Power Reserve Management by Two-Stage Stochastic Programming

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Abstract

Although bilateral trades and spot markets aim to balance power supply and demand, real-time imbalances may still occur due to nonanticipated supply and demand behavior. The real-time balance is the responsibility of the power system operator and is achieved by means of regulation purchased in a corresponding market. To ensure sufficient regulation in the market, the system operator has the possibility of reserving regulating power in advance. As reserves are however purchased prior to actual operation reserve decisions are naturally subject to supply and demand uncertainty. In contrast, regulation decisions can be deferred until uncertainty has been observed and the system is operating. In the present paper this is formalized by formulating the regulating reserve management problem as a two-stage stochastic

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program. A case study that concerns the regulating reserve management problem of the system operator of Western Denmark is discussed in detail.

Keywords: Power reserves; uncertainty modeling; stochastic programming

1 Introduction

In a power system that comprises several participants on both the supply and the demand side, it is the task of the system operator to balance production and consumption by means of regulation. To do the balancing, regulation reserve management becomes important. Previous studies indicate that power optimization problems can be handled successfully by mathematical programming. With this in mind, we present an application of stochastic programming to a power reserve management problem.

Very often power optimization problems involve uncertainty and to deal with the uncertainty, stochastic programming may come into play. In the literature, power production planning and power operation problems within stochastic programming have attracted considerable attention. Examples on hydro-thermal power production planning are [4], [6] and [9]. Both problems are two-stage stochastic programs. Whereas [4] and [9] seek to find a unit commitment plan, [6] seeks a schedule that can be compensated for in the future. Hydro-power production planning problems within multi-stage stochastic programming can be found in [8], [11] and [16]. Whereas [8] and [11] handle water scheduling through space and time respectively, [16] considers unit commitment. Hydro-thermal and purely thermal power operation problems are handled in [2], [6] and [10] using multi-stage stochastic programming. Recently, also simultaneous optimization of power production and physical trading have appeared in stochastic programming. [13] presents a multi-stage stochastic program in which all stages allow for spot market disposals and purchases. [15] and [7] incorporates spot market bidding in a two-stage stochastic program. For another power trading problem within stochastic programming, which involves more than one market, see also [18].

To our knowledge, prior work on power reserve management is limited. However, the authors of [20] formulate a stochastic optimization problem for the coordination of bidding strategies in day-ahead and reserves markets. In contrast to the present problem, the problem is formulated from the perspective of the supplier, which explains the link between day-ahead and reserve market exchange that is caused by production capacity limits. Nevertheless, the problem has some similarities to the problem of this paper in that both volumes and market prices are determined within the corresponding model. In the same spirit, the authors of [19] determine pricing and procurement of reserves in a power market. A stochastic model based on social welfare maximization allows for a so-called capacity-reliability analysis that relates the available reserve capacity to the probability of reserves shortage.

In the present paper we address the problem of regulating reserve management faced by the power system operator. The problem arises in the process of maintaining the balance between power demand and supply. The system operator corrects imbalances by regulation procured in the regulating market and sufficient amounts are not necessarily available unless reserved in advance. The major challenge of reserving regulation is that of uncertain supply and demand. To handle this uncertainty, the power reserve management problem will be analyzed by means of stochastic programming.

2 Power reserves

The project grew out of a collaboration with the former Eltra[1], which is the power system operator of Western Denmark. Due to decentralization of the power generation and deregulation of the power markets, many procedures either have been modified recently or will be within the near future. In particular, Eltra have made plans to improve the model on which power reserve management is based, which makes reserves a topic of current interest.

It is necessary to distinguish between different types of reserves

(i) Automatic regulation reserves: Reserves that cover imbalances from the time of appearance until a regulation bid is activated. The reserves are provided by running plants capable of adjusting upwards and consumers capable of adjusting downwards. Activation begins automatically within two to three minutes.

(ii) Manual regulation reserves: Reserves in the form of regulation resources that suppliers are obligated to sell in the regulating market. Activated manually within 10 minutes.

(iii) Running and available plants: Reserves for ensuring supply in spite of transmission lines or units falling out. Consist of available plants that can be started, running plants that can adjust upwards and consumers that can adjust downwards. Running and available plants plants are activated either automatically or manually.

(iv) Emergency start plants: Reserves reestablishing the system in case of blackout.

This paper considers manual regulation reserves. The reason for considering such reserves should be clear from the following discussion. In a typical power system the operator is

[1]now a part of the overall Danish power and gas system operator Energinet.dk, `www.energinet.dk`

responsible for balancing supply and demand. Prior to operation, predicted supply and demand are balanced by resources provided by the bilateral trades and the spot market trades. Supply and demand however often differ from the predictions and imbalances still occur when the system is operating. To overcome such real-time imbalances, the system operator compensates suppliers and consumers for adjusting production and demand accordingly. The imbalances are covered by so-called regulation resources provided by the regulating market, which is established by system operators for the purpose of real-time power trading. Suppliers either increase or decrease power production above or below the amounts committed through the bilateral contracts and the spot market contracts and offer the adjustments to the regulating market as so-called up- and down-regulation, respectively. Consumers offer power demand adjustments to the regulating market in a similar fashion. The system operator purchases up regulation in the case of excess demand and down regulation in the case of excess supply. In some cases, the amounts of resources provided by the regulating market are insufficient to fully cover imbalances. This may happen if

(i) Imbalances are substantial due to extreme supply and demand behavior caused by failure in supplying, unforeseen weather changes leading to unpredicted wind production or nonanticipated heat demand etc.

(ii) Spot market prices are sufficiently high to prevent market participants from saving resources for trading in the less secure regulating market.

(ii) Considerable failures occur during transmission, e.g. important transmission lines fall out.

To ensure that sufficient amounts of resources are available even when facing the above situations, regulation can be reserved prior to trading in the regulating market. The system operator may agree with reserve suppliers for the right to purchase regulation. This is done by activating a reserve bid that obligates the supplier to bid an amount of regulation into the regulating market. The right is a type of option as the system operator obtains the possibility of purchasing regulation in the regulating market but is not forced to do so. As a result, the system operator faces the trade-off between purchasing regulation at the market price only, thereby risking insufficiency of resources, and paying both the market price and an additional fixed price to ensure regulation is available.

When maintaining the power balance and managing regulating reserves the system operator must consider the uncertainties of the power system. Sources of uncertainty include supply and demand as well as regulating prices and volumes. As system imbalances are caused by demand and supply the focus is chosen to be supply and demand uncertainty. The major problem of managing regulating reserves is that reserves must be purchased prior to balancing. If reserve capacity turns out to be insufficient additional capacity must be procured elsewhere, often at a considerable price, or the system simply breaks down. On the other hand, reserves constitute serious costs which makes excess reserve capacity unwanted. Stochastic programming provides a tool for determining reserve levels that takes the nonanticipated supply and demand behavior into account.

As already explained, different power markets come into play in the correction of imbalances. To fully understand the daily work of the system operator, consider the following time schedule for acting in these markets. The system operator purchases reserves for a longer time period at the regulating reserve market (in Western Denmark, a formal market has not yet been established). Currently, the length of this period is one month or longer though a reduction to 24 hours is planned. The remaining actions concern a 24-hour operation day. By noon bids must be submitted to the spot market (in the Danish case, the Nordic market Nord Pool[2]). Having balanced predicted supply and demand, activated bids are announced by 14:00. Finally, from 24:00 to 24:00 actual supply and demand imbalances are continuously corrected by trading in the real-time regulating market (in this case, still a local market, although the integration in the Nordic market is planned). For an illustration, see Fig. 7.1.[Insert Fig. 1]

The paper is organized as follows. The power reserve management problem is presented in Section 3. Section 4 explains how uncertainty affects reserve management and the problem is stated as a two-stage stochastic program. By assuming a discrete distribution of the random data, the problem is transformed into a large-scale mathematical program that is solved by a specially designed solution procedure in Section 5. A specific instance of the power reserve management problem is addressed using data from the power system operator of Western Denmark, Eltra, and computational results are reported in Section 6.

3 The power reserve management problem

A given planning horizon is considered. In practice, the power balance should be maintained at every time point but to facilitate computations the time is discretized. The regulation bids to the regulating marked have duration of a number of full hours. Accordingly, the planning horizon is discretized into hourly time intervals and the finite number of such intervals are denoted by $\mathcal{T} = \{1, \ldots, T\}$. The planning horizon for purchasing regulating reserves may range from one to several months. In Western Denmark some

[2]`www.nordpool.no`

regulating reserve contracts have long durations, whereas some contracts are traded regularly on a monthly basis. Regarding the case study, the planning horizon is chosen to be one month, i.e. $T = 24 \times 30 = 720$. However, to increase the flexibility of the system it is intended to reduce it as is the case for Eastern Denmark that trades regulating reserve contracts on a daily basis.

3.1 Reserves

According to the present situation, the system operator of Western Denmark purchases regulation reserves mostly locally. Although not fully formalized, regulation reserves are traded on a monthly auction or as individual contracts of a longer duration. A regulation reserve bid consists of an offer period, a volume to be available throughout the offer period, a variable price that applies to the portion of the volume that is activated and a fixed price for activation of the bid. We assume for the application that regulation reserves are traded only on a monthly auction so that the offer period of a regulation reserve bid is always one month. The system operator pays the variable price for the actual amount of regulation used and the fixed price for the availability of regulation.

Regulation reserves are modeled as follows. Regulation divides into up- or down-regulation, and so do regulation reserves. Thus, the superscripts *up* and *do* are used. The indices $\mathcal{I} = \{1, \ldots, I\}$ are adopted to represent different reserve bids. The variables $\delta_i^{up}, \delta_i^{do} \in \{0, 1\}, i \in \mathcal{I}$ indicate whether the reserve bids are activated or not and the prices for activation are denoted $c_i^{up}, c_i^{do}, i \in \mathcal{I}$.

Then fixed regulation reserve purchase costs compute as

$$\sum_{i \in \mathcal{I}} (c_i^{up} \delta_i^{up} + c_i^{do} \delta_i^{do})$$

3.2 Regulation

The regulating market serves as a tool for the system operators to balance power supply and demand during operation. A larger power system may share a common regulating market. The Nordic system operators, the Swedish Svenska Kraftnät, the Norwegian Statnett and the Danish Energinet.dk, have established such a common market, in which Western Denmark was the last part of the system to be integrated in January 2006.

The suppliers to the regulating market are power balance providers that submit bids to the market. Regulation bids divide into up-regulation or down-regulation bids. Upward regulation make consumers decrease demand or suppliers increase production (system operators 'buys' power) and downward regulation make suppliers decrease production or consumers increase demand (system operators 'sells' power). A regulation bid consists of an offer period, a price and a volume. The offer period may be a number of full hours. During the offer period the volume is constant whereas the price may vary between hours. We however assume that regulation bids have an offer period of only one hour and thus both the volume and the price is constant. Generally, the up-regulation price is specified as the system spot price (assuming no grid congestion) and a raise, i.e. the up-regulation price is always above the system price. Similarly, the down-regulation price is calculated as the system price (assuming no grid congestion) and a deduction, i.e. the down-regulation price is always below the system price. Prices are usually given as positive numbers unless the system operator sells up-regulation or buys down-regulation. We assume prices are always positive.

Only recently the regulating market of Western Denmark has begun to restructure. Western Denmark trades regulation mostly locally although a full integration to the Nordic regulating market is on its way. Before July 2006, the regulating market of Western

Denmark was a pay-as-bid market, whereas now the general rule is to use local marginal prices as market prices. Nordic marginal prices should be fully in use by January 2008. If regulation is purchased outside Western Denmark, transmission capacity limits may apply. Such limits are due to physical limitations or political agreements. Since, for the current application, regulation is mostly purchased locally, we have however omitted transmission capacity limits.

Regulation comprises purchases in the regulating market that have been and have not been reserved in advance. In the case of direct purchases, $\mathcal{J} = \{I+1, \ldots, I+J\}$ are included to index different bids. Volumes are denoted $\bar{q}_{it}^{up}, \bar{q}_{it}^{do}, i \in \mathcal{J}, t \in \mathcal{T}$ and corresponding prices are denoted $\bar{p}_{it}^{up}, \bar{p}_{it}^{do}, i \in \mathcal{J}, t \in \mathcal{T}$. In the case of reserved purchases, recall that the indices $\mathcal{I} = \{1, \ldots, I\}$ are included to represent different bids. Volumes are denoted $\bar{q}_{i}^{up}, \bar{q}_{i}^{do}, i \in \mathcal{I}$. We assume all reserved purchases will be available at the regulating market, that is, there is no failure of supply. Note that whereas for direct purchases bids are time dependent, for reserved purchases bids are time independent. Corresponding prices are denoted $\bar{p}_{it}^{up}, \bar{p}_{it}^{do}, i \in \mathcal{I}, t \in \mathcal{T}$. For direct purchases prices can vary freely, whereas for reserved purchases prices should stay between limits that are agreed upon when reserving regulation. A bid is not necessarily activated completely. Actual purchases are represented by the variables $q_{it}^{up}, q_{it}^{up} \in \mathbb{R}_+, i \in \mathcal{I} \cup \mathcal{J}, t \in \mathcal{T}$.

3.2.1 Pay-as-bid pricing

Costs of purchasing regulation, whether reserved or direct, consist of up-regulation expenses and down-regulation income

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I} \cup \mathcal{J}} (\bar{p}_{it}^{up} q_{it}^{up} - \bar{p}_{it}^{do} q_{it}^{do})$$

The following bounds concern reserved purchases

$$q_{it}^{up} \leq \bar{q}_i^{up}\delta_i^{up}, \;\; q_{it}^{do} \leq \bar{q}_i^{do}\delta_i^{do}, \;\; i \in \mathcal{I}, t \in \mathcal{T} \tag{3.1}$$

whereas direct purchases must submit to the bounds

$$q_{it}^{up} \leq \bar{q}_{it}^{up}, \;\; q_{it}^{do} \leq \bar{q}_{it}^{do}, \;\; i \in \mathcal{J}, t \in \mathcal{T} \tag{3.2}$$

3.2.2 Marginal pricing

For each hour the marginal price is determined as the price of the last bid that has been activated in the direction the system is regulated. That is, if the system is up-regulated, the marginal price is the highest price of the up-regulation bids that have been activated. Likewise, if the system is down-regulated, the marginal price is the lowest price of the down-regulation bids that have been activated. The variables $\gamma_{it}^{up}, \gamma_{it}^{do} \in \{0, 1\}, i \in \mathcal{I} \cup \mathcal{J}$ indicate whether the regulation bids are activated or not for both reserved and direct regulation. Moreover, the variables $p_t^{up}, p_t^{do} \in \mathbb{R}_+^{n_1}$ represent the marginal prices. In the case of up-regulation, the marginal price is $p_t^{up} = \max\{\bar{p}_{it}^{up}\gamma_{it}^{up} : i \in \mathcal{I} \cup \mathcal{J}\}$ and in the case of down-regulation, the marginal price is $p_t^{do} = \max\{\bar{p}_{it}^{do}\gamma_{it}^{do} : i \in \mathcal{I} \cup \mathcal{J}\}$.

Costs of purchasing reserved and direct regulation amount to

$$\sum_{t \in \mathcal{T}} (p_t^{up} \sum_{i \in \mathcal{I} \cup \mathcal{J}} q_{it}^{up} - p_t^{do} \sum_{i \in \mathcal{I} \cup \mathcal{J}} q_{it}^{do}) \tag{3.3}$$

Evidently, (3.3) is nonlinear. In order to be consistent with a mixed-integer linear formulation, the variables $\rho_t^{up}, \rho_t^{do} \in \mathbb{R}_+^{n_1}, t \in \mathcal{T}$ can be introduced and (3.3) can be replaced by

$$\sum_{t \in \mathcal{T}} (\rho_t^{up} - \rho_t^{do})$$

and

$$\rho_t^{up} \geq \bar{p}_{it}^{up} \sum_{i \in \mathcal{I} \cup \mathcal{J}} q_{it}^{up} - M(1 - \delta_{it}^{up}), \;\; i \in \mathcal{I} \cup \mathcal{J}, t \in \mathcal{T} \tag{3.4}$$

$$\rho_t^{do} \leq \bar{p}_{it}^{do} \sum_{i \in \mathcal{I} \cup \mathcal{J}} q_{it}^{do} + M(1 - \delta_{it}^{do}), \;\; i \in \mathcal{I} \cup \mathcal{J}, t \in \mathcal{T} \tag{3.5}$$

with

$$M = \max\{\bar{p}_{it}^{up}(\sum_{i \in \mathcal{J}} \bar{q}_{it}^{up} + \sum_{i \in \mathcal{I}} \bar{q}_t^{up}), \bar{p}_{it}^{do}(\sum_{i \in \mathcal{J}} \bar{q}_{it}^{do} + \sum_{i \in \mathcal{I}} \bar{q}_t^{do}) : i \in \mathcal{I} \cup \mathcal{J}, t \in \mathcal{T}\}$$

For both reserved and direct purchases, the volumes activated cannot exceed the volumes bid

$$q_{it}^{up} \leq \bar{q}_{it}^{up} \gamma_{it}^{up}, \;\; q_{it}^{do} \leq \bar{q}_{it}^{do} \gamma_{it}^{up}, \;\; i \in \mathcal{I} \cup \mathcal{J}, t \in \mathcal{T} \tag{3.6}$$

Reserved regulation cannot be activated unless the regulation reserve bids are activated

$$\sum_{t \in \mathcal{T}} \gamma_{it}^{up} \leq \delta_i^{up}, \sum_{t \in \mathcal{T}} \gamma_{it}^{do} \leq \delta_i^{do}, \;\; i \in \mathcal{I} \tag{3.7}$$

3.3 Balancing

For the system operator to balance power, demand and supply imbalances are considered during operation. If the actual demand exceeds actual supply, the system operator purchases up-regulation, if the actual supply exceeds actual demand, the system operator purchases down-regulation. Imbalances between demand and supply is usually fully covered by regulation purchased directly or reserved in advance. In the case of insufficient regulation however excess demand and supply may occur. Excess demand and supply may result in irregular in- and out-flows from abroad that are penalized hardly. Such in- and out-flows, however, can be avoided by forcing consumers to decrease demand, by forcing power plants to shut down generating units or by stopping wind turbines, in which cases severe costs must be paid. Let the variables $e_t^{up}, e_t^{do}, t \in \mathcal{T}$ denote excess demand

and supply. If $b_t^{up}, b_t^{do}, t \in \mathcal{T}$ denote penalty costs, excess demand and supply give rise to the following costs

$$\sum_{t \in \mathcal{T}} (b_t^{up} e_t^{up} + b_t^{do} e_t^{do}) \tag{3.8}$$

The power balance constraints are the following

$$\sum_{i \in \mathcal{I} \cup \mathcal{J}} (q_{it}^{up} - q_{it}^{do}) + e_t^{up} - e_t^{do} = D_t - S_t, \;\; t \in \mathcal{T} \tag{3.9}$$

where $D_t, S_t, t \in \mathcal{T}$ denote demand and supply. Note that supply include central production, decentral production and wind production as well as import and demand consists of national consumption and export.

3.4 Market integration

When Western Denmark is fully integrated in the Nordic regulating market, regulation will no longer be purchased mostly locally, but the following situation will apply, as is already the case for Eastern Denmark. A power system often forms a part of a larger system and as concerns the present application, the power system of Western Denmark is connected to systems of Sweden, Norway and Germany. It should be remarked however that Western Denmark is not connected to Eastern Denmark. The larger power system can be modeled as a network in which the nodes $\mathcal{N} = \{1, \ldots, N\}$ represent uncongested power systems that exchange power with the neighboring systems. The nodes are connected to the remaining nodes by edges representing transmission lines. A common network operator maintains the balance between supply and demand by purchasing regulating power. Imbalances are covered in part by local purchases and in part by foreign exchange. In the case of foreign exchange, transmission capacity limits may come into play. With grid congestion different price zones are established. We assume no grid congestion so that a common Nordic marginal price applies. Due to high price levels and rather inflexible trading conditions, regulating power exchange with Germany is rare. Thus, in this application, the connection between Western Denmark and Germany is ignored. Accordingly, $\mathcal{N} = \{1, 2, 3\}$ where 1 represents Western Denmark, 2 Sweden and 3 Norway. The common Nordic network operator is the Norwegian system operator, Statnett.

Divide the regulation bids according to the location of balance provider, that is $\mathcal{I} = \cup_{n\in\mathcal{N}}\mathcal{I}_n$ and $\mathcal{J} = \cup_{n\in\mathcal{N}}\mathcal{J}_n$. Then the regulation reserve management problem of the common network operator involves only a few changes in modeling. (3.8) and (3.9) are replaced by

$$\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}}(b^{up}_{tn}e^{up}_{tn} + b^{do}_{tn}e^{do}_{tn})$$

and

$$\sum_{i\in\mathcal{I}_n\cup\mathcal{J}_n}(q^{up}_{it} - q^{do}_{it}) + e^{up}_{tn} - e^{do}_{tn} = D_{tn} - S_{tn}, \;\; t\in\mathcal{T}, n\in\mathcal{N} \tag{3.10}$$

4 The stochastic programming problem

The regulation reserve management problem presented in the preceding sections is a deterministic problem. The problem, however, involves incomplete information on some of the data and such uncertainties should be taken into consideration. This can be achieved by means of stochastic programming. Uncertainties concern regulating prices and volumes that can be changed until an hour prior to operation. Moreover, demand and supply uncertainty arises because of failure in supplying, unforeseen weather changes leading to unpredicted wind production or nonanticipated heat demand. As the system imbalances are caused by demand and supply, we assumed that only demand and supply is uncertain. That is, we implicitly assume that regulation prices and volumes are known in advance and cannot be changed.

A stochastic program is based on an alternating process of decisions and information and the most basic one is the two-stage stochastic program. Here, first-stage decisions are made immediately and do not anticipate the future outcome of uncertainty, whereas second-stage decisions are deferred until uncertainty has been disclosed and utilize the additional information. The most obvious optimization criterion is to minimize the sum of deterministic first-stage costs and expected second-stage costs. For an introduction to stochastic programming, see [1], [12] and [17].

Although information evolves over time and a multi-stage stochastic program could be relevant, we approximate the problem by a two-stage stochastic program. We find this approximation sufficient to capture the interplay between reserves and regulation purchases. Since reserves must be purchased up at least a month in advance, reserve decisions are first-stage decisions. Decisions have to be made before operation and thus with incomplete knowledge of future supply and demand. On the contrary, regulation bids have an activation period of at most ten minutes and therefore can be purchased very close to operation which makes regulation decisions second-stage decisions. The objective is to minimize reserve costs and and expected future regulation and penalty costs.

The uncertain data is represented by a stochastic process on some probability space. To make the problem computationally tractable, we assume a discrete multivariate distribution. Outcomes of uncertainty will be referred to as scenarios indexed by $\mathcal{S} = \{1, \ldots, S\}$ and denoted by $(D_t^s - S_t^s)_{t \in \mathcal{T} s \in \mathcal{S}}$. The corresponding probabilities will be denoted by $\pi^s, s \in \mathcal{S}$. First-stage reserve decisions are $\delta_j^{up}, \delta_j^{do} \in \{0, 1\},\ \ j \in \mathcal{J}$, whereas second-stage regulation decisions are indexed $q_{it}^{up,s}, q_{it}^{do,s}, p_{it}^{up,s}, p_{it}^{do,s} \geq 0,\ \ i \in \mathcal{I} \cup \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S}$ etc.

The two-stage stochastic programming formulation of the regulating reserve management is the following problem, depending on whether pay-as-bid or marginal pricing applies. The extension to market integration should be straightforward.

Pay-as-bid pricing

$$\begin{aligned} \min \quad & \sum_{j\in\mathcal{J}}(c_j^{up}\delta_j^{up} + c_j^{do}\delta_j^{do}) && (4.1)\\ & + \sum_{s\in\mathcal{S}}\pi^s \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}(\bar{p}_{it}^{up,s}q_{it}^{up,s} - \bar{p}_{it}^{do,s}q_{it}^{do,s}) \\ & + \sum_{s\in\mathcal{S}}\pi^s \sum_{t\in\mathcal{T}}(b_t^{up}e_t^{up,s} + b_t^{do}e_t^{do,s}) \\ \text{s.t.} \quad & (3.1)-(3.2),(3.9) \\ & \delta_j^{up},\delta_j^{do} \in \{0,1\}, \quad j\in\mathcal{J} \\ & q_{it}^{up,s}, q_{it}^{do,s}, p_{it}^{up,s}, p_{it}^{do,s}, e_t^{up,s}, e_t^{do,s} \geq 0, \quad i\in\mathcal{I}\cup\mathcal{J}, t\in\mathcal{T}, s\in\mathcal{S} \end{aligned}$$

Marginal pricing

$$\begin{aligned} \min \quad & \sum_{j\in\mathcal{J}}(c_j^{up}\delta_j^{up} + c_j^{do}\delta_j^{do}) && (4.2)\\ & + \sum_{s\in\mathcal{S}}\pi^s \sum_{t\in\mathcal{T}}(\rho_t^{up,s} - \rho_t^{do,s}) \\ & + \sum_{s\in\mathcal{S}}\pi^s \sum_{t\in\mathcal{T}}(b_t^{up}e_t^{up,s} + b_t^{do}e_t^{do,s}) \\ \text{s.t.} \quad & (3.4)-(3.7),(3.9) \\ & \delta_j^{up},\delta_j^{do} \in \{0,1\}, \quad j\in\mathcal{J} \\ & q_{it}^{up,s}, q_{it}^{do,s}, p_{it}^{up,s}, p_{it}^{do,s}, \rho_t^{up,s}, \rho_t^{do,s}, e_t^{up,s}, e_t^{do,s} \geq 0, \quad i\in\mathcal{I}\cup\mathcal{J}, t\in\mathcal{T}, s\in\mathcal{S} \end{aligned}$$

The scenario generation has been kept rather simple. The differences between demand and supply data constitute a time series and, thus, has been analyzed by means of the field. In order to capture the behavior of demand and supply differences and in particular model the differences as a stochastic process, historical data profiles have been used. In that demand and supply show strong correlations over time, the stochastic process is chosen as an autoregressive process which, to keep things as simple as possible, is chosen to be of order one. The autoregressive stochastic process, cf. [3] is the following

$$D_t - S_t = \phi(D_{t-1} - S_{t-1}) + \epsilon_t, \quad t \in \mathbb{Z} \tag{4.3}$$

where $\{\epsilon_t\}_{t\in\mathbb{Z}}$ is a Gaussian white noise process. Scenarios of demand and supply differences $\{D_t^s - S_t^s\}_{t\in\mathcal{T},s\in\mathcal{S}}$ are generated by sampling from (4.3). To reflect the true probability distribution, a large number of scenarios has been generated by the use of Monte Carlo sampling.

5 Solution procedure

The problems (4.1) and (4.2) can be considered large-scale mixed-integer linear problems solvable by standard software packages or mixed-integer linear stochastic programs amenable to decomposition algorithms such as progressive hedging [5] or dual decomposition [14]. Being able to solve the problems as mixed-integer linear problems is valuable since the approach is very flexible. Adding further linear constraints is uncomplicated. In particular, the approach becomes relevant with constraints that introduce decencies between hours. However, with the current simplicity of the model, it can be solved by a specially designed procedure that utilizes the structure of the problem. The solution procedure is motivated by current practice of the system operator.

If formalized, the problems (4.1) and (4.2) can be stated as

$$\min\{c^T\delta + \sum_{s\in\mathcal{S}} \pi^s \sum_{t\in\mathcal{T}} Q_t^s(\delta), \delta \in \mathbb{B}^n\}$$

$$Q_t^s(\delta) = \min\{p^T q | Wq = h_t^s - T\delta, q \in \mathbb{R}^{n_1} \times \mathbb{B}^{n_2}\}$$

where $\delta \in \mathbb{B}^n$ represent the first-stage decisions, $q \in \mathbb{R}^{n_1} \times \mathbb{B}^{n_2}$ represent the second-stage decisions and data vectors and matrices are derived from the problems.

Procedure 5.1. *Enumerate*

Step 1 (Initialization) Let $\bar{z} = \infty$.

Step 2 (Enumeration) Choose a first-stage solution, δ.

Step 3 (Evaluation) Let $\bar{z} = \min\{\bar{z}, c^T\delta + \sum_{s\in\mathcal{S}} \pi^s \sum_{t\in\mathcal{T}} Q_t^s(\delta)\}$, *where* $Q_t^s(\delta)$ *is calculated as in Procedure 5.2 for* $t \in \mathcal{T}, s \in \mathcal{S}$. *Return to step 2.*

Procedure 5.2. *Merit order*

Step 1 (Initialize) If $D_t^s - S_t^s > 0$, *the system must be down-regulated. Let*

$$\mathcal{I}^{up} = \{i \in \mathcal{I} : \delta_i^{up} = 1\}$$

index the activated reserve bids. Available regulation bids are then indexed by $\mathcal{I}^{up} \cup \mathcal{J}$.

Likewise, if $D_t^s - S_t^s < 0$, *the system must be up-regulated. Let*

$$\mathcal{I}^{do} = \{i \in \mathcal{I} : \delta_i^{do} = 1\}$$

index the activated reserve bids. Available regulation bids are then indexed by $\mathcal{I}^{do} \cup \mathcal{J}$.

Step 2 (Ranking) In the case of up-regulation, activate (fully unless the imbalance is covered by less) the regulation bid $(\bar{p}_{it}^{up}, \bar{q}_{it}^{up,s})$ *with the lower price from the bids indexed by* $\mathcal{I}^{up} \cup \mathcal{J}$. *Delete the bid from the set* $\mathcal{I}^{up} \cup \mathcal{J}$. *If* $\mathcal{I}^{up} \cup \mathcal{J} = \emptyset$, *the remaining imbalance is excess demand. If the imbalance is covered, stop. Otherwise, return to Step 2.*

In the case of down-regulation, activate (fully unless the imbalance is covered by less) the regulation bid $(\bar{p}_{it}^{do}, \bar{q}_{it}^{do,s})$ *with the higher price from the bids indexed by* $\mathcal{I}^{do} \cup \mathcal{J}$. *Delete the bid from the set* $\mathcal{I}^{do} \cup \mathcal{J}$. *If* $\mathcal{I}^{do} \cup \mathcal{J} = \emptyset$, *the remaining imbalance is excess supply. If the imbalance is covered, stop. Otherwise, return to Step 2.*

6 Computation results

As already stated, the case study concerns the regulating reserve management problem of the Western Denmark system operator. The data dates from June 2006, just prior to the transition from pay-as-bid pricing to marginal pricing. Hence, we solve the problem with both pay-as-bid pricing (4.1) and local marginal pricing (4.2). Reserve bids comprise bids to the auction of June as well as individual contracts that may have a longer duration. As the system operator intends to reduce the offer period of reserve bids, we assume that such individual contracts have an offer period of only one month. The reserve bids consist of seven up-regulation bids and one down-regulating bid. The volumes and the fixed prices of the reserve bids are released by Energinet.dk[3]. The variable prices have been randomly generated based on the announced regulating market prices. As regards regulating bids to the market, ten bids have been constructed. Both volumes and prices have been randomly

[3]www.energinet.dk

generated based on the regulating market prices and the total amounts of up- and down-regulation bid into the market. From this, the imbalances between demand and supply has been calculated. The data has been provided by Nord Pool[4]. The penalties for excess demand and supply are both set sufficiently high to prevent uncovered imbalances on a regular basis.

With the current data, the problem (4.1) contains eight binary variables and no constraints in the first stage and 53,280 continuous variables and 13,680 constraints in the second stage. The problem (4.2) contains the same number of variables and constraints in the first stage and 54,720 continuous variables and 39,608 constraints in the second stage.

The Procedures 5.1 and 5.2 were implemented in C++ and computations were carried out on an Intel Xeon 2.67 GHz processor with 4 GB RAM.

We have solved the problems (4.1) and (4.2) with the Procedures 5.1 and 5.2 and listed the results. For a varying number of scenarios, Table 1 displays the average optimal values and CPU times of ten different runs. Obviously, marginal pricing results in a higher optimal value than pay-as-bid pricing. The first column of Table 2 shows the total balancing costs divided into reserve costs, regulation costs and penalty costs. Recall that regulation costs consist of up-regulation expenses and down-regulation income and costs may therefore be both positive and negative. The second column of Table 2 gives the total imbalances divided into regulation and excess supply and demand along with the reserved regulation that is available but not necessarily activated. Regulation consists of both up- and down-regulation. All numbers are based on 100 scenarios and are averages of ten different runs. It is clear that both for pay-as-bid pricing and marginal pricing reserves are highly necessary in covering imbalances in an optimal fashion. Finally, Table

[4]www.nordpool.no

4 lists the reserve bids and indicate activation or not. All ten runs show the same result and indeed support the use of reserves.

To compare the stochastic programming approach to a deterministic approach, we have solved the expected value problem, in which stochastic demand and supply differences have been replaced by their expected values. Moreover, we have computed the expected results of using the expected value solution (EEV). The average EEVs and CPU times of ten different runs are displayed in Table 1. As can be shown is always the case, the EEVs exceed the optimal values of the stochastic programs. In Table 3 the total balancing costs and the total imbalances are divided into reserves, regulation and excess supply and demand and Table 4 indicate activation or not of the reserve bids. Since imbalances often cancel out on average, no reserve bids are activated in the deterministic case. The expected result of using the expected value solution however is a need for a larger amount of direct regulation and if not available, larger excess demand and supply. This is indeed reflected in higher regulation costs, much higher penalty costs and thus higher total costs. The percentual values of the stochastic solutions, that is, the percentual saving in costs of using the stochastic solutions rather than the deterministic solutions, are significant as the numbers are in the range of 36-38 percent. In conclusion, stochastic programming has its relevance in the regulation reserve management problem. [Insert Table 1-4]

7 Further research

It could be argued that regulating reserve management affects spot market trading in that purchasing regulating reserves prevent suppliers from disposing of production in the spot market. As the system operator reserves regulation, less production capacity becomes available for the spot market. We have implicitly assumed that production capacity for the spot market is not seriously affected. The assumption is justified if producers allocate production capacity for the spot market and the regulating market separately. However, it would be valuable to further investigate the matter. For instance, the model of the present paper could be incorporated as a part of a larger model that also includes the spot market.

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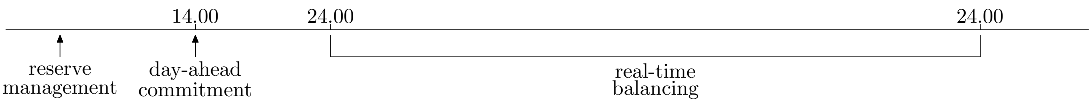

Figure 7.1: Time schedule.

Table 1: Computational results, 10 runs

S	Opt. val. Pay-as-bid pricing	Opt. val. Marginal pricing	CPU/s
100	$4.99e{+}07$	$5.22e{+}07$	51.37
500	$4.94e{+}07$	$5.18e{+}07$	257.33
1000	$4.94e{+}07$	$5.18e{+}07$	512.00
EEV	$7.89e{+}07$	$8.13e{+}07$	1.04

Table 2: Computational results, 100 scenarios, 10 runs

		Costs/DKK	Volume/MW
Pay-as-bid	Total	4.99e+07	86060.23
pricing	Reserve	3.46e+06	94320.00
	Regulation	1.32e+06	81545.20
	Excess supply and demand	4.52e+07	4515.03
Marginal	Total	5.22e+07	86060.23
pricing	Reserve	3.46e+06	94320.00
	Regulation	3.68e+06	81545.20
	Excess supply and demand	4.51e+07	4515.03

Table 3: Computational results, EVP, 10 runs

		Costs/DKK	Volume/MW
Pay-as-bid	Total	7.89e+07	86226.71
pricing	Reserve	0e+00	0.00
	Regulation	1.56e+06	78496.00
	Excess supply and demand	7.73e+07	7730.71
Marginal	Total	8.13e+07	86226.71
pricing	Reserve	0e+00	0.00
	Regulation	3.94e+06	78496.00
	Excess supply and demand	7.73e+07	7730.71

Table 4: Computational results, 10 runs

					Up-regulation			
Price/DKK		288000	625000	384000	330000	10298880	714000	966450
Volume/MW		12	25	16	11	298	21	30
Activation	100 sce.	*yes*	*yes*	*yes*	*yes*	*no*	*yes*	*yes*
	EVP	*no*	*no*	*no*	*no*	*no*	*no*	*no*
					Down-regulation			
Price		152000						
Volume		16						
Activation	100 sce.	*yes*						
	EVP	*no*						