Nonlinear Oscillations of Acoustic Shock Waves in a Cylindrical Tube

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Summary. We investigate nonlinear and dissipative acoustic waves in a tube driven by a sinusoidal driver at one end and impossing a fixed wall boundary condition at the opposite end. For driving amplitudes close to resonances in the tube, we have found multiple oscillating shock waves in a weakly nonlinear thermoviscous acoustic model. For slow off resonance driving, we observed a nearly linear oscillating ground state superimposed by bursts of oscillating schock waves. The small amplitude nearly harmonic oscillations are slow, whereas the oscillating shock waves exhibit fast propagation back and forth in the tube.

Higher order nonlinear acoustic wave equation

In nonlinear acoustics the wave fields for the flow velocity or pressure will not be simple harmonic waves. Localized shock waves may be generated or acoustic streaming may appear. The resulting flow patterns results from balancing dissipation and nonlinear effects. In the literature oscillating solitons or solitary waves have been studied in a wide range of systems as long Josephson junctions and optical fiber systems. Motivated by these investigations we shall here study oscillating solitary shock waves in a tube driven by a harmonic driver at one end while imposing a fixed wall at the opposite end. We shall asume plane waves in a cylindreical tube of length ℓ.

Mathematical model

A number of model equations for weakly nonlinear acoustic wave propagation have been derived in the literature [1]. Here we use a model for acoustic waves in a Newtonian, viscous and heat conducting gas. Our model is based on the dynamical equations for the fluid motion, continuity, the heat transfer and entropy together with an equation of state. Introducing the velocity potential $\psi = \psi(x, t)$ as function of position x along the tube center axis and at time t, our one dimensional plane wave model in dimensionless variables and coordinates reads [2]

$$\psi_{tt} - \psi_{xx} = \psi_t\psi_{xx} + (\gamma - 2)\psi_{tt}\psi_t + 2\psi_{xt}\psi_x + b\psi_{xxt} \ , \quad (1)$$

where subscripts x and t denote partial derivatives with respect to the space and time variables. The fluid flow velocity $u(x, t)$ is given by the potential through $u = -\psi_x$ and the fluid pressure p is given by $p = \psi_t$. The term $b\psi_{xxt}$ models dissipation and γ equals C_p/C_v with C_p beeing the specific heat at constant pressure and C_v is the specific heat at constant volume.

Boundary conditions

At the left end ($x = 0$) of the tube a sound generator is mounted and at the right end ($x = \ell$) we have a fixed wall. The boundary conditions become

$$u(0, t) = -\psi_x(0, t) = D\sin(\omega t) \quad \text{and} \quad u(\ell, t) = -\psi_x(\ell, t) = 0 \ . \quad (2)$$

The parameter D is the driver amplitude and the driving frequency is denoted by ω. Initially we take a fluid at rest corresponding to $\psi(x, 0) = 0$ and $\psi_t(x, 0) = 0$. We solve Eq. (1), together with the initial conditions (2), by a semi difference method discretizing to second order in space and integrating in time using a 4-5 order Runge Kutta method. Integration is conducted until steady state has emerged. The following parameters are kept fixed $\ell = 1$, $b = 5 \cdot 10^{-4}$ and $\gamma = 1.4$.

Numerical results

For the driver parameters $D = 0.01$ and $\omega = 2\pi$ the left panel of Fig. 1 shows a three dimensional plot of the fluid velocity field $u(x, t)$ as function of x and t. The driver frequency corresponds to the eigenfrequency of the second harmonic of the linearized model (1). This means we drive the nonlinear equation at a resonance frequency. However, due to damping and the nonlinear terms the emerging steady state solution consists of two oscillating shock waves, travelling forth and back in opposite directions.

The right panel of Fig. 1 shows a plot of the fluid velocity $u(x, t)$ driven at the nonresonant frequency $\omega = 0.1$ and with driver amplitude $D = -0.125$. The simulations reveal the surprizing result that the slowly varying ground state oscillation is superimposed a fast back and forth oscillating shock wave. We observe that during one driver cycle the shock wave oscillations appear for decreasing $u(0, t)$ corresponding to compression of the fluid and disappears for increasing $u(0, t)$ corresponding to decompression.

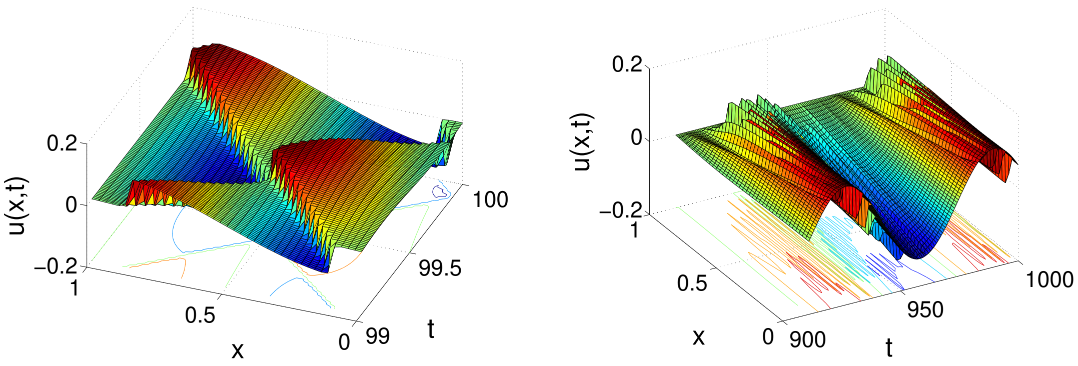

Figure 1: Left: Resonant oscillations of two counter propagating shock waves. Right: Non resonant oscillations superimposed fast travelling shock waves.

Conclusion

Driving nonlinear acoustic plane waves at resonance in a cylindrical tube leads to oscillating shock waves. A driving frequency corresponding to the n'th linear excitation mode in the linearized model of Eq. (1) leads to n oscillating fully nonlinear shocks. However, an upper limit for the number of oscillating shocks is expected given by the width of the shocks and the space available in the tube. For the nonresonant driving case full numerical simulations revealed excitation of a nearly linear ground state superimposed oscillating shock waves in bursts.The shock waves oscillates fast back and forth in comparison to the slow ground state wave oscillation.

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This work is dedicated to the memory of Yuri B. Gaididei.

References

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