m^p - (q_m^p - \Delta p_m^p) = \sum_{k \in \mathcal{K}_m} p_{mk} - (q_{mk}^p - \Delta q_{mk}^p), \forall (m, n) \in \mathcal{L}, \forall n \in \mathcal{N}_e\) \tag{21}

\[
q_m^p - (q_m^p - \Delta q_m^p) = \sum_{k \in \mathcal{K}_m} q_{mk}^p - (q_{mk}^p - \Delta q_{mk}^p), \forall (m, n) \in \mathcal{L}, \forall n \in \mathcal{N}_e\)

\[
U_m' = U_m - 2(p_m^p R_m + q_m^p X_m) + I_m (Z_m), \forall (m, n) \in \mathcal{L}, \forall n \in \mathcal{N}_e\]

\[
U_m' \geq (p_m^p)^2 + (q_m^p)^2, \forall (m, n) \in \mathcal{L}, \forall n \in \mathcal{N}_e\)

\[
0 \leq I_m' \leq T, \forall (m, n) \in \mathcal{L}, U_m' \leq U_m, \forall n \in \mathcal{N}_e\)

\[
(1 - \omega) p_m^p \leq p_m^p \leq p_m^e, \tan \theta_m^e p_m^e \leq q_m^e \leq \tan \theta_m^e p_m^e, \forall n \in \mathcal{N}_e\)

\[
0 \leq p_m^e \leq p_m^e, 0 \leq q_m^e \leq q_m^e, \forall (m, n) \in \mathcal{L}\)

\[
0 \leq p_m^e \leq p_m^e, 0 \leq q_m^e \leq q_m^e, \forall n \in \mathcal{N}_e\)

\[
p_m^p = p_m^e, \forall n \in \mathcal{N}_e\)

\[
p_m^p = p_m^e + p_m^c, \forall n \in \mathcal{N}_e\]

The objective function in (20) includes the production cost of DGs, the purchasing cost from the upper grid, and the penalty cost of load shedding. Eqs. (21) and (22) are nodal power balancing conditions, where \(\Theta(n)\) denotes the set of child buses of bus \(n\). Eq. (23) is the forward voltage drop equation. Eq. (24) represents a convex SOC relaxation to the original equality of the apparent power, and will be exact under specific conditions [29]. The limits of the nodal voltage and line current are defined in (25). In (26), the active power outputs of the DGs are bounded, while their reactive power output can be adjusted within the range of power factor angles [30], i.e., \(\tan \theta_m^c, \tan \theta_m^e\). Eq. (27) limits the power flow in each line. Load shedding is possible and limited in (28). The nodal active power load determined by a fixed demand at each bus \(n \in \mathcal{N}_e\) and charging load at a CS bus \(n \in \mathcal{N}_e\) are given by (29)-(30). Note that the DLMPs, i.e., \(\lambda' = [\lambda_1 \ldots \lambda_{|\mathcal{N}_e|}]\), are the dual variables of the nodal power balance equation (21). For brevity, the SOC-based formulation defined in (20)-(30) can be written in the following compact matrix form:

\[
\min \mathcal{F}_\text{PN}(y) = y^T Q y + q^T y \tag{31}
\]

s.t. \(\text{CusPN} \cup \mathcal{F}_\text{PN}(y)\)

with variable \(y \in \mathbb{R}^n\) and problem parameters given by \(Q \in \mathbb{R}^{n \times n}, q \in \mathbb{R}^n, A \in \mathbb{R}^{n \times m}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^m, D \in \mathbb{R}^d, E \in \mathbb{R}^{n \times d}, e \in \mathbb{R}^d, G \in \mathbb{R}^m\) and \(g \in \mathbb{R}\).

For subsequent analysis, the dual variables of the problem in (31) are \(\mu \in \mathbb{R}^m, \nu \in \mathbb{R}_+, \alpha' \in \mathbb{R}^n_+\) and \(\beta \in \mathbb{R}^d_+\). Since the pair \((\alpha', \beta)\) is associated with the constraint, \(\|\alpha'\|_1 \leq \beta\), holds for all \(\ell \in \mathcal{L}\). If the SOC in \(\mathbb{R}^n\) is defined as,

\[
\mathcal{K}_{n+1} = \{ (\alpha', \beta') | \alpha' \in \mathbb{R}^{n_+}, \beta' \in \mathbb{R}_+^d, \|\alpha'\|_1 \leq \beta \}
\]

then we have \((\alpha', \beta) \in \mathcal{K}_{n+1}, \forall \ell \in \mathcal{L}\).

B. EV flow to charging load mapping

The EV traffic flow passing a CS leads to a charging load, thereby spatially coupling the TN and PDN. The charging power of station can be denoted as a linear mapping of EV flows on the associated charging arcs \(a \in \mathcal{A}_{CS}\) as follows,

\[
p_e^C = \sum_{a \in \mathcal{A}_{CS}} e_a, \forall n \in \mathcal{N}_{CS}\]

To facilitate the analysis below, the mapping function (32) can be rewritten using the path flows:

\[
p^C = \sum_{a \in \mathcal{A}_{CS}} E_a \sum_{a \in \mathcal{A}_{CS}} h_a, \forall n \in \mathcal{N}_{CS}\]

The mapping (33) can also be written in a matrix form as,

\[
\text{CusM} \triangleq \{ y = \vartheta \cdot h \} : \pi
\]

where the coefficient matrix \(\vartheta \in \mathbb{R}^{|\mathcal{N}_{CS}| \times |\mathcal{P}|}\) maps EV traffic flows in the TN to the charging loads of each CS in the PDN, and the corresponding dual variable is denoted by \(\pi\).

V. COLLABORATIVE PRICING PROBLEM OF PTCN

At the system level, the ISO aims to coordinate electricity prices at CSs and congestion tolls on both roads and CSs while minimizing the total cost of the overall system. In this regard, the pricing strategies in the PTCN can be derived from an optimal power-traffic flow problem [2], which is hereafter referred to as a CPM:

\[
\min \Gamma_{\text{So}}(h, y) \triangleq \mathcal{F}_\text{TN}(h) + \mathcal{F}_\text{PN}(y) \tag{35}
\]

s.t. \{ CusTN \cup \text{CusPN} \cup \text{CusM} \}

where \(\mathcal{F}_\text{TN}(h) = \sum_{a \in \mathcal{A}} x_a \cdot \zeta_a, \zeta_a \triangleq M \tau_x(x_a)\).

Note that the DLMP is determined based on the charging demand at the different nodes of the grid, which is affected by EV driving patterns. As proved in [2], [14], efficient DLMPs and congestion prices can be obtained from the dual variables of the model (32). By observing the model formulation, there are three main challenges when attempting to solve it efficiently:

1) the highly nonlinear functions (1) and (2);
2) the bilinear terms in $\mathcal{F}_{TN}(h)$:

3) the dual variables $\lambda^*$ as the DLMPs.

Remark 1: The nonlinearity makes the model into a non-convex optimization problem that is generally computationally intractable. A common way to establish tractable approximations of the nonlinearity in (1)-(2) and $\mathcal{F}_{TN}(h)$ is to perform PWL approximations, e.g., [8]-[10], [22], by introducing massive binary variables that severely weaken the model convexity. Another important point to note is that DLMPs have to be retrieved from the dual variables. Currently, solving such a problem can be performed using decentralized algorithms. More specifically, these algorithms first solve the TN subproblem with the given DLMP to obtain the traffic flow pattern and charging demand. Then, it solves the optimal power flow subproblem in the PDN with fixed charging demand, and extracts related dual variables as DLMPs. This process will be repeated until the iteration converges. Given the non-convexity problem, this type of algorithm has challenges for both theoretical analysis and efficient computation. To address the challenges mentioned, the CPM is cast into the VI formulation in the following subsection.

A. CPM as a variational inequality

In this work, the collaborative pricing optimization problem is reformulated into a VI. Firstly, the Lagrangian $\mathcal{L}_{SO}$ associated with the CPM is presented as follows:

$$
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) = \Gamma_{SO}(h, y) - \mu^T (Ay - C) - \nu^T (By - D) - \pi^T (y - \vartheta h)
$$

which is defined on $\Omega_{SO} = (\Omega_T \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_t^l \times \mathcal{K} \times \mathbb{R}^m)$. Specifically, the TN variables $h$ are constrained by the feasible region $\Omega_T$ which is linear and convex, while the PDN variables $y$ belong to the set of real number $\mathbb{R}^n$. The following definitions in the notation are employed for brevity: $\alpha \subseteq \bigcup_{\ell \in \mathcal{L}} \alpha^\ell$, $\beta \subseteq \bigcup_{\ell \in \mathcal{L}} \beta^\ell$, and $\mathcal{K} \subseteq \bigcup_{\tau \in \mathcal{K}} \mathcal{K}^\tau$.

Let $z^* \in \mathbb{R}^n$ be a saddle point of the Lagrange function $\mathcal{L}_{SO}(z)$. According to saddle point theorem, the following conditions hold:

$$
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) = \mathcal{L}_{SO}(h^*, y^*, \mu^*, \nu^*, \alpha^*, \beta^*, \pi^*)
$$

which is equivalent to a system of inequalities:

$$
\left\{ \begin{array}{l}
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) - \mathcal{L}_{SO}(z) \geq 0, \forall h \in \Omega_T \\
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) - \mathcal{L}_{SO}(z) \leq 0, \forall y \in \mathbb{R}^n \\
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) - \mathcal{L}_{SO}(z) \leq 0, \forall \mu \in \mathbb{R}^m \\
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) - \mathcal{L}_{SO}(z) \leq 0, \forall \nu \in \mathbb{R}^m \\
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) - \mathcal{L}_{SO}(z) \leq 0, \forall (\alpha, \beta) \in \mathcal{K} \\
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) - \mathcal{L}_{SO}(z) \leq 0, \forall (\alpha, \beta) \in \mathcal{K} \\
\mathcal{L}_{SO}(h, y, \mu, \nu, \alpha, \beta, \pi) - \mathcal{L}_{SO}(z) \leq 0, \forall \pi \in \mathbb{R}^n
\end{array} \right.
$$

Therefore, finding a saddle point of the problem is equivalent to finding $z^* \in \Omega_{SO}$, such that:

$$
\left\{ \begin{array}{l}
(h - h^*)^T [\nabla \mathcal{F}_{TN}(h^*) - E^T \pi^*] \geq 0, \forall h \in \Omega_T \\
(y - y^*)^T [\nabla \mathcal{F}_{TN}(y^*) - A^T \mu^* - B^T \nu^* - \sum_{\ell \in \mathcal{L}} (E^\ell y + e^\ell) + G^\ell \beta^\ell)] \geq 0, \forall y \in \mathbb{R}^n \\
(\alpha - \alpha^*)^T (Ay^* - C) \geq 0, \forall (\alpha^*) \in \mathcal{K} \\
(\nu - \nu^*)^T (By^* - D) \geq 0, \forall (\nu^*) \in \mathbb{R}^m \\
(\beta - \beta^*)^T (Gy^* + g) \geq 0, \forall (\beta^*) \in \mathcal{K} \\
(\pi - \pi^*)^T (y^* - \vartheta^* h^*) \geq 0, \forall (\pi^*) \in \mathbb{R}^n
\end{array} \right.
$$

for all $z^* \in \mathbb{R}^n$.

Consequently, (36) can be compactly reformulated into the following VI:

$$
\text{VI}(\mathcal{F}_{SO}, \Omega_{SO}) : \quad (z - z^*)^T \mathcal{F}_{SO}(z^*) \geq 0, \forall z \in \Omega_{SO}
$$

where $\Omega_{SO} = (\Omega_T \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_t^l \times \mathcal{K} \times \mathbb{R}^m)$.

Remark 2: DLMPs $\lambda^\ell \in \mathcal{L}$ are naturally included in $\text{VI}(\mathcal{F}_{SO}, \Omega_{SO})$ among the variables because of $\lambda \subseteq \mathbb{R}$. This implies that the DLMPs can be explicitly solved under the proposed VI formulation.

Accordingly, the proposed CPM, which includes conic constraints, is cast as a VI problem. Notably, the VI formulation in (37) includes the original and dual variables but without needing to introduce relaxations or integer variables, thereby reducing the computational complexity.

B. Existence of a Unique Solution

The nonlinearity of the CPM makes the analysis of solution properties difficult. By using VIs, the existence and uniqueness of the solution can be easily analyzed (guaranteed). To derive the desired results, it is first noted that a VI problem $\text{VI}(\mathcal{F}(z), \Omega)$ is equivalent to a projection problem, stated as below (see [24] for the proof):

$$
z^* = \Pi_{\Omega}(z^* - \gamma \mathcal{F}(z^*))
$$

where $\Pi_{\Omega}(\cdot)$ is the minimum norm projection onto the set $\Omega$, $\gamma$ is a given positive constant, and $z^*$ is the problem solution. The projection in the form of (38) is theoretically convenient because it can be viewed as a fixed-point problem.

1) Existence: According to Brouwer’s theorem, the
existence of a solution to a fixed-point problem follows the
continuity of the mapping operator, provided that the feasible
set is compact. However, this theorem is not immediately
applicable in the case of the unbounded feasible set \( \Omega_{so} \) in
the problem as present here. To address this issue, a bounded
convex set \( \Omega_R \) is introduced, which is a union of \( \Omega_{so} \) and
a closed ball \( B(0, R) \) with a large-enough radius \( R > 0 \) at point 0,
i.e., \( \Omega_R \equiv B(0, R) \cap \Omega_{so} \). As proved in existing research [25], if
\( \mathcal{VI}(\mathcal{F}_{so}, \Omega_R) \) already possesses a solution, then \( \mathcal{VI}(\mathcal{F}_{so}, \Omega_{so}) \)
does as well. Clearly, it follows from Brouwer’s fixed point
theorem that \( z^* = \Pi_{\Omega_R}(z^* - \gamma_{so} \mathcal{F}_{so}(z^*)) \) admits a solution,
only by observing that \( \mathcal{F}_{so}(z) \) is continuous with \( z \) and \( \Omega_R \) is
a compact set, so is \( \mathcal{VI}(\mathcal{F}_{so}, \Omega_R) \). Therefore, \( \mathcal{VI}(\mathcal{F}_{so}, \Omega_{so}) \) in
(37) has a solution, as well.

2) Uniqueness: According to [24], the solution uniqueness
of a VI can be easily guaranteed if the operator (i.e., \( \mathcal{F}_{so}(z) \)
in this case) is strictly monotone. For \( z_i > z \), the following can be derived:

\[
(z_i - z)^	op (\mathcal{F}_{so}(z_i) - \mathcal{F}_{so}(z)) = \left[ \begin{array}{c} h_i - h \nonumber \end{array} \right] \left[ \begin{array}{c} \nabla \mathcal{F}_{so}(h_i) - \nabla \mathcal{F}_{so}(h) \\
\nabla \mathcal{F}_{so}(y_i) - \nabla \mathcal{F}_{so}(y) \end{array} \right].
\]

Noting that \( \nabla \mathcal{F}_{so}(\cdot) \) and \( \nabla \mathcal{F}_{so}(\cdot) \) in the problem as both
being strictly monotonically increasing functions generates the below:

\[
(z_i - z)^	op (\mathcal{F}_{so}(z_i) - \mathcal{F}_{so}(z)) \geq 0\]

Therefore, \( \mathcal{F}_{so}(z) \) is also strictly monotonically increasing
and \( \mathcal{VI}(\mathcal{F}_{so}, \Omega_R) \) has exactly one solution.

c).

Improved projection-contraction algorithm

Although the decentralized methods like the BRDA
perform well in specific cases, it is hard to draw theoretical
conclusions on its convergence. As such, the CPM is cast into
a VI, which is convenient for both developing the algorithm
and conducting the convergence analysis.

Solving such a nonlinear monotone VI is traditionally
performed with prediction-correction (PC) methods [31], e.g.,
extragradient method (EGM) [32], for which the basic
iteration for solving \( \mathcal{VI}(\mathcal{F}_{so}, \Omega_{so}) \) can be stated as:

**EGM:**

\[
\begin{aligned}
z_i &+ 1 = \Pi_{\Omega_{so}}(z_i - \gamma_{so} \mathcal{F}_{so}(z_i)) \\
(z_i &+ 1) = \Pi_{\Omega_{so}}(z_i - \gamma_{so} \mathcal{F}_{so}(z_i))
\end{aligned}
\]

where \( z_i \) represents a predictor at \( k \)-th iteration.

In order to solve the CPM more efficiently, an improved
prediction-correction algorithm (IPCA) is proposed in this
paper based on its VI formulation. The improvements include:
1) an adaptive stepsize \( \{ s_k \} \) instead of a constant one, and 2) a
relaxation step for generating new iterations. The corresponding
analysis is presented below.

In the prediction step, a common assumption is that the
operator is Lipschitz continuous. Thus, in the projection for
obtaining \( z_i \), the parameter \( \gamma_{so} \) should be chosen such that,

\[
\gamma_{so} \| \mathcal{F}_{so}(z_i) - \mathcal{F}_{so}(z^*) \| \leq L \cdot \| z_i - z^* \|,
\]

with \( L \in (0, 1) \). To make the new iteration point \( z_{i+1} \) closer to
the solution \( z^* \), a descent direction of Euclidean distance
\( \| z_i - z^* \| \) between \( z^* \) and the current iteration point \( z_i \) can
be used in the correction step.

By using the basic properties of the projection operator
(see Eq.(3) in [33]), the predictor \( z_i \) updated by the prediction
step satisfies the condition,

\[
[z_i - z^*] \| (z_i - \gamma_{so} \mathcal{F}_{so}(z_i)) - z_i \|^2 = 0
\]

According to the VI definition, since \( z_i \in \Omega_{so} \) the
following formula with \( z = z_i \) holds,

\[
(z_i - z^*) \| (z_i - \gamma_{so} \mathcal{F}_{so}(z^*)) \geq 0
\]

Summing the above inequalities and recalling that \( \mathcal{F}_{so}(\cdot) \)
is monotone, the one can be obtained:

\[
[z_i - z^*] \| (z_i - z_i^*) \| \geq 0
\]

By defining \( \mathcal{D}(z_i, z_i^*) \equiv (z_i - z_i^*) \| (z_i - \gamma_{so} \mathcal{F}_{so}(z_i) - \mathcal{F}_{so}(z_i^*)) \| \)

Consequently, by combining (39) and (43), we get,

\[
(z_i - z^*) \| \mathcal{D}(z_i, z_i^*) \| \leq \| z_i - z_i^* \|^2 \geq 0
\]

The results in (44) state that, under the conditions (39),
\( \mathcal{D}(z_i, z_i^*) \) is an descent direction for the distance function
\( \frac{1}{2} \| z_i - z^* \|^2 \) at the point \( z_i \). This indicates a new formula for
\( \mathcal{D}(z_i, z_i^*) \) to update iterations, which naturally extends the
correction step in EGM, i.e., \( z_{i+1} = \Pi_{\Omega_{so}}(z_i - s_k \mathcal{D}(z_i, z_i^*)) \),
in which \( s_k \) is the stepsize at the \( k \)-th iteration. Additionally, a
relaxation step is introduced into each iteration that yields an
improved iterate scheme:

**IPCA:**

\[
\begin{aligned}
z_i &+ 1 = \Pi_{\Omega_{so}}(z_i - \gamma_{so} \mathcal{F}_{so}(z_i)) \quad \text{(Prediction)}
\Psi_k &+ 1 = \Pi_{\Omega_{so}}(z_i - \gamma_{so} \mathcal{F}_{so}(z_i)) \quad \text{(Correction)}
s_k &+ 1 = \gamma_{so} z_i + (1 - \gamma_{so}) \Psi_k \quad \text{(Relaxation)}
\end{aligned}
\]

where \( \gamma_{so} \) is the relaxation parameter and \( 0 \leq \gamma_{so} \leq 1 \).

Clearly, the ideal stepsize \( s_k \) can generate a new iteration
point closer to the solution. It is observed that the reduction of
the square of the distance is a function of \( s_k \), namely,

\[
\Delta_k = \| z_i - z^* \|^2 - \| z_{i+1} + s_k \mathcal{D}(z_i - z^*) \|^2
\]

According to the cosine theorem and the properties of the
projection, we have,

\[
\| (1 - \gamma_{so}) \Psi_k - (z^* - s_k \mathcal{D}(z_i - z_i^*)) \|^2 \leq \| (1 - \eta_{so}) \| (z_i - s_k \mathcal{D}(z_i - z_i^*)) \|^2 - \| (1 - \gamma_{so}) (z_i - s_k \mathcal{D}(z_i - z_i^*)) \|^2
\]

where \( \mathcal{D}(z_i, z_i^*) \) for convenience. By a manipulation, we get,

\[
\begin{aligned}
\Delta_k &+ 1 = \Pi_{\Omega_{so}}(z_i - \gamma_{so} \mathcal{F}_{so}(z_i)) \quad \text{(Prediction)}
\Psi_k &+ 1 = \Pi_{\Omega_{so}}(z_i - \gamma_{so} \mathcal{F}_{so}(z_i)) \quad \text{( Correction)}
s_k &+ 1 = \gamma_{so} z_i + (1 - \gamma_{so}) \Psi_k \quad \text{( Relaxation)}
\end{aligned}
\]
IEEE TRANSACTIONS ON POWER SYSTEMS

\[
\Delta_k \geq \|z_k - z^*\|^2 - \|\left((1 - \theta_k)(z_k - s_k D_k) - (z^* - \vartheta_k z_k)\right)\|^2 \\
+ \|\left((1 - \theta_k)(z_k - s_k D_k) - \vartheta_k z_k\right)\|^2 \\
= \|\vartheta_k - (1 - \vartheta_k) z_k\|^2 + 2(1 - \vartheta_k)(z_k - z^*)^T s_k D_k \\
+ 2(1 - \vartheta_k)(z_k - z^*)^T s_k D_k \\
= \|\vartheta_k - (1 - \vartheta_k) z_k\|^2 - 2(1 - \vartheta_k)(z_k - z^*)^T s_k D_k \\
+ 2(1 - \vartheta_k)(z_k - z^*)^T s_k D_k \\
\]

Since we have no knowledge of \(z^*\), computing the maximum of \(\Delta_k\) is not trivial. With the assistance of (43), we can obtain,

\[
\Delta_k \geq \left(-(1 - \vartheta_k)^2 s_k^2 D_k^2 + 2(1 - \vartheta_k)(z_k - z^*)^T s_k D_k\right) \equiv \zeta_k(s_k) \tag{48}
\]

The implication of (48) is that \(\zeta_k(s_k)\) is the infimum of \(\Delta_k\). Therefore, one can turn to maximize \(\zeta_k(s_k)\) to find the desired stepsize \(s_k^*\), which reaches its maximum at

\[
s_k^* = \arg \max \zeta_k(s_k) = \frac{(z_k - z_k^*)^T D_k}{(1 - \vartheta_k) D_k^2} \tag{49}
\]

Remark 3: In comparison with the EGM, the stepsize in the IPCA, i.e., \(s_k^*\), can be chosen adaptively at each iteration, thereby accelerating the convergence.

Accordingly, the procedure of the IPCA is given as follows:

**Algorithm 1: IPCA**

**Step 0:** Identify an initial solution \(z_0\) such that \(z_0 \in \Omega_{SO}\); Let \(\gamma_0 = 1, L \in (0, 1)\) and \(v_0 \in (0, 1)\); set suitable sequences \(\{\vartheta_k\}\). Let \(k = 1\) and \(z_k \leftarrow z_0\).

**Step 1: Prediction step.**

1a: Compute a predictor using projection and current \(l_k\):

\[\tilde{z}_k^* = \Pi_{\Omega_{SO}}(z_k - \gamma_k \cdot \mathcal{F}_SO(z_k))\]

\[l_k = \gamma_k \frac{\mathcal{F}_SO(z_k) - \mathcal{F}_SO(z_k^*)}{\|z_k - z_k^*\|}\]

1b: Find \(\gamma_k\) to ensure the Lipschitz continuity of:

while \((l_k > L)\)

update: \(\gamma_k = \frac{3}{2} \gamma_k \min\{1, 1/l_k\}\)

\[\tilde{z}_k^* = \Pi_{\Omega_{SO}}(z_k - \gamma_k \cdot \mathcal{F}_SO(z_k))\]

\[l_k = \gamma_k \frac{\mathcal{F}_SO(z_k) - \mathcal{F}_SO(z_k^*)}{\|z_k - z_k^*\|}\]

end

**Step 2: Correction step.**

2a: Compute the ascent direction by the predictor \(\tilde{z}_k^*\):

\[D(z_k, \tilde{z}_k^*) = (z_k - \tilde{z}_k^*) - \gamma_k [\mathcal{F}_SO(z_k) - \mathcal{F}_SO(z_k^*)]\]

2b: Obtain the optimal stepsize by:

\[s_k^* = \frac{(z_k - z_k^*)^T D(z_k, \tilde{z}_k^*)}{(1 - \theta_k) \|D(z_k, \tilde{z}_k^*)\|^2}\]

2c: Compute the corrector \(\Psi(z_k)\) by projection:

\[\Psi(z_k) = \Pi_{\Omega_{SO}}(z_k - s_k^* D(z_k, \tilde{z}_k^*))\]

2d: Update \(\gamma_k\): \(\text{if } (l_k \leq R_0), \gamma_k = \frac{3}{2} \gamma_k\) end

**Step 3: Update new iterate:**

\[z_{k+1} = \vartheta_k z_k + \left(1 - \vartheta_k\right) \Psi(z_k)\]

**Step 4: Stopping test:** If \(\|z_{k+1} - z_k\| \leq \epsilon_k\), stop and declare \(z^* \approx z_{k+1}\). Otherwise update \(k = k+1\) and go to Step 1.

In the following, the convergence of the proposed IPCA is proved. To do this, it is first observed that using (39) and the definition of \(D_k\), yields

\[s_k^* = \frac{1}{2(1 - \theta_k)} \left(\frac{\mathcal{F}_SO(z_k) - \mathcal{F}_SO(z_k^*)}{2(1 - \theta_k)} \right) \geq 0\]

which implies that \(s_k^* \geq \frac{1}{2(1 - \theta_k)}\). Setting \(s_k = s_k^*\) in (48), the followings can be further obtained:

\[\Delta_k(s_k) = 2(1 - \theta_k)(z_k - z_k^*)^T s_k D_k - \frac{1}{2} (s_k^*)^2 D_k^2\]

Consequently, it follows immediately from (39) that,

\[\|z_{k+1} - z_k\|^2 \leq \|z_k - z^*\|^2 - \frac{1}{2} (1 - L) \|z_k - z_k^*\|^2\]

According to Theorem 2.1 in [33], the generated sequence \(\{z_k\}\) from (52) converges to a solution point, thereby concluding the proof.

VI. CASE STUDIES

![Fig. 3](image-url) A PTCN for the case study. (a) A real-world 56-node PDN in the service territory of Southern California Edison; (b) The real Sioux falls TN.
The proposed model and algorithm are verified using a real-scale PT
can be obtained. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2022.3162861, IEEE Transactions on Power Systems

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2022.3162861, IEEE Transactions on Power Systems
Case C: The pricing scheme with only road tolls considered, where the charging price is fixed (i.e., $\lambda_c = b_r$).

Case D: The pricing scheme with congestion tolls on roads and CSs considered, where charging price is fixed (i.e., $\lambda_c = b_r$).

Table VI shows that Case A achieves at least 6.76% savings in the total cost compared with other cases, indicating that the collaborative pricing scheme can realize more economical operation strategies for the entire system. By comparing the results of Cases A and B, it can be seen that if the congestion tolls are not included, the TN cost increases by 11.20%. This is due to the unguided selfish user behavior, which leads to a MUE state without exploiting the flexibility of traffic flow. Moreover, compared to Cases C and D with a fixed charging cost, it is found that using DLMPs has potential to alleviate power congestion and thus reduce the PDN operation cost as in Table VI. From the results, it is also observed that the cost saving achieved by congestion tolls can be higher than that by DLMPs only (7.50% v.s. 3.64%), meaning the tolling signal has a stronger ability to influence traveler behaviors.

Table VI provides further insights into EV flow patterns as well as tolling schemes on CSs. It can be observed that, under the MUE state of Case B where drivers are unguided, a large proportion of the EV charging load is concentrated at CS 1 while CS 2 is underutilized. The situation can be improved by the congestion pricing in Case A, which plays a guiding role for EVs and can potentially transfer the charging loads from CSs with higher cost (CS 1 and CS 4) to those with a lower cost (CS 2) in driver’s self-interest. From this perspective, the proposed scheme can mitigate the congestion at heavy load buses. As such, the DLMPs at CS 1 and CS 4 are reduced from 77.933 to 76.400 and 77.091 to 76.399, respectively.

C. Technical Performance of the PDN under different cases

Figs. 6-7 illustrate the road congestion toll strategies and the resultant traffic flow patterns. Clearly, the tolls are mainly imposed on the arcs with large amounts of traffic, to affect the users’ travel behavior, e.g., arcs 8, 38-39, 59-60, 79-82, etc. Because of the road tolls, part of the traffic flows on those arcs transfers to other arcs, which consequently alleviates congestion and reduces the overall social cost. Therefore, when the road pricing strategy is adopted, more road arcs are utilized and the flow on congested arcs is reduced, compared to the flow distribution in a state of user-equilibrium.

Next, we investigate the performance of different pricing schemes in the PDN operation. To this end, the voltage profiles at all nodes of the 56-node PDN are illustrated in Fig. 8. In our experiment, the nodal voltages are limited between 94% and 106% of the nominal voltage. As shown in Fig. 8, EV charging load can lead to voltage drop at CS nodes, especially when a large part of charging demand is concentrated (e.g., node 10). Note that the pricing schemes without CS tolls (i.e., Cases B and C) generally result in a lower voltage level compared to Cases A and D. Although the voltages are maintained at a relatively high level in Case D, its fluctuations across all nodes are significant. In contrast, the collaborative pricing scheme (i.e., Case A), in blue in Fig. 8, exhibits a smoother voltage profile within the imposed limits.
The total active power losses of the PDN are reported for all cases in Fig. 9. The active power losses in Case A are reduced by 22.42%, 50.20%, and 21.71% compared to Cases B, C, and D, respectively. This result highlights that the joint introduction of DLMPs and CS congestion tolls has a significant impact on reducing the power losses in distribution systems. However, imposing road tolls independently has little or no effect on the reduction of power losses, according to the results in Case C.

VII. CONCLUSION

This paper proposes a new collaborative pricing scheme for the power-transportation coupled network. The proposed CPM considers DLMPs and congestion tolls on traffic roads and at CSs jointly to coordinate the coupled networks. Tractable computation is realized by recasting the proposed CPM into the equivalent VI formulation which includes the DLMPs as a part of variables. With this reformulation, the existence and uniqueness of solutions can be analyzed rigorously and conveniently. Applying the developed IPCA, the collaborative pricing strategies can be obtained by solving the CPM in the VI form, which theoretically guarantees the theoretical convergence. Moreover, an adaptive stepsize and a relaxation step are designed to ensure faster convergence of the solution algorithm. Simulation results on a real-scale case show the computation benefits and the good economic performance of the proposed pricing scheme. Compared with the existing algorithms, the proposed IPCA shows a considerable reduction in computation time. Besides, compared to other separate pricing schemes, the proposed pricing schemes can take full advantage of guiding both EVs and GVs to alleviate the congestion in both networks and consequently reduce the overall social cost.

The EV demand is deterministic in this model. In future work, we will explore enhanced optimization techniques to incorporate the stochasticity of user behaviors into the proposed model. Furthermore, introducing the dynamic traffic flow to the model also has practical significance.

REFERENCES


IEEE TRANSACTIONS ON POWER SYSTEMS


Shiwei Xie (Member, IEEE) received the Ph.D. degree in electrical engineering from Wuhan University, Wuhan, China, in 2021. From 2019 to 2020, he was a Research Assistant with the School of Electrical and Electronic Engineering (EEE), Nanyang Technological University, Singapore. He is currently a Lecturer (Associated Researcher) with the School of electrical engineering and automation, Fuzhou University. His current research interests include variational inequality theory, distributed optimization, robust optimization, and their applications in power and transportation systems.

Qiwei Wu (Senior Member, IEEE) received the Ph.D degree in Power System Engineering from Nanyang Technological University, Singapore, in 2009. He has been a tenured Associate Professor with Tsinghua-Berkeley Shenzhen Institute, Tsinghua Shenzhen International Graduate School, Tsinghua University since Jan. 2022. He was a senior R&D engineer with Vestas Technology R&D Singapore Pte Ltd from Mar. 2008 to Oct. 2009. He was working at Department of Electrical Engineering, Technical University of Denmark (DTU) from Nov. 2009 to Aug. 2021 (PostDoc Nov. 2009-Oct. 2010, Assistant Professor Nov. 2010-Aug. 2013, Associate Professor Sept. 2013-Aug. 2021). He was a visiting scholar at the Department of Industrial Engineering & Operations Research (IEOR), University of California, Berkeley, from Feb. 2012 to May 2012 funded by the Danish Agency for Science, Technology and Innovation (DASTI), Denmark. He was a visiting professor named by Y. Xue, an Academician of the Chinese Academy of Engineering, at Shandong University, China, from Nov. 2015 to Oct. 2017. He was a visiting scholar at the School of Electrical Engineering and Applied Sciences, Harvard University from Nov. 2017-Oct. 2018 funded by the Otto Mønsted Fond. He was a visiting Associate Professor at Tsinghua-Berkeley Shenzhen Institute, Tsinghua University from Aug. 2021 to Dec. 2021. His research interests are decentralized/distributed optimal operation and control of power systems with high penetration of renewables, including distributed wind power modelling and control, decentralized/distributed congestion management, voltage control and load restoration of active distribution networks, and decentralized/distributed optimal operation of integrated energy systems. He is an Associate Editor of IEEE Transactions on Power Systems and IEEE Power Engineering Letters. He is also the deputy editor-in-chief and an Associate Editor of International Journal of Electrical Power and Energy Systems, and Journal of Modern Power Systems and Clean Energy. He is a subject editor for IET Generation, Transmission & Distribution, and IET Renewable Power Generation.

Yachao Zhang (Member, IEEE) received the B.E. and M.E. degrees from Huazhong University of Science and Technology, Wuhan, China in 2007 and 2010, respectively, and the Ph.D. degree from Wuhan University, Wuhan, China in 2017. From 2010 to 2014, he was with Power Generation Company of China Southern Power Grid. He is currently an Associate Professor at School of Electrical Engineering and Automation, Fuzhou University, Fuzhou, China. His current research interests include distributed control and optimization of power systems, renewable energy integration and microgrid modeling and control. Dr. Xu is Associate Editor for the IET Renewable Power Generation, IET Smart Grid and IET Generation, Transmission & Distribution.

Yinliang Xu (Senior Member, IEEE) received the B.S. and M.S degrees in Control Science and Engineering from Harbin Institute of Technology, China, in 2007 and 2009, respectively, and the Ph.D. degree in Electrical and Computer Engineering from New Mexico State University, Las Cruces, NM, USA, in 2013. He is now an Associate Professor with Tsinghua-Berkeley Shenzhen Institute, Tsinghua Shenzhen International Graduate School, Tsinghua University, Shenzhen, China. His research interests include distributed control and optimization of power systems, renewable energy integration and microgrid modeling and control. Dr. Xu is Associate Editor for the IET Renewable Power Generation, IET Smart Grid and IET Generation, Transmission & Distribution.

Nikos D. Hatziargyriou (Life Fellow, IEEE) is currently a Professor in Power Systems with the National Technical University of Athens. He has over ten years of industrial experience as the Chairman and CEO of the Hellenic Distribution Network Operator and as the Executive Vice-Chair of the Public Power Corporation. He was the Chair and currently the Vice-Chair of the EU Technology and Innovation Platform on Smart Networks for Energy Transition (ETIP-SNET) representing E.DSO. He has participated in more than 60 RD&D projects funded by the EU Commission, electric utilities and manufacturers for both fundamental research and practical applications. He is author of the book, Microgrids: Architectures and Control and of more than 250 journal publications and 500 conference proceedings papers. He is included in the 2016, 2017, and 2019 Thomson Reuters lists of the top 1% most cited researchers and he is Globe Energy Prize laureate 2020. He is an Honorary Member of CIGRE and the past Chair of CIGRE SC C6 “Distribution Systems and Distributed Generation”. He is the past Chair of the Power System Dynamic Performance Committee (PSDPC) and currently the Editor-in-Chief of the IEEE Transactions on Power Systems.