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A New Notion of Reserve for Power Systems with High Penetration of Storage and Flexible Demand

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***Abstract*—Modern power systems face important demand uncertainties due to increasing penetration of behind-the-meter renewable generation. System operators need to account for such uncertainties when solving the unit commitment and economic dispatch problem. The research literature has proposed advanced methods for decision making under uncertainty but, in practice, actual system operators put more trust in the tried-and-true approach of dealing with future uncertainty by committing reserves. In this paper, the unit commitment and economic dispatch problem is formulated for a system with high penetration of storage and the inadequacy of methods based on the traditional notion of reserves is exposed. Namely, in contrast to a generator, a storage unit can provide reserve capacity in a number of timeslots but it cannot provide an analogous reserve activation in all of those timeslots due to the battery's energy being depleted. After discussing two plausible but inadequate approaches, a new, generalized notion of reserves is proposed, which addresses these issues while not abandoning the practical, reserve-based approach for the operator's problem, thus making the best of both worlds. The proposed scheme enables storage units to provide reserves, without putting the system at risk of energy scarcity, which is shown to result in substantial cost savings.**

***Index Terms*—unit commitment, economic dispatch, chance constraints, reserves**

I. Introduction

Solving the unit commitment and economic dispatch (UC-ED) problem is one of the main responsibilities of power system operators. The UC-ED is solved several hours before the operational period (e.g. day) begins and refers to scheduling which generators will be ON at each timeslot and at what generation level. These decisions are based on forecasts about the system's demand. Forecast inaccuracies, though, are inevitable and operators traditionally deal with such uncertainties by making sure that enough spare capacity is available and ready to balance any possible demand deviation. This spare capacity is commonly referred to as "reserves".

In modern power systems with high penetration of behind-the-meter renewables, the net demand can be substantially more volatile and recent research literature has proposed sophisticated methods for solving the UC-ED problem under uncertainty. Indicatively, for systems without high storage penetration, [1] presented a two-stage stochastic model for the Swiss reserve market, while [2] opted for a chance-constrained program and discussed the limitations of scenario-based stochastic market-clearing models such as [3] and [4]. For a comprehensive treatment of uncertainty-informed UC-ED problems, the reader is referred to [5].

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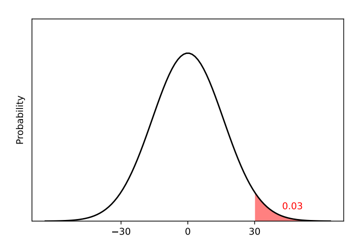
A. Practical Problems and Research Motivation

Stochastic market clearing and chance-constrained programs have not yet been adopted by system operators, who opt for the well-tested practical approach of addressing future uncertainty by committing reserves. While the standard reserve-based approach has served well for traditional generator-dominated systems, in systems with high penetration of storage and demand-response resources it faces an important complication. This complication is best illustrated by example:

Example 1. *Consider the operation of a three-generators/one-storage system in a 2-timeslots horizon, where the inflexible demand is* 100 *in each of the two timeslots. Suppose that generator A is a bulk generator with capacity* 100 *and generators B and C are small generators with capacities* 10 *and* 7.5 *respectively, while both bear a technical minimum of* 2.5*. The storage unit's state of energy is* 30*. Assume that, without considering uncertainties/reserves, the optimal dispatch is to have generator A serve the demand and not commit generators B and C at all.*

Let us now consider probabilistic real-time imbalances. Assume that the operator wishes to restrict the probability of unmanageable real-time deviations to be at most 0.03 in each timeslot, and that an upwards reserve of 30 *is sufficient to ensure that, as shown in the figure below.*



It could be said that the storage unit can fully cover the necessary reserves and the dispatch does not need to be altered. This approach is implicitly adopted in [6]. The final commitment decisions would read as in Table I.

However, if during the real-time operation the demand indeed deviates by 30 *in timeslot* 1*, the storage will need to discharge the whole of its energy and there would be zero upwards reserve for timeslot* 2*; this could lead to a loss of*

TABLE I: Example 1 (imprudent) decisions

	Capacity / State of Energy	Generation in t_1	Generation in t_2	Reserve in t_1	Reserve in t_2
Gen. A	100	100	100	0	0
Gen. B	10	0	0	0	0
Gen. C	7.5	0	0	0	0
Storage	30	0	0	30	30

load if an upwards demand deviation occurs in timeslot 2 as well.

The above example exposes the issue that, while a storage unit can provide reserves up to its state-of-energy at *any* timeslot, if the unit is called to provide upwards reserve activation in one timeslot of real-time operation, then that amount of stored energy will be depleted and the unit will not be able to fulfill its reserve commitment for the subsequent timeslots. A straightforward solution to this issue (which however creates a different problem) is to split the reserve provision of the storage unit, such that its total reserves across timeslots is no higher than its state of energy, as considered in [7]. This approach is demonstrated in the next example:

Example 2. *For the system of Example 1, suppose now that the operator allocates to the storage unit a reserve of at most* 15 *in each timeslot. To maintain feasibility, both generators B and C need to be committed, as shown in Table II.*

TABLE II: Example 2 (conservative) decisions

	Capacity / State of Energy	Generation in t_1	Generation in t_2	Reserve in t_1	Reserve in t_2
Gen. A	100	95	95	5	5
Gen. B	10	2.5	2.5	7.5	7.5
Gen. C	7.5	2.5	2.5	5	5
Storage	30	0	0	12.5	12.5

Now the storage unit commits an amount of 12.5 for reserve in each timeslot, which it can actually fulfill in both timeslots. However, notice that the probability of the system needing to activate all of its reserves in *both* timeslots is very low; while for 12 or 24-timeslot horizons of actual systems, the probability of actually activating all of the reserve capacity in all of the timeslots and in the same direction is virtually zero. This renders this example's solution unnecessarily conservative and inefficient. The next subsection sketches the proposed solution and presents the paper's contributions.

B. Proposition and Contributions

For the system of the previous subsection none of the two solutions provided was satisfactory. Intuitively, the best solution would lie somewhere between the solutions of the two examples above, indicatively as in Table III.

TABLE III: Efficient decisions

	Capacity / State of Energy	Generation in t_1	Generation in t_2	Reserve in t_1	Reserve in t_2
Gen. A	100	95	95	5	5
Gen. B	10	5	5	5	5
Gen. C	7.5	0	0	0	0
Storage	30	0	0	20	20

Notice that the solution of Table III commits one generator less than the one of Table II, at the expense of allocating an amount of 20 reserves to the storage unit in each timeslot while the storage can only provide a total balancing energy of 30. This may seem somewhat imprudent, similarly to the case of Example 1 (although less so), but the probability of needing to activate the whole of the upward reserve in every timeslot naturally diminishes with the number of timeslots. Based on this observation, notice that: if the probability of needing a total upwards reserve activation of more than 50 *across* the two timeslots is sufficiently low, the solution of Table III is acceptable. That is because a total balancing energy of 50 across the two timeslots can be covered by generator A offering 10, generator B offering another 10, and the storage offering the remaining 30, which is exactly its state of energy. This observation leads to the main idea of this paper:

Core Idea: If the storage is allowed to offer its whole energy as reserve in each timeslot (as in Example 1), but an additional notion of reserve is introduced to make sure that there are sufficient *energy* reserves across timeslots to maintain the probability of balancing energy scarcity below a desired threshold, then the operator can find a dispatch (such as the one in Table IV below) that satisfies the security constraints at a fraction of the conservative solution's (Example 2) cost.

TABLE IV: Optimal decisions

	Capacity / State of Energy	Generation in t_1	Generation in t_2	Reserve in t_1	Reserve in t_2	Energy reserve
Gen. A	100	95	95	5	5	10
Gen. B	10	5	5	5	5	10
Gen. C	7.5	0	0	0	0	0
Storage	30	0	0	30	30	30
System		100	100	40	40	50

In this paper, this idea is formalized, analyzed, and experimentally validated for a chance-constrained UC-ED problem with elaborate resource models across multiple timeslots. The link between the chance-constrained program and the deterministic equivalent is analyzed theoretically, and it is shown how the introduction of the proposed energy reserves concept arises naturally from the problem's mathematical properties.

The rest of the paper is organized as follows. Section II serves as a preliminary, by presenting the chance-constrained UC-ED problem for a traditional, generators-dominated system and demonstrating how the operator can manage the demand uncertainty by solving a reserves-based deterministic optimization problem. Section III presents the chance-constrained program for a system with generators *and* storage units, along with two benchmark solutions for reserve procurement: one imprudent (in the sense of Example 1) and one overly conservative (in the sense of Example 2). Section IV presents the proposed solution which entails conceptualizing a new notion of reserves. Section V compares the proposed solution with the two benchmarks empirically in a setting tailored to make the different effects of each scheme prominent, while Section VI applies the three schemes to a standardized case study and compares their performance. Finally, Section VII concludes the paper.

II. Unit Commitment & Economic Dispatch - Generators Only

Consider a UC-ED problem for a period $\mathcal{T}=\{1,2,...,\mathrm{T}\}$ of discrete timeslots of equal duration. The power system serves a net demand which refers to the difference between the system's inflexible load and the (behind-the-meter) renewable energy generation. The net demand's expected value (forecast) in timeslot t is denoted by D_t, and its real-time deviation X_t is a random variable which has a probability density function $f(X_t)$. The system operator is responsible for committing and dispatching the system's energy resources beforehand, such that the system operates safely and economically under the uncertain demand D_t+X_t.

Let us first consider a set of resources consisting only of generators and denote the set of generators by $\mathcal{G}$. A generator's $g\in\mathcal{G}$ commitment decision

$$u_{g,t}\in\{0,1\},\quad g\in\mathcal{G},t\in\mathcal{T}, \tag{1}$$

refers to whether the generator will be ON ($u_{g,t}=1$) or OFF ($u_{g,t}=0$) at t. The generator's transition from an OFF to an ON state is captured by the binary start-up variable $y_{g,t}$, whereas the respective shutdown variable is denoted as $z_{g,t}$:

$$y_{g,t},z_{g,t}\in\{0,1\},\quad g\in\mathcal{G},t\in\mathcal{T}. \tag{2}$$

The logic that connects the three binary variables is

$$u_{g,t}-u_{g,t-1}=y_{g,t}-z_{g,t},\quad \forall g\in\mathcal{G},t\in\mathcal{T}, \tag{3}$$

where $u_{g,0}$ is set to an initial state. When ON, the generator can be dispatched at an output $p_{g,t}$, which is divided into a set $\mathcal{L}$ of levels/segments, as in

$$p_{g,t}=\sum_{l\in\mathcal{L}}p_{g,l,t},\quad \forall g\in\mathcal{G},t\in\mathcal{T}, \tag{4}$$

with each segment's output constrained by an upper bound $\overline{\mathrm{p}}_{g,l}$, as in:

$$0\le p_{g,l,t}\le u_{g,t}\overline{\mathrm{p}}_{g,l},\quad \forall g\in\mathcal{G},l\in\mathcal{L},t\in\mathcal{T}. \tag{5}$$

A minimum-up time constraint captures the technical requirement that, when a generator is started, it cannot be shut down before a number $\mathrm{q}_g^{\mathrm{up}}$ of timeslots passes:

$$u_{g,t}\ge\sum_{\tau=\max\{0,t-\mathrm{q}_g^{\mathrm{up}}+1\}}^{t}y_{g,\tau},\quad \forall g\in\mathcal{G},t\in\mathcal{T}. \tag{6}$$

Similarly, a minimum-down time parameter $\mathrm{q}_g^{\mathrm{dn}}$ constraints the generator from starting up shortly after a shutdown:

$$1-u_{g,t}\ge\sum_{\tau=\max\{0,t-\mathrm{q}_g^{\mathrm{dn}}+1\}}^{t}z_{g,\tau},\quad \forall g\in\mathcal{G},t\in\mathcal{T}, \tag{7}$$

while constraints

$$p_{g,t-1}-\mathrm{r}_g\le p_{g,t}\le p_{g,t-1}+\mathrm{r}_g,\quad \forall g\in\mathcal{G},t\in\mathcal{T}, \tag{8}$$

implement the generator's ramp restrictions. Finally, a generator bears a cost $C_{g,t}=c(u_{g,t},y_{g,t},z_{g,t},p_{g,l,t})$ which is given as a function of its operational status and will be specifically modeled in the case studies.

The system's aggregated generation must meet the demand forecast, i.e.,

$$\sum_{g\in\mathcal{G}}p_{g,t}=\mathrm{D}_t,\quad \forall t\in\mathcal{T}, \tag{9}$$

while the system's security-of-supply requirement dictates that, at every timeslot, the committed generation capacity should suffice to serve the random real-time demand. More specifically, the probability of not having enough capacity available to meet the possible demand, at any timeslot, should be below a given threshold ε, i.e.,

$$\mathtt{Pr}\Big[\mathrm{D}_t+X_t\ge\sum_{g\in\mathcal{G}}u_{g,t}\cdot\overline{\mathrm{p}}_g\Big]\le\varepsilon,\quad \forall t\in\mathcal{T}. \tag{10}$$

A similar constraint applies for the case of not having enough demand to absorb the technical minimum generation of the committed generators, i.e.,

$$\mathtt{Pr}\Big[\mathrm{D}_t+X_t\le\sum_{g\in\mathcal{G}}u_{g,t}\cdot\underline{\mathrm{p}}_g\Big]\le\varepsilon,\quad \forall t\in\mathcal{T}. \tag{11}$$

Under these considerations, the chance-constrained UC-ED problem with generators only (CC-G) reads as

$$\min_{\mathcal{V}_{\mathrm{CC-G}}}\ \Big\{\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}}C_{g,t}\Big\} \tag{CC-G}$$
$$\text{subject to}$$
$$\begin{aligned}\text{Generators' constraints}:&\quad (1)-(8),\\ \text{UC-ED Power Balance}:&\quad (9),\\ \text{Chance constraints}:&\quad (10),(11),\end{aligned}$$

where

$$\mathcal{V}_{\mathrm{CC-G}}=\Big((u_{g,t},y_{g,t},z_{g,t},p_{g,t},C_{g,t})_{g\in\mathcal{G},t\in\mathcal{T}},(p_{g,l,t})_{g\in\mathcal{G},l\in\mathcal{L},t\in\mathcal{T}}\Big)$$

are the problem's decision variables.

Depending on how the random variable X_t is modeled, the chance constraints (10), (11) may feature an analytic reformulation. For example, if X_t is modeled as a normally distributed random variable, and independent for each timeslot, Eqs. (10), (11) feature a convex reformulation which means that problem (CC-G) can be brought to a form that is solvable by standard solvers. However, such an approach raises issues of complexity, non-intuitiveness, and transparency, while there is no standard way of tuning the formulation to handle generic (possibly non-parametric) distributions of X_t.

For such reasons, system operators traditionally opt for a deterministic equivalent of problem (CC-G), in which the uncertainty is accommodated by procuring an amount of *reserves* for each timeslot. Specifically, let $r_{g,t}^{\uparrow}$ and $r_{g,t}^{\downarrow}$ denote a generator's upwards and downwards committed reserve in t respectively. The committed upwards reserve

$$r_{g,t}^{\uparrow}\ge 0,\quad \forall g\in\mathcal{G},t\in\mathcal{T}, \tag{12}$$

is at most equal to the generator's spare capacity, as in

$$r_{g,t}^{\uparrow}\le\overline{\mathrm{p}}_g-p_{g,t},\quad \forall g\in\mathcal{G},t\in\mathcal{T}, \tag{13}$$

and it is zero if the generator is OFF, i.e.

$$r^{\uparrow}_{g,t} \leq u_{g,t} \cdot (\overline{\mathrm{p}}_g - \underline{\mathrm{p}}_g), \quad \forall g \in \mathcal{G}, t \in \mathcal{T}. \tag{14}$$

Similarly, for the downwards reserve it is

$$r^{\downarrow}_{g,t} \leq p_{g,t} - \underline{\mathrm{p}}_g, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{15}$$

$$r^{\downarrow}_{g,t} \leq u_{g,t} \cdot (\overline{\mathrm{p}}_g - \underline{\mathrm{p}}_g), \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{16}$$

$$r^{\downarrow}_{g,t} \geq 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}. \tag{17}$$

The operator deals with real time demand uncertainty by procuring adequate amounts $\mathrm{R}^{\uparrow}_t, \mathrm{R}^{\downarrow}_t$ of upwards and downwards reserves in the UC-ED problem, as in

$$\sum_{g \in \mathcal{G}} r^{\uparrow}_{g,t} \geq \mathrm{R}^{\uparrow}_t, \quad \forall t \in \mathcal{T}, \tag{18}$$

$$\sum_{g \in \mathcal{G}} r^{\downarrow}_{g,t} \geq \mathrm{R}^{\downarrow}_t, \quad \forall t \in \mathcal{T}. \tag{19}$$

Under these considerations, the deterministic equivalent, generators-only, problem (DE-G), reads as

$$\min_{\mathcal{V}_{\mathrm{DE-G}}} \quad \Big\{ \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} C_{g,t} \Big\} \tag{DE-G}$$

$$\text{subject to}$$

$$\begin{aligned} \text{Generators' constraints}: &\quad (1)-(8), \\ \text{UC-ED Power Balance}: &\quad (9), \\ \text{Generators' reserves}: &\quad (12)-(17), \\ \text{Reserve requirements}: &\quad (18)-(19), \end{aligned}$$

where $\mathcal{V}_{\mathrm{DE-G}} = \mathcal{V}_{\mathrm{CC-G}} \cup (r^{\uparrow}_{g,t}, r^{\downarrow}_{g,t})_{g \in \mathcal{G}, t \in \mathcal{T}}$. One advantage of formulation (DE-G) over (CC-G) is the intuitiveness of the notion of "reserve" variables, instead of introducing auxiliary variables with no physical meaning to handle the chance constraints. A second advantage is that, even in case of arbitrary (possibly non-parametric) distributions for X_t, the problem can still be managed effectively by finding appropriate values for $\mathrm{R}^{\uparrow}_t, \mathrm{R}^{\downarrow}_t$, a task for which there is mature know-how (cf [8], [9]), in contrast to (CC-G) for which there is no straightforward way to calculate a decent solution under arbitrary demand distributions. Thus, problem (DE-G) is the formulation that actual system operators typically use for their UC-ED problems. The practicality of formulation (DE-G) may mislead one to think that (DE-G) is just a heuristic approximation of (CC-G). The following Lemma suggests otherwise:

Lemma 1. *If X_t is independent for each timeslot, there are values $\widehat{\mathrm{R}}^{\uparrow}_t, \widehat{\mathrm{R}}^{\downarrow}_t$ for which problem* (DE-G) *is an exact reformulation of* (CC-G).

Proof. In Eqs. (10), (11), let us expand the terms $u_{g,t} \cdot \overline{\mathrm{p}}_g$ and $u_{g,t} \cdot \underline{\mathrm{p}}_g$ as

$$u_{g,t} \cdot \overline{\mathrm{p}}_g = u_{g,t} \cdot (p_{g,t} + \overline{\mathrm{p}}_g - p_{g,t}) \tag{20}$$

and

$$u_{g,t} \cdot \underline{\mathrm{p}}_g = u_{g,t} \cdot (p_{g,t} - p_{g,t} + \underline{\mathrm{p}}_g). \tag{21}$$

Substituting (20) and (21) into (10) and (11) respectively yields

$$\mathtt{Pr}\Big[\mathrm{D}_t + X_t \geq \sum_{g \in \mathcal{G}} \Big(u_{g,t} \cdot p_{g,t} + u_{g,t} \cdot (\overline{\mathrm{p}}_g - p_{g,t}) \Big) \Big] \leq \varepsilon, \quad \forall t \in \mathcal{T}, \tag{22}$$

and

$$\mathtt{Pr}\Big[\mathrm{D}_t + X_t \leq \sum_{g \in \mathcal{G}} \Big(u_{g,t} \cdot p_{g,t} - u_{g,t} \cdot (p_{g,t} - \underline{\mathrm{p}}_g) \Big) \Big] \leq \varepsilon, \quad \forall t \in \mathcal{T}. \tag{23}$$

Because of (9) and (5), we have that

$$\mathrm{D}_t = \sum_{g \in \mathcal{G}} u_{g,t} \cdot p_{g,t}, \tag{24}$$

which yields

$$\mathtt{Pr}\Big[X_t \geq \sum_{g \in \mathcal{G}} \Big(u_{g,t} \cdot (\overline{\mathrm{p}}_g - p_{g,t}) \Big) \Big] \leq \varepsilon, \quad \forall t \in \mathcal{T}, \tag{25}$$

and

$$\mathtt{Pr}\Big[X_t \leq \sum_{g \in \mathcal{G}} \Big(- u_{g,t} \cdot (p_{g,t} - \underline{\mathrm{p}}_g) \Big) \Big] \leq \varepsilon, \quad \forall t \in \mathcal{T}. \tag{26}$$

By introducing auxiliary variables $r^{\uparrow}_{g,t}, r^{\downarrow}_{g,t}$, which can be interpreted as the upwards and downwards reserves, (25) and (26) can be formulated as

$$\mathtt{Pr}\Big[X_t \geq \sum_{g \in \mathcal{G}} r^{\uparrow}_{g,t} \Big] \leq \varepsilon, \qquad \forall t \in \mathcal{T}, \tag{27}$$

$$r^{\uparrow}_{g,t} \leq \overline{\mathrm{p}}_g - p_{g,t}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{28}$$

$$r^{\uparrow}_{g,t} \leq u_{g,t} \cdot (\overline{\mathrm{p}}_g - \underline{\mathrm{p}}_g), \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{29}$$

$$r^{\uparrow}_{g,t} \geq 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{30}$$

and

$$\mathtt{Pr}\Big[X_t \leq \sum_{g \in \mathcal{G}} -r^{\downarrow}_{g,t} \Big] \leq \varepsilon, \qquad \forall t \in \mathcal{T}, \tag{31}$$

$$r^{\downarrow}_{g,t} \leq p_{g,t} - \underline{\mathrm{p}}_g, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{32}$$

$$r^{\downarrow}_{g,t} \leq u_{g,t} \cdot (\overline{\mathrm{p}}_g - \underline{\mathrm{p}}_g), \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{33}$$

$$r^{\downarrow}_{g,t} \geq 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{34}$$

respectively. Because X_t is independent for each timeslot, the probabilities $\mathtt{Pr}\Big[X_t \geq \sum_{g \in \mathcal{G}} r^{\uparrow}_{g,t}\Big]$ and $\mathtt{Pr}\Big[X_t \leq \sum_{g \in \mathcal{G}} -r^{\downarrow}_{g,t}\Big]$ can be written as

$$\mathtt{Pr}\Big[X_t \geq \sum_{g \in \mathcal{G}} r^{\uparrow}_{g,t} \Big] = 1 - F_{X_t}(\sum_{g \in \mathcal{G}} r^{\uparrow}_{g,t}), \tag{35}$$

$$\mathtt{Pr}\Big[X_t \leq \sum_{g \in \mathcal{G}} -r^{\downarrow}_{g,t} \Big] = F_{X_t}(\sum_{g \in \mathcal{G}} -r^{\downarrow}_{g,t}), \tag{36}$$

where

$$F_{X_t}(x) = \int_{-\infty}^{x} f(X_t) dX_t \tag{37}$$

is the cumulative distribution function of X_t. Setting

$$\sum_{g \in \mathcal{G}} r^{\uparrow}_{g,t} \geq \mathrm{R}^{\uparrow}_t, \quad \forall t \in \mathcal{T}, \tag{38}$$

$$\sum_{g \in \mathcal{G}} r^{\downarrow}_{g,t} \geq \mathrm{R}^{\downarrow}_t, \quad \forall t \in \mathcal{T}, \tag{39}$$

one can use numerical tools or available tables to assess the values $\widehat{\mathrm{R}}_t^{\uparrow}, \widehat{\mathrm{R}}_t^{\downarrow}$ for which

$$\mathtt{Pr}\Big[X_t \geq \widehat{\mathrm{R}}_t^{\uparrow}\Big] = \varepsilon, \quad \forall t \in \mathcal{T}, \tag{40}$$

$$\mathtt{Pr}\Big[X_t \leq -\widehat{\mathrm{R}}_t^{\downarrow}\Big] = \varepsilon, \quad \forall t \in \mathcal{T}. \tag{41}$$

This means that it suffices to have enough reserves to cover the critical values $\widehat{\mathrm{R}}_t^{\uparrow}, \widehat{\mathrm{R}}_t^{\downarrow}$, which allows us to replace Eqs. (27), (31) with

$$\sum_{g\in\mathcal{G}} r_{g,t}^{\uparrow} \geq \widehat{\mathrm{R}}_t^{\uparrow}, \quad \forall t \in \mathcal{T}, \tag{42}$$

$$\sum_{g\in\mathcal{G}} r_{g,t}^{\downarrow} \geq \widehat{\mathrm{R}}_t^{\downarrow}, \quad \forall t \in \mathcal{T}. \tag{43}$$

These reformulations bring problem (CC-G) to the form of (DE-G), which completes the proof. □

Summing up this section, we have that the formulation (DE-G) is equivalent to (CC-G) for independent demand deviations, while it offers practical ways to deal with arbitrary demand distributions and maintaining intuitiveness at the same time. Thus, system operators have good reasons for using (DE-G) in practice. However, in the face of high penetration of storage resources in addition to generators, this approach faces an important challenge as will be discussed in the next Section.

III. Unit Commitment & Economic Dispatch - Generators and Storage

Consider now that a set $\mathcal{S}$ of storage units is also available for dispatch. A storage unit can charge (or discharge) an amount of $p_{s,t}^{\mathtt{ch}}$ (or $p_{s,t}^{\mathtt{dis}}$) at t, with its mode determined by a binary variable

$$m_{s,t} \in \{0,1\}, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}. \tag{44}$$

The logic determining the unit's operational mode reads as

$$0 \leq p_{s,t}^{\mathtt{dis}} \leq \mathbb{M} \cdot m_{s,t}, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \tag{45}$$

$$0 \leq p_{s,t}^{\mathtt{ch}} \leq \mathbb{M} \cdot (1 - m_{s,t}) \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \tag{46}$$

where $\mathbb{M}$ is a sufficiently large constant. Charging and discharging operations affect the battery's state of energy $E_{s,t}$ which is defined as

$$E_{s,t} = \mathrm{E}_{s,0} - \sum_{\tau=1}^{t} (p_{s,\tau}^{\mathtt{dis}} \frac{1}{\mathrm{b}^{\mathtt{dis}}} - p_{s,\tau}^{\mathtt{ch}} \mathrm{b}^{\mathtt{ch}}), \tag{47}$$

with $\mathrm{E}_{s,0}$ being its initial state and $\mathrm{b}^{\mathtt{ch}}, \mathrm{b}^{\mathtt{dis}} \in [0,1]$ denoting charge and discharge efficiencies, while it is good practice to deliver a final state of energy equal to the initial one, i.e.,

$$E_{s,\mathrm{T}} = \mathrm{E}_{s,0} \quad \forall s \in \mathcal{S}. \tag{48}$$

The storage unit bears an operational cost $C_{s,t}$ equal to

$$C_{s,t} = \mathrm{w}_s^{\mathrm{st}} \cdot (p_{s,t}^{\mathtt{ch}} + p_{s,t}^{\mathtt{dis}}), \forall s \in \mathcal{S}, t \in \mathcal{T}, \tag{49}$$

and the battery's state of energy is bounded as

$$\underline{\mathrm{E}}_s \leq E_{s,t} \leq \overline{\mathrm{E}}_s, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}. \tag{50}$$

The power-balance constraint (9) becomes

$$\sum_{g\in\mathcal{G}} p_{g,t} + \sum_{s\in\mathcal{S}} (p_{s,t}^{\mathtt{dis}} - p_{s,t}^{\mathtt{ch}}) = \mathrm{D}_t, \quad \forall t \in \mathcal{T}. \tag{51}$$

A. Chance-Constrained Program, Generators & Storage

In the presence of storage, the system's ability to counteract a real-time deviation X_t depends also on the amounts of battery storage used in previous timeslots. Thus, for the chance-constrained program, we impose that the probability of not being able to meet the energy demand across any period $[i,t] \in \mathcal{T}^2 : i \leq t$ is less than ε. The event of an infeasible demand profile in $[i,t]$ can occur in two cases; either the total demand $\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau)$ is higher than the total capacity of committed generators for these timeslots plus the total energy stored in batteries *at the beginning* of the period, as in

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau) > \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}\overline{\mathrm{p}}_g + \sum_{s\in\mathcal{S}}(E_{s,i} - \underline{\mathrm{E}}_s)\mathrm{b}^{\mathtt{dis}}\Big] < \varepsilon, \quad \forall [i,t] \in \mathcal{T}^2 : i \leq t, \tag{52}$$

or the total demand $\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau)$ plus the amount that the storage can absorb (by charging) $\sum_{s\in\mathcal{S}}(\overline{\mathrm{E}}_s - E_{s,i})$ in period $[i,t]$ is lower than the generation technical minimum, i.e.,

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau) < \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}\underline{\mathrm{p}}_g - \sum_{s\in\mathcal{S}}(\overline{\mathrm{E}}_s - E_{s,i})\frac{1}{\mathrm{b}^{\mathtt{ch}}}\Big] < \varepsilon, \quad \forall [i,t] \in \mathcal{T}^2 : i \leq t. \tag{53}$$

Under these considerations, the chance-constrained UC-ED problem (CC-GS), with generators and storage, takes the form

$$\min_{\mathcal{V}_{\mathrm{CC-GS}}} \Big\{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{G}\cup\mathcal{S}} C_{n,t}\Big\} \tag{CC-GS}$$

subject to

Generators' constraints : (1) – (8),
Storage constraints : (44) – (50),
UC-ED Power Balance : (51),
Chance constraints : (52), (53),

$$\mathcal{V}_{\mathrm{CC-GS}} = \mathcal{V}_{\mathrm{CC-G}} \cup \Big\{(E_{s,t}, m_{s,t}, p_{s,t}^{\mathtt{dis}}, p_{s,t}^{\mathtt{ch}})_{s\in\mathcal{S},t\in\mathcal{T}}\Big\}.$$

Obtaining a deterministic, reserve-based formulation for (CC-GS) calls for special caution. The following two subsections present two simplistic approaches and discuss their shortcomings before presenting the proposed approach.

B. Imprudent Reserve Provision

This subsection presents an imprudent (non-cautionary) way to include the presence of storage, by keeping the same framework (DE-G) that was suitable for traditional, generators-dominated systems. Let $r_{s,t}^{\uparrow}$ and $r_{s,t}^{\downarrow}$ denote the reserve commitments of a storage unit and impose

$$0 \leq r_{s,t}^{\uparrow} \leq (E_{s,t} - \underline{\mathrm{E}}_s)\mathrm{b}^{\mathtt{dis}}, \qquad \forall s \in \mathcal{S}, t \in \mathcal{T}, \tag{54}$$

$$0 \leq r_{s,t}^{\downarrow} \leq (\overline{\mathrm{E}}_s - E_{s,t})\frac{1}{\mathrm{b}^{\mathtt{ch}}}, \qquad \forall s \in \mathcal{S}, t \in \mathcal{T}, \tag{55}$$

i.e. dictate that a storage unit, at each timeslot, can provide an amount of upwards (downwards) reserve at most equal to the amount of energy stored in its battery (to the amount that it can charge before reaching its battery capacity). The reserve requirements Eqs. (18), (19) are updated as

$$\sum_{n\in\mathcal{G}\cup\mathcal{S}} r^{\uparrow}_{n,t} \geq \widehat{\mathrm{R}}^{\uparrow}_{t}, \quad \forall t\in\mathcal{T}, \tag{56}$$

$$\sum_{n\in\mathcal{G}\cup\mathcal{S}} r^{\downarrow}_{n,t} \geq \widehat{\mathrm{R}}^{\downarrow}_{t}, \quad \forall t\in\mathcal{T}, \tag{57}$$

to include the reserve commitments of the storage units. Then, the imprudent formulation for the UC-ED problem with generators and storage (Impr-GS) reads as

$$\min_{\mathcal{V}_{\mathrm{Impr-GS}}} \Big\{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{G}\cup\mathcal{S}} C_{n,t}\Big\} \tag{Impr-GS}$$

subject to

$$\begin{aligned} \text{Generators' constraints}: &\quad (1)-(8),\\ \text{Storage constraints}: &\quad (44)-(50),\\ \text{UC-ED Power Balance}: &\quad (51),\\ \text{Generators' reserve constraints}: &\quad (12)-(17),\\ \text{Storage reserve constraints}: &\quad (54)-(55),\\ \text{Reserve requirements}: &\quad (56)-(57),\end{aligned}$$

$$\mathcal{V}_{\mathrm{Impr-GS}} = \mathcal{V}_{\mathrm{DE-G}} \cup (E_{s,t}, m_{s,t}, p^{\mathtt{dis}}_{s,t}, p^{\mathtt{ch}}_{s,t}, r^{\uparrow}_{s,t}, r^{\downarrow}_{s,t})_{s\in\mathcal{S},t\in\mathcal{T}}.$$

The imprudence of this approach was demonstrated in Example 1, and suggests that the system's reserves might be dangerously inadequate. The following subsection fixes this issue by taking the opposite end.

C. Conservative Reserve Provision

Towards dealing with the security issue of the approach described in the previous subsection, the operator can make sure that the reserve schedule of a storage unit is always implementable, i.e. the unit can always fulfill its reserve commitments in full. This guarantee can be implemented by conceptualizing an upper ($E^{\uparrow}_{s,t}$) and a lower ($E^{\downarrow}_{s,t}$) envelope for the battery's energy state as

$$E^{\uparrow}_{s,t} = E^{\uparrow}_{s,t-1} - p^{\mathtt{dis}}_{s,t}\frac{1}{\mathrm{b}^{\mathtt{dis}}} + p^{\mathtt{ch}}_{s,t}\mathrm{b}^{\mathtt{ch}} + r^{\downarrow}_{s,t}\mathrm{b}^{\mathtt{ch}}, \quad \forall s\in\mathcal{S}, t\in\mathcal{T}, \tag{58}$$

$$E^{\downarrow}_{s,t} = E^{\downarrow}_{s,t-1} - p^{\mathtt{dis}}_{s,t}\frac{1}{\mathrm{b}^{\mathtt{dis}}} + p^{\mathtt{ch}}_{s,t}\mathrm{b}^{\mathtt{ch}} - r^{\uparrow}_{s,t}\frac{1}{\mathrm{b}^{\mathtt{dis}}}, \quad \forall s\in\mathcal{S}, t\in\mathcal{T}, \tag{59}$$

and demanding that the whole area enclosed between the two envelopes stays within the battery's bounds, as in

$$\underline{\mathrm{E}}_s \leq E^{\uparrow}_{s,t} \leq \overline{\mathrm{E}}_s, \quad \forall s\in\mathcal{S}, t\in\mathcal{T}, \tag{60}$$

$$\underline{\mathrm{E}}_s \leq E^{\downarrow}_{s,t} \leq \overline{\mathrm{E}}_s, \quad \forall s\in\mathcal{S}, t\in\mathcal{T}, \tag{61}$$

Thus, the conservative UC-ED program for generation and storage, (Cons-GS) reads as

$$\min_{\mathcal{V}_{\mathrm{Cons-GS}}} \Big\{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{G}\cup\mathcal{S}} C_{n,t}\Big\} \tag{Cons-GS}$$

subject to

$$\begin{aligned} \text{Generators' constraints}: &\quad (1)-(8),\\ \text{Storage constraints}: &\quad (44)-(50),\\ \text{UC-ED Power Balance}: &\quad (51),\\ \text{Generators' reserve constraints}: &\quad (12)-(17),\\ \text{Storage reserve constraints}: &\quad (54)-(55),\\ \text{Reserve requirements}: &\quad (56)-(57),\\ \text{Storage robustness constraints}: &\quad (58)-(61),\end{aligned}$$

$$\mathcal{V}_{\mathrm{Cons-GS}} = \mathcal{V}_{\mathrm{Impr-GS}} \cup (E^{\uparrow}_{s,t}, E^{\downarrow}_{s,t})_{s\in\mathcal{S},t\in\mathcal{T}}.$$

Notice that such an approach effectively constraints the reserves of the storage unit to be implementable for the worst case scenario in which the unit is called to provide the full of its reserve commitment at *every* timeslot. In comparison to the chance constraint problem (CC-GS) this is an overly conservative approach and it results in unnecessary under-utilizations, since the probability of actually activating all of the reserve capacity in all of the timeslots and in the same direction is extremely low.

Concluding the discussion on these two simplistic approaches, it bears highlighting that none of the two is equivalent to (CC-GS), with the former (imprudent) formulation failing to maintain the security guarantees of the chance constraints (52)-(53), and the latter (conservative) formulation yielding an overly conservative, sub-optimal solution. Towards formulating a deterministic, reserve-based UC-ED for the case of generators and storage, while at the same time correcting the issues of this Section's approaches, the next Section describes a proposition which includes a generalized notion of reserve.

IV. A Generalized Notion of Reserve

A. Proposition

As exposed in the so far descriptions, a battery bears the special feature that it can potentially provide reserve capacity in a number of timeslots but it cannot provide reserve *activation* in all of these timeslots. The problem is not one of power capacity, but one of *energy* capacity. Motivated by this observation, this paper's proposition is to formulate the UC-ED problem so that the operator ensures not only the necessary power reserve capacity at each timeslot, but also the necessary *energy* reserve capacity *across* timeslots.

Let us conceptualize a variable $r^{\uparrow}_{n}[i\rightarrow t]$ (and, similarly, $r^{\downarrow}_{n}[i\rightarrow t]$) to represent the amount of upwards (downwards) *energy* reserve committed by a resource $n\in\mathcal{G}\cup\mathcal{S}$ throughout the period $[i,t]$, with $i,t\in\mathcal{T}$ and $i\leq t$. For the energy reserve commitments of a generator, it is simply

$$0 \leq r^{\uparrow}_{g}[i\rightarrow t] \leq \sum_{\tau=i}^{t}(u_{g,\tau}\cdot\overline{\mathrm{p}}_g - p_{g,\tau}), \quad \forall g\in\mathcal{G}, (i,t)\in\mathcal{T}^2 : i\leq t, \tag{62}$$

$$0 \leq r^{\downarrow}_{g}[i\rightarrow t] \leq \sum_{\tau=i}^{t}(p_{g,\tau} - u_{g,\tau}\cdot\underline{\mathrm{p}}_g), \quad \forall g\in\mathcal{G}, (i,t)\in\mathcal{T}^2 : i\leq t, \tag{63}$$

i.e., the generator's available energy reserve across the period $[i,t]$ is simply the sum of its single-timeslot reserve capacities. For a storage, however, it is

$$0 \leq r^{\uparrow}_{s}[i\rightarrow t] \leq (E_{s,t} - \underline{\mathrm{E}}_s)\mathrm{b}^{\mathtt{dis}},$$

$$\forall s \in \mathcal{S}, (i,t) \in \mathcal{T}^2 : i \leq t, \tag{64}$$

$$0 \leq r_s^{\downarrow}[i \to t] \leq (\overline{\mathrm{E}}_s - E_{s,t})\frac{1}{\mathrm{b}^{\mathrm{ch}}}, \quad \forall s \in \mathcal{S}, (i,t) \in \mathcal{T}^2 : i \leq t, \tag{65}$$

i.e., a storage unit s, within a period $[i,t]$, can only offer as much energy reserve as it has left in the battery by the *end* of the period (and an analogous argument applies for the downwards energy reserve). In the presence of this newly proposed energy reserves concept, the operator's reserve requirements (for all periods $[i,t]$) are fulfilled by setting

$$\sum_{n\in\mathcal{G}\cup\mathcal{S}} r_n^{\uparrow}[i \to t] \geq \mathrm{R}^{\uparrow}[i \to t], \quad \forall (i,t) \in \mathcal{T}^2 : i \leq t, \tag{66}$$

$$\sum_{n\in\mathcal{G}\cup\mathcal{S}} r_n^{\downarrow}[i \to t] \geq \mathrm{R}^{\downarrow}[i \to t], \quad \forall (i,t) \in \mathcal{T}^2 : i \leq t. \tag{67}$$

Notice that for $i = t$, the above constraints reduce to the standard reserve requirements (56), (57). Thus, this proposition can be thought of as a generalization of the traditional single-timeslot power reserves to procuring adequate energy reserves for all the possible time windows of any length.

The proposed Power and Energy Reserves UC-ED formulation, with generators and storage, (PER-GS) reads as

$$\min_{\mathcal{V}_{\mathrm{PER-GS}}} \quad \Big\{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{G}\cup\mathcal{S}} C_{n,t}\Big\} \tag{PER-GS}$$

subject to

$$\begin{aligned}
\text{Generators' constraints}: &\quad (1) - (8),\\
\text{Storage constraints}: &\quad (44) - (50),\\
\text{UC-ED Power Balance}: &\quad (51),\\
\text{Generators' energy reserves}: &\quad (62) - (63),\\
\text{Storage energy reserves}: &\quad (64) - (65),\\
\text{Energy reserve requirements}: &\quad (66) - (67),
\end{aligned}$$

$$\mathcal{V}_{\mathrm{PER-GS}} = \mathcal{V}_{\mathrm{CC-G}} \cup \Big\{(r_n^{\uparrow}[i\to t], r_g^{\downarrow}[i \to t])_{n\in\mathcal{G}\cup\mathcal{S}, t\in\mathcal{T}}, (E_{s,t}, m_{s,t}, p_{s,t}^{\mathrm{dis}}, p_{s,t}^{\mathrm{ch}})_{s\in\mathcal{S}, t\in\mathcal{T}}\Big\}.$$

The main result follows.

Theorem 1. *If X_t is independent for each timeslot, there are values for $\mathrm{R}^{\uparrow}[i \to t]$ and $\mathrm{R}^{\downarrow}[i \to t]$ for which problem* (PER-GS) *is equivalent to* (CC-GS).

Proof. Refer to the chance constraints (52) - (53). Using Eq. (47), to expand the term $E_{s,i}$, they become

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau) > \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}\overline{\mathrm{p}}_g + \sum_{s\in\mathcal{S}}\big(E_{s,0} - \sum_{\tau=1}^{i}(p_{s,k}^{\mathrm{dis}}\frac{1}{\mathrm{b}^{\mathrm{dis}}} - p_{s,k}^{\mathrm{ch}}\mathrm{b}^{\mathrm{ch}}) - \underline{\mathrm{E}}_s\big)\mathrm{b}^{\mathrm{dis}}\Big] < \varepsilon, \tag{68}$$

and

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau) < \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}\underline{\mathrm{p}}_g - \sum_{s\in\mathcal{S}}\Big(\overline{\mathrm{E}}_s - \big(E_{s,0} - \sum_{\tau=1}^{i}(p_{s,k}^{\mathrm{dis}}\frac{1}{\mathrm{b}^{\mathrm{dis}}} - p_{s,k}^{\mathrm{ch}}\mathrm{b}^{\mathrm{ch}})\big)\Big)\frac{1}{\mathrm{b}^{\mathrm{ch}}}\Big] < \varepsilon, \tag{69}$$

respectively. But sums of the form $\sum_{\tau=1}^{i}(\cdot)$ can be written as $\sum_{\tau=1}^{i}(\cdot) = \sum_{\tau=1}^{t}(\cdot) - \sum_{\tau=i}^{t}(\cdot)$, yielding

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau) > \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}\overline{\mathrm{p}}_g + \sum_{s\in\mathcal{S}}\Big(E_{s,0} - \sum_{\tau=1}^{t}(p_{s,k}^{\mathrm{dis}}\frac{1}{\mathrm{b}^{\mathrm{dis}}} - p_{s,k}^{\mathrm{ch}}\mathrm{b}^{\mathrm{ch}}) + \sum_{\tau=i}^{t}(p_{s,k}^{\mathrm{dis}} - p_{s,k}^{\mathrm{ch}}) - \underline{\mathrm{E}}_s\Big)\mathrm{b}^{\mathrm{dis}}\Big] < \varepsilon, \tag{70}$$

and

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau) < \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}\underline{\mathrm{p}}_g - \sum_{s\in\mathcal{S}}\Big(\overline{\mathrm{E}}_s - \big(E_{s,0} - \sum_{\tau=1}^{t}(p_{s,k}^{\mathrm{dis}}\frac{1}{\mathrm{b}^{\mathrm{dis}}} - p_{s,k}^{\mathrm{ch}}\mathrm{b}^{\mathrm{ch}}) + \sum_{\tau=i}^{t}(p_{s,k}^{\mathrm{dis}}\frac{1}{\mathrm{b}^{\mathrm{dis}}} - p_{s,k}^{\mathrm{ch}}\mathrm{b}^{\mathrm{ch}})\big)\Big)\frac{1}{\mathrm{b}^{\mathrm{ch}}}\Big] < \varepsilon. \tag{71}$$

Based on Eq. (47), it is

$$E_{s,0} - \sum_{\tau=1}^{t}(p_{s,k}^{\mathrm{dis}}\frac{1}{\mathrm{b}^{\mathrm{dis}}} - p_{s,k}^{\mathrm{ch}}\mathrm{b}^{\mathrm{ch}}) = E_{s,t}. \tag{72}$$

Substituting (72) into Eqs. (70), (71), yields

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau) > \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}\overline{\mathrm{p}}_g + \sum_{s\in\mathcal{S}}\big(E_{s,t} + \sum_{\tau=i}^{t}(p_{s,k}^{\mathrm{dis}}\frac{1}{\mathrm{b}^{\mathrm{dis}}} - p_{s,k}^{\mathrm{ch}}\mathrm{b}^{\mathrm{ch}}) - \underline{\mathrm{E}}_s\big)\mathrm{b}^{\mathrm{dis}}\Big] < \varepsilon, \tag{73}$$

and

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t}(\mathrm{D}_\tau + X_\tau) < \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}\underline{\mathrm{p}}_g - \sum_{s\in\mathcal{S}}\Big(\overline{\mathrm{E}}_s - \big(E_{s,t} + \sum_{\tau=i}^{t}(p_{s,k}^{\mathrm{dis}}\frac{1}{\mathrm{b}^{\mathrm{dis}}} - p_{s,k}^{\mathrm{ch}}\mathrm{b}^{\mathrm{ch}})\big)\Big)\frac{1}{\mathrm{b}^{\mathrm{ch}}}\Big] < \varepsilon. \tag{74}$$

Similarly to the proof in Lemma 1, by adding and subtracting $u_{g,\tau}p_\tau$ from the term $u_{g,\tau}\underline{\mathrm{p}}_g$ and using the power balance equation (51), we obtain

$$\mathtt{Pr}\Big[\sum_{\tau=i}^{t} X_\tau > \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t} u_{g,\tau}(\overline{\mathrm{p}}_g - p_{g,\tau}) + \sum_{s\in\mathcal{S}}(E_{s,t} - \underline{\mathrm{E}}_s)\mathrm{b}^{\mathrm{dis}}\Big] < \varepsilon, \tag{75}$$

and

$$\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_\tau < \sum_{g\in\mathcal{G}}\sum_{\tau=i}^{t}\Big(-u_{g,t}\cdot(p_{g,t}-\underline{\mathrm{p}}_g)- \sum_{s\in\mathcal{S}}\Big(\overline{\mathrm{E}}_s - E_{s,t}\Big)\frac{1}{\mathrm{b^{ch}}}\Big] < \varepsilon. \tag{76}$$

By introducing auxiliary variables $r_n^{\uparrow}[i \to t], r_n^{\downarrow}[i \to t]$, which can be interpreted as upwards and downwards energy reserves, (75) and (76) can be reformulated as

$$\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_t > \sum_{g\in\mathcal{G}} r_g^{\uparrow}[i\to t] + \sum_{s\in\mathcal{S}} r_s^{\uparrow}[i\to t]\Big] \le \varepsilon, \tag{77}$$

$$0 \le r_g^{\uparrow}[i\to t] \le \sum_{\tau=i}^{t}(u_{g,\tau}\cdot\overline{\mathrm{p}}_g - p_{g,\tau}), \tag{78}$$

$$0 \le r_s^{\uparrow}[i\to t] \le (E_{s,t}-\underline{\mathrm{E}}_s)\mathrm{b^{dis}}, \tag{79}$$

and

$$\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_t < \sum_{g\in\mathcal{G}} -r_g^{\downarrow}[i\to t] + \sum_{s\in\mathcal{S}} -r_s^{\downarrow}[i\to t]\Big] \le \varepsilon, \tag{80}$$

$$0 \le r_g^{\downarrow}[i\to t] \le \sum_{\tau=i}^{t}(p_{g,\tau} - u_{g,\tau}\cdot\underline{\mathrm{p}}_g), \tag{81}$$

$$0 \le r_s^{\downarrow}[i\to t] \le (\overline{\mathrm{E}}_s - E_{s,t})\frac{1}{\mathrm{b^{ch}}}, \tag{82}$$

respectively. By setting

$$\sum_{n\in\mathcal{G}\cup\mathcal{S}} r_n^{\uparrow}[i\to t] \ge \mathrm{R}^{\uparrow}[i\to t], \tag{83}$$

$$\sum_{n\in\mathcal{G}\cup\mathcal{S}} r_n^{\downarrow}[i\to t] \ge \mathrm{R}^{\downarrow}[i\to t], \tag{84}$$

Eqs. (77), (80) become

$$\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_t \ge \mathrm{R}^{\uparrow}[i\to t]\Big] \le \varepsilon, \tag{85}$$

$$\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_t \le -\mathrm{R}^{\downarrow}[i\to t]\Big] \le \varepsilon. \tag{86}$$

Similarly to Lemma 1, the probabilities $\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_t \ge \mathrm{R}^{\uparrow}[i\to t]\Big]$ and $\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_t \le -\mathrm{R}^{\downarrow}[i\to t]\Big]$ can be written as

$$\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_t \ge \mathrm{R}^{\uparrow}[i\to t]\Big] = 1 - F_{Z_{it}}(\mathrm{R}^{\uparrow}[i\to t])$$

$$\texttt{Pr}\Big[\sum_{\tau=i}^{t} X_t \le -\mathrm{R}^{\downarrow}[i\to t]\Big] = F_{Z_{it}}(-\mathrm{R}^{\uparrow}[i\to t]),$$

where $F_{Z_{it}}$ is the cumulative distribution function of the random variable $Z_{it} = \sum_{\tau=i}^{t} X_t$. Using the same procedure as in Lemma 1, the original chance constraints (52), (53) can be reformulated as

$$\sum_{n\in\mathcal{G}\cup\mathcal{S}} r_n^{\uparrow}[i\to t] \ge \widehat{\mathrm{R}^{\uparrow}}[i\to t], \tag{87}$$

$$0 \le r_g^{\uparrow}[i\to t] \le \sum_{\tau=i}^{t}(u_{g,\tau}\cdot\overline{\mathrm{p}}_g - p_{g,\tau}), \tag{88}$$

$$0 \le r_s^{\uparrow}[i\to t] \le (E_{s,t}-\underline{\mathrm{E}}_s)\mathrm{b^{dis}}, \tag{89}$$

$$\sum_{n\in\mathcal{G}\cup\mathcal{S}} r_n^{\downarrow}[i\to t] \ge \widehat{\mathrm{R}^{\downarrow}}[i\to t], \tag{90}$$

$$0 \le r_g^{\downarrow}[i\to t] \le \sum_{\tau=i}^{t}(p_{g,\tau} - u_{g,\tau}\cdot\underline{\mathrm{p}}_g), \tag{91}$$

$$0 \le r_s^{\downarrow}[i\to t] \le (\overline{\mathrm{E}}_s - E_{s,t})\frac{1}{\mathrm{b^{ch}}}, \tag{92}$$

which completes the proof.

□

The next subsection briefly discusses some important aspects and extensions of the proposed framework before proceeding to the empirical evaluation of Section V.

B. Remarks, Extensions, and Policy Implications

Demand Response participation was not explicitly presented since it would follow the same logic as storage participation, i.e., a "state of energy" variable for each Demand Response aggregator would be conceptualized, as described in [10], [11] and the rest of the model follows as is. Nevertheless, capturing the modeling details (and uncertainties) of flexible loads into the proposed model is an interesting direction for future work. Provisionally, one would use aggregation methods similar to the ones proposed in [12], [13] or [14], but replacing the traditional reserve market model with the proposed one.

Ramp Constraints also create an issue with reserve procurement: if not accounted for, a generator's reserve activation might be infeasible due to insufficient ramping capability. One way around this, is to include constraints of the form

$$p_{g,t} + r_{g,t}^{\uparrow} - p_{g,t+1} - r_{g,t+1}^{\downarrow} \le \text{ramp},$$
$$p_{g,t+1} + r_{g,t+1}^{\uparrow} - p_{g,t} - r_{g,t}^{\downarrow} \le \text{ramp}.$$

A less conservative approach is to introduce a separate, flexible ramping product, as described in [15] and actually adopted by CAISO [16].

Different types of uncertainty, other than net demand deviation, can also be accounted for by using the proposed framework. The framework enables storage systems to provide various types of reserves (including e.g. spinning, non-spinning, contingency, etc) by generalizing the notion of reserve procurement in one timeslot, to energy reserves across timeslots and making sure that a storage system can deliver its reserve. Nonetheless, dimensioning the (across-timeslots) different reserve requirements of a system requires an involved statistical analysis (not conducted in this paper) to construct the joint probability distribution of different non-independent random variables, i.e. accounting also for correlated uncertainties such as e.g. contingencies that propagate imbalances across time.

Financial Remuneration of reserve commitment (in the face of the newly proposed reserves' framework) is an important policy issue for which this paper takes no standpoint. However, one thing that *can* be safely commented on is that, compared to the other security-informed approach (i.e. the Conservative scheme), the Proposed scheme releases the storage units from the over-conservative and inefficient envelope constraints (Eqs. 58 - 61). Thus, the storage is enabled to provide more reserves in general, existing reserve products inclusive. Thus, even if the energy reserve is not remunerated at all, it is plausible to expect that storage resources will receive extra payments in comparison to being overly restricted on (or even excluded from) providing reserves. Additionally, though, the proposed formulation motivates a new notion of reserve (i.e. the generalized *energy* reserves), which can also be interpreted as a new reserve product. It is worth noting that, if the energy reserves are remunerated and the reserve remuneration policy is not re-designed, the proposed framework would result in over-compensations for generators since they would be paid multiple times for the same reserve capacity. Therefore, future work should address this issue before considering further uptake of this paper's proposition.

Flow Constraints for the dispatch can be readily incorporated in the proposed UC-ED formulation without any complication. However, proactively ensuring that all possible reserve activations of the committed reserves should also satisfy the power flow constraints is more complicated; one demand at each node and a separate reserve requirement for each node would need to be considered, but a node can also utilize the reserves of its neighbors (up to the line capacity limit). Then, similarly to the proposed concept of energy reserves (i.e. reserves across time), the operator would need to make sure that enough "zonal" reserves exist for each combination of nodes. Nevertheless, this might be excessively complicated and unnecessary for practical purposes.

V. Empirical Evaluation

This Section presents an empirical evaluation of all three deterministic reserve-based models presented, i.e., the Imprudent, the Conservative, and the Proposed model. We are interested in the system cost of the UC-ED problem and in the total loss of load and loss of RES probabilities across the whole horizon $\mathcal{T}$. The next subsection presents the evaluation setting and subsection V-B presents the simulation results.

A. Evaluation Framework

This Section considers a small setup of 20 generators and 8 storage units within a 12-timeslots commitment period to facilitate intuition and understanding of the results. A larger case study will be conducted in the next Section. For this smaller setup, generators' minimum up/down and ramping constraints were not considered in order to rule out the possibility of infeasibilities in reserve activation due to these constraints instead of the problem at hand, since this would disrupt the clear evaluation of the schemes described. Furthermore, for simplicity, only one segment was considered per generator, and each generator's cost at t was modeled as:

$$C_{g,t} = u_{g,t}\mathrm{w}_g^{\mathtt{fx}} + y_{g,t}\mathrm{w}_g^{\mathtt{su}} + z_{g,t}\mathrm{w}_g^{\mathtt{sd}} + \mathrm{w}_g^{\mathtt{vr}} \cdot (p_{g,t})^2, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}, \quad (93)$$

which comprises four components: a fixed operational cost $u_{g,t}\mathrm{w}_g^{\mathtt{fx}}$ for being ON at t, a startup cost $y_{g,t}\mathrm{w}_g^{\mathtt{su}}$, a shutdown cost $z_{g,t}\mathrm{w}_g^{\mathtt{sd}}$ and, finally, a variable operational cost $\mathrm{w}_g^{\mathtt{vr}} \cdot (p_g)^2$ for producing $p_{g,t}$ at t. The operational cost factors $\mathrm{w}_g^{\mathtt{vr}}$ are set in increasing order from 0.1 to 2 and the startup costs $\mathrm{w}_g^{\mathtt{su}}$ are set in decreasing order from 580 to 250. This simulates a setting where larger generators have high startup cost but low marginal cost, while smaller (and faster) generators have lower startup cost but high marginal cost. Accordingly, the shutdown costs $\mathrm{w}_g^{\mathtt{sd}}$ are set to evenly spaced values between 20 and 0 and the fixed costs $\mathrm{w}_g^{\mathtt{fx}}$ to random normal values around 50, with a standard deviation of 5.

The generators' capacities $\overline{\mathrm{p}}_g$ are evenly spaced between 12 and 5 MW, whereas their lower bounds $\overline{\mathrm{p}}_g$ are evenly spaced between 5 and 1 MW.

The operational cost of storage $\mathrm{w}_s^{\mathtt{st}}$ was normally distributed around 0.01 with a standard deviation of 0.002. A system with high RES penetration was considered, with a net demand D_t exhibiting a valley, even taking negative values in the middle of the horizon (where solar energy may exceed the demand). The tolerance parameter ε was set to 2%.

The system's real-time imbalances X_t were simulated to follow a normal distribution $\mathcal{N}(0,5)$. In the results of the next subsection, the system's loss of load probability and the system's RES curtailment probability are evaluated empirically for each of the three schemes. To evaluate these probabilities empirically, each scheme's unit commitment decisions are kept fixed to the values obtained by the respective program, and the system's balancing dispatch is simulated for 1000 experiments using a different deviation profile $(X_t)_{t\in\mathcal{T}}$ in each experiment sampled from $\mathcal{N}(0,5)$.

The rest of this subsection specifies the details of the balancing energy economic dispatch problem (BE-ED). The BE-ED decides the balancing action of each participant at each timeslot, denoted as $\widetilde{p}_{n,t}$ (positive for upwards and negative for downwards). The balancing dispatch is conceptualized as the difference from the participant's *scheduled* dispatch which was decided previously by the UC-ED program. In what follows, a tilde accent $\widetilde{(\cdot)}$ is used for the BE-ED decision variables and a hat accent $\widehat{(\cdot)}$ for the values of the UC-ED problem's variables (which remain fixed parameters for the BE-ED problem).

Let a generator's (fixed) commitment and scheduled dispatch decisions be denoted as $\widehat{u}_{g,t}$ and $\widehat{p}_{g,t}$. Given the scheduled decisions $\widehat{u}_{g,t}, \widehat{p}_{g,t}$, the generator's balancing energy $\widetilde{p}_{g,t}$ is upper bounded by the spare capacity $\overline{p}_g - \widehat{p}_{g,t}$ left in the generator (provided the generator is committed), as in

$$\widetilde{p}_{g,t} \leq \widehat{u}_{g,t} \cdot (\overline{p}_g - \widehat{p}_{g,t}), \quad \forall g \in \mathcal{G}, t \in \mathcal{T}. \quad (94)$$

Similarly, a generator's (downwards) balancing energy is bounded by the scheduled dispatch $\widehat{p}_{g,t}$, i.e. it can only down-regulate as much energy as it is scheduled to produce (minus its technical minimum):

$$\widetilde{p}_{g,t} \geq -\widehat{u}_{g,t} \cdot (\widehat{p}_{g,t} - \underline{p}_g), \quad \forall g \in \mathcal{G}, t \in \mathcal{T}. \quad (95)$$

For a storage unit, let $\widetilde{E}_{s,t}$ denote the unit's final state-of-energy, after the scheduled dispatch *and* balancing actions.

Assuming, for simplicity, $\mathrm{b}^{\mathrm{ch}} = \mathrm{b}^{\mathrm{dis}} = 1$, its value is determined as

$$\widetilde{E}_{s,t} = \widehat{E}_{s,t} - \sum_{\tau \leq t} \widetilde{p}_{s,t}, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \tag{96}$$

and it should remain within the feasible bounds, i.e.:

$$\underline{\mathrm{E}}_s \leq \widetilde{E}_{s,t} \leq \overline{\mathrm{E}}_s, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}. \tag{97}$$

By introducing two variables

$$\widetilde{L}_t^{\mathtt{load}}, \widetilde{L}_t^{\mathrm{RES}} \geq 0, \quad \forall t \in \mathcal{T}, \tag{98}$$

to denote the system's lost load and lost RES respectively, the system's balancing power balance equation reads as

$$X_t - \sum_{n \in \mathcal{G} \cup \mathcal{S}} \widetilde{p}_{n,t} = \widetilde{L}_t^{\mathtt{load}} - \widetilde{L}_t^{\mathrm{RES}}, \quad \forall t \in \mathcal{T}. \tag{99}$$

The objective of the BE-ED problem[1] is to minimize the lost load (weighted by the value of lost load voll) and lost RES (weighted by the value of lost RES volr), as in

$$\min_{\mathcal{V}_{\mathrm{BE-ED}}} \left\{ \sum_{t \in \mathcal{T}} \mathrm{voll} \cdot \widetilde{L}_t^{\mathtt{load}} + \mathrm{volr} \cdot \widetilde{L}_t^{\mathrm{RES}} \right\} \tag{BE-ED}$$

$$\text{subject to}$$

$$\begin{aligned} \text{Generators' constraints :} \quad & (94) - (95), \\ \text{Storage constraints :} \quad & (96) - (97), \\ \text{BE-ED Power Balance :} \quad & (98) - (99), \end{aligned}$$

$$\mathcal{V}_{\mathrm{BE-ED}} = \left\{ (\widetilde{p}_{n,t})_{n \in \mathcal{G} \cup \mathcal{S}, t \in \mathcal{T}}, (\widetilde{E}_{s,t})_{s \in \mathcal{S}, t \in \mathcal{T}}, (\widetilde{L}_t^{\mathtt{load}}, \widetilde{L}_t^{\mathrm{RES}})_{t \in \mathcal{T}} \right\}.$$

By executing the BE-ED for 1000 different deviation profiles, the loss of load and loss of RES probabilities of each scheme can be assessed by simply counting the times when variables $\widetilde{L}_t^{\mathtt{load}}, \widetilde{L}_t^{\mathrm{RES}}$ were non-zero for the respective BE-ED problem (that uses the focal scheme's commitment and scheduling decisions as input).

B. Results

First, a comparison of the three presented models is made in terms of their system cost for various levels of storage penetration. Keeping the generators' capacities to the values described, the system's total storage capacity $\sum_{s \in \mathcal{S}} \overline{\mathrm{E}}_s$ is set equal to $\beta \cdot \sum_{g \in \mathcal{G}} \overline{\mathrm{p}}_g$, where the factor β instantiates the percentage of storage penetration in the system. For various levels of storage penetration, the system's cost for each model is presented in Fig. 1. One can observe that, for all levels of storage penetration, the cost of the conservative scheme is significantly higher than the other two schemes, while the cost of the proposed scheme is closer to the cost of the Imprudent scheme than to the cost of the Conservative scheme.

The next question is whether these differences in costs come with analogous differences in the system's loss of load probability. Fig. 2 shows that they don't; the proposed scheme achieves a loss of load probability closer to the one of the Conservative scheme (which is virtually zero) rather than the much higher one of the Imprudent scheme. This result validates the ability of the proposed scheme to achieve a loss of load probability that is low enough, without the unnecessary extra costs of the Conservative scheme (and in fact with costs that are quite close to the Imprudent scheme). Fig. 3 shows the respective loss of RES probability.

[1]Note that, since we are only interested in the probability of lost load or RES, it is not needed to consider the re-dispatch cost since it is inconsequential (assuming it is always lower than the value of lost load or RES). This allows us to use a single variable for the storage dispatch and avoid the binary m.

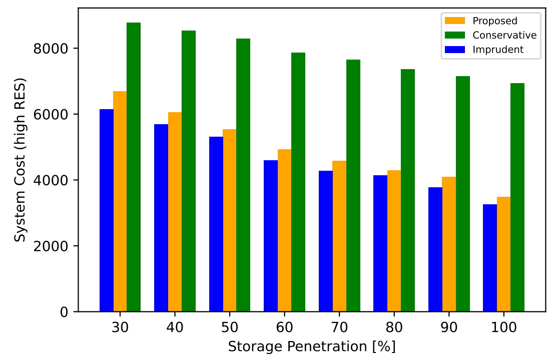


Fig. 1: System cost of the three schemes for various levels of storage penetration in the high RES case.

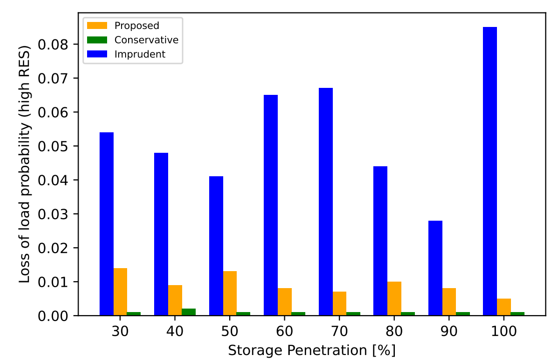


Fig. 2: System's empirical loss of load probability of the three schemes for various levels of storage penetration.

The next figure (Fig. 4) compares the three models with respect to whether they favor generators or storage units towards reserve commitments. The figure shows the up and down reserves committed by storage units (lighter shade) on top of the up and down reserves committed by generators (darker shade) for each timeslot. For the Proposed scheme, only the single-timeslot reserves ($i = t$) for each timeslot are depicted. We observe that the Conservative scheme heavily favors generators for reserves. In contrast, the Imprudent scheme favors storage units, especially for upwards reserves. The Proposed scheme stands somewhere between.

Finally, the behavior of the Proposed scheme is demonstrated with respect to longer-period energy reserves. In the horizontal axis of Fig. 5, lies the period length (namely $t - i + 1$ for timeslots t, i). The average energy reserve for each period is calculated by averaging the energy reserves over all pairs

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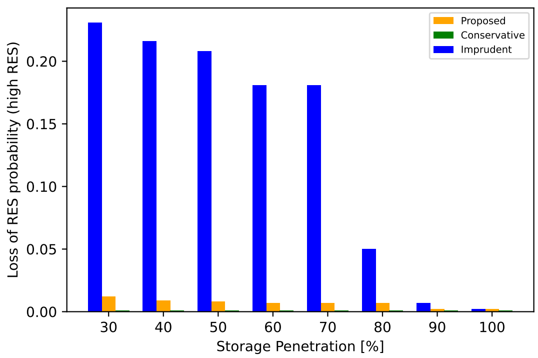

Fig. 3: System's empirical loss of RES probability of the three schemes for various levels of storage penetration.

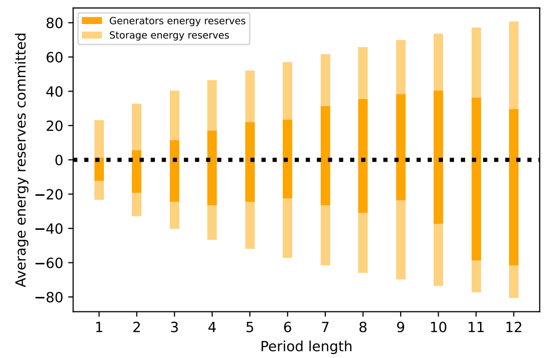

Fig. 5: Energy reserves' allocation among storage and generators as a function of the period's length.

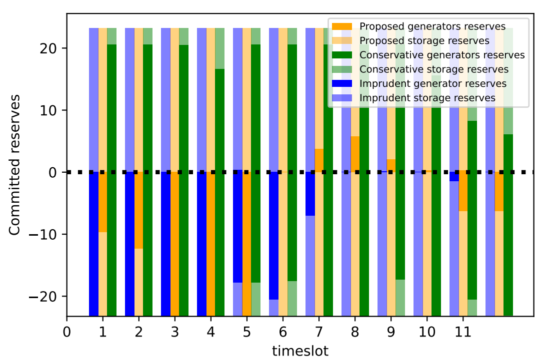

Fig. 4: Reserve allocation among storage and generators for the three schemes.

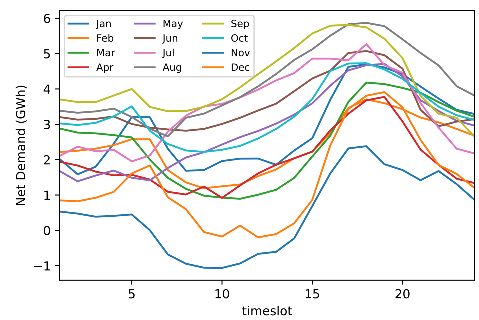

Fig. 6: Hourly net demand for one day of each month.

of timeslots with the respective period length (for example for length 10, there are three pairs of timeslots: (10,1), (11,2), and (12,3)). Notably, the figure suggests that the energy reserves are allocated to generators and storage units quite evenly.

VI. Case Study

This Section presents the results of applying the three schemes to a larger, standardized case study. The Reliability Test System of the Grid Modernization Laboratory Consortium (RTS-GMLC) [17] is used for that purpose, which is a system with all the main types of traditional generators and high penetration of RES and storage. For this case study, minimum-up / minimum-down times and ramp constraints are also included, while the generators' cost functions take a piece-wise linear form:

$$C_{g,t} = u_{g,t}\mathrm{w}_g^{\mathrm{fx}} + y_{g,t}\mathrm{w}_g^{\mathrm{su}} + \sum_{l\in\mathcal{L}} p_{g,l,t}\mathrm{h}_{g,l}\mathrm{f}_g, \quad \forall g\in\mathcal{G}, t\in\mathcal{T}, \tag{100}$$

where $\mathrm{h}_{g,l}$[Btu / MWh] is the incremental heat rate of a generator's segment (increasing for higher segments) and f_g[$ / Btu] is the generator's fuel price. All the system's parameters were set to the data available in [18].

The system was simulated for one day (the 15th) of each month. The system's net demand in each timeslot was derived by subtracting the total RES generation from the total demand (across all three regions of the system). The resulting net demand profiles (for one day of each month) are depicted in Fig. 6. For each month, hourly real-time deviations datapoints were constructed by averaging over the 5-minute deviations within each hour. Then, a number of different parametric distribution types were fit to each month's hourly deviations data, and compared in terms of the Kolmogorov-Smirnov distance, i.e. a metric indicating how well the parametric distribution describes the raw data. The Beta distribution type was found to perform best in almost all months. Fig. 7 shows each month's deviation data along with the corresponding fitted Beta distribution. The reserve requirements for each month and each scheme were determined based on the month's fitted Beta distribution.

The resulting cost of each UCED scheme for the 15th day of each month is depicted in Fig. 8. The figure verifies the tendency of the proposed scheme to achieve a system cost close to the one of the Imprudent scheme rather than the one of the Conservative scheme. Moreover, the cost savings in comparison to the Conservative scheme are massive: the system's high penetration of storage allows it to cover the net demand with only a few generators committed; however, the Conservative scheme needs to commit additional generators to cover the necessary reserves resulting in costs that, for most months, are ten times as much as the ones of the Proposed scheme. For August and September, the difference is less

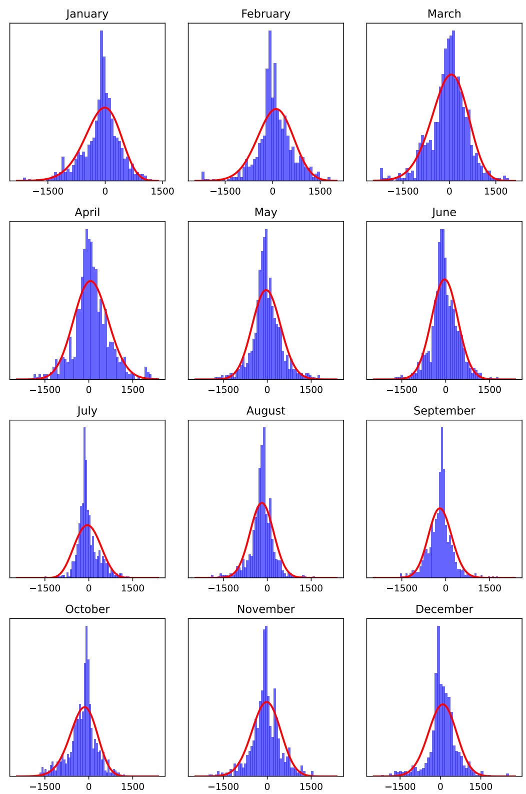

Fig. 7: Real-time hourly net demand deviations.

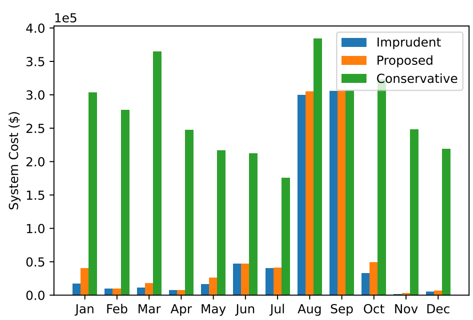

Fig. 8: System costs for a day of each month with the three schemes using the GMLC case study.

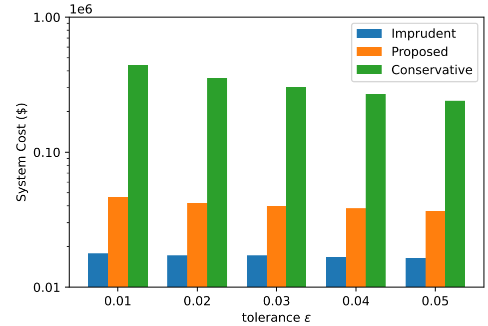

Fig. 9: System costs for a day of each month with the three schemes as a function of the tolerance ε.

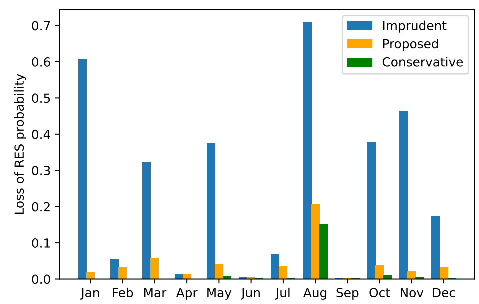

Fig. 10: Loss of RES probability for each month.

dramatic because the higher net demand in these months (cf Fig. 6) causes more generators to be committed also for the Proposed scheme, just to cover the demand. For November, on the other hand, where the RES generation is highest (even surpassing the Demand at times, cf Fig. 6), the Proposed scheme is able to fully utilize the storage resulting in a remarkably low cost. Fig. 9 shows how these costs change as a function of the tolerance parameter ε. Note that the y-axis is in logarithmic scale.

Again, the relevant question is whether these cost savings come with acceptable constraint violation probabilities. For this case study, the loss of load probability is zero for all three schemes; the reason is the fact that the energy coming from Storage units is costless, which causes the optimization algorithms to keep the necessarily committed generators as close as possible to their technical minimum. This, in turn, means that the committed generators have plenty of upward reserve available (more than actually required by the reserve constraints), resulting in zero lost load in real-time. In contrast, the downward reserve constraints *are* active, which points to the loss of RES generation probability in real-time as the relevant metric to evaluate the schemes. Indeed, as Fig. 10 shows, the probability of RES curtailments in real-time is much higher than tolerated in the case of the Imprudent scheme. The figure again verifies that the probability of constraint violation for the Proposed scheme remains within acceptable bounds and closer to the Conservative scheme.

Based on the results above, the Proposed scheme satisfies the constraint violation tolerance levels but at a cost that is (even orders of magnitude) lower than the one of the Conservative scheme. The arguably small price to pay is the introduction of $2|\mathcal{T}|^2(|\mathcal{G}| + |\mathcal{S}|)$ reserve-related variables (in contrast to the $2|\mathcal{T}|(|\mathcal{G}| + |\mathcal{S}|)$ of the Imprudent and $2|\mathcal{T}|(|\mathcal{G}| + 3|\mathcal{S}|)$ of the Conservative scheme) and a number of $2|\mathcal{T}|^2(|\mathcal{G}| + |\mathcal{S}| + 1)$ reserve-related constraints. For the experiments performed in a 3.00 GHz / 32 GB RAM laptop, using the Gurobi solver from the Pyomo environment, this resulted in the average computational times presented in Table V.

TABLE V: Schemes' computational times

	Imprudent	Proposed	Conservative
Computational time (s)	4.65	27.48	5.08

VII. Conclusions

This paper considered the unit commitment and economic dispatch problem of a system with high penetration of storage and presented a chance-constrained program and a more practical, reserve-based optimization. After exposing the issues of non-cautionary approaches, a proposition of generalizing the traditional notion of power capacity reserves to the notion of energy reserves for periods of time was presented and theoretically analyzed. The empirical evaluation validated that the proposed scheme is able to achieve a loss of load probability within acceptable bounds (and close to the one of the Conservative scheme), but at a small fraction of the cost. This encouraging result suggests that the proposition should be further examined in focused case studies of real systems to evaluate whether some actual system can benefit from substantial cost savings.

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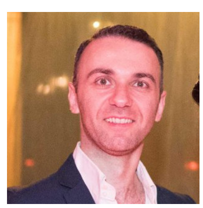

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