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Market Design for Future District Heating Systems

Linde Frölke

PhD Thesis, November 2022, Kongens Lyngby, Denmark

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Division for Power and Energy Systems (PES)
DTU Wind and Energy Systems

Market Design for
Future District Heating Systems

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DTU - Technical University of Denmark, Lyngby - November 2022

Market Design for Future District Heating Systems

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I'm getting older,
got more on my shoulders.
But I'm getting better at admitting when
I'm wrong.

— Billie Eilish

Preface

This thesis is prepared at the Department of Wind and Energy Systems of the Technical University of Denmark in partial fulfillment of the requirements for acquiring the degree of Doctor of Philosophy in Engineering. The Ph.D. project was funded by the EU H2020 project EMB3Rs (grant agreement No 847121).

This dissertation summarizes the work carried out by the author during her Ph.D. project. It started on 01st September 2019, and it was completed on 30th November 2022. During this period, she was hired by the Technical University of Denmark as a Ph.D. student at the Division for Power and Energy Systems.

The thesis is composed of 6 chapters and 3 attached scientific papers, 2 of which have been peer-reviewed and published, whereas the remaining one is currently under review.

Linde Frölke
November 2022

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As I already briefly mentioned in the previous paragraph[1], I could not have wished for a better group than E(L)MA. I was lucky to collaborate closely with Eléa, who is not only sweet and smart and strong but also bakes amazing cakes. I thank Anubhav and Amandine for interesting initial discussions on what could have been fruitful collaborations – time is not our friend. Thank you Eléa, Enrica, Peter and Yannick for reviewing parts of this thesis and providing valuable comments that helped me improve it. I am grateful to Misha for sharing his recent thesis-writing experiences and template with me. To what I consider to be the 'old group', Anubhav, Eléa, Enrica, Amandine, Jochen, Liyang, Li, Yannick: thank you for making ELMA so nice! We even had an almost-great time in lockdown with online board games and a big plan to optimize the world (Jochen and Anubhav – we need to execute this plan). I am happy to know that we will continue seeing each other. Thank you, Ingríd, for the daily morning meetings that got me through the second lockdown. A big cheer to Peter, Yannick, and Ben for their speediness and good moods that made for great DTU Runs – keep it going! To what I consider to be the 'new group', Peter, Ben, Thomas, Martina, Lesia, Andrea, Alice: I am so happy that you re-introduced office culture and hygge at DTU, thank you for that.

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[1] Because Pierre loves cross-references and footnotes.

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Linde

Kgs. Lyngby, Denmark, November 2022

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Abstract

District heating is envisioned to play an important role in future carbon-neutral energy systems, by facilitating the integration of renewable and excess heat, and providing flexibility to the electrical power system. With the integration of renewable and excess heat, there will be increased fluctuations in the available amount and the generation cost of heat. To provide heat reliably at a reasonable cost, flexibility on the demand side is needed. Attractive sources of demand-side flexibility exist both at the end-consumer and in thermal storage systems. While the increased number of small heat providers and the increasing demand-side flexibility lead to a more complex operation of the system, these developments may at the same time pave the way for liberalized heat markets. This thesis concerns the design of market mechanisms to support the more complex operation of future district heating systems, with three future system characteristics in mind: the presence of many small generators, the fact that generators and flexible loads are distributed over the system, and the presence of thermal storage systems.

First, we investigate an alternative to direct market participation of excess heat producers, where these actors are sent a price signal, based on which they self-schedule their production. In a realistic case study we quantify how suboptimal this self-scheduling is in comparison to market participation, in terms of total generation cost. We find that the self-scheduling method may be suitable under low excess heat penetration, while we advise that more sophisticated pricing signals and/or other market setups are used when excess heat covers a significant share of the heat load. Second, we propose a market mechanism that takes heating network constraints into account. We provide insight into the price formation in this network-aware market design, by including peer-to-peer trades that reveal the network losses caused by each market participant. In an illustrative case study, we show that the proposed market design can result in schedules and prices that are beneficial for network operation and reduce its cost, by effectively promoting more local heat consumption. Our third contribution sheds light on a more fundamental issue in market design for non-merchant storage. To cope with time-linking constraints that are present in such markets, it is common to make simplifying assumptions on the end-of-horizon storage level. Using illustrative examples and formal proofs, we analyze which market properties hold under such assumptions, as well as in their absence. In particular, we find that market inefficiencies may even arise given perfect decisions on the end-of-horizon storage level, because prices in subsequent market horizons may fail to reflect the value of stored energy. Finally, we propose a method for restoring market efficiency in a perfect foresight setting.

Resumé

Fjernvarme forventes at spille en vigtig rolle i fremtidige kulstofneutrale energisystemer ved at facilitere integrationen af vedvarende energikilder og overskudsvarme, samt at levere fleksibilitet til elsystemet. Med integrationen af vedvarende varmekilder og overskudsvarme vil der være øgede udsving i den tilgængelige mængde af varme og også større udsving i produktionsomkostninger. For at levere varme pålideligt til en rimelig pris er fleksibilitet på efterspørgselssiden nødvendig. Attraktive kilder af fleksibilitet på efterspørgselssiden findes både hos slutforbrugeren og i termiske lagringssystemer. Mens det øgede antal små varmeudbydere og den stigende fleksibilitet på efterspørgselssiden fører til en mere kompleks drift af systemet, kan denne udvikling samtidig bane vejen for liberaliserede varmemarkeder. Denne afhandling omhandler design af markedsmekanismer til at understøtte den mere komplekse drift af fremtidige fjernvarmesystemer i betragtning af tre fremtidige systemkarakteristika: tilstedeværelsen af mange små generatorer, det faktum, at disse generatorer og fleksible belastninger er fordelt over hele systemet, og tilstedeværelsen af termiske lagringssystemer.

Først undersøger vi et alternativ til direkte markedsdeltagelse af overskudsvarmeproducenter, hvor disse aktører får tilsendt et prissignal, ud fra hvilket de selv skemalægger deres produktion. I et realistisk casestudy kvantificerer vi, hvor suboptimal generatorernes egen skemalægning er i forhold til markedsdeltagelse, hvad angår de samlede produktionsomkostninger. Vi finder, at generatorernes egen skemalægning kan bruges under lav gennemtrængning af overskudsvarme, mens vi anbefaler, at mere sofistikerede prissignaler og/eller andre markedsmekanismer anvendes, når overskudsvarme dækker en væsentlig del af varmebehovet. For det andet foreslår vi en markedsmekanisme, der tager hensyn til begrænsninger i fjernvarmenettet. Vi giver indsigt i prisdannelsen i dette netværksbevidste markedsdesign ved at inkludere peer-to-peer handel, som giver netværkstab forårsaget af hver markedsdeltager. I et illustrativt casestudy viser vi, at det foreslåede markedsdesign kan resultere i tidsplaner og priser, der er gavnlige for netværksdriften og reducerer omkostningerne ved at fremme mere lokalt varmeforbrug. Vores tredje bidrag kaster lys over et mere grundlæggende problem inden for markedsdesign til non-merchant energilagringssystemer. For at klare tidsbindingsbegrænsninger, der er til stede i sådanne markedsdesigns, er det almindeligt at lave forenklede antagelser om lagerniveauet ved slutningen af markedshorisonten. Ved hjælp af illustrative eksempler og formelle beviser analyserer vi, hvilke markedsegenskaber holder under sådanne forudsætninger, såvel som i deres fravær. Vi finder, at markedsineffektivitet endda kan opstå givet perfekte beslutninger om lagringsniveauet ved slutningen af horisonten, fordi priserne i efterfølgende markedshorisonter muligvis ikke afspejler værdien af lagret energi. Til sidst foreslår vi en metode, som garanterer, at markedet er effektivt, når man antager, at fremtidige parametre kan forudsiges perfekt.

Chapter 1

Introduction

1.1 Context and motivation

Many countries around the world have committed to become carbon neutral by 2050 [1]. The decarbonisation of the energy system is often defined as a first step towards this goal. Studies show that *district heating* can play an important role in future carbon-neutral energy systems [2, 3]. District heating systems consist of heat generators and consumers, as well as a network of pipelines to distribute the heat. Given a sufficiently high spatial heat load density, these systems can be beneficial from both economic and energy-efficiency perspectives, compared to heat generation at the individual end-user. Therefore, district heating is traditionally more popular in densely populated areas and cooler climates, such as Northern Europe, Russia, and Northern China. There is evidence that district heating could also supply a large share of residential and commercial heat loads in many North-Western European countries, such as Germany and the United Kingdom, and even has potential in Southern European cities [4, 5]. Interest in *district cooling* as part of future sustainable energy systems is also increasing, among others in the European Union [6]. A report from the European Parliament states that 50% of the heat load in the European Union could be supplied by district heating in the future [7]. Besides potential cost and energy savings, advantages of district heating and cooling compared to local solutions include easier pollution control [8], reduced complexity and space requirements at the end user [9], improved domestic air quality [10], and greater flexibility in choosing heat sources [9].

For energy systems to become carbon neutral, the dependence on conventional, fossil-fuel based generators must come to an end. District heating contributes towards this goal among others by facilitating the integration of certain carbon-neutral heat sources that are not suitable for use in individual heating applications [11]. Such sources include, but are not limited to, excess heat, solar collector fields, and large-scale heat pumps [12, 13]. The European Commission defines excess heat as 'a by-product of another process that would be emitted into the environment, until supplied for off-site use', explicitly excluding Combined Heat & Power (CHP) plants from this definition as these are built with the intention of generating heat [14]. There are different types of excess heat sources, including excess heat from waste incineration, industrial processes, and urban sources [15]. Besides enabling the decarbonisation of the heating sector itself, district heating systems are envisioned to play a broader role in future energy systems. Studies advocate that heating systems should provide flexibility to electrical power systems, as heating systems contain several attractive sources of flexibility [16, 17].

Conventional district heating systems are of a centralized nature, with few large generators producing heat through combustion of fossil fuels and municipal waste. In the transition to carbon neutrality, heat systems must adapt to accommodate the integration of carbon neutral heat sources, which differ from conventional ones in several ways. First of all, they typically have a smaller heat generation capacity. This also implies that a greater number of these sources are needed to meet a similar heat load. Second, future heat sources may be distributed over both transmission and distribution heat networks, while conventional generators are usually located at a central location connected to the transmission network. Finally, the generation profiles of solar and excess heat sources differ from conventional sources. For one, the available heat from solar and excess heat producers is *fluctuating* depending on external factors, such as weather conditions, industrial activity, or the cooling load in a supermarket, for instance. Indirectly, these factors may again be weather dependent. As a result, both daily and seasonal variations in the maximum available heat can be expected. In addition, these sources can usually be operated with *limited flexibility*. For example, excess heat sources may be able to deviate only slightly from a baseline production profile.

Due to the rise of CHP plants and heat-pump based excess heat producers, the physical connection between district heating and power systems is becoming stronger. As a result, it becomes increasingly important to take *economic interactions* between these systems into account, even though their markets or pricing systems are usually not explicitly coupled. The electricity price strongly affects the district heating schedule [18], because it largely determines the heat generation cost of CHP plants and heat pumps. Therefore, large electricity price fluctuations due to the increased penetration of renewables in power systems result in similar fluctuations in the generation cost of heat [19]. It should also be noted that many of the foreseen changes in district heating are similar to those already occurring in power systems, such as the rise of distributed energy resources and fluctuating renewable energy sources. Extensive research has been performed in the past decades on how to accommodate these changes in the power system. Therefore, the literature on power systems can inspire the development of solutions for future heating systems.

With the expected fluctuations in the available capacity and generation cost of future heat sources, flexibility in heat consumption will be necessary to maintain reliable heat provision at a reasonable cost. Attractive sources of demand-side flexibility in heat systems include thermal energy storage, virtual storage in buildings, and pipeline storage [20]. Thermal energy storage systems are available for both short-term and seasonal applications. The latter is especially useful given the seasonality of carbon-neutral sources such as solar heat, as well as many sources of cooling-based excess heat. Second, it is expected that residential and industrial consumers will become more pro-active in their usage of heat, for example by adapting their load in response to price signals. By allowing indoor temperature to vary within a comfort range, buildings can act as a virtual storage [21]. This marks the rise of the *heat prosumer*, defined in this thesis as a pro-active consumer that may possess assets for energy generation, conversion and/or storage [22]. Third, the district heating network itself can be a source of flexibility by storing heat in the pipelines.

The operation of future heating systems will become more complex with the massive integration of distributed excess and renewable heat sources, and the need for flexibility provision to the electricity system. Whereas most electrical power systems have moved to liberalized markets, almost all existing district heating systems have a strongly regulated market, where the price of heat and/or the profit made by district heating companies is regulated [23]. Liberalization of heating markets would mean that the schedule and price of heat are determined based on a heat market clearing, with minimal regulation on the form of generator and consumer bids. The main argument against liberalization of heat markets is the lack of competition among heat generators, as a relatively small number of large producers can supply end-users in a certain area due to distance restrictions of heat transport. In this regard, heat systems are unlike power systems.

However, more liberalized heating markets may become attractive in large district heating systems, in view of the possible increase in market competition due to increasing demand-side flexibility, as well as the increasing number of excess heat producers. This is illustrated by the case of Lithuania, where a relatively advanced level of competition has been introduced in the district heating market [24, 25]. In fossil-fuel based heat systems, the marginal cost of heat fluctuates less on a daily basis, which is another reason that heat markets and variable heat prices were less essential in the past. However, with the expected increase in daily price fluctuations, a real-time heat price could be of use, for example to incentivize demand-side flexibility [19].

1.2 Challenges and research directions

As a result of these developments, the operation of district heating systems will become more complex. At the same time, the integration of new market actors may pave the way for liberalized heat markets. These markets should be designed such that they can support the more complex operation of future district heating systems and coordination of the actors involved. By coordination we mean the act of steering different actors to enable them to work together effectively. This section presents the three main challenges we identify for *market-based coordination* of actors in future district heating systems. Based on these challenges, we introduce three corresponding research directions (**RD**s) for this thesis and identify the existing research gaps in each of these directions.

Moving from a system with a few large generators and passive consumers to a setting with *many smaller generators and prosumers*, the first challenge involves the optimal coordination of these actors. Their size and multitude may make them unsuitable for full market integration, i.e. the submission of bids in wholesale markets as an individual entity [26], as the transaction cost may be too large for both the market operator and the small heat providers. From a market operator's perspective, direct market participation of many small producers may be problematic for several reasons. The communication burden increases with the number of participants, as each of them must submit bids. Furthermore, the optimization problem that is solved to determine the optimal heat schedule and price of heat becomes more complex as the number of bids increases [27], which may lead to computational issues. Therefore, current electricity markets often have a minimum bid size, which excludes many small actors from participating directly in the market. Direct market participation may also be undesirable from the perspective of excess heat producers and prosumers with local heat generation. Excess heat providers, such as energy-intensive industries or data centers, are usually companies for which heat generation is not their main business. The same holds for (residential) prosumers with local production. It may therefore be difficult for them to formulate and submit bids, as this can be a labour-intensive process that involves special competences, so that the transaction cost for selling small amounts of heat would be too high. Related to this challenge, we define our first research direction:

[RD1] Scheduling and pricing for many small heat producers

In electricity systems, the need for coordinating many small actors has arisen in the context of demand response. Similar to excess heat generators, residential (electricity) consumers are small actors that cannot participate in the market directly, as their capacity is normally smaller than the minimum bid size. Instead, the literature envisions a new market entity called an *aggregator* to bridge the gap between demand-side flexibility providers and the wholesale and/or flexibility markets [28]. Direct control of such a large number of loads by the aggregator is costly, computationally heavy, and undesirable from a privacy perspective. As an alternative method for activating demand-side flexibility, *dynamic pricing* has been investigated in the literature, and in some places implemented in practice [29]. Under dynamic-pricing schemes, the aggregator sends a time-dependent price signal to (residential) loads, who use it to self-schedule their energy consumption [30]. This strategy may also be suitable for the coordination of many small actors in the heating system, and has in fact been implemented in practice in the Stockholm district heating system [31]. However, coordination of small heat producers through price signals has not been explored in the scientific community.

The second challenge we see concerns *network operation* in future district heating systems. With the integration of heat sources in the distribution network, as well as the rise of the prosumer, these systems become increasingly *distributed*. In such a distributed system, it is more difficult to guarantee network feasibility and to ensure energy- and cost-efficient use of the network [32]. Indeed, the installation of small generation capacities across a district heating network may increase the burden for network operators [24]. Efficient network operation is important for district heating systems, since low operational cost increases their competitiveness with local solutions at the end-user, such as local heat pumps. There is thus a need for the design of heat markets that assist network operators in the integration of prosumers and distributed heat producers. Previous works emphasize the need for network-aware coordination of distributed energy resources to support reliable and efficient system operation [32–34]. In this context, studies in both heat and power systems have proposed to include network constraints explicitly in market-clearing optimization problems. This motivates our second research direction:

[RD2] Network-aware market design for distributed district heating systems

A major challenge in this field has been to model the temperature and pressure dynamics in the network accurately, while maintaining convexity of the optimization problem. Non-convexity in market-clearing formulations has many drawbacks, including the lack of optimality guarantees on solver solutions, intractability of larger problems, and the absence of meaningful dual variables. Meaningful dual variables are desirable in a market setting because of their useful interpretations for pricing. For electrical power systems, research on network-aware optimal dispatch has been ongoing since the early 1990s [35, 36]. The Alternating Current (AC) power flow equations underlying the power system are non-linear, and including these equations in the optimal dispatch results in the non-convex AC Optimal Power Flow (OPF) model. In the past decades, the approximation and convexification of this problem has been an important research topic [37, 38].

In the design of network-aware optimal dispatch mechanisms for district heating, similar difficulties arise. The equations governing the dynamics of district heating networks are highly complex and non-linear. As a consequence, the most general Optimal Thermal Flow (OTF) model with variable flow and temperature is a non-convex, nonlinear problem. In an optimal-dispatch setting, the so-called node method is often applied, which results in a Mixed-Integer Nonlinear Program (MINLP) [39, 40]. Studies on network-aware heat market design have proposed different convexification approaches in three main categories, based on the related 'control strategy' for the network variables: Variable-Flow-Variable-Temperature (VFVT) with convex relaxations, Constant-Flow-Variable-Temperature (CFVT), and Variable-Flow-Constant-Temperature (VFCT) [41–47]. Each of these strategies has its own advantages and drawbacks, and the most suitable design must be selected considering the characteristics of future district heating systems. Previous works on network-aware heat markets have not placed emphasis on the integration of flexible agents such as prosumers and storage systems. As a result, their flexibility is often not harnessed to the fullest. This shows that our different research directions are in fact closely linked, and should not be tackled in isolation.

The third challenge we observe for the operation of future district heating systems is the optimal integration of storage. In fact, there are two types of energy storage systems that are relevant in this context. First, the district heating network itself can be seen as a special kind of heat storage. This is a result of time delays that occur when transporting heat from generators to consumers. In this regard, heat networks differ from power networks, where transmission of electricity occurs almost instantaneously. In a market setting, *pipeline heat storage* becomes relevant in certain network-aware market-clearing formulations. Second, it is expected that regular *thermal energy storage* will become an essential part of future district heating systems [48]. To support the optimal operation of these systems, there is a need for suitable market mechanisms that harness the flexibility from both types of storage. It remains an open question how heat markets, and energy markets in general, should be designed to optimally integrate storage. In this context, a specific type of market design that considers *non-merchant* storage has sparked research interest in recent years [49–51]. A non-merchant storage does not submit price-quantity bids to a market, but its operational constraints are included in the market-clearing problem, so that the storage operation is co-optimized with generation and loads to achieve the highest social welfare. Not only designated storage devices can be integrated in markets as non-merchant storage: in fact, pipeline heat storage behaves as a special kind of non-merchant storage in network-aware heat markets. This motivates our third and final research direction:

[RD3] Market integration of non-merchant storage

A special feature of storage is that its optimal dispatch involves *time-linking* or *intertemporal* constraints. Indeed, network-aware heat market designs that consider network storage naturally include such time-linking constraints in the market-clearing optimization problem, showing the connection between [**RD2**] and [**RD3**]. The intertemporal constraints pose challenges to market design, in part because energy markets are cleared sequentially for subsequent finite time horizons, even though different market-clearing horizons influence one another. An overview of market design with intertemporal constraints is given in [52]. If subsequent market intervals are cleared without taking their interdependence into account, this can result in suboptimal, 'myopic' scheduling of non-merchant storage. Myopic decisions on end-of-horizon storage levels are problematic because the temporal arbitrage by the storage is reduced, and desirable market properties may be lost.

1.3 Contributions

Motivated by these developments, the main objective of this thesis is to design markets that support the optimal operation of future district heating systems. This thesis contributes to the literature through the analysis and design of market mechanisms for district heating systems, focusing on three important aspects as specified by our research directions. We cover a broad range of topics within this field, with the nature of our contributions ranging from fundamental towards more applied. First, this thesis thoroughly investigates the performance of a specific exogenous pricing solution for scheduling and pricing excess heat. Our findings highlight that more complex market-based coordination may be necessary in future, when the penetration of these sources significantly increases. Second, we improve the market-based coordination of distributed heat sources by proposing a network-aware market mechanism, with a focus on flexibility in nodal heat injections. This market mechanism can help maintain feasibility and reduce operating costs in district heating networks, by effectively promoting more local heat consumption. Our third contribution sheds light on a more fundamental issue within market design for non-merchant storage. This work draws attention to the fact that market efficiency in such formulations may be compromised, and shows why this happens using a thorough duality analysis. Furthermore, we propose a method for restoring this market property in a perfect foresight setting. Although this thesis is focused on heat, our work on non-merchant storage considers a more general formulation, so that it is applicable to other types of non-merchant energy storage as well.

Towards the first research objective regarding scheduling and pricing for many small heat producers, we focus on excess heat producers, which are expected to supply a large share of the heat load in some future district heating systems. The possible price-responsiveness of these actors also makes them more interesting to study in terms of pricing than non-dispatchable renewable heat producers. Several existing works have investigated the scheduling and pricing of excess heat producers. In these studies it is often assumed that these producers participate directly in a market by submitting marginal-cost based bids [53–56]. All of these works disregard flexibility in the production profile of excess heat producers, which illustrates the fact that bid formation for excess heat providers is non-trivial. More importantly, we have argued above that market participation may be unsuitable for small producers, as well as prosumers. As an alternative to direct or aggregated market participation, price signals could be used for the integration of small heat producers. In the literature, such price signals have already been investigated in the context of demand-side flexibility in power systems [57]. Here, common price-signal designs include time-of-use pricing, critical peak pricing, and real-time pricing [57–59].

An interesting price signal is used to integrate small heat producers in the Open District Heating system in Stockholm [31]. Here, the system operator disseminates an ambient temperature-dependent price signal, which is computed using a pre-determined decreasing function of the ambient temperature. This is motivated by the fact that heat demand usually decreases with ambient temperature too. Excess heat producers use this price signal to *self-schedule* their production, and are paid according to the price signal for each generated unit of heat energy. Although the methods seems to have successfully attracted and integrated several sources of excess heat into the Stockholm heating system, this method has not yet been verified in systems with higher shares of excess heat. To this end, **[Paper A]** aims to investigate how the performance of this particular pricing method depends on excess heat penetration. This is done through a realistic case study of the Copenhagen district heating system. **[Paper A]** thus presents a quantitative analysis of an existing market mechanism's performance, given an existing district heating system. The main contribution of this work is the quantification of sub-optimality of the price-signal method compared to the case where excess heat producers participate in the market directly. A methodological contribution of this work is its district heating system model, that is quite simple but includes electricity-price dependent bidding behavior for both CHP plants and cooling-based excess heat providers. This model can be easily applied to investigate other topics, such as quantifying the effect of forecast errors in electricity prices. Due to its simplicity, the model could easily be extended, for example to include uncertainty or network constraints.

Towards the second research objective, many studies have proposed network-aware heat market designs to support the operation of increasingly distributed heating systems. In our view, the two most common network formulations used in this context have important shortcomings. In CFVT-based heat markets that are most widely used, the flow of the heating fluid becomes a parameter in the optimization problem. This limits the exploitation of prosumer flexibility considerably, as the sign of nodal heat injections is fixed, and rather restrictive bounds are imposed on their size as well. As the harnessing of this flexibility becomes increasingly important in future district heating systems, this is a severe limitation of this formulation. A second common approach is to apply convex relaxations to the VFVT formulation [41, 42, 60]. A major drawback of this approach is that the obtained solution is in most cases infeasible to the original problem, and there exist no optimality guarantees for the feasible solution that needs to be retrieved ex-post. In addition, the relaxed problems often remain nonlinear mixed-integer problems [41, 42], and can therefore already become intractable in the presence of a moderate number of nodes. In summary, more suitable network-aware heat market designs are needed to support the operation of future distributed heating systems. [**Paper B**] aims to fill the research gap related to the design of network-aware markets with a focus on optimally harnessing prosumer flexibility. In contrast to previous studies, we use the VFCT control strategy to model the network, which allows for flexible sign and size of nodal power injections. The resulting market-clearing optimization problem is a linear program, which ensures computational tractability and the existence of meaningful duals for pricing. The proposed market design helps ensure network feasibility of resulting heat schedules. By including an approximation of heat losses in the network, this mechanism is also able to reduce operational cost of district heating. [**Paper B**] analyzes the proposed market mechanism in detail. In order to increase *transparency* of scheduling and pricing using this mechanism, peer-to-peer trades are added to the formulation. In this way, we can trace the network losses back to specific producers and consumers, which can be used to explain the price formation in this market. A final contribution of [**Paper B**] is the formulation of a suitable benchmark to evaluate our proposed network-aware market mechanism against.

In relation to the third and final research direction, we focus on end-of-horizon issues in the market integration of non-merchant storage. The main challenge is to account for the relations between subsequent market intervals, in order to avoid myopic decision-making regarding the end-of-horizon state-of-charge in the storage. If the markets are cleared myopically, taking only the current horizon into account, non-merchant storage systems will be empty by the end of the horizon, unless negative prices occur. To avoid this, a future-aware decision on the final storage level could be enforced. However, many studies on market clearing with non-merchant storage make simplifying assumptions on the final state of energy. A first contribution of [**Paper C**] is to formally show that while common simplifying assumptions ensure cost recovery for the storage, they can lead to market inefficiencies. This effect has been overlooked in existing works that consider market properties within a single horizon [51]. To remedy this, a future-aware approach would be needed. Our second contribution in [**Paper C**] is the analysis of market properties in a future-aware setting, assuming perfect foresight of future market parameters. Previous works have applied rolling-horizon methods to set a future-aware end-of-horizon level for the non-merchant storage [52, 61, 62]. In [**Paper C**], we consider a more general formulation where the final storage level is constrained to take a pre-specified value, to be determined for each market clearing while taking future market horizons into account. The second contribution in [**Paper C**] is to show that a new problem arises in this future-aware setting, even if end-of-horizon storage levels are chosen with perfect knowledge about future market horizons. This problem entails that market prices in subsequent horizons may fail to reflect the value of storage. Previous works have analyzed this issue in relation to ramping constraints in a rolling-horizon setting [61]. Through an illustrative example as well as detailed proofs, [**Paper C**] shows under which conditions the market prices in subsequent market horizons fail to reflect the value of storage. In addition, we show how this may leads to a lack of dispatch-following incentives and lack of cost recovery for the non-merchant storage. In particular, the storage system may sometimes fail to recover its cost, while it improves overall social welfare. As a final contribution, we propose an adaptation of the future-aware market with non-merchant storage where these market properties are restored, given perfect foresight about future market-clearing parameters.

Our contributions towards the three research directions are compatible, i.e. the proposed market designs can be combined. For instance, both the price-signal method from [**Paper A**] and the storage formulation from [**Paper C**] can be implemented in a network-aware market as proposed in [**Paper B**].

1.4 Thesis organization

This thesis provides an overview of the contributions made during this Ph.D. project, based on the articles written in this period. In Chapter 2, we provide context through a brief overview of existing district heating systems and markets. In addition, this chapter lays the theoretical foundations for the remainder of the thesis, by introducing the fundamentals of convex optimization and common formulations in energy markets. Chapters 3 to 5 each present the contributions to one of our three research directions. Finally, we draw conclusions and provide perspectives for future work in Chapter 6. The scientific articles that this thesis is based on are provided as appendices.

Notation: For coherence within this thesis, notations have been adapted compared to their original form in the publications. Lower-case roman letters are reserved for decision variables and for functions. We use lower-case Greek letters for all dual variables, but Greek letters may sometimes also be used for other purposes. We use bold symbols for vectors. For instance, for a double indexed value p_{gt}, the vector $\boldsymbol{p}_g$ stacks all values for different t, and the vector $\boldsymbol{p}$ is a vector stacking all the vectors $\boldsymbol{p}_g$. We use calligraphic capital letters to represents sets, e.g. $\mathcal{G}$, and the matching capital letter (in this case G) represents the size of this set. Where possible, we use matching lower-case letters as indices for elements from those sets, for example g would be the index corresponding to the set $\mathcal{G}$. We use the superscripts H and E to distinguish between heat- and electricity-related quantities. Upper and lower limits are represented using $\overline{\cdot}$ and $\underline{\cdot}$, respectively. We use $\hat{\cdot}$ to indicate that a quantity is forecasted or estimated.

1.5 List of publications

The selected publications forming the basis of this thesis are:

[Paper A] L. Frölke, I.-M. Palm, J. Kazempour, “Market integration of excess heat”, in *Electric Power Systems Research (EPSR)*, vol. 212, Article number: 108459, 2022. DOI:10.1016/j.epsr.2022.108459.

[Paper B] L. Frölke, T. Sousa, P. Pinson, “A network-aware market mechanism for decentralized district heating systems”, in *Applied Energy*, vol. 306, Article number: 117956, 2022. DOI: 10.1016/j.apenergy.2021.117956.

[Paper C] L. Frölke, E. Prat, P. Pinson, R.M. Lusby, J. Kazempour, "On the efficiency of energy markets with non-merchant storage", Submitted to *IEEE Transactions on Energy Markets, Policy and Regulation*, 2022.

Although the following publications were prepared over the course of the PhD project, they have been excluded from this thesis.

[Pub. D] C. Rasmussen, L. Frölke, P. Bacher, H. Madsen, C. Rode, "Semi-parametric modelling of sun position dependent solar gain using B-splines in grey-box models", in *Solar Energy*, vol. 195, p. 249-258, 2020. DOI:10.1016/j.solener.2019.11.023.

[Pub. E] P. Bacher, H.G. Bergsteinsson, L. Frölke, M.L. Sørensen, J. Lemos-Vinasco, J. Liisberg, J.K. Møller, H.A. Nielsen, H. Madsen. "onlineforecast: An R package for adaptive and recursive forecasting." Submitted to *The R journal*, 2022. Preprint available: www.arxiv.org/abs/2109.12915.

Chapter 2

Heat market design preliminaries

This chapter provides context on existing district heating systems and markets, and lays the theoretical foundations for the remainder of this thesis. Although this thesis focuses on heat market design, the same principles underlie the design of other energy markets. As the literature on electricity markets is more developed, it is used here to inform and inspire the discussion on future heat markets.

In Section 2.1, we describe the elements of current and future district heating systems. Next, we introduce several fundamental concepts in energy market design in Section 2.2, and discuss these concepts in the context of current European district heating markets. As optimization is an essential tool used in almost all market-clearing procedures, we present the fundamentals of (convex) optimization in Section 2.3, including common formulations used in energy markets. Energy markets may include models of network operational constraints, which we discuss in detail specifically for heat markets in Section 2.4.

2.1 Current and future district heating systems

We describe common elements in current and future district heating systems in terms of generation, consumption, storage and transport of heat. Table 2.1 provides an overview of current and future elements in each category. The development of district heating systems is often expressed in terms of so-called 'generations', depending on their operational principles [3, 48, 63]. Most current systems are considered to be of the third generation, while the requirements for future fourth and fifth generations are discussed in the literature.

2.1.1 Generation

In many systems, the majority of the heat load is supplied by Combined Heat & Power (CHP) plants that simultaneously generate heat and electricity [48, 64]. Such cogeneration is highly efficient compared to generating the different forms of energy separately. Although waste-to-energy and biomass-fueled CHP plants are common, fossil fuels still account for 90% of total heat supply in district heating systems globally [65]. Fossil-fueled and sometimes electric heat boilers are typically used to supply peak load [48]. In future carbon-neutral district heating systems, most heat should be supplied by renewable and excess heat sources [48]. While waste incineration in CHP plants is useful as a year-round reliable source of heat, recycling of waste should be prioritized over heat generation from an overall energy-efficiency perspective. Renewable heat sources include solar and geothermal heat. Small amounts of solar and geothermal heat have already been introduced in several countries, but more can be harnessed in the future [66, 67].

There is a great potential to use excess heat from industrial or urban sources [15, 48], and an increasing amount of such sources is integrated in district heating networks [65, 68]. Industrial

	Current	Future
Generation	• Heat-only and CHP plants (fossil fuels, waste, biomass) • Boilers (fossil fuel, electric) • Centralized • Controllable	• Excess heat • CHP plants (waste) • Renewable heat (solar, geothermal) • Decentralized • Seasonal & daily fluctuation
Consumption	• Inflexible • Space heating $\gg$ DHW • Seasonal differences	• Flexible • Space heating $\approx$ DHW • Reduced seasonal differences
Storage		• Short-term TES • Seasonal TES
Network	• Medium supply temperature • Centralized • Radial, unidirectional • Delay & storage	• Low supply temperature • Distributed • Perhaps circular, bidirectional • Delay & storage

Table 2.1: Overview of elements in current and future district heating systems. CHP = combined heat and power, DHW = domestic hot water, TES = thermal energy storage.

excess heat can be extracted from energy-intensive processes in for example steel and cement factories. Urban excess heat can come from a variety of sources, such as waste-water treatment plants, data centers, metro tunnels, and service-sector buildings [15, 65]. The excess heat from many urban sources comes from cooling processes, either by refrigeration systems in the service sector [69], or by cooling systems in e.g. data centers [70]. Power-to-X production facilities may also supply excess heat from cooling their electrolyzers [71]. Heat pumps are used for heat recovery in such cooling-based excess heat sources. Due to the presence of heat pumps and CHP plants, future district heating systems will remain tightly connected to power systems.

In future district heating systems, heat consumers may also have their own local heat production to complement district heating, for example using photovoltaic-thermal panels [72]. Certain excess-heat providers are also both producers and consumers of heat, such as supermarkets that both consume heat for space heating, while producing heat as a by-product of refrigerator cooling [73]. With the integration of excess and renewable heat, as well as heat production by small consumers, future district heating systems will be increasingly *decentralized*.

2.1.2 Consumption

District heating supplies residential and commercial buildings, as well as industries. Residential consumption can be subdivided into Space Heating (SH) and Domestic Hot Water (DHW) consumption. Substantial reductions in the space heating demand of future buildings are expected due to improved isolation [74]. A benefit of this is that seasonal heat load will have smaller fluctuations, as these seasonal fluctuations mainly come from the space heating demand. On the downside, heat load reductions challenge the efficient operation of district heating systems, as lower heat densities imply a higher unit cost of distribution [74].

Currently, heat demand is rather inelastic [75]. However, experimental and simulation studies have shown that buildings can provide short-term flexibility [76]. Space heating demand can be flexible by allowing deviations from a setpoint indoor temperature within an acceptable range [18, 20]. In [77], demand-side flexibility is obtained from storage tanks for domestic hot water, as well as from flexible space heating demand. The flexibility provided by buildings may be smaller than by dedicated storage devices [18], because the latter can provide medium-term storage as well. The available flexibility varies widely with building type: for example, less insulated buildings have a higher flexibility potential [78]. The flexibility can be used for peak shaving in the total system load [79], relieving congestion in distribution networks [80], integration of excess and renewable heat, and cost reductions [76]. There is thus a great potential benefit of harnessing this flexibility, but in current systems this is rarely implemented. The adoption of demand-side management requires more advanced metering and communication infrastructure in heating systems. Furthermore, effective business models are needed to make flexibility provision acceptable and profitable for consumers [18].

2.1.3 Storage systems

The term *thermal energy system* is used for heat storage in dedicated devices. Some current district heating systems make use of short-term thermal storage, typically in hot water tanks, for example to smoothen daily variations in heat load or deal with price fluctuations of CHP plants. Short-term thermal storage can also absorb daily fluctuations in renewable and excess heat sources, and reduce the use of expensive peak-load generators. With the increased electricity dependence of future heat sources, it may become profitable for short-term thermal storage to provide balancing power to electricity systems [81]. Finally, network congestion and high pumping cost during peak hours can be prevented using short-term storage [82].

Although longer-term thermal storage systems are much less common [20], several types of seasonal thermal storage are technically feasible, including large storage tanks, pit storage, and borehole storage [83]. Seasonal latent heat storage systems consist of abundant, low-cost, and non-toxic materials, but heat loss may be around 30%, and the energy density of these storages is low [84]. Still, this technology is already used commercially on large scale in e.g. Denmark, Germany, and Sweden [85], and other technologies for seasonal thermal storage are under development [84]. In Denmark, several large pit heat storages have been built in e.g. Marstall [81]. In Alberta, Canada, a borehole seasonal storage allows the local district heating system to supply over 90% of the yearly space heating load with solar energy [86]. Heat sources in future district heating systems will exhibit strong seasonality, due to seasonal patterns in solar radiation and cooling loads of excess heat providers. Therefore, the economic benefit of seasonal storage systems will increase [81].

2.1.4 Network layout and operation

Almost all modern district heating systems use water as a heat carrier [63]. Heat transport is considerably slower than electricity transmission, with delays that can amount to multiple hours in large networks [8]. District heating networks usually have a radial structure, consisting of a transmission network with distribution pipelines branching out [3]. The networks consist of a supply and a return side. Supply temperatures in second-generation networks were above $100°C$ with relatively high heat losses, but these are lowered in third-generation systems. The trend towards lower distribution temperatures is expected to continue [48], to decrease network heat loss and to allow the injection of low-temperature excess heat in the supply network. Most modern heating networks are operated with variable mass flows, to adapt to changes in demand more easily than when varying supply temperatures [41, 87]. The variable cost of heat distribution comes from pipeline heat loss and electricity used for pumping. There is a trade-off between these two cost components, as higher flow leads to increased power used for pumping, while reducing heat loss. The relative contributions of pumping cost and heat loss cost vary between studies, depending on the design of the heating system and the operating strategy [88, 89].

In future heating networks, several changes are envisioned to improve efficiency of network operation, and to accommodate developments in the generation and consumption of heat. Heating networks need to adapt to the situation where generation moves from large centralized plants to smaller, more distributed heat sources, while flexibility in heat consumption may become available. Network operators are looking for solutions to optimally integrate these decentralized (excess) heat producers and prosumers [90]. In this context, alternative structures to the existing tree structures are sometimes considered, such as a ring structure with branches [3]. In addition, while current district heating systems are operated with unidirectional pipeline flow, the possibility of bidirectional networks has been investigated in early-stage works, such as the simulation study in [91].

The possibility of utilizing the thermal energy storage present in the network itself is a popular research topic. In order to harness this flexibility, advanced control strategies are needed, and the temperature of the water in the network needs to be varied [20, 41]. However, frequent temperature variations are undesirable as they may cause premature material fatigue in steel pipelines [20]. In addition, flexibility provided by network storage has been shown to be limited compared to the flexibility potential from buildings [92]. Overall, there are disadvantages to the active utilization of pipeline heat storage, while the benefit to be gained may be small [93].

2.2 Energy market design in theory and practice

Here, we introduce important concepts in the design of energy markets, and describe how existing heating systems deal with these different aspects, with a focus on European heating markets.

2.2.1 General organization of energy markets

An electricity market design can be defined as 'the set of arrangements which govern how market actors generate, trade, supply and consume electricity and use the electricity infrastructure' [94]. Energy markets have the aim of introducing competition within and between the generation and consumer sectors, which can lead to increased cost efficiency in two main ways. First, competition can have price-lowering effects in the short term, by creating the incentive to offer the available energy for the lowest possible price. Second, markets can be designed to create efficient long-term investment signals, providing generators and consumers with incentives to invest in cheaper and/or more energy-efficient technologies [24, 95].

Two core elements of a market design are a *bidding format* and a *market-clearing procedure*. The latter can be defined as a set of rules that determines the schedule and price of energy for the considered time horizon, given bids from market participants. Typically, an optimization problem is formulated to clear the market, with the aim of finding a schedule that is optimal with respect to a specified objective. We will describe the general formulation of such optimization problems and related common bidding formats in Section 2.3.

Energy is often traded in both day-ahead and intra-day markets. In the day-ahead market, a schedule and price is determined for the following 24-hour horizon. During the day itself, intra-day adjustments can be traded in intra-day markets closer to the time of delivery. Finally, there may be balancing or real-time markets, which are used as a last resort to offset imbalances between supply and demand [30]. Day-ahead planning is useful to give generators sufficient time to plan their operation, while forecasts used (e.g. weather forecasts, load forecasts) are reasonably accurate at this stage [96]. However, there will be forecast errors and possibly unforeseen changes in plant availability, which can be accounted for in intra-day and balancing markets.

Most European countries do not have a common liberalized marketplace for heat, because heat generation and distribution is handled by a single company or organization, commonly referred to as the *district heating company*. Little information on existing scheduling procedures is available, likely because this scheduling is done by the district heating companies internally. The Greater Copenhagen area has an optimal scheduling procedure in place, supervised by Varmelast [68]. Varmelast is responsible for preparing day-ahead heating plans, and adjusting this plan three times daily, as updated heat forecasts, electricity price forecasts, and available capacities become available. The schedule is made based on a cost-minimization principle, while also considering the hydraulics of the system. However, waste incineration plants and geothermal energy are politically prioritized [97]. The exact scheduling procedure is confidential.

2.2.2 Market actors and structure

Actors on energy markets include generators, consumers, a market operator, and a network operator. There may be separate operators of distribution and transmission networks, but district heating systems commonly have a single network operator handling both transmission and distribution of heat. For fair competition, it is often considered important that generation, transport, and retail of energy are executed by separate entities. A *vertically-integrated* utility is a single entity that owns generation, transmission and distribution assets [98]. If a vertically-integrated utility owns all generation, it may have a monopoly for the supply in a certain area. When different tasks are executed by different entities, and no vertically-integrated utilities exist, the market structure is called *unbundled*. In power systems, there is typically a *wholesale market* where generators, large consumers, and energy *retailers* can trade electricity. Small consumers cannot participate there directly, because there is a minimum bid size that exceeds their needs. Instead, they buy energy from retailers on the *retail market*.

Related to unbundling and ownership is the concept of *third-party access* (TPA) or market opening. In heating systems, third-party access refers to the non-discriminatory access to networks to supply heat [99]. The name already suggests that currently, most networks do not allow all ‘third parties’, in contrast to gas and electricity systems, where market opening and third-party access have already been addressed in the past decades. The European Commission has called for the opening of district heating networks for third parties, in order to promote integration of renewable and excess heat [100]. Third-party access can also promote competition in district heating systems [99, 101]. Several possible schemes for third-party access in district heating markets as listed in [24] are 1) no TPA, 2) negotiated TPA, 3) regulated TPA, and finally 4) full TPA. Under negotiated third-party access, the district heating company is free but not obliged to negotiate contracts with third parties [24]. Under regulated third-party access, the district heating company must connect external heat producers, although it is often allowed to put some general requirements on the suppliers in place. Under regulated third-party access, the conditions of access are determined in advance, whereas these conditions are negotiated on a case-by-case basis in negotiated third-party access [99]. Finally,

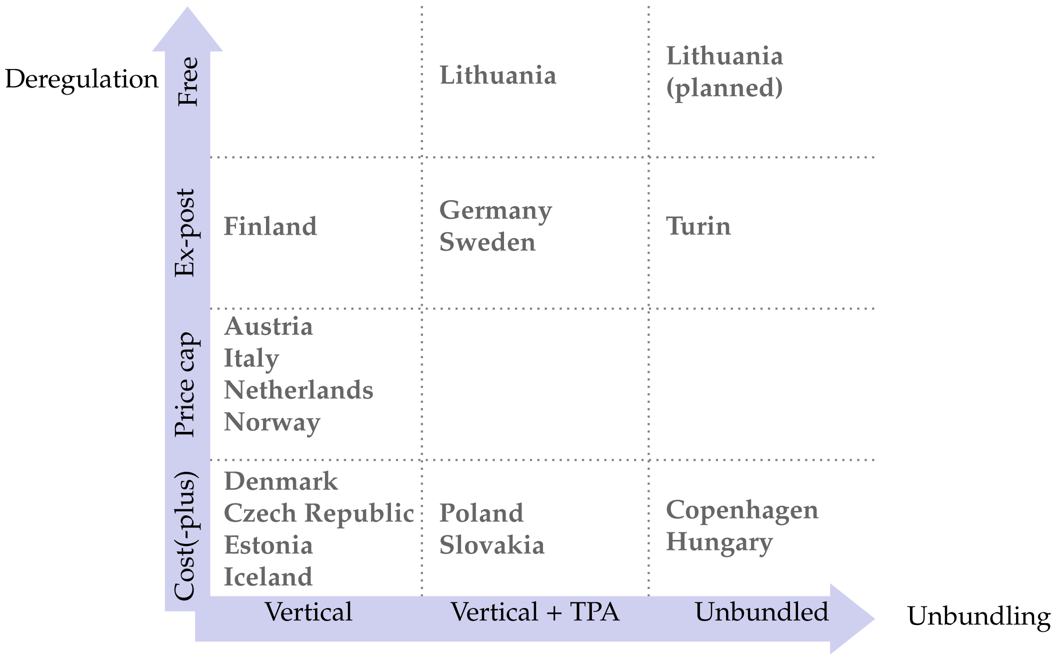


Figure 2.1: Categorization of European district heating systems based on level of unbundling and deregulation. TPA = third-party access. Adapted from [25].

full third-party access is used to describe a system in which non-discriminatory access has to be given to all producers wishing to enter the system, if they satisfy a set of pre-specified requirements, and where the network operator is a separate entity that does not own any heat generation units

In most European countries, heat generation and distribution is handled by a single, vertically-integrated district heating company. An overview of European countries and their level of unbundling and deregulation (discussed below) in heating markets is shown on the horizontal axis in Figure 2.1. The district heating company often owns both generation plants and the network, and sells heat directly to heat consumers without any retail market. In some cases, there are other heat generators present in the system, so-called *independent heat producers*, that are not owned by the district heating company. As illustrated in Figure 2.1, there are a few exceptions to this rule. In Hungary, heat production and distribution are to be executed by different entities [25]. Most Italian district heating systems are vertically integrated, with the exception of the Turin area, where unbundling has been completed [25, 102]. Denmark is listed as vertically integrated in Figure 2.1, but exceptions exist in the larger Danish networks. For example, in the district heating system of Greater Copenhagen, the distribution company is formally separated from the generators. In addition, the independent entity Varmelast is installed by the different generation companies in this system, and given the task of scheduling the heat generation [68]. Although many countries have vertically integrated structures, it should be noted that district heating companies are often publicly owned [66, 103]. Many Danish district heating companies are cooperatives owned by its own customers. In Sweden, after heat market liberalization in 1996, many district heating companies moved from municipality-owned to private- or state-owned large energy companies [23].

Lithuania has ambitious goals related to the unbundling of district heating market activities [24]. Regulated third-party access has been implemented [25], as an intermediate step towards the aim of full third-party access [24]. In Sweden, third-party access has been particularly highly debated, when liberalization on the heating market lead to price increases in some regions. Process industries, such as steel factories and waste-water treatment plants, have promoted the introduction of third-party access in Swedish heating networks [99] . Sweden had a weak form of negotiated third-party access implemented between 2009-2014, where the district heating company must pay third parties according to the avoided variable costs [99]. Since 2014, regulated third-party access has been adopted [104]. Poland has implemented full third-party access, but due to complexity of regulations, this is not effective in practice [101]. Most other countries have no specific regulation in place regarding third-party access, and we refer to [101] for an overview.

2.2.3 Market competition and regulation

The literature disagrees about the question whether or not district heating is a *natural monopoly*, which can be defined as follows: 'A firm producing a single homogeneous product is a natural monopoly when it is less costly to produce any level of output of this product within a single firm than with two or more firms. In addition, this "cost dominance" relationship must hold over the full range of market demand for this product' [105]. Although many works consider district heating as a natural monopoly, this view is challenged in several recent works, such as [24, 101]. In [101, 106] it is noted that the heating *network* may be regarded as natural monopoly, but that on the production side, a second competitor does not face high sunk costs. Therefore, a competitive heat market should in general be possible, especially in larger networks. Natural monopoly or not, in the absence of perfect competition (i.e. in practice), most markets can be subject to the execution of market power by its participants. To combat the exertion of market power, the market may need to be regulated.

The subject of competition is especially relevant in heat markets, compared to e.g. electricity and gas markets. Creating competition in electricity and gas markets is easier, as their grids cover large areas and are even interlinked on international scales. In district heating systems, however, heat losses under transport of energy restrict the size of the networks: typically, networks are built to serve a single city or urban region. As a result, the number of generators and consumers competing in the same system is also more limited [107]. In sufficiently large heating systems, where a greater number of generators can be integrated, competition on the generation side is considered feasible [24]. The authors of [24] argue that the regulatory reform in Lithuania shows the feasibility of (partial) liberalization of district heating markets. Depending on the system size, the benefits and disadvantages of liberalized and regulated markets must be carefully weighted. In strongly regulated markets, there may be a lack of incentive to invest in energy- and cost-efficient new technologies [24]. In liberalized systems, efficiency of the system may be increased through competitive pressure. However, liberalized markets do not necessarily decrease energy prices for consumers, if there is room for exertion of market power [107]. It has for example been shown that prices of heat in some regions in Sweden increased after liberalization of the heating market [75].

2.2.4 Remuneration of generators

In the literature, existing district heat pricing methods are divided into two categories: regulated or liberalized [23, 103]. Although one could argue that even liberalized markets have some type of regulation in place, we stick with this divide and define it as follows. In a liberalized heat market, the price of heat is defined between producers and consumers, with minimal additional regulation in place. In regulated heat markets, the price formation is more strongly regulated with the aim of protecting consumers from excessively high prices, by preventing monopoly pricing and inhibiting the exercise of market power by generators. Ideally, such regulation should also provide incentives for investments in cheaper and more efficient technologies. However, a drawback of several types of price regulation is the lack of such incentives [107]. Different type of regulation principles have been applied in practice [23, 103, 107]. Price regulation may be done ex-ante or ex-post, i.e. before or after the market is cleared [103, 108]. On the vertical axis of Figure 2.1, European countries are sorted based on their regulations on the pricing of heat.

One popular regime for pricing of heat generation is regulated pricing with mandatory price control [103]. Common methods for regulated pricing are non-profit and cost-plus methods, as well as the introduction of a price cap. In Denmark, a non-profit principle is applied to both generation and distribution of district heat. This means that the price of heat must cover all cost of supply and distribution, both variable and fixed. A drawback of this method is that there is little incentive for suppliers to reduce production cost, e.g. by investing in new technologies or increasing energy efficiency [107]. However, this effect is partially overcome by the public ownership of district heating companies in Denmark. In the Greater Copenhagen optimal scheduling procedure, the participating plants must submit their true cost, based on fuel prices, operation and maintenance cost, energy taxes, CO_2 quota, and electricity price forecasts. The price of heat is not determined by Varmelast, but follows from confidential long-term bilateral contracts between generators and the district heating company.

The *cost-plus* pricing mechanism is similar to the non-profit scheme, except that a limited profit is allowed. Usually, the allowed profit is specified as a fixed share of the total cost. The cost-plus method, for example used in Iceland, Poland, and Slovakia [25], is the most common method for regulated pricing in Europe [103]. As the allowed profit is proportional to a heat generator's total cost, a drawback of this method is that generators have an incentive to inflate costs [23]. In countries including The Netherlands and Norway, a price cap for district heat is set based on the price of the cheapest alternative [24, 25], although this method can be too restrictive and therefore hamper district heating expansion [103].

Another popular pricing paradigm for heat generation in Europe is liberalized pricing with ex-post price control on request [25, 103]. Suppliers are not allowed to exercise market power by charging excessively high prices, which is controlled ex-post on request, when there is reasonable evidence that excessive prices have been charged. The Swedish district heating sector was deregulated in 1996, which involved removal of the non-profit pricing principle [109]. In some areas, heat price increases followed, such as in Uppsala and Stockholm, which opened the debate whether and how to re-regulate the heat market [99, 109, 110]. In 2008, Sweden introduced transparency rules, which state that suppliers must make information about their price setting mechanisms public, and submit annual reports [23, 103, 109].

Lithuania stands out in Figure 2.1, having both liberalized pricing and high degree of unbundling. Here, a heat auction is organised by Baltpool, the operator of the energy exchange [111]. Heat suppliers have to place offers for the available quantity and a related price, and the auction determines the heat generation schedule for each supplier based on a least-cost principle, taking into account the operational constraints of the heating network [25]. The variable cost of the district heating company is used as a price ceiling for bids in this auction [24]. If a scheduled participant cannot deliver the promised quantity, it must cover the cost for an alternative heat generator.

Interestingly, peak power supply, reserve, and balancing remain functions served by the district heating company only, as it is expected that introduction of competition in these functions is not feasible for now [24]. For the district heating company supplying these monopoly markets, the cost-plus method is applied on long-term heat prices [111]. Price regulation is only applied for specific producers, such as those that supply more than one third of the heat load. It is reported that district heating prices have steadily declined since introduction of third-party access and auction in Kaunas, Lithuania [25]. As more independent heat producers have entered the Lithuanian district heating systems, the market price of heat has moved away from the price ceiling in both Kaunas and Klaipeda, which suggests that competition was successfully introduced [24].

2.2.5 Energy rates for consumers

Energy rates for consumers generally consist of different components: the actual energy price, tariffs, and taxes [112]. This is also the case for heat, where the tariff usually covers the fixed costs related to maintenance and operation [23]. In Denmark and Sweden, there is an additional cost component for consumers related to their contribution to efficiency in the district heating system, which can be estimated from the average temperature gradient or average flow at the consumer's heat exchanger [23, 113]. Only the energy price is subject to competition in liberalized energy markets, whereas grid tariffs are usually strongly regulated [112].

The price that small consumers pay for a unit of energy is usually not determined by wholesale market-clearing mechanisms directly. In liberalized electricity markets, consumers buy electricity from *retailers*, which participate on the wholesale market and resell the energy, thereby acting as intermediates [30]. Consumers should be free to choose between different retailers easily, to ensure competition on the retail market. In the case of heat, consumers usually buy from the district heating company only, so there is usually no possibility to choose between multiple heat retailers.

Electricity retailers often offer flat-price contracts for small consumers [30], although modern retailers also offer variable-price contracts. Whether or not the price is variable, the retailer absorbs the risk of deviating from consumption schedules [30, 114]. The consumer pays a premium for the retailer to absorb its risks, i.e. the price is higher than the average cost of supplied energy [114]. Variable-price or other types of contracts may help retailers to reduce this risk and the related cost, and thereby the risk premium charged to consumers could be lowered under such contracts [114] For heat, a flat rate for each year is also common, even though the system marginal cost fluctuates considerably over the year. In some countries, e.g. Sweden, the unit price of heat energy varies per season [23]. We are not aware of any countries where consumer district heating prices vary on a smaller time scale, such as in variable-price contracts for electricity. In Copenhagen, the energy component of the consumer heat price could be derived from the optimal dispatch supervised by Varmelast [68], but this price is not used. Instead, the price of heat for consumers is fixed for an entire year, based on a contract between the consumer and the local heating supply company [97].

2.3 Convex optimization for energy market design

We now present fundamental concepts of convex optimization, and proceed to show how convex optimization can be used for energy market design. Finally, we discuss desirable properties in the field of market design.

2.3.1 General formulation

In an optimization problem, the goal is to find a/the optimal value of one or more *decision variables*, given a certain *objective function* that is to be minimized or maximized. Our convention is to formulate optimization problems in the minimization form. Any problem can be converted to a maximization problem by multiplying the objective function by -1. We denote the vector that gathers all decision variables by $\boldsymbol{x}$. Optionally, one may specify a set of conditions that the variables in $\boldsymbol{x}$ should satisfy, turning the problem from a *unconstrained* into a *constrained* optimization problem. Two kinds of constraints can be specified, namely, inequality and equality constraints. In this thesis, we only use continuous optimization variables $\boldsymbol{x}$. When integer-valued optimization variables are included, the related optimization problems become Mixed-Integer (MI) Programs, which are non-convex.

The following problem presents a classical, continuous *convex* optimization problem using the notation of [115]:

$$\min_{\boldsymbol{x}} \quad f_0(\boldsymbol{x}) \tag{2.1a}$$

$$\text{s.t.} \quad f_i(\boldsymbol{x}) \leq 0 \quad : \lambda_i \qquad i = 1, \dots, m \tag{2.1b}$$

$$\boldsymbol{A}_i^\top \boldsymbol{x} = B_i \quad : \nu_i \qquad i = 1, \dots, p\,. \tag{2.1c}$$

Here, the objective function f_0 is a convex function of $\boldsymbol{x}$. The set of m inequality constraints in (2.1b) are also specified using functions f_i convex in $\boldsymbol{x}$. Finally, p linear equality constraints are specified, which must be linear in $\boldsymbol{x}$ for the problem to remain convex. The coefficients specifying each linear constraint are collected in vector $\boldsymbol{A}_i$ and scalar B_i for all $i = 1, \dots, p$. Each of the constraints in (2.1) has a *dual variable* associated to it, where we use the vector $\boldsymbol{\lambda}$ to collect the duals for inequality constraints, and $\boldsymbol{\nu}$ for equality constraints. Alternatively, constraints can be rewritten using the concept of a *feasible region*, which contains all values that satisfies the constraint. For example, (2.1b) would be rewritten to

$$\boldsymbol{x} \in \mathcal{F}_i \qquad i = 1, \dots, m\,, \tag{2.2}$$

where $\mathcal{F}_i$ is the *feasible region* defined as

$$\mathcal{F}_i = \{\boldsymbol{x} \mid f_i(\boldsymbol{x}) \leq 0\}\,. \tag{2.3}$$

Depending on the form of the functions f_0 and f_i, convex optimization problems can be divided into subcategories. For example, (2.1) is a Linear Program (LP) if f_0 and f_i for all i are linear. If all f_i are affine and the objective function is convex quadratic, (2.1) is a Quadratic Program (QP).

To generalize the convex problem in (2.1), regular inequalities (2.1b) can be replaced by so-called *generalized inequality constraints*:

$$f_i(\boldsymbol{x}) \preceq_{K_i} 0\,, \tag{2.4}$$

where K_i is a proper cone with $K_i \subseteq \mathbb{R}^{k_i}$ for a certain dimension k_i [115]. This results in a more general form of the standard convex form (2.1), as the generalized inequality has the regular inequality constraint (2.1b) as a special case. Again, the form of f_i for $i = 0, \dots, m$ leads to different types of convex optimization problems, such as conic and semidefinite programs [115].

For any optimization problem, convex or not, the Lagrange dual function and a related dual problem can be defined. The Lagrange dual function related to problem (2.1) is

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \inf_{\boldsymbol{x}} \quad \sum_{i=1}^{m} \lambda_i \, f_i(\boldsymbol{x}) + \sum_{i=1}^{p} \nu_i \left(\boldsymbol{A}_i^\top \boldsymbol{x} - b_i\right), \tag{2.5}$$

where the infimum (i.e. greatest lower bound) is taken over all feasible $\boldsymbol{x}$. For any positive $\boldsymbol{\lambda} \geq 0$ and any $\boldsymbol{\nu}$, this function provides a lower bound on the optimal objective value of problem (2.1) [115]. This property can be used to derive the *dual problem* of *primal problem* (2.1). The objective of the dual problem is to find the best lower bound that the Lagrange dual function can give. A general form of the dual problem related to primal formulation (2.1) is given by:

$$\max_{\boldsymbol{\lambda}, \boldsymbol{\nu}} \quad g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \tag{2.6a}$$

$$\text{s.t.} \quad \lambda_i \geq 0 \qquad\qquad i = 1, \ldots, m\,. \tag{2.6b}$$

For specific types of optimization problems, such as linear, quadratic, and conic problems, a general form of the dual problem can be specified in more detail, see e.g. [115].

Strong duality holds when the optimal value of the primal problem coincides with the best lower bound from the Lagrange dual, i.e. the optimal value of the dual problem. In this case, the *duality gap* is said to be zero. When strong duality holds, the dual variable associated to a constraint can be interpreted as the sensitivity of the primal objective function to a change in the bound of that constraint. If a constraint is not binding, this implies that a change in this bound would not affect the value of the objective function, and therefore the associated dual variable has a value of zero. Instead, if the constraint is binding, the associated dual variable may be nonzero.

For convex optimization problems with differentiable functions f_i for $i = 0, \ldots, m$, the Karush-Kuhn-Tucker (KKT) conditions provide a set of necessary and sufficient conditions for a tuple $(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ to be primal and dual optimal with zero duality gap. In other words, if $(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ satisfy the KKT conditions, these values of the primal and dual variables solve the primal and dual problem to optimality with zero duality gap, and vice versa. The KKT conditions do not say anything about the *uniqueness* of optima. It is possible that there are multiple solutions to an optimization problem, meaning that several points $(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ solve the primal and dual problem to optimality.

2.3.2 Optimization in energy market design

In day-ahead markets, a common market design is a two-sided auction, where both consumers and producers place bids for each time period [30]. We use index t for time periods in the market clearing, and collect all periods for a single market clearing in the set $\mathcal{T} = \{1, 2, \ldots, T\}$ of size $|\mathcal{T}| = T$. A general bidding format can be specified as follows. Each market participant i submits a feasible region $\mathcal{F}_i$ for its power injection profile $\boldsymbol{p}_i$, as well as a cost function $f_{0,i}$ that maps its power injection profile to a cost, i.e. $f_{0,i} : \mathbb{R}^T \to \mathbb{R}$. For a consumer, the power injection is negative. The market participants i are collected in the set $\mathcal{I}$. A general form of an optimization problem for clearing such a market is:

$$\min_{\boldsymbol{p}_i} \quad \sum_{i \in \mathcal{I}} f_{0,i}(\boldsymbol{p}_i) \tag{2.7a}$$

$$\text{s.t.} \quad \boldsymbol{p}_i \in \mathcal{F}_i, \quad \forall i \in \mathcal{I} \tag{2.7b}$$

$$f^{\text{grid}}(\{\boldsymbol{p}_i\}_{i \in \mathcal{I}}) \in \mathcal{F}^{\text{grid}} \tag{2.7c}$$

$$f^{\text{balance}}(\{\boldsymbol{p}_i\}_{i \in \mathcal{I}}) \in \mathcal{F}^{\text{balance}} \qquad : (\boldsymbol{\lambda})\,. \tag{2.7d}$$

Here, the optimization variables $\boldsymbol{p}_i$ are vectors of the power injections, each of size T to specify a power injection for each time period. The problem may include additional optimization variables to specify the feasible sets of the participants. Constraint (2.7b) ensures that the scheduled power is feasible according to the bid by the market participant. Optionally, grid- or network-related constraints may be specified, as represented by constraint (2.7c). We will discuss such constraints for heat markets in Section 2.4. The abstract constraint (2.7d) enforces a form of supply-demand balance, which may be specified in different ways depending on the network representation in the market.

Under the most common *uniform pricing* scheme, the market price is derived from dual variable $\boldsymbol{\lambda}$ corresponding to the balance constraint (2.7d). If network constraints are disregarded, there will be a single uniform market price λ_t for each time period t that the market is cleared for. This unit price is paid by each consumer and received by each producer. If instead, network constraints are included in the market clearing, prices may differ between the nodes (or zones) as specified by the network representation. Therefore, the price at each time t is now a vector $\boldsymbol{\lambda}_t$ of uniform prices, one for each node. These prices are also called the *locational marginal prices* (LMPs).

In a common simple form of the two-sided auction, the bidding format is of the price-quantity type. Loads l submit the maximum quantity $\overline{D}_{lt}$ they are willing to consume, and the maximum price they are willing to pay per unit U_{lt} (i.e. willingness to pay or utility per consumed unit). Generators g submit the maximum quantity they are able to generate $\overline{G}_{gt}$, along with the minimum unit price C_{gt} they are willing to sell it for. This results in linear cost and utility functions for all market participants. The bidding format may be extended with a lower production limit $\underline{G}_{gt}$ and lower consumption limit $\underline{D}_{lt}$. Loads l and generators g are collected in the respective sets $\mathcal{L}$ and $\mathcal{G}$. Given these bids, a common simple form of a pool market without network constraints is specified as follows:

$$\min_{\boldsymbol{g},\boldsymbol{d}} \quad \sum_{t\in\mathcal{T}} \left(\sum_{g\in\mathcal{G}} C_{gt}\, g_{gt} - \sum_{l\in\mathcal{L}} U_{lt}\, d_{lt} \right) \tag{2.8a}$$

$$\text{s.t.} \quad \underline{G}_{gt} \le g_{gt} \le \overline{G}_{gt}, \quad \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \qquad : (\underline{\mu}_{gt}, \overline{\mu}_{gt}) \tag{2.8b}$$

$$\underline{D}_{lt} \le d_{lt} \le \overline{D}_{lt}, \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \qquad : (\underline{\chi}_{lt}, \overline{\chi}_{lt}) \tag{2.8c}$$

$$\sum_{g\in\mathcal{G}} g_{gt} = \sum_{l\in\mathcal{L}} d_{lt}, \quad \forall t \in \mathcal{T} \qquad : (\lambda_t)\,. \tag{2.8d}$$

Here, the decision variable g_{gt} represents the scheduled generation of generator g at time t, and d_{lt} represents the scheduled demand of load l at time t. In this notation, the index g matches the variable $\boldsymbol{g}$, but the context will always make it clear which of the two is meant. The objective 2.8a is to minimize the difference between total cost and total utility, or equivalently, to maximize the *social welfare*. The uniform market price for time t is given by the dual variable λ_t corresponding to the energy balance constraint (2.8d). For each time t, the price λ_t is a scalar since there are no network constraints, and thus no nodal price differences. The dual variables corresponding to lower and upper limits of constraints (2.8b) and (2.8c) are indicated in the brackets on the right-hand side of these equations.

This basic market design has been extended in many ways, for example by including start-up and ramping constraints for generators, which introduces integer variables to the problem. Other versions of the two-sided auction for energy have been proposed, which allow for more elaborate descriptions of market participants' feasible regions, as well as the inclusion of uncertainty formulations, and complex network constraints. To allow market participants to specify more complex feasible regions, [116] proposes a price-region bidding format. Here, market participants may specify the feasible region of power injections using a set of linear constraints, using additional state variables if needed. Such bidding formats can enable more accurate representation of flexible resources, so that more flexibility can be harnessed. Uncertainty-aware markets have been proposed, for example using chance constraints [117]. Such more elaborate market formulations often require optimization frameworks beyond linear and quadratic programs. This is also the case for more elaborate network-aware market formulations, such as in [118]. The work in [119] generalizes many of these approaches by formulating a conic market, where a Second-Order Cone Program (SOCP) is used to clear the market. Conic constraints can be used to include more accurate models of both market participants and the network, and to include uncertainty, e.g. using chance constraints. A uniform pricing scheme is derived from the dual variables in this conic formulation, where each location and time is subject to its own uniform price [119].

2.3.3 Desirable market properties

In the field of market and mechanism design, four common desirable properties have been formulated, which an ideal mechanism should satisfy. The first property, *market efficiency*, holds when the dispatch determined by the market achieves the optimal value of the market's objective, and the market provides *dispatch-following incentives*. The latter means that the prices determined by the market are *dispatch supporting*, so that no market participants can benefit from deviating unilaterally from the optimal dispatch. Second, a market mechanism is *incentive compatible* if market participants have the incentive to bid truthfully. For example, this would mean that a generator cannot increase its profits by inflating its production cost in a price bid. Third, *cost recovery* (also called *individual rationality*) refers to a market in which no generator or consumer incurs a loss. This implies among others that a generator will be paid enough to cover the production cost of its schedule. The fourth and final property of *revenue adequacy* holds when the market operator does not suffer a loss when clearing the market. In other words, payments by market participants add up to at least the amount that is to be paid to (other) market participants. A special case of revenue adequacy is *budget balance*: in this case, the payments by market participants are exactly equal to the amount paid to market participants.

No market design can generally satisfy all four properties simultaneously in practice, as it is only possible for all four to hold if there are an infinite number of market participants [120]. In most common market designs, there is possibility for strategic behavior, especially when there is a lack of competition. This means that incentive compatibility does not hold. Indeed, this is the case for the market formulation in (2.8), as well as conic generalizations of this market design. If participants bid truthfully, reported utilities correspond to the true willingness to pay for loads, and reported cost correspond to the true generation cost for generators. In addition, the reported available capacities would match their real values. In case market participants do not bid truthfully, the social welfare can be reduced, so that market efficiency also fails to hold. On the other hand, cost recovery and revenue adequacy are ensured in problem (2.8).

2.4 Modeling district heating networks

In order to support the optimal operation and control of district heating networks, heat market designs can take network constraints into account. As works on heat network modeling specifically for market design is scarce, we first review modeling approaches in control and optimal dispatch in district heating systems. We close of with a discussion of network models that have been used in heat market design.

Ultimately, the temporal and spatial dynamics of fluid and energy flows in a district heating network are described by a set of nonlinear partial differential equations. These equations are the starting point for deriving (generally nonlinear) optimization models for heat networks, also called Optimal Thermal Flow (OTF) models. These models consist of a hydraulic model describing the fluid flow and pressures in the network, and a thermal model for the temperature dynamics. In a market or optimal-dispatch context, the hydraulic part of the system is assumed to be in a steady state, since hydraulic transients propagate at the speed of sound, and thus can be neglected given typical time granularity in these applications [63]. However, thermal transients propagate much slower, so that the dynamics of temperature propagation do need to be taken into account [8]. It is usually assumed that the network is radial, and that the temperature in a cross-section of a pipeline is constant.

In principle, the partial differential equations governing the district heating network dynamics could be approximated and solved using finite-difference based methods. The work [121] outlines the assumptions needed to arrive at a simplified partial differential equation, leading to a pipeline model of a single spatial dimension. Similar models are used in several previous works, including [121–123]. The partial differential equation for transport of energy through pipelines is also discussed in [124]. The authors derive a steady-state heating network simulation model, emphasizing the assumptions needed to arrive at the final result. Finite-difference based methods have not been applied in a market setting for heat, due to the high computational burden associated with these methods [125]. Instead, many network-aware heat market designs use the so-called *node method* from [126] or simplified variations of this formulation to approximate the partial differential equations. The node method discretizes the state variables in space, using only the network node locations. The outlet temperature of a pipeline at time t is modelled as a weighted sum of historic inlet temperatures. For a visualisation of which historic temperatures are used in the node method, we refer to [45]. The node method uses a set of auxiliary time index variables. If the flow is considered a variable rather than a parameter in this model, integer variables are needed to model the variable time indices, and the optimization problem becomes a Mixed-Integer Nonlinear Program (MINLP). In [127], the node method is compared to a finite-difference approach. It is concluded that the node method is superior both in terms of accuracy as well as computational cost for a given discretization size. The recent work [121] also provides a comparison of these different modeling approaches, assessing their performance using real measurements, while also discussing them qualitatively. It is concluded that the node method provides a higher accuracy for a given time discretization size. However, it contains integer variables, which leads to a higher computational burden.

The literature distinguishes three types of *control strategies*, Variable-Flow-Variable-Temperature (VFVT), Constant-Flow-Variable-Temperature (CFVT), and Variable-Flow-Constant-Temperature (VFCT), depending on whether the temperatures and flows are decision variables in the optimization problem. The complexity of heating network models lies in the so-called *mixing equation*, which describes the temperature of a mixture of several water streams of possibly different temperatures. The mixing equation is bilinear and thus non-convex, in case both the flow and the temperature are considered as continuous decision variables.

In the VFVT control strategy, flows as well as temperatures are decision variables in the optimization problem, which in most formulations results in a MINLP formulation. Iterative algorithms have been proposed to solve the resulting optimization problems [39, 40, 128]. Iterative solution methods may be suitable for control of heat systems, but have several drawbacks in practice. The convergence of the algorithms is not guaranteed, and can become computationally intractable for relatively small systems. To get around this, [129] restricts the mass-flow variable to a limited set of discrete values, whereas the temperature remains a continuous variable. This approach reduces complexity of the network problem to a Mixed-Integer Linear Program (MILP). A similar approach is taken in [130], where in this case the flow is a continuous variable, while the temperatures take a set of discrete values, again resulting in a MILP formulation. In [41], a VFVT-based optimal dispatch model is solved non-iteratively, using a simplified version of the node method. The resulting model is still a MINLP, which is converted to a Mixed-Integer Second-Order Cone Program (MISOCP) or MILP using different convexification methods. A problem with this approach is that the solution to the relaxed problem is usually infeasible to the original problem, so that a feasible solution needs to be retrieved after solving the relaxed problem. There is no bound on the optimality gap of this retrieved solution, which means that there is no guarantee that the solution is close to the optimum of the original problem. Another conic mixed-integer form of VFVT for optimal dispatch is proposed in [42], where the mixing equation is omitted, using the assumption that the mixture temperature equals the temperature of the largest inflow of each node [42]. Again, solutions to this model are certainly infeasible to the original problem.

The second category of works considers CFVT, also called *quality regulation*, because the temperature of the water is variable while the flows are parameters in the model [45, 87, 131, 132]. When the flows are fixed, so are the time delays. This greatly reduces the model complexity, resulting in an LP network model. A drawback of quality regulation is that frequent temperature changes in steel pipes may speed up material fatigue [20]. The third and final control strategy is VFCT, also called *quantity regulation*, where the flow is considered variable while the temperatures are parameters in the model. The VFCT strategy is applied in e.g. [46, 47]. Most modern district heating system are controlled using variable flows [133].

Few works have compared the different control strategies. As the VFVT control strategy allows for most flexibility, it should provide the least-cost solution, compared to VFCT and CFVT methods. This is confirmed in [134]. The comparison in [135] shows that variable-flow methods have both lower fuel cost and operational cost, but this comes at the cost of a higher computation time. When the district heating network is modeled in order to provide flexibility to power systems, the topic of network or pipeline heat storage is often studied. While such network storage can arguably be exploited when VFVT is used, see e.g. [41], we argue that this is not the case for VFCT and CFVT formulations, even though existing works sometimes suggest that CFVT formulations do store heat in the network. Our reasoning is that when the flows are fixed, a change in the amount of energy injected at a specific time is always directly linked to a change in energy extracted at another node and another time. There is no flexibility at what time the extraction takes place in this formulation, except if the loads are flexible. In that case, such flexibility should be attributed to the loads.

Specifically in the setting of network-aware market design for heat, previous works have applied the CFVT control strategy, see e.g. [19, 136–138]. This formulation is attractive because its dual variables can be readily used for pricing. In addition, these works often have the Chinese setting in mind, where district heating networks are operated with constant flow. In a market context, most studies aim to show that the heating system can provide flexibility to the power system. Pricing considerations are rarely discussed for network-aware heating market designs. An exception to this is [136], where a generalized locational marginal price for a combined heat and electricity market is proposed and analyzed in detail.

Chapter 3

Market integration of excess heat

Based on the work in [**Paper A**], this chapter presents our contribution towards the first research direction: scheduling and pricing for many small heat producers. Although solar heat could also fall into this category, we focus on excess heat producers, as they can adapt their production in response to price signals while solar heat cannot. Excess heat has the potential to cover a large share of the heat load in district heating systems. For example, a detailed study estimates that industrial excess heat could currently cover over 80% of the heat load in certain large Danish district heating networks, such as Aalborg and Kalundborg [139]. Another study estimates that 50% of Copenhagen's heat load could be supplied by excess heat from biorefineries that could be built in the area [140]. A high-level estimation in [74] suggests that the potential excess heat capacity for use in households is also large in other European countries, especially Germany, France, and The Netherlands. The integration of excess heat contributes to carbon neutrality in district heating systems, and may increase cost efficiency by increasing competitive pressure, if it results in a greater number of generators participating in the heat market [24]. Although not all processes from which excess heat can be extracted are carbon neutral, the excess heat can be considered carbon neutral if it does not lead to additional carbon emissions. Most excess heat sources remain untapped, although their integration is technically feasible, as shown in a simulation study [141] and proven by applications in several existing district heating systems [142]. One barrier to its integration is the lack of suitable methods for scheduling and pricing excess heat. Direct market participation may be undesirable, since excess heat producers may lack the specialist knowledge needed for bid submission. As a pragmatic alternative, the operator of the Stockholm Open District Heating sends excess heat producers a time-variant price signal, based on which they locally determine their preferred schedule for heat injection into the district heating system [31]. We use the term *self-scheduling* for this individual action of excess heat providers.

In this chapter, we present qualitative and quantitative comparisons of two paradigms for the price-based integration of excess heat producers: *self-scheduling* and *market participation*. The latter can be seen as an ideal benchmark that may be impractical in practice, in particular in the presence of many excess heat providers, while the former could be seen as a pragmatic solution. The aim of the comparison is to determine whether price-based self-scheduling can suffice for market integration of excess heat producers. Existing studies have only focused on direct market participation for the scheduling and pricing of excess heat [53–56]. Price signals have been studied in the context of heat demand response [97, 143], but not for the scheduling and pricing of (excess) heat. Although the self-scheduling model has its benefits in terms of practicality, the resulting total generation cost is at best equal to but likely higher than what can be achieved by co-optimizing all generation costs, as done in the market-participation model. A market clearing minimizes the total cost on the system level, while self-scheduling individuals are maximizing their own profit, which is not necessarily aligned with the system cost minimization. Therefore, the market-participation model is an ideal benchmark in terms of total system cost. As the main contribution of this chapter, we quantify the potential suboptimality of the price-based self-scheduling method using a realistic case study of the district heating system in Greater Copenhagen. In particular, we thoroughly analyze how this suboptimality depends on the market share of excess heat. We study a specific price-signal design, but our method is general in the sense that it can be applied to investigate the performance of any other price signal in any given district heating system. Our results inform the discussion of whether it is advisable to use such price-signal methods in current and future district heating systems.

In Section 3.1, we describe the self-scheduling and market-participation paradigms in more detail, and provide a qualitative comparison. We then formulate the market-participation model in Section 3.2 and the self-scheduling model in Section 3.3. The case study is presented in Section 3.4, including bidding models for the market participants, the used input parameters, numerical results, and a summary of our conclusions. Finally, we propose directions for future work in Section 3.5.

3.1 Two paradigms

We first provide clear definitions of the self-scheduling and market-participation models in Section 3.1.1, followed by a qualitative comparison in Section 3.1.2.

3.1.1 Definition

The market-participation model refers to the situation where the excess heat producers submit bids to the heat market, and is thus treated the same as any other heat generator. After receiving all the bids, the market operator clears the market, and obtains a schedule for all hours of the following day, as well as a market price for each hour. We assume that this price is used for remuneration of heat generators, although this is not currently the case in Copenhagen. Instead, the generators are remunerated according to prices as agreed in long-term contracts with the district heating company.

Under the self-scheduling model, the heat market operator broadcasts a time-varying price signal for excess heat producers, which serves as a basis for excess heat producers to self-schedule their heat generation accordingly. In principle, any price signal can be used, but we will study a specific signal that is presented in our case study. We assume for simplicity that the resulting schedule is then shared with the market operator, or equivalently, that the market operator has a perfect forecast of the total amount of heat to be injected by excess heat producers. In practice, the market operator may in fact not be informed about the self-determined schedules of excess heat producers, so that the market operator must use a forecast. Given the excess-heat schedules, the market operator schedules the remaining producers in the subsequent market clearing, which is equivalent to the market clearing under the market-participation paradigm. The excess heat providers are remunerated according to the price signal sent to them, whereas the generators participating directly in the heat market are still remunerated according to the hourly market price. The self-scheduling method mimics the setup used in the Stockholm Open District Heating system [31], which we will explain in detail in our case study description in Section 3.4.

3.1.2 Comparison of the two paradigms

We compare the two paradigms based on several characteristics, as summarized in Table 3.1 and elaborated on in the following. First, we discuss optimality of the resulting schedules in

	Market Participation	Self-scheduling
Optimal scheduling	✓	✓ / ×
Relevance for small producers	×	✓
Price formation	Endogenous	Exogenous
Problem type	Linear program	Linear program

Table 3.1: Comparison of the two paradigms for scheduling and pricing excess heat. Adapted from [**Paper A**].

terms of total heat generation cost. In the market-participation model, the schedule of excess heat producers is co-optimized with that of other generators with the objective of achieving minimal total generation cost. Therefore, the market-participation model can be seen as an ideal benchmark with respect to total generation cost. Generally, the self-scheduling model results in a suboptimal schedule in terms of total generation cost. Theoretically, it is sometimes possible to design an efficient price signal such that the resulting dispatch and total generation cost equal those of market-participation model. However, it is not guaranteed that such a price signal exists, and in practice it would be nearly impossible to design such a 'perfect' price signal. It can be expected that the suboptimality of the self-scheduling method increases with the share of excess heat in the system. However, it is not trivial to foresee how steep this increase is, and whether this steepness changes. For example, one may expect to see some plateauing behavior. It is this dependence that we aim to reveal in our case study.

A second point of comparison of these methods is their relevance for scheduling and pricing heat from small producers, for whom heat production is not their main business. Direct market participation of excess heat producers has some drawbacks in practice. Most importantly, it may be difficult for small excess heat producers to decide on market bids, as heat production is by definition not their main business, and it may not make sense for them to have specialized staff to deal with bid submission. They may therefore prefer to receive a price signal, sine the resulting self-scheduling may be easier to automate. Compared to self-scheduling, the market participation of these producers implies that the operator receives many bids, clears a more complex market with an increased number of variables and constraints, and then shares the resulting schedules with each market participant. Therefore, the self-scheduling paradigm may be preferred in practice from the perspective of the market operator as well, as it is a rather simple and computationally cheap way of scheduling and pricing excess heat, with lower IT requirements. Given the advantages that the price-signal method brings, a small optimality gap with respect to total generation cost could be justified, if this method attracts more low-cost excess heat to the system.

The nature of their price-formation process is a third difference between the two methods. Under the market-participation scheme, the price paid to excess heat producers (and other generators) is a result of bids submitted by market participants, including the excess heat producers. As the price originates from the bids, this price formation is called *endogenous*. In contrast to this, the price formation in the self-scheduling case is *exogenous*, as the price received by the excess heat producers is set by the market operator, unrelated to the cost parameters of these agents. However, the price for the other generators under the self-scheduling paradigm is still formed endogenously. This means that the price received by the excess heat producers under self-scheduling in general does not represent the marginal system cost. The price signal can of course be designed with different properties in mind. For example, the market operator could forecast the marginal price that would result from the market clearing, and use this as a price signal. In Stockholm, the hourly

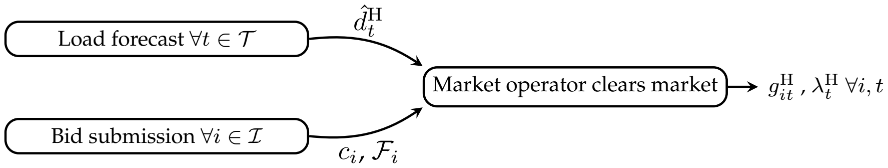


Figure 3.1: Overview of the information flow in the market-participation model. Adapted from **[Paper A]**.

price signal for excess heat producers is a predetermined, decreasing function of the ambient temperature. This temperature-dependent price function is known to businesses that consider to become providers of excess heat, which has the advantage that it is straightforward for them to estimate whether the expected profits can cover their investment and operational cost.

Finally, we emphasize that we formulate both the market-participation and self-scheduling models as linear programs without binary or integer variables. In the next two sections, we specify the corresponding optimization problems.

3.2 Market-participation model

Here, we formulate the market design that was used for the case study simulations in this chapter. We assume a market-clearing setup similar to the current Copenhagen heat market at the day-ahead stage, which is a marginal-cost based dispatch without network constraints. Network feasibility of the dispatch is checked after market clearing [144]. This market is cleared before the electricity market. The market-participation model assumes submission of bids from all generators, including excess heat providers. The heat load is a parameter and heat loads do not submit bids, which is also the case in the current heat market clearing in Copenhagen. Instead, the market operator uses a forecast of the total heat load at each time period, $\hat{D}_t^{\mathrm{H}}$, to clear the market. The market formulation in this chapter is network agnostic, unlike in the following Chapter 4, so that all market participants are subject to a single uniform price λ_t^{H} for each hour. We denote the set of all market participants by $\mathcal{I} = \mathcal{E} \cup \mathcal{G}$, where the disjoint sets $\mathcal{E}$ and $\mathcal{G}$ collect excess heat producers and the remaining generators, respectively. We use the index e for excess heat producers and g for other generators, whereas the index i may refer to any market participant.

The market design is of the general form (2.7), without network constraints. The time periods for a single market interval are collected in the set $\mathcal{T}$ of size $|\mathcal{T}| = T$. The bidding format is given by $\big(\mathcal{F}_i, c_i(\cdot)\big)$ for each market participant i. Here, $\mathcal{F}_i$ is the convex feasible region of the heat generation profile vector $\boldsymbol{g}_i^{\mathrm{H}}$, which collects the production for all time periods t. The bid $c_i : \mathcal{F}_i \to \mathbb{R}$ is the convex cost function of participant i given a production profile $\boldsymbol{g}_i^{\mathrm{H}}$. The location of bids is not relevant in the current network-agnostic framework. Figure 3.1 gives an overview of the information flow in the market-participation model.

Similar to constraint (2.7b), generation profiles must be contained in the provided feasible regions:

$$\boldsymbol{g}_i^{\mathrm{H}} \in \mathcal{F}_i \qquad \forall i \in \mathcal{I}\,. \tag{3.1}$$

The energy-balance constraint in the current market design is given by

$$\sum_{g\in\mathcal{G}} g_{gt}^{\mathrm{H}} + \sum_{e\in\mathcal{E}} \big(g_{et}^{\mathrm{H}} - w_{et}\big) = \hat{D}_t^{\mathrm{H}} - u_t \qquad : \lambda_t^{\mathrm{H}}\,, \tag{3.2}$$

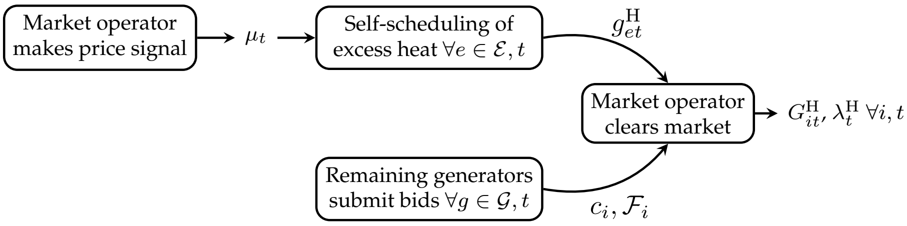


Figure 3.2: Overview of the information flow in the self-scheduling model. Adapted from [**Paper A**].

which ensures that at every time t, there is a balance between the predicted heat load $\hat{D}^{\mathrm{H}}_t$ and the scheduled generation. If needed, the load can be shed by an amount u_t at a constant cost C^{U} per unit of lost load. Similarly, excess heat producer e can be curtailed at any time t by an amount w_{et}, without a cost associated to it. The need for curtailment arises sometimes when the feasible region for excess heat providers are such that their lower bounds for production are non-zero. The uniform market price at time t is given by the dual variable λ^{H}_t related to this balance constraint.

The objective in the market is to minimize total cost over one entire market horizon, given by objective function

$$\sum_{g \in \mathcal{G}} c_g \left(\boldsymbol{g}^{\mathrm{H}}_g \right) + \sum_{e \in \mathcal{E}} c_e \left(\boldsymbol{g}^{\mathrm{H}}_e - \boldsymbol{w}_e \right) + C^{\mathrm{U}} \sum_{t \in \mathcal{T}} u_t \,, \tag{3.3}$$

where the total cost includes generation cost and cost of lost load. The excess heat producers are only paid per unit of actually scheduled heat production, given by $g^{\mathrm{H}}_{et} - w_{et}$.

The set of optimization variables is $\Gamma = \{\boldsymbol{g}^{\mathrm{H}}, \boldsymbol{w}, \boldsymbol{u}\}$.

3.3 Self-scheduling model

Figure 3.2 provides an overview of the information flow for the self-scheduling paradigm. First, the market operator determines a price signal μ^{H}_t for each t in the upcoming market period $\mathcal{T}$, and broadcasts it to excess heat producers. Although we investigate a specific price-signal design in this chapter's case study, our method is general in the sense that it can be used for any other price signal. Next, each self-scheduling excess heat producer $e \in \mathcal{E}$ solves optimization problem (3.4) to determine an optimal schedule for the upcoming market horizon $\mathcal{T}$:

$$\min_{\boldsymbol{g}^{\mathrm{H}}_e} \quad c_e \left(\boldsymbol{g}^{\mathrm{H}}_e \right) - (\boldsymbol{\mu}^{\mathrm{H}})^\top \boldsymbol{g}^{\mathrm{H}}_e \tag{3.4a}$$

$$\text{s.t.} \quad \boldsymbol{g}^{\mathrm{H}}_e \in \mathcal{F}_e \,. \tag{3.4b}$$

The objective (3.4a) for the excess heat producer is to minimize its net cost, given by the difference between the cost of generation and the revenue from selling the generated heat at the price $\boldsymbol{\mu}^{\mathrm{H}}$. This price is a parameter in the self-scheduling model. The only constraint in the self-scheduling optimization problem is feasibility of the dispatch in (3.4b).

At the same time, all remaining generators $g \in \mathcal{G}$ have submitted bids in the same way as in the market-participation model. The market operator receives those bids, as well as the determined optimal schedules $\boldsymbol{G}^{\mathrm{H}}_e$ from each excess heat producers. Given these inputs, the market operator clears the market using the following market-clearing optimization problem:

$$\min_{\boldsymbol{g}_g^{\mathrm{H}}, \boldsymbol{u}} \quad \sum_{g \in \mathcal{G}} c_g\left(\boldsymbol{g}_g^{\mathrm{H}}\right) + C^{\mathrm{U}} \sum_{t \in \mathcal{T}} u_t \tag{3.5a}$$

$$\text{s.t.} \quad \boldsymbol{g}_g^{\mathrm{H}} \in \mathcal{F}_i, \quad \forall g \in \mathcal{G} \tag{3.5b}$$

$$\sum_{g \in \mathcal{G}} g_{gt}^{\mathrm{H}} + \sum_{e \in \mathcal{E}} G_{et}^{\mathrm{H}} - w_t = \hat{D}_t^{\mathrm{H}} - u_t, \quad \forall t \in \mathcal{T} \qquad : (\lambda_t^{\mathrm{H}}), \tag{3.5c}$$

which is an adapted version of the market-clearing under the market-participation model. The objective function no longer minimizes the cost of excess heat producers. The feasible region constraint for the excess heat producers is also omitted. In the energy balance constraint (3.5c), the excess heat production $g_{et}^{\mathrm{H},*}$ is now a parameter. The wasted excess heat w_t is still a variable to be determined by the market clearing, but it is now determined as a single value, rather than for each excess heat producer individually.

In the self-scheduling model, the excess heat providers are not given the opportunity to minimize the wasted excess heat w_t, as they are in the market-participation model. Therefore, they are paid at price μ_t^{H} for any self-scheduled unit of generated heat, regardless of whether it is wasted or not.

3.4 Simulation study

We consider the district heating system of Copenhagen, which currently includes 13 CHP plants [145], including waste-to-energy and fossil-fueled plants, with a total installed capacity of $2050\,\mathrm{MW}$ [68]. The Copenhagen system also includes reserve generators, but these are outside the scope of this study. We vary the excess heat capacity added to this system from 0 to $2100\,\mathrm{MW}$. The excess heat is assumed to be of the cooling-based type, as would usually be found in the service sector, among others. More specifically, we assume that the excess heat is a by-product of cooling supermarket refrigeration cabinets using local heat pumps.

3.4.1 Bidding models

We specify the bidding behaviour of CHP plants and excess heat producers by providing models for their feasible regions $\mathcal{F}$ and cost functions $c(\cdot)$. An underlying assumption in these bidding models is that the heat market is cleared before the electricity market, as is the case in Copenhagen. Therefore, these bids must rely on a forecasted day-ahead electricity price $\hat{\boldsymbol{\lambda}}^{\mathrm{E}}$, rather than realized electricity prices.

The scheduled heat production for a CHP affects its limits for electricity generation, and vice versa. The assumed shape of the feasible region for CHP plants is depicted in Figure 3.3. The lower bound on this region is due to the minimum power-to-heat ratio R_i, which represents the minimum amount of power generation needed in order to extract one unit of heat. Thus, the slope of this line is R_i. The upper bound of the region is a result of the maximum fuel consumption $\overline{F}_i$ of the plant. There may additionally be a vertical border on the right of this region, given by an upper bound on heat generation $\overline{G}^{\mathrm{H}}$ as given by the dotted line in Figure 3.3. If this bound is not given, the crossing of the two sloped borders defines a maximum heat generation. As depicted, the lower bound on the heat generation is zero, regardless of the scheduled electricity generation. As the heat market is cleared before the electricity market, all dispatches within the feasible region

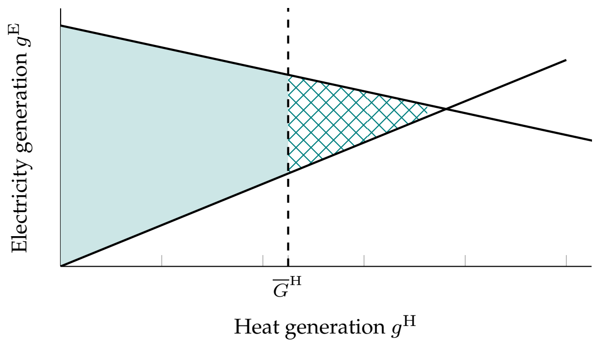


Figure 3.3: Assumed shape of feasible region for the electricity and heat production of CHP plants. The region is formed by the two sloped lines, and optionally the vertical line given by the upper limit on heat production $\overline{G}^{\mathrm{H}}$. If there is no upper limit on heat production, the hatched area is also included in the feasible region.

are still feasible at the time of bidding. This implies that the feasible region for CHP i at time t can be written as

$$\mathcal{F}_{it} = \left\{ g_{it}^{\mathrm{H}} \;\middle|\; 0 \leq g_{it}^{\mathrm{H}} \leq \min\left(\overline{G}_i^{\mathrm{H}}, \frac{\overline{F}_i}{\rho_i^{\mathrm{H}} + R_i\, \rho_i^{\mathrm{E}}} \right) \right\} , \tag{3.6}$$

where the upper bound is the minimum of $\overline{G}^{\mathrm{H}}$ or the crossing of the sloped borders. Here, ρ_i^{E} and ρ_i^{H} are the fuel efficiency for electricity and heat, respectively. Note that in our CHP model, the feasible region for the entire market horizon can be split in individual feasible regions for each time period. This is possible due to the lack of intertemporal constraints (for example, ramping constraints) in our CHP model.

As the heat market is cleared before the electricity market, the heat schedule affects the feasible region of the CHP in the electricity market. The cost bid of the CHP plants is designed to take this interaction into account. We use the CHP cost bid as derived in [146] for CHP plants in a sequential heat and electricity market setting. The marginal cost of heat depends on the (forecasted) electricity price $\hat{\lambda}^{\mathrm{E}}$ and (constant) fuel price α_i as follows:

$$C_{it}^{\mathrm{H}} = \begin{cases} \alpha_i \left(\rho_i^{\mathrm{E}}\, R_i + \rho_i^{\mathrm{H}}\right) - \hat{\lambda}_t^{\mathrm{E}}\, R_i & \text{if } \hat{\lambda}_t^{\mathrm{E}} \leq \alpha_i \rho_i^{\mathrm{E}} \\ \hat{\lambda}_t^{\mathrm{E}}\, \frac{\rho_i^{\mathrm{H}}}{\rho_i^{\mathrm{E}}} & \text{if } \hat{\lambda}_t^{\mathrm{E}} > \alpha_i \rho_i^{\mathrm{E}} . \end{cases} \tag{3.7}$$

The cost function is split into two cases depending on the forecasted electricity price. If the forecasted electricity price does not exceed a certain threshold, CHP plants bid their fuel cost minus the income from electricity sale, as represented by the first case in (3.7). If instead the forecasted electricity price is above this threshold, CHP plants bid the lost opportunity cost from selling heat instead of electricity, as in the second case in (3.7). This implies that the CHP plants submit the following linear cost function:

$$c_g\left(\boldsymbol{g}_g^{\mathrm{H}}\right) = C_{gt}^{\mathrm{H}}\, g_{gt}^{\mathrm{H}} . \tag{3.8}$$

The feasible region of the considered cooling-based excess heat producers is

$$\mathcal{F}_e = \left\{ \boldsymbol{g}_e^{\mathrm{H}} \;\middle|\; (21) - (26) \text{ from Appendix A.2 of } [\textbf{Paper A}] \right\} . \tag{3.9}$$

The considered constraints are all linear, as specified in Appendix A.2 of [**Paper A**]. The constraints include a model of the heat pump, as well as a model of cooling-cabinet temperature dynamics.

There is flexibility in the excess heat production of these agents, because limited fluctuations of the temperature in the cooling cabinets around a setpoint are allowed. In contrast to the feasible region in our CHP model, the feasible region of the excess heat producers cannot be split into independent feasible regions for each $t \in \mathcal{T}$, due to the presence of intertemporal constraints for temperature dynamics in the cooling cabinets.

We assume that excess heat producers bid at zero cost, corresponding to the following cost function:

$$c_e^{\mathrm{H}}(\boldsymbol{g}_e^{\mathrm{H}}) = 0 \quad \forall e\,, \tag{3.10}$$

which is reasonable given that the heat pump would be used to cool the cabinets anyway, so this cost is not to be attributed to the heat production.

3.4.2 Input parameters

The two models are simulated for a full year in an hourly time resolution. Our online repository [147] contains a detailed description of the used input parameters and code, which can be used to reproduce our results. For a description of chosen model parameters for both CHP plants and excess heat producers, we refer to [**Paper A**].

A central input parameter to this case study is the price signal $\boldsymbol{\mu}^{\mathrm{H}}$ that is disseminated by the market operator. We study specifically the 'Spotvärme prima' price signal used in Stockholm's Open District Heating program [148], which is fully determined by the ambient temperature T^{A}. We were only able to obtain this price signal for discrete values of the outdoor temperature. In order to obtain a price signal as a continuous function of the ambient temperature, we fit an approximate pricing function as

$$\mu(T_t^{\mathrm{A}}) = \begin{cases} 380 \cdot 0.92^{T_t^{\mathrm{A}}} & \text{for } T_t^{\mathrm{A}} < 17.5°\mathrm{C} \\ 0 & \text{for } T_t^{\mathrm{A}} \geq 17.5°\mathrm{C}\,. \end{cases} \tag{3.11}$$

As seen in Figure 3.4, the function μ is monotone decreasing with the ambient temperature, as the base of the exponent is non-negative and below 1.

The case study also requires forecasts of ambient temperature, electricity prices, and heat load for each $t \in \mathcal{T}$. We take historical data of these inputs as perfect forecasts, see [**Paper A**] for specifics on data sources. The ambient temperature is needed to determine $\boldsymbol{\mu}^{\mathrm{H}}$ using the function μ from (3.11). Furthermore, we model the Coefficient of Performance (COP) of the heat pumps as ambient-temperature dependent functions.

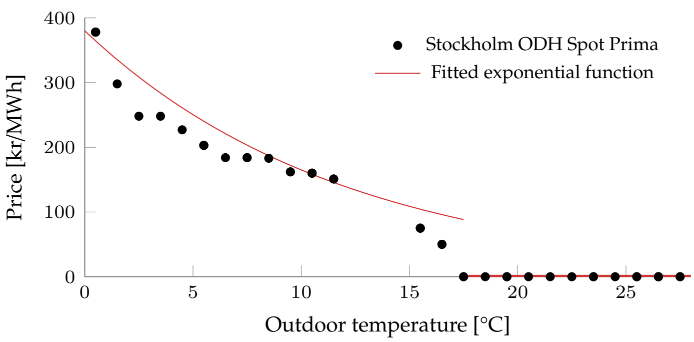
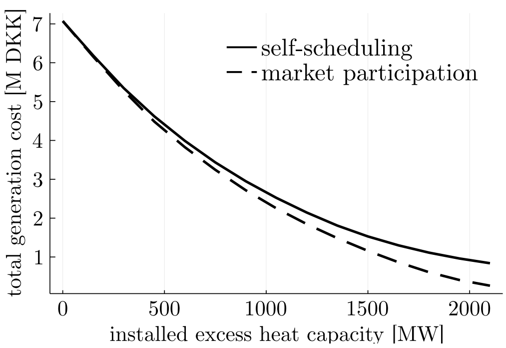


Figure 3.4: Price signal for excess heat producers. Black dots are real prices obtained from Stockholm Open District Heating. The prices used in our case study are given by the red line. Adapted from [149].

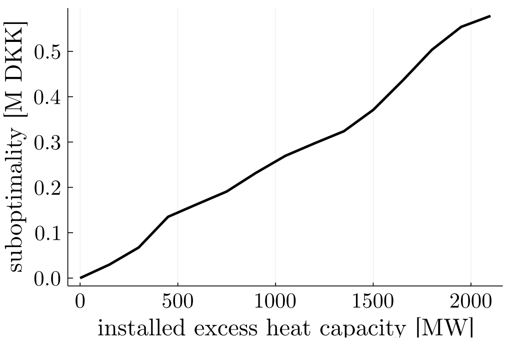
3.4.3 Main results

Figure 3.5 displays our main result, the evolution of self-scheduling suboptimality under increasing excess heat penetration, in three different ways. We study the suboptimality in terms of the total generation cost of CHP plants. The upper Figure 3.5(a) shows that the total generation cost initially decreases steeply for both the self-scheduling and market participation scheme, but this decrease flattens out afterwards. As expected, the suboptimality increases with the installed excess heat capacity. The *absolute* suboptimality is the increase in total generation cost of CHP plants, compared to the market participation model. Figure 3.5(b) shows that for the considered range of excess heat capacity, the suboptimality increases approximately linearly. There is no plateauing behavior seen in this range. However, the *relative* suboptimality, i.e. the suboptimality normalized by the total generation cost of the market participation benchmark, increases faster than linear. This can be observed in Figure 3.5(c), where the relative suboptimality increases faster with greater installed excess heat capacity. This is due to the fact that the total generation cost of the market-participation benchmark also decreases with the installed excess heat capacity.

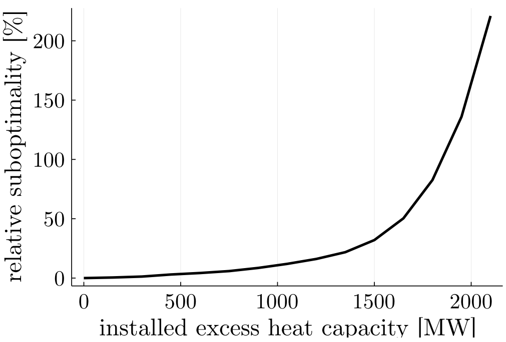
To gain insight in the cause of the suboptimality, we plot the suboptimality and wasted excess heat on a monthly resolution in Figure 3.6, for three selected levels of excess heat penetration. It shows that the suboptimality is largest in those months where excess heat capacity has the potential to cover the full load in some hours, but only does so if it is scheduled at the correct times (i.e. corresponding to the load). When excess heat almost fully covers the heat load in a certain period,



(a) Total generation cost

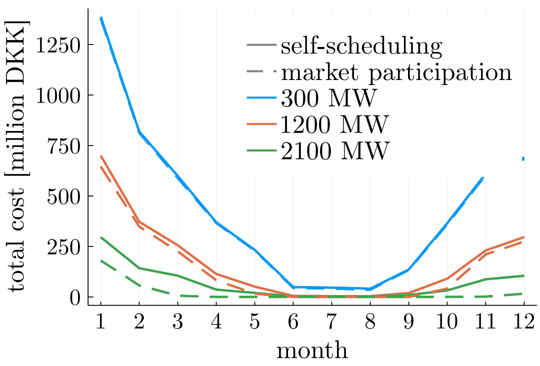


(b) Absolute suboptimality i.t.o. generation cost

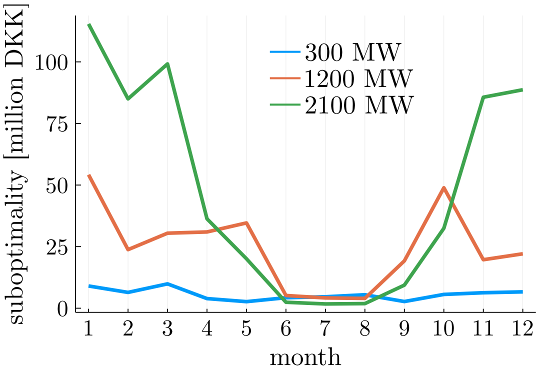


(c) Relative suboptimality i.t.o. generation cost

Figure 3.5: Comparison of total generation cost in self-scheduling and market participation models aggregated over full year, as a function of the installed excess heat capacity. Figure 3.5(a) and 3.5(b) are taken from **[Paper A]**.



(a) Total generation cost



(b) Difference in generation cost

Figure 3.6: Monthly suboptimality of self-scheduling compared to market integration in terms of total generation cost of all CHP plants in the system. Taken from **[Paper A]**.

the suboptimality decreases. This happens because the most expensive generators are pushed out of the market by cheaper excess heat, even if the excess heat is scheduled suboptimally. From this observation, it is likely that the suboptimality curve in Figure 3.5(b) will plateau when the excess heat capacity is further increased, and that the curve will bend downward at an even higher installed excess heat capacity.

Finally, we plot the monthly wasted excess heat in Figure 3.7, for the same selected levels of excess heat capacity. For both scheduling paradigms, the total wasted excess heat increases steadily with installed capacity. The increase happens first in the months with lower heat demand. There may also be waste of excess heat under the market participation scheme, because the excess heat producers have limited flexibility and may have a minimum generation capacity that exceeds the heat load. This is seen more frequently when the installed capacity increases. Under the self-scheduling mechanism, waste of excess heat is in addition caused by mismatches between scheduled excess heat production and heat load, which could be avoided if the excess heat producers would participate in the market.

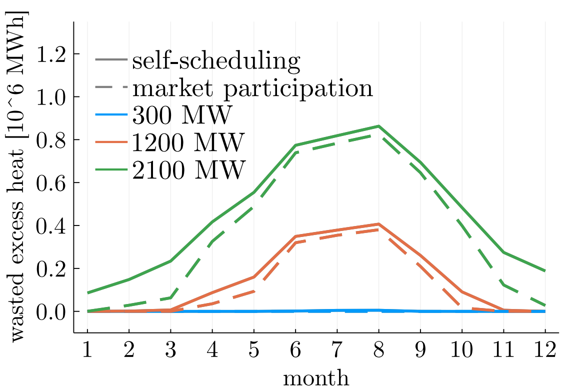


Figure 3.7: Monthly values for excess heat scheduled volume and wasted volume for self-scheduling compared to market integration. Taken from **[Paper A]**.

3.4.4 Conclusion

When scheduling and pricing excess heat producers, there is a trade-off between different objectives: simplicity and transparency in the self-scheduling model, versus complexity and optimality in the market-participation model. The self-scheduling model offers an attractive, simple solution for both market operator and excess heat producers, and it can be a transparent way of communicating prices to producers. Based on our analyses we conclude that self-scheduling may suffice at lower excess heat penetration, but is no longer adequate at higher levels. This is due to two main effects:

1. **Expensive scheduling**: Under self-scheduling, excess heat producers are not aware of the CHP plants' production cost over time, and there is no incentive to adapt their self-scheduled profile to this cost either. Therefore, the self-scheduled excess heat will generally not replace the most expensive CHP plants, so that total CHP generation cost is higher under self-scheduling than it is under market participation.

2. **Wasted excess heat**: Under the self-scheduling paradigm, excess heat producers have no incentive to match the heat load, so that the self-scheduled excess heat may exceed the load and must be wasted.

We advise that more sophisticated pricing signals and/or other market setups are used when excess heat covers a significant share of the heat load. Under lower penetrations of excess heat, self-scheduling can be considered suitable, especially in periods of high heat load. The small suboptimality compared to a perfect benchmark could be accepted, if this method would successfully attract more excess heat producers to connect to the district heating network. The total system generation cost can in some cases be lowered by the presence of excess heat producers, even when they are scheduled suboptimally.

3.5 Future perspectives

We concluded that the studied self-scheduling model no longer suffices for systems where excess heat can supply a large share of the heat load. Future work should propose and study new methods for better scheduling and pricing heat from this large number of small generators, for whom heat generation is not their main business. It would be important that scheduling procedures account for the heat load in the system, as well as the generation cost of other generators, to avoid the two negative effects that we found for the self-scheduling method studied in this chapter. One suitable option could be market participation through an *aggregator* for excess heat providers, a proposal that has been studied extensively in the context of scheduling distributed energy resources in electricity systems, see for instance [150] for a review. An aggregator would place bids in the heat market, while incentivizing excess heat providers to schedule themselves according to those bids, by sending them a price signal. In such a setup, this new market actor would be responsible for determining a suitable price signal. Different price-signal design methods have been proposed in the literature, for example using reinforcement learning [151] or bi-level optimization [152]. A specific type of aggregator that has been proposed is called a Virtual Power Plant (VPP), which are usually assumed to have access to the operational parameters and constraints of the resources it aggregates [153, 154]. Although aggregators may enable many small generators to enter the market and thereby increase competition, an aggregator with too large market share may also harm competition [150]. Furthermore, a for-profit aggregator earns a profit that could be earned by the consumers themselves, if they could organize themselves without the need for an intermediate actor.

Otherwise, if excess heat providers in the future would be able to participate in the market directly, a direction of study could be the design of a suitable bidding format for excess heat providers. The bidding format should be such that the full flexibility of these providers can be harnessed, see for example [116]. As excess heat producers are generally located in the distribution networks, the market participation of such agents may necessitate the inclusion of network considerations in the optimal dispatch. This topic is addressed in Chapter 4.

In the context of systems with a large installed capacity of excess heat, it would be interesting to investigate the benefits of installing (seasonal) heat storage. Storage systems may decrease the wasted excess heat, so that a higher share of the load could be supplied by excess heat year-round. It may also improve the performance of the self-scheduling method, as the storage could compensate for suboptimally scheduled excess heat.

For systems with low penetration of excess heat, the self-scheduling model can still be of interest. In future work, the self-scheduling model from this chapter can be improved and extended. In particular, it would be interesting to compare the performance of the Stockholm price signal to other price signals to be designed. Price signals can be designed to minimize the expected total cost, such as for instance done in [152]. Furthermore, the price that results from clearing the heat market without the excess heat producers, i.e. $\boldsymbol{\lambda}^{\mathrm{H}}$ from (3.2), would be a clear candidate for further investigation. Other price-signal designs to be used for self-scheduling could be inspired by the literature on aggregators in electricity systems.

We see several meaningful ways to improve and extend the analysis done in this chapter. First, we only analyzed the benefits of the two paradigms from a system perspective. To extend our analysis, future work should look into the individual generator perspective. Parameters of interest would be revenues and profits for excess heat producers under different scheduling and pricing methods, and payback periods of investments. In relation to this, suitable business models for these actors should be identified, similar to the work in [155] for supermarket excess heat. Second, it would be interesting to quantify the relative contribution of the two main effects causing the suboptimality as listed in the case study conclusions. Third, we considered cooling-based excess heat production only. It is likely that the system would benefit from a diversified excess heat stock in terms of daily and seasonal patterns in heat production, especially from adding non-seasonal excess heat sources such as energy intensive industries. Future work should include a more diverse set of excess heat sources. Our study could also be extended by adding heat storage to the system and quantifying its benefits.

Finally, we could extend individual modeling choices to be more detailed. For one, the modeled feasible region for CHP plants could be more complex, see e.g. [64] for an overview. In the market-participation model, the cost function for excess heat producers could be reformulated to more accurately represent their cost. There exists a profile $\hat{\boldsymbol{G}}_e^{\mathrm{opt}}$ that minimizes the cost of electricity used by the heat pump, given $\hat{\boldsymbol{\lambda}}^{\mathrm{E}}$. By definition, the scheduled excess heat profile leads to an electricity cost greater than or equal to this minimum cost. To account for this gap, the cost bid for the excess heat producer could alternatively be defined as

$$C_e^{\mathrm{H,alt}}\left(\boldsymbol{g}_e^{\mathrm{H}}\right) = (\hat{\boldsymbol{\lambda}}^{\mathrm{E}})^{\top}\left(\boldsymbol{g}_e^{\mathrm{H}} - \hat{\boldsymbol{G}}_e^{\mathrm{opt}}\right), \tag{3.12}$$

which is the difference between the (forecasted) electricity cost related to the profile scheduled by the market, and the minimum electricity cost this producer could obtain.

Chapter 4

Network-aware heat market design

This chapter presents our contribution towards the second research direction, market design for distributed future heating systems. It is based on the work in [**Paper B**]. District heating systems become more distributed with the integration of prosumers, including excess heat producers and pro-active residential and commercial consumers. The installation of many small generation capacities across the system is found to increase the burden for the network operator to balance the network hydraulics [24]. Indeed, distributed energy resources increase complexity of operation and control of energy systems, as a larger number of sources distributed over wider network areas needs to be managed [153], and the number of possible system configuration increases rapidly. Furthermore, the location of heat injection has a considerable impact on heat losses in the network [101]. However, failing to coordinate distributed energy generators may lead to network operation problems [33]. The challenge is to thus ensure reliable and efficient operation of these more complex systems, by coordinating the many agents involved and harnessing various sources of flexibility. Market mechanisms can be designed to support this more complex operation, by creating the right incentives for generators and end-consumers. To align the objectives of individual users with the objective of the system operator, previous studies have proposed to introduce network constraints in energy market clearing problems. For electricity markets, [33] finds that markets with explicit network constraints are necessary under a very high penetration of distributed energy sources. As the location of heat injection has considerable impact on network heat loss [101], network-aware heat market designs should also consider the minimization and allocation of costs associated to these losses [101].

In this chapter, we present our network-aware heat market design that is based on the Variable-Flow-Constant-Temperature (VFCT) control strategy. The VFCT formulation has been applied in an optimal dispatch setting, to prevent congestion in a distribution network with a single point of heat injection and several flexible consumers [47]. However, regarding the use of the VFCT control strategy in a market setting, several questions remain to be answered, in particular related to pricing and network losses. To fill this gap, we explore who *causes* losses in the network, which generator should *compensate* for the losses, and who should *pay* for the losses. We use *peer-to-peer trades* in our formulation to reveal the cause of network loss, and to enforce specific generators to compensate for these losses. This opens up for different ways to account for heat loss in both dispatch and pricing. In this way, we provide insight into the price formation in a VFCT based market, so that the prices become more *transparent*. For a meaningful evaluation of our proposed market design, we formulate a benchmark. Using an illustrative case study, we perform a detailed analysis of our VFCT-based heat market and show its benefits for the support of optimal operation of district heating networks.

First, we set the scene in Section 4.1 by describing the system setup considered in this chapter. Next, we motivate our choice for the VFCT formulation in Section 4.2. The VFCT model equations are then presented in Section 4.3. Our market design consists of an optimal dispatch mechanism

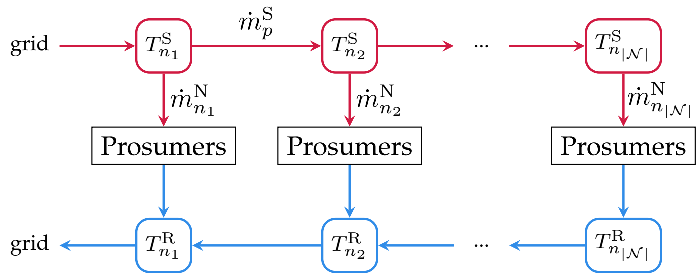


Figure 4.1: Representation of the heating system setup considered in this chapter. The district heating network consists of a supply-side network in red, and a return-side network in blue. Prosumers connect the supply and return side. The root node is connected to a greater heating grid. Nodal temperatures, nodal flows, and pipeline flows are indicated. Taken from [**Paper B**].

and a pricing method. We introduce different optimal dispatch strategies in Section 4.4, and pricing methods in Section 4.5. These sections also present our proposed benchmark market design. In Section 4.6, we present a case study to show how this network-aware market affects dispatch and pricing. Finally, Section 4.7 concludes this chapter and provides perspectives for future work.

4.1 System description

We focus on distribution-level heating networks, as it is here that the operation of distributed agents will become relevant with the integration of excess heat producers and pro-active consumers. With this in mind, we make several simplifying assumptions regarding the district heating network. A first assumption is that the district heating network is *unidirectional*, which means that the fluid flow in pipelines is unidirectional. It is important in our formulation that nodal flow is *bidirectional*, to harness flexibility from prosumer nodes. At the nodes, fluid can flow from the supply to the return side or vice versa, so that nodes are free to be net generators or net loads. Existing district heating networks are operated with unidirectional flow, so our assumption matches current practice. Although bidirectional flow is discussed as a vision for future district heating, this concept is still in an early stage of research and development [91]. Second, we assume that the heat network is *radial*, which means that no cycles exist. Of course, there are cycles when considering flow between supply and return side as well – the network is thus considered radial on the supply side and return side individually. Figure 4.1 shows an example of the considered network setup. This particular example heat network has no branches, but in general our setup does include branched networks.

In our setup, the *root node* of the local heat distribution system is connected to a greater heat grid, that may supply heat for a given import price. Due to the unidirectionality, heat cannot be exported to the greater grid. The heat import from the grid is assumed unlimited. However, this quantity is limited indirectly as a result of network constraints.

The district heating network consists of a supply and a return side, as also indicated in Figure 4.1. The prosumers in this network may be net generators or consumers at different points in time. A net generator injects heat by heating up cold fluid while it is flowing from the return side to the supply side. The supply pipelines then transport the heated fluid to net consumers, which consume heat by extracting heat from the hot fluid while it flows from the supply-side to the return-side network. The supply side of the district heating network is described by a directed graph $(\mathcal{N}, \mathcal{P})$, with a set of nodes $\mathcal{N}$ connected by pipes $\mathcal{P} \subseteq \mathcal{N} \times \mathcal{N}$. It is assumed that the supply and return networks are identical, so that it suffices to describe the supply-side network configuration only. Any pipe $p \in \mathcal{P}$ is defined by its supply-side start and end node, i.e., if $p = (n_1, n_2)$, then the flow in the supply side of pipe p goes from node n_1 to node n_2. For some parameters and variables, separate values for supply and return side are needed. For example, nodal supply and return temperatures will differ. Therefore, we use superscripts S and R to denote variables and parameters related to the supply and return side of each node and pipe.

The system state is described by nodal supply T_n^{S} and return temperatures T_n^{R}, nodal mass flow rates $\dot{m}_n^{\mathrm{N}}$ (our convention is that these are positive when flowing from supply to return side), and unidirectional pipeline mass flow rates $\dot{m}_p^{\mathrm{S}}$. Due to mass conservation and symmetry of the network, the mass flow rates in the supply and return side of a pipe are equal. Therefore it suffices to consider only the supply-side flow. These quantities are indicated in Figure 4.1.

4.2 Choice of network model and control strategy

The different network modeling approaches and control strategies as discussed in Section 2.4 each have their advantages and weaknesses. Therefore, the modeler should define what properties are most important in the envisioned application, and choose the most suitable model accordingly. In a market setting, convex models are preferred. The main reason is that the dual variables in such formulations have meaningful interpretations for pricing, and can be used to formulate a simple and transparent pricing mechanism with attractive market properties. Furthermore, in non-convex models there is no guarantee that current commercial solvers can find a global optimum, and the problems can quickly become computationally intractable for larger systems. For these reasons, we choose not to use the Variable-Flow-Variable-Temperature (VFVT) control strategy in our market framework. The drawbacks of convex relaxations of the VFVT formulation were discussed in more detail in Section 2.4.

We aim to design a network-aware heat market that integrates *distributed generators* and *prosumers*, while minimizing operational cost. Thus, the chosen network model must harness flexibility from distributed producers and prosumers, as well as estimate and minimize heat losses in the network. Considering these specifications, we choose to use a VFCT control strategy in our optimization. This strategy allows for much more flexible production and consumption at distributed network nodes, compared to the Constant-Flow-Variable-Temperature (CFVT) formulation with fixed flows. In the latter, both the sign and size of nodal flows are fixed before the optimization, so that it is decided in advance whether the node will be net producer or consumer, and a minimum heat injection or extraction is also assigned. We argue that this lack of flexibility is not acceptable in future heating systems. The choice for constant temperatures is justified by several studies. In [124] it is found that for mass flow rates in the usual range, heat loss is nearly independent of mass flow rate. This supports the choice for the VFCT control strategy, because it results in constant loss factors between any pair of nodes, as further discussed in Section 4.3.4. Experimental data from e.g. [77] also supports the choice for constant nodal temperatures in the model. The VFCT formulation is able to capture network heat losses in a simplified manner. Therefore, we call the market *loss-aware*. To allow for distributed heat injections, we must omit pressure and pumping power constraints, as the linearization used in [47] is only suitable for systems with a single injection point.

This formulation does not exploit the network storage effect, so this source of flexibility is not utilized. Leaving out the network-storage effect is justified by the finding that network storage flexibility is limited compared to the flexibility potential from buildings [92]. Furthermore, we argue that the CFVT formulation only includes a limited form of network storage. As the flows are fixed in that case, the delays are fixed too. Therefore, it is known for each supplier at what time the injected heat arrives at a certain consumer. Thus, the only way to change the amount of energy in the network while meeting a non-flexible load is by scheduling a supplier that is located a longer distance away.

4.3 Network model

We now present our VFCT-based heating network model, to be used in the market design in the next section. First, we discuss the simplifying assumptions needed to derive this network model from a full VFVT model. Next, we formulate the model equations. As the choice of temperature parameters in this model has a big effect on the outcome, we devote a section to this. Finally, we derive the multiplicative losses.

4.3.1 Assumptions for derivation

The VFCT network model can be derived from a full VFVT model using several simplifying assumptions, similar to the way the DCOPF is derived from the ACOPF to model power grid dynamics.

On the supply side of a radial heat network, constant nodal temperatures follow from two assumptions. First, it is assumed that pipeline heat loss is independent of the size of the flow. This means that no matter how fast the fluid is flowing through pipeline p, it will cool down by temperature gradient ΔT_p. We add a second assumption stating that the injection temperature of supplied heat is fixed for each node. These nodal injection temperatures can be fixed in such a way that the pipeline temperature gradients ΔT_p^{S} are respected. This is done by selecting the injection temperature at a single node, and deriving all other nodal temperatures using the pipeline gradients. For the supply side, the constant-temperature formulation can be derived from these two assumptions.

On the return side, these assumptions do not suffice, because due to the tree-like structure of the network and the reversed flow directions, mixing of fluid flowing from different pipelines may occur. One way to ensure constant temperatures on the return side would be to assume absence of return-side pipeline losses, combined with a nodal return-side injection temperature equal for all nodes. One may also derive a consistent set of return-side nodal temperatures by specifying ΔT_p^{R} for each pipeline, and setting the return-side temperature of the root node. The rest of the nodal temperatures for the return side can then be derived. In this way, the mixing equation becomes trivial, because it is always a mixture of several flows of the same temperature.

Finally, we omit pressure considerations, by leaving out the equation that relates pipeline flow and pressure loss, as well as the pumping power equation. In [77], an approximation of the pumping power equation is formulated, but this is only suitable for networks with a single generating node. For our purposes, we assume that the energy used for pumping (overcoming pressure loss) is negligible compared to heat losses.

4.3.2 Model equations

We consider a market horizon with time periods t collected in the set T. The network consists of a set of nodes n collected in $\mathcal{N}$ and pipelines p collected in $\mathcal{P}$. The network model variables include the nodal flows $\dot{m}^{\mathrm{N}}_{nt}$ for each node n and time t, and the pipeline flows $\dot{m}^{\mathrm{S}}_{pt}$ for each supply pipeline p and time t. Due to mass conservation and network symmetry, the mass flow rates in the supply and return side of a pipe are equal, so that we only need to model one of them. We collect the heating network variables in the set $\Gamma^{\mathrm{DHN}} = \{\dot{\boldsymbol{m}}^{\mathrm{N}}, \dot{\boldsymbol{m}}^{\mathrm{S}}\}$. The nodal supply and return side temperatures, T^{S}_n and T^{R}_n for all $n \in \mathcal{N}$ are parameters in our model. These quantities do not have a time index, because they are assumed to be *constant in time*. As a result, time delays can be neglected, which greatly simplifies the model as it eliminates the need for binary variables.

The model variables are bounded. As the pipeline flow is unidirectional, it must be positive. Furthermore, an upper bound is specified:

$$0 \leq \dot{m}^{\mathrm{S}}_{pt} \leq \overline{\dot{M}}^{\mathrm{S}}_{p} \,. \tag{4.1}$$

The bidirectional nodal flows can be above and below zero. In both directions, the size of this flow is upper bounded:

$$-\underline{\dot{M}}^{\mathrm{N}}_{n} \leq \dot{m}^{\mathrm{N}}_{nt} \leq \overline{\dot{M}}^{\mathrm{N}}_{n} \,. \tag{4.2}$$

We take the convention that the nodal flow $\dot{m}^{\mathrm{N}}_{nt}$ is positive towards the return side.

Conservation of mass at each node is ensured by the *mass-flow balance equation*:

$$\sum_{p \in \mathcal{S}^-_n} \dot{m}^{\mathrm{S}}_{pt} - \sum_{p \in \mathcal{S}^+_n} \dot{m}^{\mathrm{S}}_{pt} = \dot{m}^{\mathrm{N}}_{nt} \,, \tag{4.3}$$

where $\mathcal{S}^-_n$ and $\mathcal{S}^+_n$ are the respective sets of pipelines whose supply sides end and start at node n.

Every node has a heat exchanger that injects locally produced heat into the network in case the node is a net generator, and extracts heat from the network in case the node is a net consumer. The nodal power injection (negative when heat is extracted) and nodal flow are related as

$$\sum_{i \in \mathcal{I}_n} p^{\mathrm{H}}_{it} = -C_{\mathrm{f}}\, \dot{m}^{\mathrm{N}}_{nt} \, (T^{\mathrm{S}}_n - T^{\mathrm{R}}_n) \,, \tag{4.4}$$

where C_{f} is the heat capacity of the fluid, and p^{H}_{it} is the heat injection from prosumer i at time t. The latter variable will be further discussed in relation to our case study, where we provide a prosumer model. We generally use the superscript H to refer to heat-related quantities, whereas the superscript E is used for electricity-related quantities. This equation contains the assumption that nodes must inject heat at T^{S}_n when producing, and after extracting heat the fluid must be cooled down to T^{R}_n.

4.3.3 Choice of nodal temperatures

When using the VFCT formulation, it is important to carefully select meaningful values of T^{S}_n and T^{R}_n, with the following two principles in mind. First, the supply-side temperature at a node must be greater than its return-side temperature. Second, as the nodal temperatures determine temperature losses in each pipeline, the pipeline connections and their directions must be taken into account when choosing nodal temperatures values. In particular, nodal temperatures should be non-increasing along the flow direction of the supply-side pipelines, and the same holds for the return side. For example, if there is a supply pipeline connecting n_1 and n_2, then the downstream node n_2 should have a supply temperature lower than upstream node n_1. The nodal temperatures may be selected based on measurements, as in [47]. Otherwise, one may estimate temperature losses as in [156] (Equations 7 and 8), or use average mass flow and temperature loss equations as in [128] (Equation 6).

4.3.4 Multiplicative losses

We show below that the VFCT formulation leads to multiplicative heat loss in the network. This means that it is fixed what share of injected heat in a certain pipeline is lost while the heat carrier flows to the next node. We add equations to our network model, in order to provide more insight in pipeline heat loss. In the next section, we will use this derivation to formulate a peer-to-peer model, that allows us to allocate losses to market participants.

Heat is lost in pipelines when the fluid travels between nodes. Equations (4.3) and (4.9) implicitly model those losses. Let nodes n_1 and n_2 be two arbitrary nodes from $\mathcal{N}$. If $n_1 = n_2$, there is no loss related to the energy sold, so assume $n_1 \neq n_2$, and assume n_1 is upstream from n_2. In that case, generated energy from n_1 can be sold to n_2. If a generated amount $g^{\mathrm{H}}_{n_1}$ is injected at n_1, we have

$$g^{\mathrm{H}}_{n_1} = C_{\mathrm{f}}\, \dot{m}_{n_1} \, (T^{\mathrm{S}}_{n_1} - T^{\mathrm{R}}_{n_1}) \, . \tag{4.5}$$

By conservation of mass, $\dot{m}_{n_1} = -\dot{m}_{n_2}$. Thus, a heat load $d^{\mathrm{H}}_{n_2}$ of

$$d^{\mathrm{H}}_{n_2} = -p^{\mathrm{H}}_{n_2} = -C_{\mathrm{f}}\, (-\dot{m}_{n_1}) \, (T^{\mathrm{S}}_{n_2} - T^{\mathrm{R}}_{n_2}) \, . \tag{4.6}$$

is consumed at n_2. The share of energy lost on the way, given by *loss factor* $W_{n_1 n_2}$, is thus

$$\begin{aligned} W_{n_1 n_2} &= \frac{g^{\mathrm{H}}_{n_1} - d^{\mathrm{H}}_{n_2}}{g^{\mathrm{H}}_{n_1}} = \frac{C_{\mathrm{f}}\, \dot{m}_{n_1} \, (T^{\mathrm{S}}_{n_1} - T^{\mathrm{R}}_{n_1}) - C_{\mathrm{f}}\, \dot{m}_{n_1} \, (T^{\mathrm{S}}_{n_2} - T^{\mathrm{R}}_{n_2})}{C_{\mathrm{f}}\, \dot{m}_{n_1} \, (T^{\mathrm{S}}_{n_1} - T^{\mathrm{R}}_{n_1})} \\ &= 1 - \frac{T^{\mathrm{S}}_{n_2} - T^{\mathrm{R}}_{n_2}}{T^{\mathrm{S}}_{n_1} - T^{\mathrm{R}}_{n_1}} \, . \end{aligned} \tag{4.7}$$

In the final expression, the flow cancelled out, so that the share of energy lost between the two nodes is independent of this variable. Note that the loss factor is *negative* if n_2 is upstream from n_1. Equivalently, we can denote the relationship between the generation and load by

$$(1 - W_{n_1 n_2}) \, g^{\mathrm{H}}_{n_1} = d^{\mathrm{H}}_{n_2} \, . \tag{4.8}$$

We can conclude that, as a result of the constant nodal temperature assumption, the losses become *multiplicative*. The loss factor for each pair of nodes is constant, i.e., it does not depend on any variables.

4.4 Optimal dispatch formulation

Here, we proceed to present our proposed optimal dispatch formulations. A schematic overview of the general format of the optimization problem and its components is given in Figure 4.2. We specify the bidding format, including the form of generator and consumer feasible regions, in Section 4.4.1. In Section 4.4.2, the peer-to-peer trades are discussed. The network model was

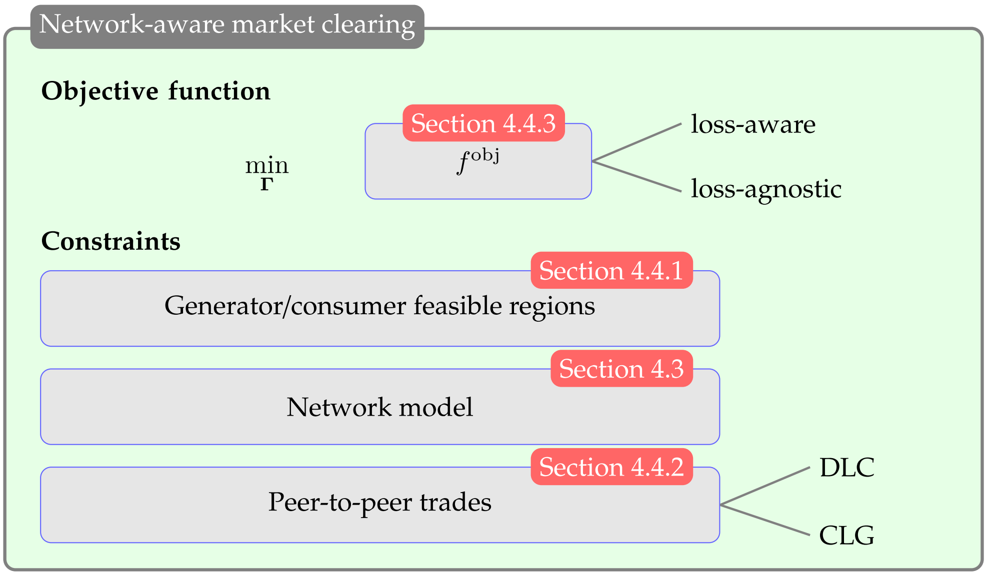


Figure 4.2: Schematic overview of the network-aware market formulations and its components. For each component, the relevant section is indicated. Γ is the set of decision variables and f^{obj} is the objective function.

presented in the previous section. For a summary of the full optimization problems, we refer to [**Paper B**].

By varying two settings in the formulation, we propose three different optimal dispatch strategies. All three are *network-aware*, meaning that their dispatch is feasible in the network (assuming that our network model is accurate). The first setting to be varied is Centralized Loss Generation (CLG) versus Decentralized Loss Generation (DLG), determining which generator(s) inject heat to compensate for network losses. This is discussed in Section 4.4.2 in relation to the formulation of peer-to-peer trades, which allow us attribute network losses to particular agents. The second setting that can be varied is *loss-awareness* versus *loss-agnosticism*, which can be specified in the objective function, as discussed in Section 4.4.3. An overview of the resulting three strategies is given in Table 4.1. Although these two settings could produce four different dispatch strategies, we exclude one of the four options, as indicated in Table 4.1. The loss-agnostic Decentralized Loss Generation (DLG) strategy is unlikely to be applicable in reality, does not mimic any existing market designs, and results in counter-intuitive dispatch and prices. The latter is due to the fact that the loss generation is competing with energy generation for the limited capacity local generators.

The aim in this chapter is to show the benefits of the loss-aware dispatch. The loss-agnostic dispatch is used as a *benchmark*, which also resembles current practice in heat markets, as these usually disregard network constraints and operational cost. However, if the benchmark would disregard network constraints completely, a fair comparison to our proposed loss-aware market would not

	DLG	**CLG**
loss-aware	loss-aware DLG	loss-aware CLG
loss-agnostic	loss-agnostic DLG	loss-agnostic CLG

Table 4.1: Overview of considered dispatch strategies. Taken from [**Paper B**].

be possible, as the benchmark dispatch would not respect network constraints. Therefore, we formulate a benchmark with the same feasible space as the proposed dispatch, but with a different, loss-agnostic objective function.

4.4.1 Bidding format

In this section, we specify the bid format used in our market design for market participants. The set $\mathcal{I}$ contains all market participants (excluding the grid agent) that are present in the considered heating network. Each agent i has an associated location $n_i \in \mathcal{N}$, and the set $\mathcal{I}_n$ collects all agents located at node n. The index g is used for the grid agent. All market participants are to submit generation and load bids of a price-quantity format. The load bids are extended in our formulation to include load shifting flexibility, as we will specify below.

We model the total heat injection p_{it}^{H} for each market participant as the difference between hourly heat generation g_{it}^{H} and load d_{it}^{H}:

$$p_{it}^{\mathrm{H}} = g_{it}^{\mathrm{H}} - d_{it}^{\mathrm{H}} \tag{4.9}$$

Here, the heat load is modelled to consists of two parts: an inflexible domestic hot water load $\hat{D}^{\mathrm{DHW}}$ and a partially flexible space heating load d^{SH},

$$d_{it}^{\mathrm{H}} = d_{it}^{\mathrm{SH}} + \hat{D}_{it}^{\mathrm{DHW}}\,. \tag{4.10}$$

The flexibility in space heating load is represented using a reference profile $\hat{D}_{it}^{\mathrm{SH}}$ and a maximum flexibility $\overline{F}_i$. At every time step, the space heating load of agent i deviates by at most $\overline{F}_i$ from the reference profile, i.e.

$$\max\{\hat{D}_{it}^{\mathrm{SH}} - \overline{F}_i, 0\} \leq d_{it}^{\mathrm{SH}} \leq \hat{D}_{it}^{\mathrm{SH}} + \overline{F}_i\,, \tag{4.11}$$

where the lower bound also ensures non-negativity of the space heating load. In addition, an energy budget constraint ensures that space heating load may be shifted to a different hour, but is satisfied at some point within the considered market horizon:

$$\sum_{t\in\mathcal{T}} d_{it}^{\mathrm{SH}} = \sum_{t\in\mathcal{T}} \hat{D}_{it}^{\mathrm{SH}}\,. \tag{4.12}$$

Finally, the load bid must be accompanied by a utility function $u_{it}(d_{it}^{\mathrm{SH}})$, which specifies for each time t the utility of space heating consumption. In summary, the bid format for heat load is $\big(\hat{D}_{it}^{\mathrm{DHW}}, \hat{D}_{it}^{\mathrm{SH}}, \overline{F}_i, u_{it}(\cdot)\big)$.

Related to generation, the market participants have to submit upper bounds $\overline{G}_{it}^{\mathrm{H}}$, which are enforced by the constraint

$$0 \leq g_{it}^{\mathrm{H}} \leq \overline{G}_{it}^{\mathrm{H}}\,. \tag{4.13}$$

The generation bid must include a cost function $c_{it}^{\mathrm{H}}(g_{it}^{\mathrm{H}})$ that specifies the generation cost at each time. The bid format for heat generation is thus $\big(\overline{G}_{it}^{\mathrm{H}}, c_{it}^{\mathrm{H}}(\cdot)\big)$.

The variables related to market participants are collected in the set $\Gamma^{\mathrm{agent}} = \{\boldsymbol{p}^{\mathrm{H}}, \boldsymbol{d}^{\mathrm{H}}, \boldsymbol{d}^{\mathrm{SH}}, \boldsymbol{g}^{\mathrm{H}}\}$.

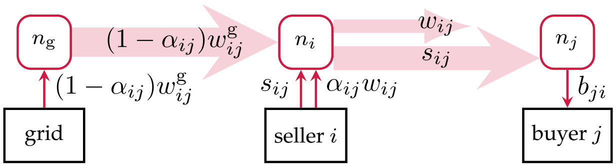


Figure 4.3: Visualisation of peer-to-peer variables related to the trade $\tau_{ij} = s_{ij} = b_{ji} = -\tau_{ji}$. If $\alpha = 1$, the seller generates the losses, while this is done by the grid in case $\alpha = 0$. Taken from **[Paper B]**.

4.4.2 Peer-to-peer trades

We include peer-to-peer trades in our market design with the main aim of relating specific generators and consumers to heat loss in the network. In general, peer-to-peer trades have also been used in the literature to enable market participants to negotiate bilateral trades without interference of a market operator. The framework presented here can also be used for such bilateral negotiation. Our peer-to-peer formulation extends a common peer-to-peer setup, as described in for example [157]. To this formulation we add an explicit loss representation, as well as a set of constraints to prevent arbitrage. We emphasize that the addition of these peer-to-peer trades does not change the optimal dispatch, as we do not consider any preferences with respect to trading partners. The same dispatch could therefore be obtained using an equivalent pool market formulation. The peer-to-peer trades are here added solely for the purpose of connecting heat losses in the network to specific trading partners.

A trade τ_{ijt} between agent i and j at time t is defined to be positive if i sells heat to j, and negative if the reverse holds. Trades with the grid agent g are also possible. We decompose τ_{ijt} into sales $s_{ijt} \geq 0$ and buys $b_{ijt} \geq 0$ as

$$\tau_{ijt} = s_{ijt} - b_{ijt} \,. \tag{4.14}$$

The following constraint ensures trade reciprocity.

$$s_{ijt} = b_{jit} \,. \tag{4.15}$$

Self-consumption of produced energy is given by the variables $b_{iit} = s_{iit}$.

In our definition, trades τ equal the heat energy received by the buyer, *excluding* any network heat loss that may be associated to that trade. The estimated losses w_{ij} related to the trade must be generated by some agent. We propose two different ways of selecting this producer: *distributed loss generation* (DLG) and *centralized loss generation* (CLG). The binary *parameter* $\alpha_{ij} \in \{0, 1\}$ indicates whether the seller of trade τ_{ij} will produce the associated losses ($\alpha_{ij} = 1$) or whether grid import will compensate for these losses ($\alpha_{ij} = 0$). This binary parameter allows us to capture CLG and DLG in a single set of equations. Figure 4.3 visualizes the relationship between the different trade-related variables, including losses w and w^g that we introduce next.

The network losses related to trade τ_{ij} can be computed using the nodal loss factor $W_{n_i n_j}$ from Equation 4.7:

$$w_{ijt} = W_{n_i n_j} \, s_{ijt} \,, \tag{4.16}$$

where w_{ijt} is the energy that is lost between n_i and n_j. This quantity is used in the DLG formulation. Losses must be positive,

$$w_{ijt} \geq 0\,. \tag{4.17}$$

As a result, w_{ij} is the network loss between n_i and n_j associated with the trade τ_{ij} in case the energy is flowing from n_i to n_j, i.e., i is the seller. If instead energy is flowing from n_j to n_i and i is the buyer, then $w_{ij} = 0$ while now $w_{ji} \geq 0$ represents the loss associated with this trade.

Under the CLG formulation, the grid agent must compensate for this loss by injecting an amount of energy w^{g}_{ijt} at the grid connection node. Intuitively, this injection of the grid agent should result in an amount of w_{ijt} arriving at the seller node, as shown in Figure 4.3. If $w_{ijt} > 0$, i is the seller of the trade τ_{ijt}. Therefore, an amount of w_{ijt} must arrive at node i. This means that the grid agent must produce

$$w^{\mathrm{g}}_{ijt} = \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, w_{ijt} = \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, W_{ij}\, s_{ijt}\,.$$

Again, the loss must be positive, i.e.

$$w^{\mathrm{g}}_{ijt} \geq 0\,. \tag{4.18}$$

So far, this peer-to-peer formulation allows for arbitrage. If nonzero pipeline losses are considered, this arbitrage would only occur between different agents at a single node. However, we would like to exclude arbitrage completely, so that the resulting trades give insight in the energy that actually flows between agents. To prevent arbitrage, we include the constrain that agents must buy an amount exactly equal to their heat consumption,

$$\sum_{j\in\mathcal{I}} b_{ijt} = d^{\mathrm{H}}_{it}\,, \tag{4.19}$$

where we emphasize that the sum includes self-consumption b_{ii}. As a result, reselling of purchased energy (i.e. arbitrage) is prevented. Similarly, we formulate an arbitrage-preventing constraint for generation:

$$\sum_{j} s_{ijt} + \alpha_{ij} w_{ijt} = g^{\mathrm{H}}_{it}\,. \tag{4.20}$$

The grid agent must produce

$$\sum_{ij}(1-\alpha_{ij})w^{\mathrm{g}}_{ijt} + \sum_{j}(s_{\mathrm{g}jt} + \alpha_{ij}\, w_{\mathrm{g}jt}) = g^{\mathrm{H}}_{\mathrm{g}t}\,, \tag{4.21}$$

where w^{g}_{ij} is the amount the grid agent must inject to compensate for the losses in the trade τ_{ij}. The primal variables related to the peer-to-peer trading are collected in the set $\Gamma^{\mathrm{p2p}} = \{\tau, b, s, w, w^{\mathrm{g}}\}$.

4.4.3 Loss-aware and loss-agnostic dispatch

Our loss-aware and loss-agnostic dispatch differ only in the objective function f^{obj} of the respective optimization problems. Both markets have the objective to maximize some form of social welfare. In the loss-aware dispatch, the cost of producing losses is included in the objective function, so that this is minimized alongside with production cost, while the loss-agnostic dispatch does not take these costs into account. This leads to the following definition of the loss-aware objective function:

$$f^{\text{awa}} = \sum_{t\in\mathcal{T}} \left(c^{\text{H}}_{\text{gt}}(g^{\text{H}}_{\text{gt}}) + \sum_{i\in\mathcal{I}} \left(-u_{it}(d^{\text{SH}}_{it}) + c^{\text{H}}_{it}(g^{\text{H}}_{it}) \right) \right) , \tag{4.22}$$

where c^{H}_{gt} is the cost function of grid imports at time t. This expression includes total production cost consisting of the cost of energy sold to a consumer and the cost of producing losses.

For the loss-agnostic benchmark, we artificially remove the cost of produced losses from the objective function, to consider the production cost of consumed load only:

$$\begin{aligned} f^{\text{agn}}(g^{\text{H}}_{it}, w_{ijt}) = &\sum_{t\in\mathcal{T}} c^{\text{H}}_{\text{gt}} \left(g^{\text{H}}_{\text{gt}} - \sum_{j\in\mathcal{I}} w_{\text{g}jt} - \sum_{i,j\in\mathcal{I}} (1-\alpha_{ij})\, w^{\text{g}}_{ijt} \right) + \\ &\sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{I}} \left(-u_{it}(d^{\text{SH}}_{it}) + c^{\text{H}}_{it} \left(g^{\text{H}}_{it} - \sum_{j\in\mathcal{I}} \alpha_{ij} w_{ijt} \right) \right) . \end{aligned} \tag{4.23}$$

After loss-agnostic dispatch, the cost of losses can be determined a posteriori, which will be discussed in more detail in Section 4.5. If the agent that should generate the losses is not specified in the loss-agnostic case, there will in general not be a unique solution to the optimization problem. This is another reason for us to specifically assign 'loss generators' using the peer-to-peer trades.

4.5 Pricing and loss allocation

After finding the optimal schedule using one of the dispatch strategies, an *allocation mechanism* is needed to determine payments and revenues for all market participants. We propose two allocation mechanisms, that differ in the way they allocate the cost of network heat losses. Both mechanisms are based on the principle of nodal marginal pricing. First, we discuss how the cost of generated losses can be determined for the three different dispatch mechanisms. Next, we propose two *loss allocation mechanisms*, i.e., mechanisms for allocating this cost to the market participants.

Determining the cost of loss

Nodal marginal prices are denoted by π^{N}_{nt}, and seller i and buyer j marginal price by π^{s}_{it} and π^{b}_{jt} respectively. We derive expressions for the nodal prices in the Appendix of [**Paper B**]. It is shown that the price per unit of loss and price per unit of consumed energy are equal. Since the heat loss associated to a unit of consumed energy increases with distance from the generator, so does the the loss cost per unit of energy consumed. In Appendix A.2 of [**Paper B**], it is derived that seller i receives unit price $\mu^{\text{S}}_{it} + \mu^{\text{inj}}_{it}$, whereas buyer j pays $-\mu^{\text{B}}_{jt} + \mu^{\text{inj}}_{jt}$ per unit consumed. For loss-aware DLG, it is shown that these prices relate as

$$\pi^{\text{b}}_{jt} = (1 + W_{ij})\pi^{\text{s}}_{it} . \tag{4.24}$$

From this, we derive that the cost of loss $C^{\text{L,DLG}}_{ijt}$ connected to energy sale s_{ijt} on the loss-aware DLG market is given by

$$C^{\text{L,DLG}}_{ijt} = w_{ijt}\, \pi^{\text{s}}_{it} . \tag{4.25}$$

Similarly, in Appendix A.3 of paper [**Paper B**] it is derived for the loss-aware CLG market that the prices of seller and buyer relate as

$$\pi^{\mathrm{b}}_{jt} = \pi^{\mathrm{s}}_{it} + \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}} W_{ij}\pi^{\mathrm{s}}_{\mathrm{g}t}\,. \tag{4.26}$$

Thus, the cost of loss $C^{\mathrm{L,CLG}}_{ijt}$ connected to sale s_{ijt} in the loss-aware CLG market is given by

$$C^{\mathrm{L,CLG}}_{ijt} = w^{\mathrm{g}}_{ijt}\pi^{\mathrm{s}}_{\mathrm{g}t}\,. \tag{4.27}$$

This means that the price of loss is equal to the nodal price at the grid connection.

For loss-agnostic CLG, a derivation in Appendix A.4 of [**Paper B**] shows that in the absence of network congestion, the buyer and seller of any trade experience the same marginal price, no matter the distance between them. The formulation thus successfully removes the connection between the buyer and the cost of losses it causes. In the loss-agnostic case, the cost of loss is not represented by any dual variable, but it can be computed in hindsight. Similarly to the loss-aware markets, we set the price of loss in the loss-agnostic case to the nodal price of the generator of the loss, which in this CLG setting is the grid node. As a result, the ex-posteriori computation of loss cost connected to sale s_{ijt} is according to (4.27).

Allocating the cost of loss

After an optimal schedule is determined, the cost of losses can be computed, and this cost needs to be allocated to market participants. One desirable property of a loss allocation scheme is *budget balance*, which means that the payments for loss add up to the cost of loss. In this way, the network operator does not suffer a loss or earn a profit.

We propose two budget balanced loss allocation policies: *individual* and *socialized*. In the former, loss cost are allocated to the heat consumer that is causing them, while loss cost are shared evenly under the latter policy. The socialization could be done in different ways. Here, we define the socialized loss cost per unit consumed as the average over the entire considered market horizon, which results in a single unit price of loss for all time periods t. This choice mimics common network and loss charges in practice, where the network and loss costs are usually even averaged over an entire year and then socialized in a constant grid tariff. Our socialized loss allocation policy is a *pro rata* method, as described in [158] for electrical losses. In our case, 100% of the loss cost is allocated to the consumers. Any pro-rata procedure is network-agnostic, as loads near generating nodes pay the same loss price per unit consumed as distant loads [158].

For our loss-aware markets, the individual loss allocation policy follows naturally from the nodal prices. Therefore, the payments from this allocation policy support the schedule resulting from the loss-aware dispatch. This is not the case for the socialized loss allocation. This means that the loss-aware dispatch would be different if the market participants would be able to anticipate the socialized loss allocation. For the loss-agnostic market, the lack of dispatch support holds for both individual and socialized loss allocation. For simplicity, we assume in this thesis that the agents are unable to anticipate the loss allocation post-processing.

The socialized cost of loss per consumed unit of heat for DLG is

$$\pi^{\mathrm{L,DLG}} = \frac{\sum_{i,j,t} w_{ijt}\pi^{\mathrm{N}}_{n_i t}}{\sum_{i,t} d^{\mathrm{H}}_{it}}\,, \tag{4.28}$$

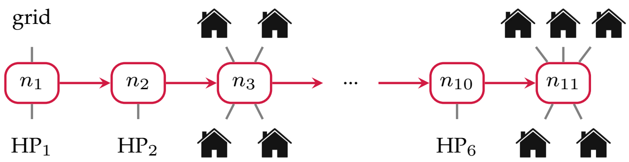


Figure 4.4: Layout of the case study district heating system. Nodes $1, 2, 4, 6, 8, 10$ each contain a single prosumer with a heat pump (marked with HP). Nodes $3, 5, 7$ each contain 4 flexible consumers, and nodes $9, 11$ each contain 5 flexible consumers. There is a grid connection at node 1. The flow is unidirectional, directed from the supply side of node 1 to 11. The return side is equal to the displayed supply network, with reversed flow directions. This figure is taken from [**Paper B**].

and for CLG it is

$$\pi^{\mathrm{L,CLG}} = \frac{\sum_{i,j,t} w^{\mathrm{g}}_{ijt} \pi^{\mathrm{N}}_{n_{\mathrm{g}}t}}{\sum_{i,t} d^{\mathrm{H}}_{it}} \,. \tag{4.29}$$

4.6 Illustrative case study

In this case study we aim to illustrate the properties of our proposed network- and loss-aware market frameworks. While [**Paper B**] includes two variants of this case study, we focus on a single variant here. First, Section 4.6.1 present the input parameters used. We have chosen input parameters that clearly show the possible effects of loss-aware dispatch. Although these parameters therefore do not correspond to any specific real system, their values are within realistic ranges. We present and discuss the case study results in Section 4.6.2. Our online repository [159] contains an implementation of the three market variations in Julia [160]. This case study can be reproduced using the repository. It also includes an example analysis of the market outcomes.

4.6.1 System description

This case study considers an hourly day-ahead market consisting of 24 hours. Figure 4.4 shows the considered supply network topology. A set of 11 nodes is connected by a pipeline network without any branches, although branches are allowed in our framework. The constant nodal temperatures are fixed by setting the supply temperature at n_1 to $T^{\mathrm{S}}_{n_1} = 90°\mathrm{C}$, the return temperature at n_{11} to $T^{\mathrm{S}}_{n_1} = 40°\mathrm{C}$, combined with the assumption of $0.1\,\mathrm{K\,m^{-1}}$ temperature loss on the supply side and $0.05\,\mathrm{K\,m^{-1}}$ on the return side.

We distribute 28 prosumers over the nodes in the system. Six of these prosumers may generate heat using local heat pumps, while the remaining 22 prosumers only consume heat. All even nodes and node 1, i.e. node $1, 2, 4, 6, 8, 10$, contain one of the generating prosumers, as indicated in Figure 4.4. The remaining agents without production are distributed over the remaining nodes. On the consumption side, we use measurements from the Nordhavn neighbourhood in Copenhagen, collected in the EnergyLab Nordhavn project [161], to create unique Domestic Hot Water (DHW) and space-heating reference profiles for each prosumer. The following quadratic utility functions are applied:

$$u_{it}\left(d^{\mathrm{SH}}_{it}\right) = -\tilde{u}_{it}\left(d^{\mathrm{SH}}_{it} - \hat{D}^{\mathrm{SH}}_{it}\right)^2 \,, \tag{4.30}$$

i.e., the utility is inversely proportional to the squared deviation from the reference space heating profile.

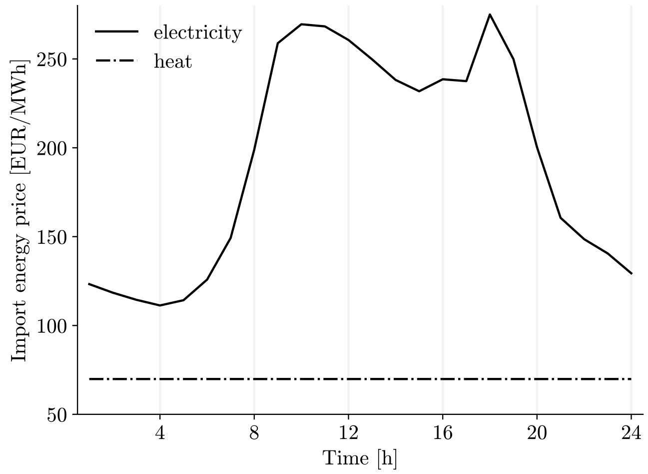


Figure 4.5: Heat and electricity import prices used in case study. Taken from [**Paper B**].

The generated heat from the heat pump of agent i is linearly proportional to its electricity load $d_{it}^{\mathrm{E,hp}}$:

$$g_{it}^{\mathrm{H}} = \mathrm{COP}_i \, d_{it}^{\mathrm{E,hp}} \,, \tag{4.31}$$

where the proportionality is given by the Coefficient of Performance (COP) of this heat pump. The COPs are considered constant in this chapter. As a result, the cost bid of these agent linearly depends on the electricity price $c_{\mathrm{g}t}^{\mathrm{E}}$ as

$$c_{it}\left(g_{it}^{\mathrm{H}}\right) = \frac{1}{\mathrm{COP}_i} \, c_{\mathrm{g}t}^{\mathrm{E}} \, g_{it}^{\mathrm{H}} \,. \tag{4.32}$$

The maximum generation capacity is the same for all six generating prosumers, but their heat pump COP differ. The six heat pump COP values range linearly from 3.27 to 3.46. The heat pumps are placed so that the marginal production cost increases (and thus, COP decreases) with distance from the grid connection. The heat source at node 1 will thus have the lowest generation cost, followed by node 2, node 4, etc.

The used electricity and heat import price curves are plotted in Figure 4.5. Consumers in many countries, including Denmark, can enter contracts with a variable electricity price, and this is assumed in the current work. The used electricity price profile is from Nord Pool day-ahead market Elspot on January 8$^{\text{th}}$ 2021, multiplied by a factor 2.55 to account for taxes and transport cost. The heat import price is constant at $524\,\mathrm{DKK/MWh}$ or $69.87\,€/\mathrm{MWh}$, as was the case for consumers in Copenhagen in 2021 [162].

We assume that the agents are non-strategic and regulation-agnostic, which means that they do not change bidding behaviour in the dispatch because of the hindsight payments.

4.6.2 Numerical results

We illustrate how network- and loss-awareness affects the schedule and (nodal) prices, as well as the quantity and cost of the resulting network losses. We focus on the comparison between the loss-aware DLG market with individual loss allocation, compared to the loss-agnostic CLG

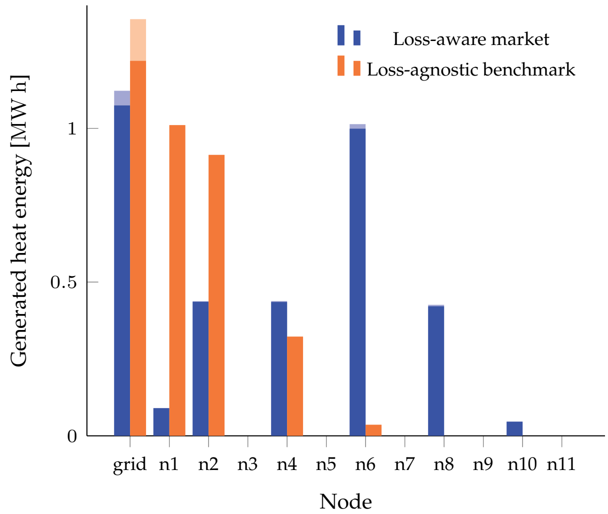


Figure 4.6: Total scheduled nodal generation. Generated *losses* are shaded. Adapted from [**Paper B**].

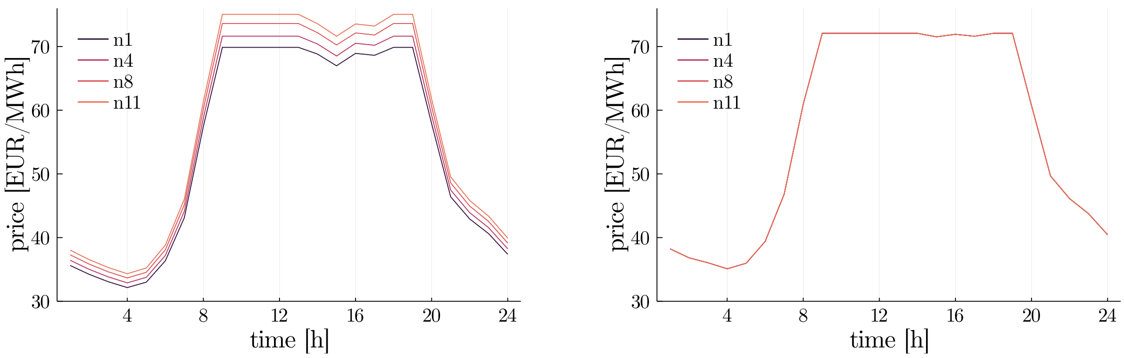
market with socialized loss allocation. The former is our proposed market design with the pricing that suits it most naturally, while the latter is our benchmark, and resembles current practice. Before proceeding to numeric results, we focus on a benefit of network-aware markets that is not visible in the simulations: the fact that the resulting dispatch is feasible in the network, given that the considered network model is accurate. This holds for both our loss-aware and loss-agnostic markets.

The most important effect on the dispatch is that loss-aware markets may increase the scheduled volume of distant generators, i.e. generators located further away from the grid connection point, thereby moving generation closer to consumption. This effect is seen when comparing the total nodal dispatch between the loss-aware DLG and loss-agnostic CLG markets in Figure 4.6. The more local heat consumption is a result of minimizing the cost of pipeline heat loss alongside the cost of consumed heat. While the production cost of generators at distant nodes is higher in this illustrative case study, these generated have a smaller associated network loss, due to the reduced distance between generator and consumer.

Directly related to this more local loss consumption is the fact that the loss-aware markets are guaranteed to be have a lower or equal amount of heat losses in the network than the loss-agnostic market. This effect is quantified in Table 4.2. Admittedly, the size of these reductions is case dependent. As a result of this reduced heat loss, the total generation cost in loss-aware markets is lower or equal to that in the loss-agnostic market, as shown also shown in the table. Here, we compute the total generation cost for the loss-agnostic market after clearing the market.

	loss-agnostic benchmark	loss-aware market *(% decrease)*
cost	201.7 M€	↓ **1.3 %**
heat loss	137.3 kW	↓ **47 %**

Table 4.2: Comparison of total generation cost (i.e., including loss costs) and heat loss in the loss-agnostic and loss-aware markets.



(a) loss-aware market, *individual* loss allocation

(b) loss-agnostic benchmark, *socialized* loss allocation

Figure 4.7: Locational marginal prices as a function of time.

Next, we point out that loss-awareness in most cases leads to variations in nodal marginal prices between nodes, even when there is no congestion. This effect is shown in Figure 4.7(a). In contrast, the loss-agnostic benchmark with socialized loss allocation produces equal prices for all nodes in the absence of congestion, as shown in Figure 4.7(b).

4.7 Future perspectives

The proposed network-aware market design can potentially support the operation of distributed heating networks by reducing network heat loss and the related cost. Regarding network-awareness in heat markets, our approach has been to simplify the network representation compared to many works in the literature. Based on several simplifying assumptions, we formulated a VFCT-based heat market. An advantage of our formulation compared to more complex network-aware heat markets is the transparency and intuitive interpretability of the prices that result from our market design. It should be noted that the VFCT model assumption of flow-independent temperature losses is only verified on system with single injection point [77]. Experimental verification of the validity of this assumption in heat distribution systems with multiple injections should be performed. To use this market design in practice, it is important to choose the fixed nodal temperatures accurately and fairly, as these values have a large effect on the resulting market outcomes. This may be the greatest challenge for practical application of the proposed network-aware market design.

Although it is likely that the assumed network model is not accurate enough for real-time operation of heating networks, it may be sufficiently realistic for the scheduling phase. However, it should be checked in experiments that the proposed market can provide near-feasible schedules. Generally, the purpose of an energy market is to bridge the gap between network and system operation and the many actors interacting within this system. Inclusion of network constraints in a market setting is only useful if the resulting formulation leads to improved operation of the system, by providing the right incentives for market participants to support this operation. Compared to our formulation, more complex network models for optimal dispatch have been proposed, for instance in [41] and [42]. However, as none of these models has been experimentally verified, it is not guaranteed that this complexity will lead to actual improved heat system operation compared to our approach. Future research should focus on the interaction between network-aware heat markets and the control strategies in the corresponding heating networks. The benefits of network-aware dispatch could then be quantified in real experiments, and the use of different network formulations in market designs could be compared.

In the current work, we have not considered uncertainty related to heat generation and consumption. On the distribution level, there is large uncertainty associated with non-aggregated load profiles. At the same time, excess heat production profiles may be subject to uncertainty, for example due to weather dependence. It would be important to investigate how this uncertainty can be taken into account in optimal dispatch and market design.

Our network formulation focused on the minimization of operating cost related to network heat loss, while we did not include operating costs and constraints related to water pumping in the network. A meaningful extension of our work would be to add estimates of pumping power and its related cost. For a single point of heat injection, [47] includes a first order approximation of the bilinear expression for pumping energy. This approximation is not suitable for direct use in the presence of multiple injection points, but can perhaps be generalized to the multiple injection setting. Another option would be to include approximate pumping costs after market clearing, and respond to these cost as a recourse action.

Finally, the design of network-aware markets suitable for bidirectional and/or meshed heat distribution networks could be a topic for future investigations. It may be possible to adapt our proposed market design to accommodate meshed networks. To allow for bidirectional flows, the multiplicative loss formulation could be used, but it is non-trivial how nodal temperatures and pipeline flows could remain part of such model.

Chapter 5

Market integration of storage

This chapter presents our contribution towards the third and final research direction: market integration of non-merchant storage. It is based on the work in [**Paper C**]. Previous studies have highlighted the need for large-scale energy storage in systems with very high penetration of intermittent and stochastic renewables [163]. The International Energy Agency foresees a great increase in utility-scale battery storage, from less than 20 GW in 2020 to over 3000 GW by 2050 [164]. However, market-design and regulatory issues currently still form a barrier to the economic profitability of large-scale storage [165, 166]. The need for market reforms is illustrated by a 2018 FERC order, which called for system operators to facilitate market participation of electric storage, and to provide them with fair compensation, while taking the physical and operational characteristics of these assets into account [167]. Various ways of doing so are discussed in e.g. [168]. Here, we study the option of including *non-merchant* storage, which entails that the market includes physical and operational storage constraints, but no price-related bid for the storage. This formulation is especially relevant for heat markets, as non-merchant storage arises naturally in network-aware district heating market designs with variable temperatures [41, 45]. Furthermore, such designs have been considered in the context of financial storage rights [49, 50], or market integration of storage in general [51, 168]. The intertemporal nature of storage constraints poses a challenge in a market context, as they introduce connections between subsequent market-clearing horizons. Uncertain parameters from future market-clearing horizons therefore affect the optimal decisions in preceding horizons. In the case of storage, the state of energy in the storage at the end of a market horizon is such a decision, depending on both current and future market parameters.

In this chapter, we address the end-of-horizon issue in markets with non-merchant storage. In the literature, it is common to make simplifying assumptions on the final state of energy in non-merchant storage, in order to bypass the end-of-horizon issues. Our aim is to show the importance of dealing with end-of-horizon effects properly. We convey two key messages in a rigorous manner, which are backed up by proofs in [**Paper C**]. First, we show that common simplifying assumptions introduce market inefficiencies and lead to the loss of dispatch-following incentives for the storage operator. Therefore, our first message is that it is important to set a future-aware end-of-horizon storage level, rather than making simplifying assumptions. We move on to a setting where we set a future-aware level, assuming perfect foresight of future market horizons. That is, the end-of-horizon storage level is set to its optimal value. We show that in this setting, a new problem arises: market prices may fail to reflect the value of the energy that was in the storage at the beginning of the market horizon. In particular, we provide a set of mathematical conditions under which market inefficiencies will occur. The mildness of these conditions show that this problem is likely to occur in practice. Our second message is therefore that this problem must be addressed, as it leads to the loss of market efficiency and cost recovery for storage. Our final contribution is a reformulation of the market to retain the value of storage. Here, the market properties of efficiency and cost recovery are restored, given perfect foresight about future market parameters.

The remainder of this chapter is organized as follows. Section 5.1 presents an introduction to non-merchant storage, and discusses bidding formats for storage in current European and US electricity markets. In Section 5.2, we discuss various ways of making end-of-horizon decisions in markets with non-merchant storage. We present the considered stylized energy market model with non-merchant storage in Section 5.3. In Section 5.4, we show how common simplifying assumptions affect the market properties. In Sections 5.5 and 5.6, we place ourselves in a future-aware setting with perfect foresight. In Section 5.5, we show which market properties are guaranteed to hold in this setting, and which may fail to hold. Remaining in a perfect foresight setting, Section 5.6 presents our approach for restoring those market properties. Finally, we draw conclusions and present directions for future work in Section 5.7.

5.1 Market design for integration of storage

We outline existing approaches, from research and practice, for integrating storage in energy market designs. Most of these approaches come from electricity market design, but are applicable to heat markets too. Inspired by [168], we categorize the designs based on the way they represent storage in the objective function and in the constraints of the market-clearing formulation. An overview of the three categories is provided in Figure 5.1.

The first market design considers *self-managed merchant storage* of the storage (self-schedule in [168]), implying that the storage submits price-quantity bids for charging and discharging decisions. The storage operator itself is responsible for ensuring the feasibility of its scheduled (dis-)charging profile, and should take this into account when formulating its bids. Separate (linear) cost functions may be submitted for charged energy $b^+_{st} \geq 0$ and discharged energy $b^-_{st} \geq 0$ of storage s at time t. The market would then include the storage cost curves f_s in the objective function given linear cost parameters C^-_{st} and C^+_{st} as

$$\sum_{s\in\mathcal{S}} f_{0,s}(\boldsymbol{b}^-_s, \boldsymbol{b}^+_s) = \sum_{s\in\mathcal{S}} \left(\sum_{t\in\mathcal{T}} C^-_{st}\, b^-_{st} - C^+_{st}\, b^+_{st} \right) . \tag{5.1}$$

The first term in the right-most expression is positive, because a discharging storage can be seen as a generator. Limits on the net charging of energy would be specified as

$$-\underline{B} \leq b^+_{st} - b^-_{st} \leq \overline{B} \qquad \forall s, t\,. \tag{5.2}$$

The second management option is *market-managed merchant storage* (coined SOC-management-lite in [168]). In this case, the storage can submit operational parameters, in addition to the cost function for charging and discharging. The market will include operational constraints for the storage, keeping track of the state of energy and its feasibility. The storage operator is able to influence its schedule through the offer curve, and has the possibility to be strategic here.

The third and final option is equal to our definition of *non-merchant storage* (called ISO-SOC-Management in [168]). Here, the storage operational constraints are taken into account in the market clearing, but there is no option to submit a cost curve. Note that market-managed merchant storage is equal to the non-merchant storage formulation if the storage operator decides to bid a zero cost curve. It is claimed in [168] that this third option would in theory be the most economically efficient, since storage does not have exogenous marginal cost besides possibly

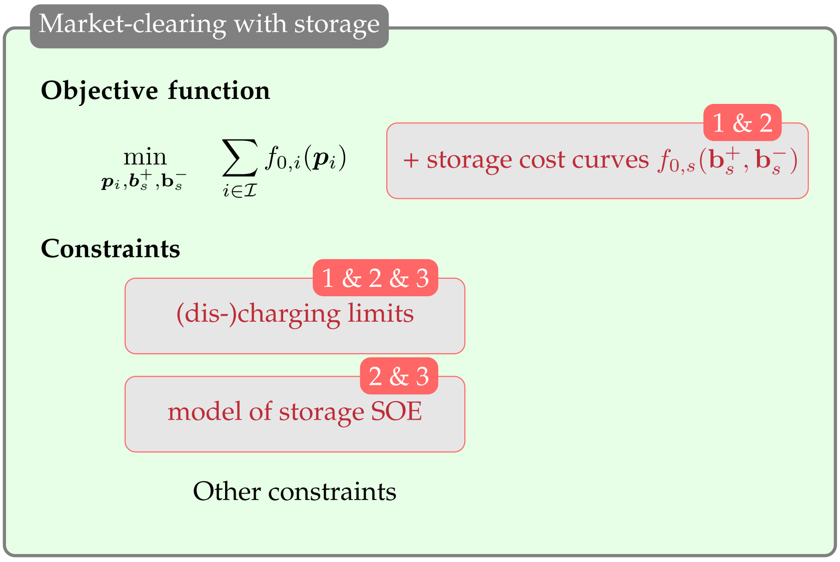


Figure 5.1: An overview of the way different market designs represent storage in the objective function and constraints. We distinguish self-managed merchant storage (1), market-managed merchant storage (2) and non-merchant storage (3). SOE = state of energy. Index s is used for storage systems, i for remaining market participants. Charging and discharging variables are b^+ and b^-, respectively. Recall that the power-injection profiles of other market participants are denoted $\boldsymbol{p}_i$, and cost curves by f_0.

degradation cost. The non-merchant storage formulation can be adapted to include degradation cost [168]. On the other hand, strategic price-maker storage could have negative effects on the market properties, depending on its bidding strategy. The impact of different bidding formats for a strategic price-marker storage is studied in [169]. It is found that under increased uncertainty, complex bid structures involving storage operational constraints are beneficial for both storage owners and the system.

In US markets, it is common to design resource-specific bidding formats, whereas European markets typically consider a general bidding format available for all different types of market participants. Several US markets already consider adapted market designs and bidding formats for pumped-hydro storage, where the feasibility of storage dispatch is ensured by the system operators [168, 170]. Instead of price-quantity bids, the storage would in that case submit its technical parameters, accompanied by a cost curve [170]. The market clearing thus includes the storage cost curve in the objective function, as well as the storage-specific operational constraints, corresponding to the market-managed merchant storage approach for integration of storage. In most European markets, there are no bid formats specifically designed for storage, so that storage operators must choose between existing bid formats like any other market participant [170]. This means that regular price-quantity bids can be submitted like in the first market format, i.e. self-managed merchant storage. Alternatively, to ensure feasibility of the market schedule resulting from its bids, storages may opt to use block orders [170]. Such an offer would include a charging block and a discharging block, making sure that the storage is not scheduled to discharge without its charging offer being accepted first. In the EPEX Spot market, a so-called 'loop order' has been introduced, which allows bids of exactly two blocks to be executed or rejected together [170]. Both these bidding formats would however represent the operational constraints of the storage in a rather restricted way, limiting the flexibility that can be harnessed.

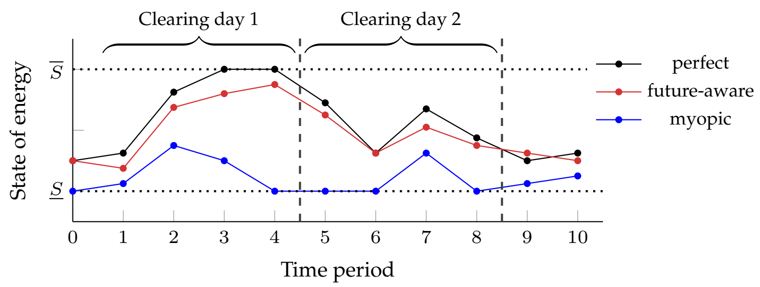


Figure 5.2: Illustrative example: the effect of different end-of-horizon decisions on the state of energy in the storage over several market-clearing horizons. Adapted from [**Paper C**].

5.2 End-of-horizon considerations

The splitting of a market clearing with time-linking constraints into multiple clearing horizons can result in sub-optimal, 'myopic' scheduling of non-merchant storage. This effect is shown for an illustrative example in Figure 5.2. The black curve represents an optimal state-of-energy profile that may be found when clearing current and (relevant) future market horizons simultaneously, assuming perfect foresight over future horizons. In this case, the storage would in general be non-empty by the end of a market clearing horizon (in this case at $t = 4$).

When instead, subsequent market horizons are cleared separately, as is done in practice, a different optimal state-of-energy profile may be found. As the value that could be obtained in future market horizons is not visible in the previous market horizon, it would be locally optimal to use as much energy from the storage as possible, unless negative prices occur [168]. As a result, the storage level would be at its lower bound by the end of each horizon, as shown by the blue curve in Figure 5.2 for $t = 4$ and $t = 8$. Many existing works with non-merchant storage do not consider the end-of-horizon issue, thereby neglecting the relation between subsequent market horizons [49–51, 171]. Other works by-pass the need for making future-aware decisions by assuming equal storage levels at the beginning and end of each horizon [172–174]. If this level is set to the lower bound $\underline{S}$, this approach would also lead to the blue curve in Figure 5.2, otherwise this curve would be shifted in the vertical direction. Sometimes a reasonable level of this initial and final storage level is determined [172], but in many works, this level seems to be chosen arbitrarily.

In formulations with non-merchant storage, additional considerations are needed in order to avoid myopic decisions about the energy left in the storage at the end of the market-clearing horizon. Such *future-aware* decisions about end-of-horizon storage levels can be enforced in different ways [168]. For example, a desired final storage level may be included in the bidding format and enforced in a hard constraint. Another option would be to allow the storage owner to offer a value for each unit of energy left in the storage by the end of the horizon, and add a term to the objective function. Rolling-horizon market formulations have also been used to support non-myopic end-of-horizon decisions [52, 61, 62]. Each of these approaches could lead to a future-aware state-of-energy profile such as the red profile in Figure 5.2, if end-of-horizon decisions were well-chosen.

In this chapter, we do not deal with the question of *how* to optimally determine end-of-horizon decisions. We consider a rather general formulation for enforcing non-myopic end-of-horizon decisions. Our formulation is general in the sense that the final state of energy can be determined using any desired method, e.g. rolling horizon, online learning, etcetera. Instead, we highlight a problem that arises regardless of how such decisions are made, even under perfect foresight. We show that even when *perfect* final storage levels are enforced, several desirable market properties may no longer be guaranteed. If a storage performs arbitrage between different horizons, the value of energy present in the storage may be lost in subsequent market horizons. In other words, subsequent market-clearing horizons are unaware of losses and gains incurred by market participants in past horizons [61]. This problem has been addressed in a rolling-horizon setting, and with a focus on ramping constraints [61, 175]. Here, we analyze the problem specifically for markets with non-merchant storage, and without the rolling-horizon approach.

5.3 A stylized market design with non-merchant storage

Our market design includes a storage model, which we will present first. In the remainder of this chapter, we consider a single storage system. As we will not include network constraints, the results would not change when including multiple storage systems. For our current purposes, we keep the non-merchant storage model in the market design as simple as possible. This allows us to show that a minimal set of constraints can already lead to problems when clearing the markets sequentially. The considered stylized model of the storage system is:

$$0 \leq e_t \leq \overline{S}, \quad \forall t \in \mathcal{T} \qquad : (\underline{\nu}_t, \overline{\nu}_t) \tag{5.3a}$$

$$e_t = e_{t-1} + b_t, \quad \forall t \in \mathcal{T} \setminus \{1\} \qquad : (\rho_t) \tag{5.3b}$$

$$e_1 = E^{\text{init}} + b_1 \qquad : (\rho_1) \tag{5.3c}$$

$$e_T = E^{\text{end}} \qquad : (\xi)\,. \tag{5.3d}$$

The state of energy in the storage system e_t is a decision variable for each time t in the considered time horizon $\mathcal{T} = \{1, 2, \ldots, T\}$. The first constraint (5.3a) ensures that the state of energy stays within the physical bounds of the storage system, which is bounded below by 0 and above by $\overline{S}$. The state of energy changes over time depending on the amount of charged energy b_t, and this dependence is described by (5.3b) and (5.3c). The decision variable b_t is positive when charging, and negative when discharging. A single decision variable for (dis-)charging suffices in our model, because we do not model charging and discharging losses. The addition of such losses would lead to more complex notation in the primal and dual problems, but would not alter our main message, and are therefore omitted. The final constraint (5.3d) fixes a decision for the end-of-horizon storage level e_T, to a pre-specified parameter E^{end}. This formulation contains approaches from the literature as special cases. For example, E^{end} can be chosen equal to E^{init}. In a future-aware approach, E^{end} would be identified separately for each market horizon, depending on knowledge of future market-clearing horizons. Our formulation does not prescribe any method for doing so, and therefore includes e.g. rolling horizon methods as a special case. In a myopic market clearing, (5.3d) would be omitted, or E^{end} would be chosen equal to E^{init}.

The market formulation that we consider in this chapter resembles the common pool day-ahead market as presented in (2.8) of Chapter 2, though with the addition of the storage constraints. We consider two versions of this market clearing, *constrained* and *free*, depending on the in- or exclusion of the end-of-horizon constraint (5.3d). The *constrained* market-clearing optimization problem for set of time periods $\mathcal{T}$, which includes the end-of-horizon constraint, is denoted $\mathbf{C}(\mathcal{T})$ and given by:

$$\max_{\boldsymbol{d},\boldsymbol{g},\boldsymbol{b},\boldsymbol{e}} \quad \sum_{t\in\mathcal{T}} \left(\sum_{l\in\mathcal{L}} U_{lt}\, d_{lt} - \sum_{g\in\mathcal{G}} C_{gt}\, g_{gt} \right) \tag{5.4a}$$

$$\text{s.t.} \quad \sum_{l\in\mathcal{L}} d_{lt} + b_t - \sum_{g\in\mathcal{G}} g_{gt} = 0, \quad \forall t \in \mathcal{T} \qquad : (\lambda_t) \tag{5.4b}$$

$$0 \le g_{gt} \le \overline{G}_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \qquad : (\underline{\mu}_{gt}, \overline{\mu}_{gt}) \tag{5.4c}$$

$$0 \le d_{lt} \le \overline{D}_{lt}, \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \qquad : (\underline{\chi}_{lt}, \overline{\chi}_{lt}) \tag{5.4d}$$

$$\text{(5.3a)} - \text{(5.3d)}\,. \tag{5.4e}$$

We assume linear cost functions for loads l and generators g, which are specified by bid and offer prices U_{lt} and C_{gt} in the objective function (5.4a). If participants bid truthfully, these quantities correspond to the willingness to pay for loads, and the marginal generation cost for generators. The energy balance (5.4b) now includes the charging variable b_t, besides the load and generation. The dual variables are collected in the set $\Gamma = \{\boldsymbol{\lambda}, \boldsymbol{\rho}, \xi, \underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}}, \underline{\boldsymbol{\nu}}, \overline{\boldsymbol{\nu}}, \underline{\boldsymbol{\chi}}, \overline{\boldsymbol{\chi}}\}$.

For purposes of pricing and analysis of the mechanism, we derive the dual problem $\mathbf{CD}(\mathcal{T})$ of (5.4) as

$$\min_{\Gamma} \quad \sum_{t\in\mathcal{T}} \left(\sum_{g\in\mathcal{G}} \overline{G}_{gt}\overline{\mu}_{gt} + \sum_{l\in\mathcal{L}} \overline{D}_{lt}\overline{\chi}_{lt} + \overline{S}\overline{\nu}_t \right) + E^{\text{init}}\rho_1 - E^{\text{end}}\xi \tag{5.5a}$$

$$\text{s.t.} \quad C_{gt} - \underline{\mu}_{gt} + \overline{\mu}_{gt} - \lambda_t = 0, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \qquad : (g_{gt}) \tag{5.5b}$$

$$- U_{lt} - \underline{\chi}_{lt} + \overline{\chi}_{lt} + \lambda_t = 0, \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \qquad : (d_{lt}) \tag{5.5c}$$

$$- \rho_t + \lambda_t = 0, \quad \forall t \in \mathcal{T} \qquad : (b_t) \tag{5.5d}$$

$$- \underline{\nu}_t + \overline{\nu}_t + \rho_t - \rho_{t+1} = 0, \quad \forall t \in \mathcal{T} \setminus \{T\} \qquad : (e_t) \tag{5.5e}$$

$$- \underline{\nu}_T + \overline{\nu}_T + \rho_T - \xi = 0 \qquad : (e_T) \tag{5.5f}$$

$$\underline{\mu}_{gt}, \overline{\mu}_{gt}, \underline{\nu}_t, \overline{\nu}_t, \underline{\chi}_{lt}, \overline{\chi}_{lt} \ge 0\,. \tag{5.5g}$$

A more compact dual formulation could be derived by combining the non-negativity constraints (5.5g) with the other constraints. However, as we use the constraints in their equality form in subsequent proofs, we present the dual formulation in this loner form. We choose the signs of the free variables ρ_1 and ξ such that (5.5e) and (5.5f) have a similar form for all t, including $t = T$.

The *free* market-clearing problem $\mathbf{F}(\mathcal{T})$ is obtained from the constrained problem (5.4) by removing the end-of-horizon constraint (5.3d). The dual problem $\mathbf{FD}(\mathcal{T})$ of the free market clearing is obtained from (5.5) by setting $\xi = 0$.

A thorough analysis of the dual problem and an interpretation of the dual variables is available in [51]. We use λ_t as the uniform hourly market price for all market participants, including the storage system. Previous works sometimes use other dual variables for pricing [49–51], but in [50, 51] it is shown that the two pricing systems are equivalent for the free market clearing with non-merchant storage, i.e $\mathbf{F}(\mathcal{T})$. The dual variable ρ_t represents the marginal value of an additional unit of energy in the storage at time t, i.e. increasing e_t by one unit. By (5.5d), it holds that $\rho_t = \lambda_t$ for all t, so that both these dual variables represent the hourly uniform marginal price. The relation between ρ_t and λ_t would be changed if limits on the rate of charge b_t were included in the storage model (5.3). In that case, the equality would only hold when the rate-of-charge limits were not active. If charging and discharging losses were added as a constant proportion of the amount charged or discharged, the relation between ρ_t and λ_t as given by (5.5d) would change by a constant factor.

Constraints (5.5e) and (5.5f) specify the relation between the market price at consecutive time periods. The price only changes over time if the state of charge in the storage system is at a

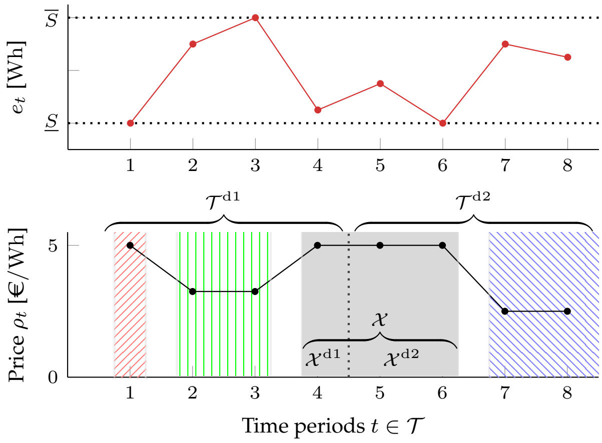


Figure 5.3: Bottom: Example of market-clearing price dividing the market horizon into four time zones. Top: Storage state-of-energy (e_t) profile related to the below market price signal. Taken from [**Paper C**].

bound. More specifically, the market price decreases from time t to $t+1$ if $e_t = \underline{S}$, and increases if $e_t = \overline{S}$. This effect motivates the definition of a *time zone* similar to a spatial zone that may arise in network-aware markets when network constraints are inactive.

Definition 1 (Time zone) *A time zone is the longest possible set of consecutive time steps with the same market price. That is, a set of time steps $\mathcal{Z}$ with $\min_{t\in\mathcal{Z}} t = z_0$ and $\max_{t\in\mathcal{Z}} t = Z$ is a time zone if and only if*

1. *$\mathcal{Z}$ only includes consecutive time steps*
2. *For all $t \in \mathcal{Z}$ it holds that $\lambda_t = K$ for some constant K*
3. *$\rho_{z_0-1} \neq c$ and $\rho_{Z+1} \neq c$.*

We illustrate this definition in Figure 5.3. The bottom plot shows four time zones with constant market price $\lambda_t = \rho_t$. A example state-of-energy profile for the storage that is in accordance with the given time zones is shown in the upper plot. For example, the storage must be at a lower bound at $t = 1$, as the market price ρ decreases from $t = 1$ to $t = 2$.

5.4 Simplified end-of-horizon decisions

Here, we investigate the effect of the following two common end-of-horizon simplifications for non-merchant storage on desirable market properties. **Assumption 1** entails that the final storage level is unconstrained. As a result, the storage will be scheduled to maximize social welfare within the current market-clearing horizon. Unless negative prices occur, this will lead to the end-of-horizon storage level hitting its lower bound. Under **Assumption 2**, the final storage level is constrained the initial storage level.

5.4.1 Market properties

In [**Paper C**], we show that either assumption guarantees cost recovery for the storage owner. This cost recovery is even ensured within each market-clearing horizon (and therefore also in the long run). However, each of the simplifying assumptions can lead to loss of market efficiency in the long run. In particular, social welfare can be negatively impacted, and the storage may have an incentive to deviate from the market clearing outcomes. For the details of these derivations we refer to [**Paper C**]. Here, we provide intuition for these results using an illustrative example.

5.4.2 Illustrative example I

In this illustrative example, we consider two sequential market-clearing intervals by the sets $\mathcal{T}^{\mathrm{d1}} = \{1, 2\}$ and $\mathcal{T}^{\mathrm{d2}} = \{3, 4\}$, i.e. a market-clearing interval consists of two time periods. There is a non-merchant storage with a maximum capacity of $\overline{S} = 2.5\,\mathrm{Wh}$. In addition, there is a single load and two generators, a cheaper one and a more expensive one. The price-quantity bids of the load and generators can be found in the Appendix of [**Paper C**]. Our code for the illustrative examples in this chapter is available in our online repository [176].

We compare three cases with different assumptions on the initial and end-of-horizon storage level E^{init} and E^{end}:

1. On both day 1 and day 2, there is no constraint on the final level, corresponding to Assumption 1. On day one, $E^{\mathrm{init}} = 0$.
2. The initial and final levels are equal as in Assumption 2. Specifically, $E^{\mathrm{init}} = E^{\mathrm{end}} = 1.25\,\mathrm{Wh}$, i.e. half of the storage capacity.
3. The final storage level is optimal w.r.t. the storage profit over these two clearing horizons. This corresponds to $E^{\mathrm{end}} = 2.5\,\mathrm{Wh}$, i.e. the maximum capacity. On the second day, the end-of-horizon storage level is unconstrained.

The optimal state-of-energy profiles for the storage system in these different cases are shown in Figure 5.4. It shows that the temporal arbitrage by the storage is limited in Cases 1 and 2, compared to the future-aware approach in Case 3. This is a result of the simplifying end-of-horizon assumptions, as they do not allow temporal arbitrage between different market-clearing intervals. For Cases 1 and 2, the charging and discharging pattern is the same, but this is not a general result. The missed opportunity in terms of arbitrage is reflected in the social welfare for the different cases. In Cases 1 and 2, the total social welfare obtained is €46, whereas it is €55.5 in Case 3.

To evaluate the cost recovery and dispatch-following incentives for the storage system, we compare the storage system's profit in the three cases in Table 5.1. The profit is nonnegative for each day in Cases 1 and 2, which is consistent with our finding that cost recovery is ensured within each market clearing under the simplifying assumptions. This does not hold for Case 3, where the profit

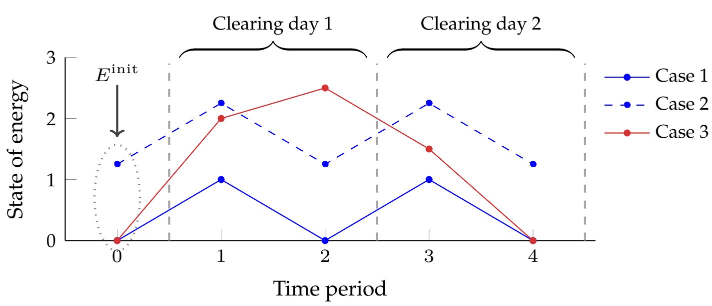


Figure 5.4: Storage state of energy over time for the three cases in illustrative example I. Blue lines are myopic approaches, whereas the red line represents a future-aware approach.

	t	Case 1	Case 2	Case 3
$\mathcal{T}^{\mathrm{d1}}$	1	-4	-4	-10
	2	4	4	-2.5
$\mathcal{T}^{\mathrm{d2}}$	3	-11	-11	6
	4	11	11	9
Total $\mathcal{T}^{\mathrm{d1}}$		0	0	-12.5
Total $\mathcal{T}^{\mathrm{d2}}$		0	0	15
Total		0	0	2.5

Table 5.1: Profit (€) of the storage system for the three cases in illustrative example I. Adapted from [**Paper C**].

is negative on the first day. However, the storage benefits from the temporal arbitrage between the two days in Case 3, as it earns a greater profit. In Cases 1 and 2, the storage operator can improve its profit by changing its schedule to that of Case 3, which shows that the simplifying assumptions indeed do not ensure dispatch-following incentives for the storage.

5.5 Future-aware end-of-horizon decisions

We move on to investigate market properties in the case where a future-aware decision is made for the energy in the storage at the end of a market-clearing horizon. The main aim is to show the effect of splitting a market clearing in multiple intervals on the optimal primal and dual solutions. We place ourselves in a *perfect foresight* setting, in order to prove that the effects to be shown are not caused by suboptimal end-of-horizon decisions, but purely by the act of splitting itself. More specifically, by perfect foresight we mean that the storage operator decides on the future-aware E^{end} with perfect foresight of all future market parameters.

5.5.1 Definition of full- and split-horizon problems

To investigate the effect of splitting the market clearing, we define a *full-horizon* problem as a perfect baseline for comparison. In the full-horizon problem, the market outcomes are determined in a single clearing for the entire *finite* horizon $\mathcal{T}$. To allow for fair comparison, we restrict ourselves to this finite time period and disregard what happens after time $t = |\mathcal{T}| = T$. Therefore, we do not constrain the end-of-horizon level for the storage at $t = T$, which means that the full-horizon optimization model is given by $\mathbf{F}(\mathcal{T})$ from (5.4). Optimal values of variables in this model are denoted by $*$. In a finite-horizon world, the full-horizon problem ensures cost recovery, dispatch-following incentives, and market efficiency [51]. Therefore, we can consider this full-horizon market clearing as a perfect baseline in our comparison.

In the *split-horizon* problem on the other hand, the market outcomes for $\mathcal{T}$ are determined in two separate, sequential market-clearing problems. The set $\mathcal{T}$ is divided into two 'days' of equal length denoted $\mathcal{T}^{\mathrm{d1}} = \{1, \dots, H\}$ and $\mathcal{T}^{\mathrm{d2}} = \{H+1, \dots, 2H\}$, where $T = 2H$. On the first day, the storage operator is to specify a future-aware end-of-horizon storage level E^{end}, so that the optimization problem for the first day is given by $\mathbf{C}(\mathcal{T}^{\mathrm{d1}})$. The storage level at the end of the second day is unconstrained in the split-horizon problem too, to ensure fair comparison to the full-horizon problem. The split-horizon optimization model for the second day is therefore $\mathbf{F}(\mathcal{T}^{\mathrm{d2}})$. Optimal values of variables in the split-horizon model are denoted by $'$. The term split-horizon problem refers to the combination of both optimization problems. They can be combined in a single optimization problem that is solved at once, because the two problems are independent. Of course, the E^{end} for the first day has to equal E^{init} for the second day for this combined formulation to make physical sense. When combining the first- and second-day objectives into a single objective function, this function equals the objective function of the full-horizon problem.

Finally, we define the *split-horizon model with perfect foresight*. In this case, the final storage level E^{end} on $\mathcal{T}^{\mathrm{d1}}$ is chosen equal to the optimal storage level e^*_H obtained from solving the full-horizon problem.

5.5.2 Market properties

Here, we give an overview of three main theoretical results from [**Paper C**], and provide intuition for these results using an illustrative example. For rigorous proofs of these results we refer to the paper.

Our first result relates to the effect of the split-horizon clearing on primal solutions. In [**Paper C**] we prove that the primal solution(s) of the split-horizon problem with perfect foresight are equal to those from the full-horizon problem. This correspondence is formalized in the following lemma.

Lemma 1 *Let e^*_H be part of an optimal solution $\boldsymbol{x}^*$ to the full-horizon problem. Then $\boldsymbol{x}^*$ is an optimal primal solution to the full-horizon problem if and only if $\boldsymbol{x}^*$ is an optimal primal solution to the split-horizon problem with $E^{\mathrm{end}} = e^*_H$.*

Next, we analyze how splitting the market clearing affects the dual solutions. In this case, the correspondence between the full- and split-horizon problems is weaker, i.e. it can only be shown in one direction. Our second theoretical result is that any optimal primal and dual solution to the full-horizon problem links to a corresponding solution to the split-horizon problem. The following lemma specifies what this means exactly.

Lemma 2 *Any optimal primal and dual solution pair $\{\boldsymbol{x}^*, \boldsymbol{\zeta}^*\}$ to the full-horizon problem is also an optimal solution to the split-horizon problem, with additional split-horizon variable ξ' taking the value $\xi' = \rho^*_{H+1}$.*

While previous works such as [61] have noted the existence of problematic solutions to the split-horizon problem, they do not mention the fact that the dual solution to the full-horizon problem always remains a solution to the split-horizon problem. Therefore, we find it important to emphasize that a dispatch-supporting dual solution to the split-horizon problem always exists.

Our third and final result shows that the correspondence between dual solutions to the full- and split-horizon problems does not hold in the other direction. Indeed, there may exist additional dual solutions to the split-horizon problem, that are not optimal and even infeasible to the full-horizon problem. A problem with such additional dual solutions is that they can lead to loss of market properties. Before stating our theorem about their existence, we provide a formal definition of such problematic solutions.

Definition 2 (Weak dual) *A dual solution to the split-horizon problem is* ***weak*** *if the resulting price is not dispatch supporting for the non-merchant storage. Such a weak dual exists if and only if one of the following situations occurs.*

1. *$e_H \in (\underline{S}, \overline{S})$ and $\rho'_H \neq \rho'_{H+1}$*
2. *$e_H = \underline{S}$ and $\rho'_H < \rho'_{H+1}$*
3. *$e_H = \overline{S}$ and $\rho'_H > \rho'_{H+1}$.*

The storage has incentive to deviate from its schedule if such deviations can improve its profit. The following two cases show that weak solutions provide this incentive to deviate:

1. If $\rho'_H > \rho'_{H+1}$, the storage can improve its profit by decreasing e'_H. This is possible in case $e_H \neq \underline{S}$, i.e. both in situations 1) and 3) of Definition 2.
2. If $\rho'_H < \rho'_{H+1}$, the storage can improve its profit by increasing e'_H. This is possible in case $e_H \neq \overline{S}$, i.e. both in situations 1) and 2) of Definition 2.

Under a weak dual solution, the storage operator may even earn a negative profit, and thus, cost recovery is not ensured. This would for example be the case if the storage system charged an amount e'_H at price ρ'_H and is scheduled to sell the same amount for a lower ρ'_{H+1}, and no other trades occur.

In Theorem 1 we provide sufficient conditions for a weak solution to exist.

Theorem 1 *Assume all cost and utility bids are unique. More specifically, $U_{lt} = U_{l't'} \iff l = l' \wedge t = t'$, and $C_{gt} = C_{g't'} \iff g = g' \wedge t = t'$, and $U_{lt} \neq C_{gt'}\ \forall t, t', l, g.$.*
If the optimal solution to the full-horizon problem is such that $H + 1$ and T are in different time zones, and

$$\rho^*_H = \rho^*_{H+1}\,, \tag{5.6}$$

then the split-horizon problem with perfect foresight admits a weak dual solution.

We refer to [**Paper C**] for the exact proof, and discuss the reasoning behind this result more informally in this chapter. The proof is constructive: we show how a weak dual solution for the split-horizon problem can be constructed from a solution to the full-horizon problem. This is possible due to a 'freedom' that arises in the split-horizon problem as a result of the splitting. The primal split-horizon problem includes an additional constraint compared to the full-horizon problem, namely:

$$e_H = E^{\text{end}} \quad : (\xi) \tag{5.7}$$

which also introduces an additional dual variable ξ. Whereas in the full-horizon problem, the following dual relation must hold:

$$-\underline{\nu}_H + \overline{\nu}_H + \rho_H - \rho_{H+1} = 0\,, \tag{5.8}$$

this is replaced by the following dual relation for the split-horizon problem:

$$-\underline{\nu}_H + \overline{\nu}_H + \rho_H - \xi = 0\,. \tag{5.9}$$

Thereby, the additional dual variable ξ introduces freedom in the relation between market prices ρ_H and ρ_{H+1} in the time periods where the market is split.

The sufficient conditions in Theorem 1 ensure that this potential freedom can be exploited. If cost and utility bids were not unique, it could happen that all market participants' power injections are scheduled to be on a bound of their feasible region, but there is still no freedom in the related dual variables. To exclude such exceptional cases, we assume uniqueness of the cost and utility bids. The remaining conditions ensure that the full-horizon optimal solution has a time zone that extends over both days, but does not extend over the entire second day. In the split-horizon case, this time zone would be split over two market-clearing intervals, and at most one of the two parts of the split time zone would have a marginal load or generator. The other will lack a marginal load or generator, which leaves room for the optimal market-clearing price to take multiple values. However, if $t = T$ was included in that time zone, while $e_T^* \in (\underline{S}, \overline{S})$, the freedom in ρ_{H+1} would be lost.

5.5.3 Illustrative example II

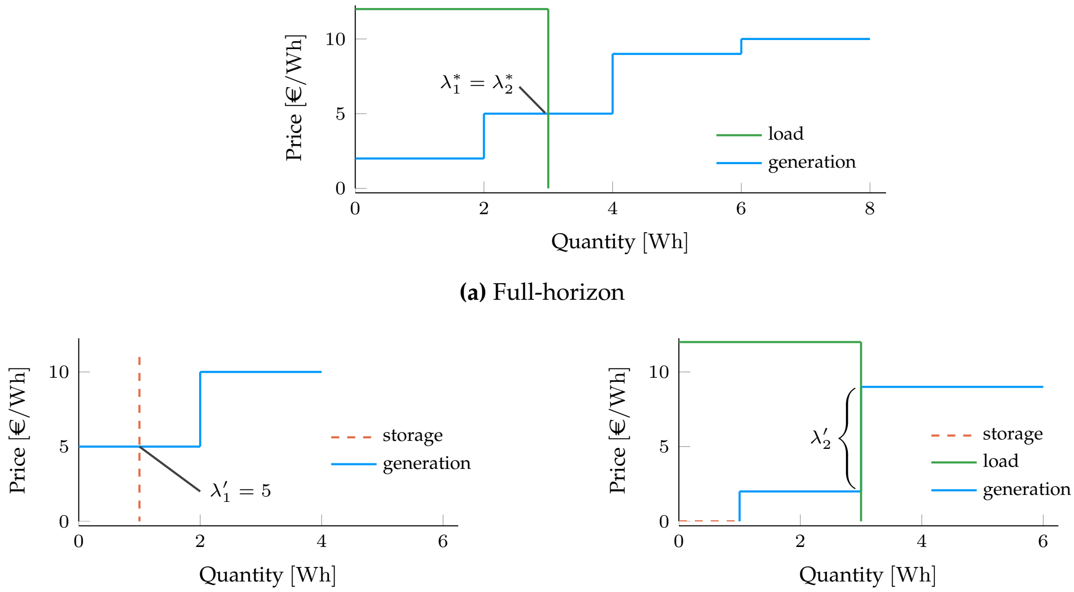
In this example, we apply the full-horizon and split-horizon optimization problem to the same case and compare the results. We consider a time horizon $\mathcal{T} = \{1, 2\}$ consisting of two days, $\mathcal{T}^{\mathrm{d1}} = \{1\}$ and $\mathcal{T}^{\mathrm{d2}} = \{2\}$. A single load, two generators, and a non-merchant storage with capacity $\overline{S} = 2.5\,\mathrm{Wh}$ participate in the market. The assumed price-quantity bids for the load and generators are listed in Table 5.2.

We first clear the market using the full-horizon problem, to obtain the results for this ideal benchmark. For the unique optimal solution, the storage is charged to $e_1^* = 1\,\mathrm{Wh}$ on the first day, and discharged fully on the second day. The storage is thus not at a bound after the first hour. Therefore, there is a single time zone for the two days, and the clearing can be represented in a single market-clearing diagram as done in Figure 5.5(a). The unique optimal values for the market prices are $\boldsymbol{\lambda}^* = [5, 5]€/\mathrm{Wh}$. Performing temporal arbitrage, the storage buys one unit of generation at $t = 1$ from a generator with cost of $5\,€/\mathrm{Wh}$, and sells it at $t = 2$. Thereby, the presence of the storage helps avoid the scheduling of a more expensive generator of $9\,€/\mathrm{Wh}$ at $t = 2$.

When we clear the market using the split-horizon problem with perfect foresight, we enforce on the first day that $E^{\mathrm{end}} = e_1^* = 1\,\mathrm{Wh}$. As a result, the primal solutions of the two problems are equal, which is a general result as proven in Lemma 1. The individual market-clearing diagrams for the two days under the split-horizon problem are shown in Figure 5.5(b) and 5.5(c). The first day is subject to a unique optimal market price equal to that of the full-horizon problem, i.e. $\lambda_1' = 5\,€/\mathrm{Wh} = \lambda_1^*$. However, price multiplicity arises on the second day, where all values λ_2' in the range $[2, 9]€/\mathrm{Wh}$ are optimal. As stated in Lemma 2, the optimal dual price from the full horizon problem is still a solution to the split-horizon problem too, i.e. $5 = \lambda_2^* \in [2, 9]$. The values of $\lambda_2' < 5$ are part of weak dual solutions, since the storage has incentives to deviate from the schedule. The storage paid $\lambda_1' = 5$ for the stored energy, and therefore loses profit for each stored unit: it could thus increase its profit by decreasing e_1. In this particular example, the storage earns a negative profit, so that cost recovery is lost. This example also proves that the conditions in Theorem 1 are not *necessary* for weak dual solutions to exist, because one of the conditions is now not met. Namely, the final time period $t = T = 2$ is included in the time zone that spans the two days.

t	$\overline{D}_{1t}(\mathrm{Wh})$	$\overline{P}_{1t}(\mathrm{Wh})$	$\overline{P}_{2t}(\mathrm{Wh})$	$U_{1t}(€)$	$C_{1t}(€)$	$C_{2t}(€)$
1	-	2	2	-	5	10
2	3	2	2	12	2	9

Table 5.2: Price-quantity bids for one load and two generators in illustrative example II. Taken from the Appendix of [**Paper C**].



(a) Full-horizon

(b) Split-horizon for $t = 1$

(c) Split-horizon for $t = 2$

Figure 5.5: Market-clearing diagrams for illustrative example II. Subfigures (a) and (c) are adapted from **[Paper C]**.

To sum up, this example shows that the optimal social welfare is obtained by the split-horizon problem with perfect foresight, and the system benefits from using the storage. However, there exist optimal solutions for which the storage earns negative profit and has incentive to deviate from the schedule.

5.6 Restoring market efficiency in future-aware setting

We present a method to restore the market properties of the split-horizon problem with perfect foresight. This is done by ensuring that the split-horizon problem admits *only* the dual solutions of the full-horizon problem. Remaining in a perfect-foresight setting, we now also assume perfect foresight of the optimal dual variables to the full-horizon problem, including ρ^*_{H+1}. We propose a *future-aware-plus* method which is a modification of the split-horizon problem. The modification restores the link between ρ_H and ρ_{H+1} that is present in the full-horizon problem given by Equation (5.8). This is done by interpreting end- and start-of-horizon storage levels as variables, as well as adding related terms to the objective function to steer these new variables to their correct values.

For the first day of the split-horizon problem, this means that e_H becomes a variable, the end-of-horizon constraint (5.3d) is omitted, and the term $e_H \, \rho^*_{H+1}$ is added to the objective. The latter term could be interpreted as the expected value to be gained if an amount of e_H is stored, so that the expected value of storage in the next market clearing is considered. In the current perfect foresight setting, ρ^*_{H+1} is the perfect prediction of this dual variable, and therefore equal to the optimal value from the full-horizon problem. As a result of these modifications, dual relation (5.9) in the first day of the split-horizon problem is replaced by

$$- \underline{\nu}_H + \overline{\nu}_H + \rho_H - \rho^*_{H+1} = 0 \,. \tag{5.10}$$

We denote optimal values obtained using the future-aware-plus method by $''$. In illustrative example II, the optimal value $\rho_2^* = 5\,€/\mathrm{Wh}$ would be known at the time of market clearing for day 1. Using this value, the optimal values $e''_H = 1\,\mathrm{W\,h}$ and $\rho''_H = 5\,€/\mathrm{Wh}$ are obtained.

For the second day of the split-horizon problem, the initial storage level e_H is turned into a variable, and the term $e_H\,(\rho''_H - \underline{\nu}''_H + \overline{\nu}''_H)$ is added to the objective function, where the optimal values obtained in the first day are used. The addition of this variable e_H adds the following constraint to the split-horizon dual problem for the second day:

$$-\rho_{H+1} + \rho''_H - \underline{\nu}''_H + \overline{\nu}''_H = 0\,. \tag{5.11}$$

In illustrative example II, this would add the constraint $\rho_2 = \rho''_1 = 5\,€/\mathrm{Wh}$, so that there is a single dual solution for ρ''_2 equal to the solution in the full-horizon problem. Thus, the added term in the objective function makes the market aware of the value of the energy that was in the storage at the start of the market-clearing horizon.

In all, the future-aware-plus method restores the full-horizon optimal solution(s) in the split-horizon problem under perfect foresight. Therefore, the market properties of cost recovery and market efficiency are guaranteed to hold as long as there is perfect foresight of ρ^*_{H+1}. The future-aware-plus method is impractical for application under imperfect foresight, i.e. in practice. Errors in the prediction of ρ^*_{H+1} would generally lead to different values of e_H being obtained in the clearings for the first and second day. As a result, the dispatch determined for the different market intervals could be infeasible.

5.7 Conclusions and future perspectives

Table 5.3 summarizes this chapter's theoretical results. The simplified end-of-horizon decisions on the storage level guarantee cost recovery for the non-merchant storage, even *within* a single market-clearing horizon. However, this comes at the cost of suboptimal dispatch compared to future-aware approaches, both for the storage itself, and in terms of social welfare. Therefore, market designs with these simplifying assumptions are inefficient. The future-aware market with perfect foresight does restore optimality in terms of social welfare, but due to the possible existence of weak dual solutions, cost recovery and dispatch-following incentives are not guaranteed. In our proposed future-aware-plus market design, we restore these market properties in a perfect foresight setting. Cost recovery does not necessarily hold within each market interval, as the storage may perform arbitrage across market intervals, but it is guaranteed in the long term. Under imperfect foresight, however, our proposed solution may produce infeasible market outcomes when there is an error in the prediction of ρ^*_{H+1}. Although we did not analyze the market properties formally in this case, we expect that cost-recovery and market efficiency would not be guaranteed to hold due to possible infeasibility of market outcomes.

In Theorem 1 we provided sufficient conditions for the existence of weak solutions to the split-horizon problem with perfect foresight. The mildness of these conditions show that this problem is not just technically possible, but actually likely to occur in practice. We emphasize that the provided conditions are not *necessary*, i.e. weak dual solutions to the split-horizon problem can also exist while these conditions do not hold. It is possible that the conditions could be further specified and restricted to make them both necessary and sufficient. However, this would probably lead to less intuitively attractive conditions, as several special 'miscellaneous' cases would need to be excluded.

		(perfect foresight)		(imperfect foresight)
	$E^{\text{init}} = E^{\text{end}}$	future-aware	future-aware-plus	
cost recovery within clearing	✓	✗	✗	?
across clearings	✓	✗	✓	?
dispatch-following incentives	✗	✗	✓	?
optimal social welfare	✗	✓	✓	?
market efficiency	✗	✗	✓	?

Table 5.3: Overview of market properties for different end-of-horizon decisions and market designs for non-merchant storage. ? = to be investigated.

A natural direction for future work is the design of a future-aware market with non-merchant storage that is suitable in an *imperfect*-foresight setting. It would be a first priority for market outcomes to be feasible, which is not ensured in the future-aware-plus method. A question that arises in this setting is whether each market property should be guaranteed *in expectation*, or in absolute terms. The answer to this question may be different for each market property. Market efficiency and optimal social welfare would in an imperfect foresight setting never be guaranteed in absolute terms. For cost recovery on the other hand, we could envision a market design that guarantees this property in absolute terms under imperfect foresight. The design of such a market would be an interesting first extension of this work. However, it is possible that mechanisms that guarantee cost recovery only in expectation can actually improve the expected profit of the storage. Future works should investigate this trade-off.

The results in this chapter have been obtained assuming linear cost and utility functions, which are common in practice. We expect that the pricing problem can also arise in the nonlinear convex case, for similar reasons as in the linear case. However, the exact conditions in the nonlinear convex case need to be analyzed in more detail. This would be an interesting extension of our current analysis. Another extension could be to allow the non-merchant storage to provide services in several markets, for example, both energy and frequency regulation markets. The authors of [177] emphasize the importance of providing multiple services for the cost-effectiveness of storage, and the existence of market and policy barriers that prevent storage from doing so.

Given a suitable market design in the imperfect foresight setting, one could investigate how closely a real storage operator could approach perfect operation of the storage. This depends on the methods used for determining end-of-horizon decisions, i.e. the optimal final storage level E^{end}. Future work should propose and compare different methods. A popular approach has been the use of rolling-horizon markets, where the immediate dispatch is determined in an optimization that includes a longer time horizon. Forecasts of future market horizon parameters are needed to apply this method. In [165], forecasts of future market prices are used to determine the optimal storage dispatch in the current market horizon. An implicit assumption of using price forecasts directly is that the storage is a price taker. Other works apply stochastic programming methods, using scenarios of parameters in future market horizons. All of these methods rely on forecasts to indirectly determine optimal end-of-horizon decisions. Alternatively, one could consider to forecast the end-of-horizon decision directly from available data. Different forecasting techniques could be applied, for example online (reinforcement) learning.

In network-aware market designs for district heating, non-merchant storage is implicitly present. By changing the temperature of the water in the network, the amount of energy contained can be varied. In many studies with variable network temperatures, these temperatures are initialized at a level above their lower bound. Due to the lack of end-of-horizon constraints, these temperatures will be at their lower bound by the end of the simulated market clearing. This results in an overestimated value of network storage that many such works compute, see e.g. [45]. In practical applications, the lack of future-aware end-of-horizon decisions on the network temperature is likely to lead to suboptimal heat schedules, and may even result in problems with meeting inflexible heat loads in the start of a market-clearing horizon.

Chapter 6

Conclusions

In this thesis, we presented our contributions to three research directions in the field of market design for district heating systems, focusing on different aspects of future district heating systems. The first research direction involved the design of scheduling and pricing mechanisms for systems with a large number of small heat producers, where we focused on price-responsive excess heat providers. Next, we considered network-aware market design, which becomes important in more distributed district heating systems. In the third research direction, we investigated market properties in energy markets with non-merchant storage.

6.1 Key findings

Related to the integration of excess heat providers in district heating systems (**RD1**), we investigated whether self-scheduling based on a price signal can be a suitable alternative to direct market participation. The performance of the self-scheduling model was expected to depend strongly on the market share of excess heat providers. Therefore, we quantified the suboptimality resulting from self-scheduling for varying levels of excess heat penetration in a realistic case study of the Copenhagen district heating system. The case study included realistic bidding models of the Combined Heat & Power (CHP) plants and cooling-based excess heat providers. We showed that the absolute suboptimality increases approximately linearly with the installed excess heat capacity. However, the relative suboptimality increases more dramatically, as the generation cost of the perfect benchmark also decreases with installed excess heat capacity. We found that this suboptimality is caused by two main effects. Firstly, the excess heat is not optimally scheduled to replace the CHP plants when these are most costly, leading to suboptimal scheduling of CHP plants. Second, the self-scheduling excess heat producers are not aware of the heat load and have no incentive to match this load. Therefore, a larger amount of self-scheduled excess heat is wasted. We concluded that the suboptimality of self-scheduling may be acceptable under low excess heat penetration, given the advantages of this method in terms of simplicity and transparency. However, our findings suggested that more advanced coordination is needed when a higher market share of excess heat is achieved.

Towards the second research direction (**RD2**), we proposed a network-aware heat market mechanism based on the Variable-Flow-Constant-Temperature (VFCT) control strategy. This network formulation was chosen after careful evaluation of network modeling approaches for district heating systems. The variable-flow was preferred over the variable-temperature formulation, as it allows for more flexibility in both the direction and size of nodal heat injections, which is important for the integration of prosumers. Due to the complexity of more detailed and potentially more accurate heating network models, those formulations may be more suitable for the *control* of district heating systems, while simpler formulations may be more suitable for heating *markets*. Markets can be used to bridge the gap between the needs of the system operator and the needs of actors that are part of this system. On the one hand, suitable market designs support system operation, by providing schedules that are likely (near-)feasible in the network and result in reasonable operating cost. On the other hand, the market should provide interpretable and transparent price signals to market participants. Our proposed VFCT-based heat market was designed with this trade-off in mind. We analyzed how the addition of VFCT-based network constraints affects the scheduling and pricing of heat. Using peer-to-peer trades, we were able to trace network losses back to the producer and consumer related to these losses. A dual analysis revealed how modeled heat losses in the network affect nodal prices. We performed an illustrative case study to compare the proposed network- and loss-aware market to a network-aware but loss-agnostic benchmark, which was formulated to allow a fair comparison. By estimating and pricing heat losses in the network, we showed that our network-aware market mechanism promotes a more local heat consumption, which can result in reduced losses and total cost. We concluded that the proposed market mechanism can reduce operating costs while integrating distributed prosumers.

Towards the research direction related to energy markets with non-merchant storage (**RD3**), we addressed fundamental end-of-horizon issues that are often overlooked in the literature. These issues are often by-passed by making simplifying assumptions on the end-of-horizon storage level, in particular, to assume equal final and initial states of energy, or to disregard the final level altogether. We showed using an illustrative case study and formal derivations that these common simplifying assumptions, while ensuring cost recovery for the non-merchant storage within each market interval, can lead to market inefficiencies. Therefore, we concluded that it is important to set a future-aware end-of-horizon storage level, instead of making simplifying assumptions. However, we showed that in the absence of the simplifying assumptions, the prices in subsequent market horizons may fail to reflect the value of storage. Our aim was to draw attention to this pricing problem, which is important to address in practice, as it can lead to lack of dispatch-following incentives, and possibly lack of cost recovery for the storage. Intuition for the mechanism behind this pricing problem was provided using an illustrative example. In our main theorem, we provided sufficient conditions for the pricing problem to occur. The mild conditions showed that this problem is actually likely to occur in practice. Finally, we proposed a market-clearing procedure with non-merchant storage which is efficient and ensures cost recovery, under perfect foresight about future market-clearing parameters.

6.2 Perspectives for future research

The investigations in this thesis open up directions for future research from several perspectives. While more detailed, technical directions for future work were discussed in Chapters 3-5, in this section we take a broader view, and draw connections between the different chapters.

From a technical modeling perspective, we made several assumptions that could be relaxed in future work. An assumption common to all chapters has been that of perfect foresight. Our work towards each research direction could be extended by relaxing this assumption and considering uncertainty in specific parameters. When modeling the price-based self-scheduling of excess heat producers, the electricity prices and ambient temperature were assumed to be perfectly forecasted. In an extension of this work, we could quantify how uncertainty in electricity prices affects the performance of the self-scheduling and market-participation models. In network-aware heat market designs, it can be useful to consider uncertainty in the feasible region of generators and loads. This is especially the case on a distribution level, where there is high uncertainty in residential load. In addition, there could be uncertainty in the generation profile from excess and solar heat producers. Inspired by recent works on uncertainty in network-aware electricity markets, our network-aware heat market could be extended to be robust with respect to uncertainty. It could furthermore be investigated how the related cost of robustness could be allocated to the sources of uncertainty. In market designs with non-merchant storage, end-of-horizon decisions need to be made under uncertainty about the parameters in the next market horizon. Methods for optimally determining end-of-horizon decisions related to the storage system's state of energy are needed. Finally, market designs with non-merchant storage need to be adapted to deal with decisions that turn out suboptimal in hindsight, and to represent the value of storage at the start of a market clearing in an imperfect-foresight setting. We envision a reformulation of markets with non-merchant storage to ensure cost recovery for the storage operator, by tracing the end-of-horizon energy in the storage back to specific time periods where this energy was bought.

For our network-aware heat market design, we relied on several assumptions to derive a simplified model of a district heating network. Future research should continue to study how to best represent district heating networks in a market setting, bearing in mind the trade-off between accuracy, computational tractability, and interpretability of the resulting prices. Generally, in the literature on network-aware heat market design, we find that the chosen network model is assumed to be accurate, and that the set-points found by clearing the market can be directly used in the control of the system. In practice, errors will be made by the network models used in market design, which are usually less detailed and apply a lower time resolution than network models used for system control. Although some network models, such as the model specified by the node method, have been tested in practice, many others have not. It would be interesting to quantify the benefit of different network-aware heat market designs in practice, with a related control policy in mind. This would show the real benefit of using these market designs, compared to network-agnostic ones. It is not sure that the heating system would actually benefit from market designs with more detailed and complex network models of the system, if it has not been proven that such models lead to improvements in practice. Related to the work done in this thesis, it would be interesting to test the proposed VFCT-based market design in a real system, accompanied by a suitable control strategy.

From a broader modeling perspective, the availability of realistic, high-quality data is an important aspect that enables research and technological innovation. Such data is lacking in several areas related to district heating. For example, network data is usually not public, which makes it difficult to build realistic case studies for network-aware heat markets. Similar to IEEE test cases for power networks, it would be beneficial to have standardized test cases for district heating networks. This would also allow comparison between different network-aware heat market designs proposed in the literature. In addition, information regarding market-clearing procedures in existing district heating systems is difficult to find. The literature extensively covers current practices and conditions in the district heating systems of e.g. Denmark, Sweden, and China, but other countries are not covered in the same level of detail. Future research should diversify with regard to the studied district heating systems, and provide new insights on best practices from other regions.

From a system operation and market design perspective, this thesis focused on market design to support district heating system operation in the day-ahead stage. Related to the aforementioned uncertainty considerations, it would be interesting to study the design of intra-day and real-time heat markets, as well as their interaction with day-ahead markets. A related open question is the inclusion of non-merchant storage when it participates in both day-ahead and intra-day markets. Another important direction for future work from the system operation perspective remains the integration of many small, distributed heat producers and prosumers. We have shown that price-signal-based self-scheduling may not suffice when these participants cover a large share of the market, and new methods for their integration are needed. It is possible that market-based coordination of flexible consumers and excess heat providers can be achieved by similar methods. This coordination could be done with different aims in mind, such as integrating excess and renewable heat, supporting network operation, relieving congestion, or peak shaving. The question is how this coordination of prosumers and small producers should be organized. Several proposals from the power system literature, regarding integration of distributed energy resources, could be interesting to investigate in a heat setting. First, heat aggregators could be introduced as a new market participant responsible for the coordination of distributed prosumers and small producers. Future work should develop strategies for a heat aggregator, both for bidding in the market, as well as for price-signal design to provide the aggregated agents with the right incentives. As a second paradigm, decentralized trading platforms for heat could be established, for example in the form of energy communities for heat.

From a policy perspective, it is important to consider that district heating systems are more local and less interconnected than power systems, and that different systems therefore may require different solutions. Liberalized heat markets are likely not suitable for small district heating systems. The market mechanisms that were proposed and investigated in this thesis can still be applied, though in a more regulated setting. In larger district heating systems, it will be important to create enough competition to enable liberalization of their district heating markets, so that competitive forces can lead to increased system efficiency. To stimulate competition, policy must facilitate the integration of excess heat sources in district heating systems, and the activation of flexibility in heat consumption. As a side benefit, independent heat producers and prosumers may experience increased autonomy when given market access, as they can influence the heating system by changing their own behavior in response to market signals. To support policy-making, it would be useful to have a tool to evaluate whether a liberalized market could be advantageous in a given district heating system. While we in this thesis assumed perfect competition and disregarded strategic behaviour, future research could extend our work by modeling the effect of strategic behaviour in the proposed market designs. Such models can be used to simulate the effect of strategic behavior on the market outcomes, and thereby to evaluate whether it is advisable to liberalize the heat market in a given system. With the Lithuanian transition towards a liberalized heat market in mind, one could consider self-scheduling based on a price signal as a suitable intermediate step to attract more heat generators, and to activate demand-side flexibility. Once a sufficient amount of independent heat producers are integrated, a liberalized market could be introduced to coordinate these many small actors.

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Collection of relevant publications

[Paper A] L. Frölke, I.-M. Palm, J. Kazempour, "Market integration of excess heat", in *Electric Power Systems Research (EPSR)*, vol. 212, Article number: 108459, 2022. DOI:10.1016/j.epsr.2022.108459.

[Paper B] L. Frölke, T. Sousa, P. Pinson, "A network-aware market mechanism for decentralized district heating systems", in *Applied Energy*, vol. 306, Article number: 117956, 2022. DOI: 10.1016/j.apenergy.2021.117956.

[Paper C] L. Frölke, E. Prat, P. Pinson, R.M. Lusby, J. Kazempour, "On the efficiency of energy markets with non-merchant storage", Submitted to *IEEE Transactions on Energy Markets, Policy and Regulation*, 2022.

[Pub. A] Market integration of excess heat

Market Integration of Excess Heat

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***Abstract*—Excess heat will be an important heat source in future carbon-neutral district heating systems. A barrier to excess heat integration is the lack of appropriate scheduling and pricing systems for these producers, which generally have small capacity and limited flexibility. In this work, we formulate and analyze two methods for scheduling and pricing excess heat producers: self-scheduling and market participation. In the former, a price signal is sent to excess heat producers, based on which they determine their optimal schedule. The latter approach allows excess heat producers to participate in a market clearing. In a realistic case study of the Copenhagen district heating system, we investigate market outcomes for the two excess heat integration paradigms under increasing excess heat penetration. An important conclusion is that in systems of high excess heat penetration, simple price signal methods will not suffice, and more sophisticated price signals or coordinated dispatch become a necessity.**

***Index Terms*—Heat market, open district heating, excess heat, pricing.**

I. Introduction

District heating is expected to play an important role in future carbon-neutral energy systems, especially in urban areas [1]. District heating networks facilitate the decarbonization of heat generation, for instance by allowing for distribution of *excess heat* to consumers. Examples of excess heat producers include energy intensive industries such as metal and cement factories, sources in the service sector that produce heat as a by-product of their refrigeration systems [2], and data centers that produce heat from cooling their servers [3]. In many cities, excess heat has the potential to cover a large share of total heat demand. For example, [4] finds that excess heat could cover over half of Greater Copenhagen's heat demand in 2050. Excess heat could also cover over 80% of demand in several other large Danish district heating networks [5]. Current district heating systems typically rely on few large generators, such as Combined Heat and Power (CHP) plants and waste incinerators. Compared to these conventional sources, excess heat producers are generally of smaller capacity and lower flexibility. Integration of excess heat producers would therefore result in a more distributed heating system, with a large number of small heat sources. In such a system, coordination and scheduling of heat generation becomes more challenging.

A major barrier to excess heat injection in district heating networks is the lack of suitable methods for *scheduling* and *pricing* excess heat. Most potential excess heat sources are untapped, even though it has been shown that their integration decreases both fuel usage and operational cost of the system [6], and the integration of excess heat has proven feasible in simulation studies and in practice [7], [8]. It remains an open question how heat scheduling and pricing systems can be designed to optimally integrate excess heat producers.

Most existing district heating systems do not have a liberalized market. In Greater Copenhagen, the daily heat dispatch is determined by Varmelast.dk, which is a regulated heat market that dispatches generators based on submitted price-quantity bids. While the scheduled quantities are determined by this market, the prices that could be derived from the market are not used. Instead, the price of heat is fixed in advance in contracts between suppliers of district heating and distributors/transmitters. Excess heat providers currently do not have the possibility to participate in this dispatch procedure. See e.g. [9] for more details on Varmelast.dk.

Only few existing works have studied excess heat producers in a market setting. In [6], the impact of excess heat producers on the heat market in Espoo, Finland, is investigated. References [10] and [11] study the potential effect of dynamic pricing on marginal cost of district heating systems, for a system including excess heat producers. In both works, it is assumed that the excess heat production profile is constant over each month or over the entire year, and that this profile is fully inflexible. Furthermore, the price bidding behaviour of CHPs is not modeled accurately, either disregarding electricity price dependence [11], or disregarding the dependence on opportunity cost in the electricity market [10]. Reference [12] applies marginal-cost pricing to a case study in the Netherlands to assess whether different producers can recover their fixed costs from market revenues. Their market clearing consists of a combined unit commitment and economic dispatch. Excess heat from industrial processes is included, but its flexibility is not modeled. None of these works investigate the effect of increasing excess heat penetration.

In this work, we investigate whether price signals to be disseminated by the heat market operator can suffice for market integration of excess heat producers. To the best of our knowledge, this is the first paper in the literature that explores the integration of excess heat producers through price signals. Given a price signal, excess heat producers self-schedule their production. We evaluate this *self-scheduling* model by comparison to an ideal benchmark, namely direct *market participation* of excess heat producers. This comparison is done by performing a realistic case study of the Copenhagen district heating system, which currently includes 13 CHP plants. We evaluate the success of the self-scheduling method by adding an increasing number of excess heat producers to the Copenhagen system. We aim to show the consequences of integrating cooling-based excess heat producers under these two paradigms, including how the suboptimality of the self-scheduling model evolves under increased penetration of excess heat. Our main finding is that price signals can be used as an alternative for market participation of excess heat producers, but their success depends highly on the quality of the signal, as well as the penetration of excess heat. As long as the installed excess heat capacity is sufficiently low, a simple price signal may suffice, but as excess heat penetration increases there may be significant downsides to this approach.

The remainder of this article is structured as follows. In Section II, we discuss the self-scheduling and market participation models in more detail. We provide model formulations, and outline the bidding behavior of different market participants. The real case study of the Copenhagen district heating system is presented in Section III, including numerical results. In Section IV we conclude with further discussions regarding the implications of both methods and provide several recommendations for future work.

II. Market participation versus self-scheduling

One natural way of integrating excess heat producers in heat markets, is by direct market participation. For example in the district heating system of Copenhagen, a marginal-cost based heat market is operated by Varmelast[1]. This paradigm will further be referred to as *market participation*. Another option is for heat market operators to publish a time-varying price signal for excess heat producers, which optimally self-schedule their heat generation accordingly, and share the resulting schedule with the market operator. We further refer to this paradigm as *self-scheduling*. In the Open District Heating system of Stockholm, small excess heat producers are successfully integrated in this way using an ambient temperature-dependent price signal[2].

We first discuss the advantages and benefits of each model in Section II-A, and then present the model formulations in Sections II-B and II-C. Finally, Section II-D outlines CHP and excess heat provider bidding behaviour.

A. Comparison of market integration and self-scheduling

We compare the two paradigms in Table I. First of all, they differ in the formation of the price received by excess heat participants. Under the market participation scheme, the market-clearing price follows from the bids submitted by market participants (including excess heat producers), while the price for excess heat is set exogenously by the market operator in the self-scheduling case. As also indicated in Table I, market participation would provide incentives to optimally schedule excess heat production, as it minimizes total generation cost. If the price signal for self-scheduling is designed perfectly, the resulting schedules may be the same as in a market setting. Otherwise, the schedule resulting from price signals will be suboptimal. This suboptimality may increase with the penetration of excess heat producers.

[1]www.varmelast.dk/

[2]www.opendistrictheating.com

	Market Participation	Self-scheduling
Price formation	Endogenous	Exogenous
Optimal scheduling	✓	✓ / ×
Relevance for small producers	×	✓
Problem type	Linear program	Linear program

TABLE I: Comparison of the two paradigms for scheduling and pricing excess heat

Although market participation of excess heat producers would be optimal from a cost minimization perspective, it has some drawbacks in practice for a heat system with many small (excess) heat producers. For small excess heat producers, it may be difficult to decide on market bids, and they may therefore prefer to receive a price signal. For the market operator, the market participation of many small excess heat producers poses a communicational challenge: the operator receives bids from many participants, clears a more complex market, and then needs to send the individual schedules to each small market participant. Therefore, the self-scheduling paradigm may be preferred in practice, as it is a rather simple and computationally cheap way of scheduling and pricing excess heat, with lower IT requirements. We consider the market participation scheme as *ideal benchmark*, and explore the success of self-scheduling scheme in comparison to this ideal benchmark. Both the market participation and the self-scheduling model are Linear Programs, in which no binary variables are used.

B. Model formulation for market participation

In the market participation scheme, excess heat producers participate in a market clearing. We formulate a heat market clearing without network constraints as a linear optimization problem. The market clearing results in a uniform market price and scheduled quantities for all participants, including excess heat producers. All market participants submit price-quantity bids. The market then dispatches generators to minimize total generation cost, i.e., according to the merit order. Following EU electricity market design, we do not consider unit commitment constraints. This implies that unit commitment constraints should be internalized into the bids. We also choose not to enforce network constraints in the optimal dispatch, because the current optimal dispatch mechanism in Copenhagen does not include such constraints either. Instead, hydraulic conditions are checked after clearing the market [9].

The objective of the market clearing is to minimize total heat generation cost, given by the function

$$f(\Gamma^{\mathrm{mp}}) = \sum_{t\in\mathcal{T}}\left(\sum_{e\in\mathcal{E}} c_{et}(G^{\mathrm{H}}_{et} - W_{et}) + \sum_{i\in\mathcal{G}} c_{it}G^{\mathrm{H}}_{it} + c^{\mathrm{U}}U_t\right), \tag{1}$$

where Γ^{mp} is the set of optimization variables for the market participation model, c_{it} is the bid price of CHP i at time t, c_{et} is the bid price of excess heat producer e, G^{H}_{it} is the generated heat by a CHP, and c^{U} represents the (constant) penalty cost per unit of unsupplied load U_t. The excess heat production G^{H}_{et} may in some cases exceed the load, so that an amount of W_{et} will have to be wasted, i.e., vented to the outside air. The

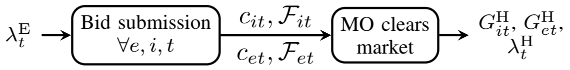


Fig. 1: Overview diagram of market participation model. Market Operator is abbreviated to MO.

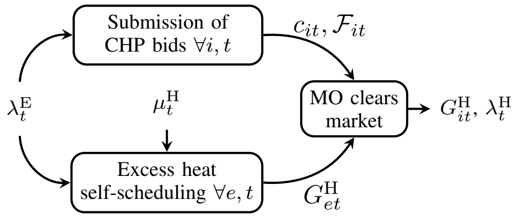


Fig. 2: Overview diagram of self-scheduling model and the following market clearing. Market Operator is abbreviated to MO.

wasted excess heat must be non-negative and cannot exceed the produced excess heat:

$$G_{et} \geq W_{et} \geq 0 \qquad\qquad \forall e, t\,. \tag{2}$$

A power balance must hold between the predicted heat load $\hat{L}_t^{\mathrm{H}}$ and the scheduled generation. If the load cannot be supplied due to insufficient installed capacity, the load can be curtailed by an amount U_t of unsupplied load:

$$\sum_{e\in\mathcal{E}} G_{et}^{\mathrm{H}} - W_{et} + \sum_{i\in\mathcal{I}} G_{it}^{\mathrm{H}} = \hat{L}_t^{\mathrm{H}} - U_t \quad : \lambda_t^{\mathrm{H}} \qquad \forall t\,. \tag{3}$$

The uniform market price λ_t^{H} is given by the dual variable corresponding to constraint (3), which is equal to the marginal price bid of the most expensive scheduled generator.

The market participants' bid includes a description of the feasible region $\mathcal{F}$ of their heat generation. These feasible regions are respected in the market clearing:

$$G_{it}^{\mathrm{H}} \in \mathcal{F}_{it} \qquad\qquad \forall i, t \tag{4}$$

$$G_{et}^{\mathrm{H}} \in \mathcal{F}_{et} \qquad\qquad \forall e, t\,. \tag{5}$$

We will define these feasible regions for CHPs and excess heat producers in Section II-D, and in more detail in Appendices A and A. The set of optimization variables in the market-clearing optimization problem is given by $\Gamma^{\mathrm{mp}} = \{G_{it}^{\mathrm{H}}, G_{et}^{\mathrm{H}}, W_{et}^{\mathrm{H}}, U_t^{\mathrm{H}}\}$. Fig. 1 shows a graphical overview of the market participation model. The forecasted electricity price λ_t^{E} is an input to the marginal cost model of both CHPs and excess heat producers, which will be discussed in Section II-D.

C. Model formulation for self-scheduling

In the self-scheduling model, the market operator broadcasts a price signal μ_t^{H} for each market period t to all excess heat producers, who self-schedule their production accordingly. The resulting schedule is submitted to the market operator, who uses the total excess heat production as a fixed input to the market clearing with conventional generators only. This market clearing is as described previously in Section II-B, except that G_{et}^{H} is now a parameter instead of a variable for all e, t. As a result, the total self-scheduled heat generation by the excess heat producers is prioritized in the heat market, and the CHPs may supply any remaining unsupplied load. If the self-scheduled excess heat exceeds the heat load at certain hours, some of the excess heat is wasted, i.e., vented to the air.

We assume an ambient-temperature-dependent price signal for excess heat producers, inspired by the Stockholm Open District Heating pricing system. In particular, the received price decreases with the ambient temperature, as heat demand often decreases with ambient temperature too. Under the self-scheduling scheme, the excess heat producers are paid as in the price signal for each unit generated, regardless of whether (part of) the produced excess heat exceeds the supplied load and needs to be wasted. Clearly, this is an undesirable effect of the self-scheduling paradigm. The CHPs are still paid at the uniform marginal price resulting from the market clearing.

During self-scheduling, excess heat producers aim to minimize total cost, given by the difference between costs for electricity used by the heat pump L_{et}^{E} bought at the (forecasted) electricity spot price λ^{E}, and the income from selling the generated heat G_{et}^{H}:

$$C_e^{\mathrm{H}} = \sum_{t\in\mathcal{T}} \left(\lambda_t^{\mathrm{E}} L_{et}^{\mathrm{E}} - \mu_t^{\mathrm{H}} G_{et}^{\mathrm{H}}\right)\,. \tag{6}$$

It is assumed that all excess heat producers and CHPs use the same forecasted electricity prices λ_t^{E}. The excess heat producers must schedule an amount of heat that respects their physical constraints:

$$G_{et}^{\mathrm{H}} \in \mathcal{F}_{et} \qquad\qquad \forall e, t\,. \tag{7}$$

The feasible region $\mathcal{F}_{et}$ will be defined in the next Section II-D, and more details can be found in Appendix A. Fig. 2 shows a graphical overview of the self-scheduling model.

D. Excess heat and CHP models

To simulate the two paradigms, the bidding behavior of different market participants needs to be modeled. In this work, we consider CHPs and cooling-based excess heat producers only. The derivation of their bidding behavior and feasible regions is presented in more detail in Appendices A and A. We assume that the heat market is cleared daily before the electricity market is cleared, as is currently the case in Copenhagen. The resulting feasible region for CHP i at time t is given by

$$\mathcal{F}_{it} = \left\{ G_{it}^{\mathrm{H}} \;\middle|\; 0 \leq G_{it}^{\mathrm{H}} \leq \min\left(\overline{G}_i^{\mathrm{H}}, \frac{\overline{F}_i}{\rho_i^{\mathrm{H}} + r_i\,\rho_i^{\mathrm{E}}}\right)\right\}\,, \tag{8}$$

where $\overline{G}_i^{\mathrm{H}}$ is the maximum heat generation, ρ_i^{E} and ρ_i^{H} are the fuel efficiency for electricity and heat, respectively, r_i is the minimum power-to-heat ratio, and $\overline{F}_i$ is the maximum fuel consumption. Note that these parameters are here considered time-invariant, but this could easily be adapted.

Reference [13] derives the optimal heat bid c_{it}^{H} for CHPs in a sequential heat and electricity market setting. The marginal cost of heat depends on the (forecasted) electricity price λ^{E} and (constant) fuel price α_i as follows:

$$c_{it}^{\mathrm{H}} = \begin{cases} \alpha_i \left(\rho_i^{\mathrm{E}}\, r_i + \rho_i^{\mathrm{H}}\right) - \lambda_t^{\mathrm{E}}\, r_i & \text{if } \lambda_t^{\mathrm{E}} \leq \alpha_i \rho_i^{\mathrm{E}} \\ \lambda_t^{\mathrm{E}}\, \frac{\rho_i^{\mathrm{H}}}{\rho_i^{\mathrm{E}}} & \text{if } \lambda_t^{\mathrm{E}} > \alpha_i \rho_i^{\mathrm{E}}\,. \end{cases} \tag{9}$$

This bidding function assumes that in case the forecasted electricity price is lower than or equal to a certain threshold, CPHs bid their fuel cost minus the income from electricity sale, as represented by the first case in (9). Otherwise, if the forecasted electricity price is relatively high, CHPs bid the lost opportunity cost from selling heat instead of electricity, as in the second case in (9).

The feasible region for excess heat producers is derived in Appendix A. The excess heat producers' flexibility in heat production is represented by modeling cooling cabinet heat dynamics as a set of linear constraints as given in Appendix A through Eqs. (21)-(26). We define the feasible region for excess heat production as the set of production setpoints that satisfy the aforementioned constraints:

$$\mathcal{F}_{et} = \left\{ G_{et}^{\mathrm{H}} \quad \middle| \quad (21) - (26) \right\}\,. \tag{10}$$

The price bid of excess heat producers in the market participation model is assumed to be at zero[3], i.e.,

$$c_{et} = 0 \qquad\qquad \forall e, t\,. \tag{11}$$

III. Copenhagen case study

We analyze the application of the market participation and self-scheduling models in the district heating system of Copenhagen, Denmark.

A. Case study description

The district heating system of Copenhagen consists of 13 CHPs. In our case study, we vary the level of excess heat capacity added to this system from 0 to 2100 MW. The excess heat is assumed to be produced as a by-product of cooling. In particular, we assume excess heat producers cool refrigeration cabinets using a local heat pump. We simulate in an hourly time resolution for a full year. In our online appendix, a detailed description of all used parameters can be found, as well as the code used to generate our results[4].

Several time series are needed as inputs for the market participation and/or self-scheduling models. The forecasted ambient temperature is an input used to model the Coefficient Of Performance (COP) of the heat pumps, and also to determine the price of waste heat in the self-scheduling model. We use hourly temperature measurements from 2019 from the Danish Meteorological Institute [14]. For the month March we use measurements from 2020, due to many missing measurements for March 2019. As the electricity price forecast, we use Nord Pool historical electricity prices for DK2 [15]. For the forecasted heat load, we use the hourly heat load in the entire Copenhagen district heating area for 2019, provided by Varmelast.

For the self-scheduling model, the excess heat price signal needs to be given. The self-scheduling pricing signal used here is inspired by Stockholm's Open District Heating Spot Prima price. We approximate their ambient-temperature dependent price function using an exponential regression on data available from their website. The resulting price signal $\mu_t^{\mathrm{H}}(\cdot)$ is defined as follows:

$$\mu_t^{\mathrm{H}}(T_t^{\mathrm{A}}) = \begin{cases} 380 \cdot 0.92^{T_t^{\mathrm{A}}} & \text{for } T_t^{\mathrm{A}} < 17.5°\mathrm{C} \\ 0 & \text{for } T_t^{\mathrm{A}} \geq 17.5°\mathrm{C}\,. \end{cases} \tag{12}$$

Note that the price is decreasing with the ambient temperature, as the base of the exponent 0.92 is non-negative and below 1.

We further require input parameters for CHP and excess heat producer models. For the CHPs, most input parameters have been obtained from [16]. The minimum power-to-heat ratio was not given there, and therefore a default value of $r = 0.45$ has been taken from [17].

The excess heat producers are assumed to have the same input parameters. The heat dynamics parameters are $A = 0.1$ and $B = \frac{1}{21}$. The temperature in the cooled room has to be within 2-8 °C, while the average temperature every 6 hours has to be within 4-5 °C. The indoor temperature at the excess heat producers is assumed to be constant at 25 °C. The heat pumps' maximum generation capacity is $\overline{G}^{\mathrm{H}} = 30$ kW. Heat pump ramping limits are set to 0.25 of its maximum generation capacity. It is assumed that the COP of the excess heat producers' heat pumps varies with the ambient temperature. The approximate ambient temperature dependence of the COP was obtained using a more detailed heat pump simulation model, under several simplifying assumptions, including a linear dependence of supply and return temperatures on ambient temperature.

To validate our CHP bidding model, we have compared our resulting heat market prices to Varmelast heat market prices for a given electricity price signal. The results showed satisfactory correspondence between our and Varmelast's heat prices.[5]

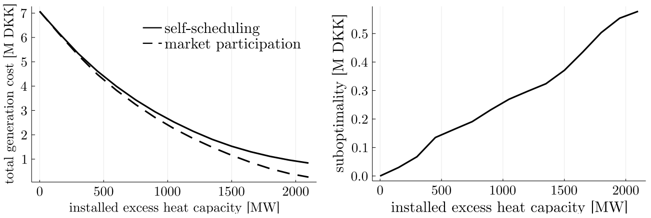
B. Results

The total heat load in 2019 in Copenhagen was around $8.3\mathrm{TW\,h}$ or $30\mathrm{PJ}$. For the different maximum capacity levels of the excess heat producers of 300, 1200, and 2100 MW participating in the heat market, we find that the excess heat providers are scheduled for 1.8, 5.8, and $8.0\mathrm{TW\,h}$, respectively. Considering that [4] reports that excess heat could cover over 50% of the Copenhagen heat load, the $1200\mathrm{MW}$ case could be realistic for the Copenhagen system in 2050.

[3]Bidding at zero is reasonable if any sold excess heat is considered extra income for these producers. However, the electricity consumption cost of a given excess heat production profile may be greater than the electricity cost of the production profile that minimizes these costs. One may therefore choose to define the price bid of a certain production profile as the difference between the electricity cost of the given profile and the minimum electricity cost this producer could obtain if it would minimize electricity cost only.

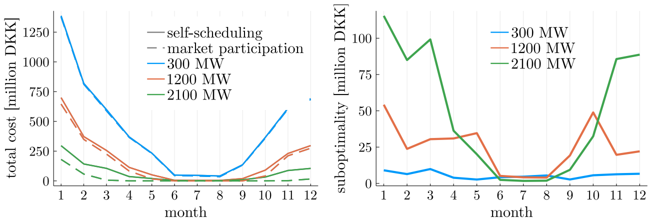
[4]www.github.com/linde-fr/excess-heat-in-market

[5]Exact results cannot be shared here due to confidentiality of Varmelast pricing data.



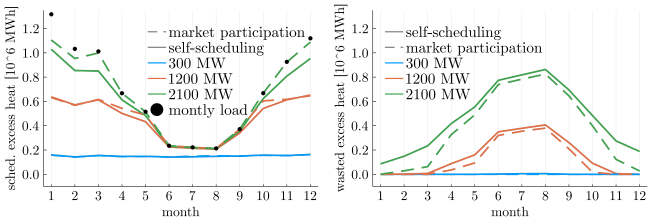
(a) Total generation cost (b) Difference in generation cost

Fig. 3: Full year suboptimality of self-scheduling compared to market integration in terms of total CHP generation cost. Note that this does *not* include excess heat generation cost.



(a) Total generation cost (b) Difference in generation cost

Fig. 4: Monthly suboptimality of self-scheduling compared to market integration in terms of total CHP generation cost.



(a) Scheduled volume (b) Wasted volume

Fig. 5: Monthly values for excess heat scheduled volume and wasted volume for self-scheduling compared to market integration

Suboptimality: When the excess heat producers are self-scheduling and the market for CHPs is cleared afterwards, the total generation cost for CHPs will be greater than it is in the case the market is cleared with excess heat producers integrated. Here we consider suboptimality in terms of the total CHP generation cost, where we treat this total cost similarly as in the objective function of the market clearing. That is, generation cost of each scheduled CHP is computed as its scheduled quantity multiplied by its price bid, and the total generation cost is obtained by summing over all CHPs. In Fig. 3, we visualize how the total suboptimality over a full year depends on the penetration of excess heat producers. In the left-hand Fig. 3a, we observe that both schemes experience a steep decrease of the total generation cost with the installation of the first 500 MW of excess heat capacity, but this decrease flattens out afterwards. Fig. 3b shows that the absolute suboptimality grows almost linearly with the installed excess heat capacity. The curve is slightly steeper when the latest installed excess heat is replacing a CHP that is expensive compared to the CHPs that bid a price just below it.

Next, we investigate how this suboptimality in total CHP generation cost is distributed over the year, by zooming in on three levels of excess heat penetration in Fig. 4. The left-hand Fig. 4a shows that the generation cost is unequally distributed over the year, as monthly heat load varies significantly over the year. As seen in the right Fig. 4b, the suboptimality is quite equally distributed over the year for a low capacity of excess heat at 300 MW (blue). For higher excess heat penetration, the suboptimality is increasingly shifted to the colder months. In the warmest summer months, i.e., from June to August, the suboptimality of the 300 MW case (blue) is relatively high compared to the 1200 MW and 2100 MW cases (orange and green). The reason for this is that from a certain level of excess heat capacity, the overcapacity in summer is very high, so that the total demand is (almost) always completely supplied by excess heat, both under self-scheduling and market participation. Therefore, suboptimality will be low in summer for a higher penetration of excess heat.

Scheduled and wasted excess heat: The scheduled excess heat varies over the year. There can be a difference between *generated* and *scheduled* excess heat, as some excess heat may have to be wasted in case it exceeds the heat load. We compare the monthly scheduled excess heat for the self-scheduling and market participation under different levels of excess heat penetration in Fig. 5a, and do the same for monthly wasted excess heat in Fig. 5b. Under the market participation model, a capacity of 2100 MW (green) is enough to supply the load fully in all but the coldest months, i.e., December to March. This is not the case for the self-scheduling model, which is due to a greater mismatch between supplied excess heat and heat load. This manifests itself in the consistently greater amount of wasted heat for the self-scheduling model. In general, differences in the self-scheduling and market participation model in total scheduled excess heat are greatest in months where excess heat is the marginal supplier in some hours, but not in all hours. With increasing excess heat capacity, the months where this is the case shift more towards the winter months with greater heat load. Note that the scheduled volume in the summer months is almost equal for the 1200 MW (orange) and 2100 MW (green) cases, which is the reason that suboptimality in summer months is similar for these cases, as we observed previously in Fig. 4b.

Finally, we highlight that the total wasted excess heat increases steadily with increasing excess heat penetration, for both scheduling paradigms. This is due to the limited flexibility of these producers, combined with the fact that these producers have a minimum heat generation that may exceed the load. To decrease the wasted excess heat and supply a higher share of the load, it would be beneficial to install heat storage as the penetration of excess heat increases.

Market prices: We compare monthly average market prices resulting from the market clearing in both the self-scheduling and market participation models in Fig. 6. Recall that we also clear the market in the self-scheduling model, but in this case the excess heat schedule is a fixed input to the market.

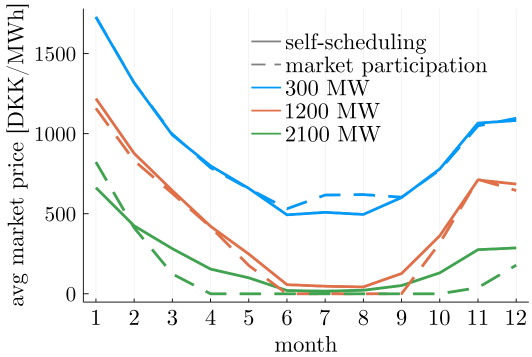


Fig. 6: Average market prices per month for self-scheduling compared to market participation, under various levels of excess heat penetration.

Under a low excess heat penetration of 300 MW (blue), the average market price only differs for the summer months. Counter-intuitively, the average market price for the market participation model is higher in this case than that in the self-scheduling model. The explanation of this is that the market participation model schedules excess heat when it can reduce total generation cost as much as possible, which means that it will spread out the excess heat schedule to avoid scheduling of more expensive generators. The effect of this spread can be that on average, the marginal generator is more expensive than in the self-scheduling case, as is the case for our case study. In other words, the minimization of total generation cost does not necessarily lead to the lowest average market prices. This is also seen for the month January for the 2100 MW case (green). However, in most cases and months the market participation model does lead to lower average market prices compared to the self-scheduling model.

As expected, the average market price decreases under increasing excess heat penetration. With a relatively high installed excess heat capacity of 2100 MW (green), the market price for the market participation model is close to zero from April up to and including October. In the case of high installed capacity, the average marginal price difference between the two models is most pronounced.

IV. Conclusions and Future Perspectives

We have investigated the consequences of integrating excess heat in district heating systems under two different scheduling and pricing paradigms: self-scheduling and market participation. The self-scheduling is attractive due to its simplicity for both market operator and excess heat producers, but may lead to suboptimal scheduling. Our main conclusion is that at higher excess heat penetration, a simple price signal is no longer adequate, and more sophisticated pricing signals and/or other market setups may be needed. In our case study, we have shown that the disadvantages of using a price signal under high excess heat penetration include:

1) **Expensive scheduling**: Excess heat is scheduled in hours where CHPs can produce relatively cheap heat instead of where CHPs produce more expensive heat. As a result, total CHP generation cost is higher under self-scheduling than the market participation.
2) **Wasted excess heat**: Excess heat production is not matched to heat load, so that a greater amount of excess heat is wasted.
3) **High market prices**: Even though market participation may lead to higher average market prices in some cases, market prices are most of the time higher in the self-scheduling model, especially under high excess heat penetration.

A. Discussion

Under increasing penetration of excess heat, market prices decrease under both paradigms, and most drastically under market participation. Market prices get close to zero during the summer time already for intermediate penetration of excess heat. This may affect the recovery of fixed cost for generators. For example, in [12] it is shown that market revenues can be insufficient for investment cost recovery for most heat producers in a case study in the Netherlands. This problem has also been encountered in electricity systems with high shares of solar and wind energy [18]. For this case, it has been suggested that marginal-cost-based market clearing is not necessarily a proper solution for systems of generators with high fixed and low marginal costs, and a rethinking of power markets is needed [18]. This problem can be expected to arise in excess heat based heat systems too.

We have concluded that more sophisticated pricing signals than the Stockholm price are needed in systems with high excess heat penetration. However, the Stockholm ambient-temperature dependent pricing signal has two attractive properties: transparency and interpretability. These properties should be considered when designing new methods for generating pricing signals.

Finally, we note that cooling-based excess heat producers provide most excess heat during the summer months, which is a mismatch with the load that is minimal in this period. This relation indicates that a seasonal storage may be a suitable supplement in systems dominated by cooling-based excess heat producers.

B. Recommendations for future work

In this work, we have designed a model of cooling-based excess heat producers with the aim was to mimic general dynamics of such producers in a convex manner. The model has been formulated after discussion with experts in more detailed heat pump modeling. However, the model has not been verified using real data of excess heat producers. This should be done in future work.

Furthermore, the model could be extended and made more realistic in several ways. In our model, flexibility of heat producers was limited using an energy budget to be respected over every six hours. This is a stylized representation of excess heat flexibility. Future reformulations of the model could focus on improving the representation of this (limited) flexibility. On the conventional generator side, the market considered here assumes that unit commitment constraints are included in the price-quantity bids. However, we have ignored unit commitment considerations in the construction of supply bids. Our work could be extended with block bids to represent bidding behavior of CHPs more realistically.

It could be argued that a scaling is needed to adapt the Stockholm heat price signal to the Copenhagen case. A sensitivity analysis for the value of scaling factor could show how this would affect our results.

We have addressed effects of excess heat integration on the day-ahead market, without considering potential sources of uncertainty. Future work could investigate how uncertainty of heat load, as well as uncertainty in excess heat production, could affect the scheduling and pricing of excess heat. We have also disregarded network considerations. Inclusion of such constraints could change our results quantitatively, as there would be a delayed arrival of CHP heat, while local excess heat would be delivered close to real time. In addition, heat transported from a distance would be accompanied by greater heat losses. Our work could be extended to include network constraints, for example using the linear formulation in [19].

Our analyses could also be extended by adding different types of market participants. For example, future work could investigate the effect of adding flexible loads to the system. It is furthermore likely that future district heating systems will include other excess heat producers that are not cooling based, such as energy intensive industries.

Acknowledgment

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Appendix

We derive the price bids and feasible regions $\mathcal{F}$ for CHPs and excess heat producers in Appendices A and A, respectively.

Our model of CHP plants, including their bidding behavior, is identical to the sequential decoupled formulation in [13]. As it is the current practice in Copenhagen, we assume sequential heat and electricity markets, where the heat market is cleared first.

The fuel intake F_{it} of CHP i is equal to the fuel used for electricity generation G^{E}_{it} and heat generation G^{H}_{it}. The fuel generation is upper bounded as

$$F_{it} = \rho^{\mathrm{E}}_i G^{\mathrm{E}}_{it} + \rho^{\mathrm{H}}_i G^{\mathrm{H}}_{it} \leq \overline{F}_i \qquad \forall i, t\,. \tag{13}$$

Recall that ρ^{E}_i and ρ^{H}_i represent the fuel efficiency for electricity and heat, respectively. A minimum power-to-heat ratio r_i relates heat and electricity production as

$$G^{\mathrm{E}}_{it} \geq r_i G^{\mathrm{H}}_{it} \qquad \forall i, t\,. \tag{14}$$

Both heat and electricity generation must be non-negative, but due to (14) limiting heat generation suffices.

$$0 \leq G^{\mathrm{H}}_{it}\,. \tag{15}$$

If drawn in a diagram with G^{E}_{it} on the y-axis and G^{H}_{it} on the x-axis, constraints (13)-(15) form a triangle with the y-axis as a base. At the tip of the triangle, the amount of generated heat is

$$G^{\mathrm{H},*} = \frac{\overline{F}_i}{\rho^{\mathrm{H}}_i + r_i\, \rho^{\mathrm{E}}_i} \qquad \forall i\,. \tag{16}$$

Additionally, the heat generation of a CHP may be upper bounded as

$$G^{\mathrm{H}}_{it} \leq \overline{G}^{\mathrm{H}}_i \qquad \forall i, t\,. \tag{17}$$

As the heat market is cleared before the electricity market, constraints (13)-(17) can be replaced by the following bounds on the generated heat quantity:

$$0 \leq G^{\mathrm{H}}_{it} \leq \min\left\{\overline{G}^{\mathrm{H}}_i, \frac{\overline{F}_i}{\rho^{\mathrm{H}}_i + r_i\, \rho^{\mathrm{E}}_i}\right\} \qquad \forall i, t\,. \tag{18}$$

Recall that we have already provided the feasible space for CHPS resulting from this constraint in Section II-D in Equation (8).

The net heat production cost for a CHP is given by the difference of fuel cost and revenue from electricity sale:

$$C^{\mathrm{H}}_{it} = \alpha_i\,(\rho^{\mathrm{E}}_i G^{\mathrm{E}}_{it} + \rho^{\mathrm{H}}_i G^{\mathrm{H}}_{it}) - \lambda^{\mathrm{E}}_t G^{\mathrm{E}}_{it} \qquad \forall i, t\,, \tag{19}$$

where α is the fuel price. In [13], the optimal heat bid c^{H}_{it} is derived for CHPs in a sequential heat and electricity market setting. The price bid depends on the (forecasted) electricity price as

$$c^{\mathrm{H}}_{it} = \begin{cases} \alpha_i\,(\rho^{\mathrm{E}}_i\, r_i + \rho^{\mathrm{H}}_i) - \lambda^{\mathrm{E}}_t\, r_i & \text{if } \lambda^{\mathrm{E}}_t \leq \alpha_i \rho^{\mathrm{E}}_i \\ \lambda^{\mathrm{E}}_t\, \frac{\rho^{\mathrm{H}}_i}{\rho^{\mathrm{E}}_i} & \text{if } \lambda^{\mathrm{E}}_t > \alpha_i \rho^{\mathrm{E}}_i\,. \end{cases} \tag{20}$$

We assume that all excess heat is produced as a by-product of cooling. In particular, we assume these agents cool their refrigeration cabinets using a local heat pump. The heat pump's heat output G^{H}_{et} relates to its electricity load L^{E}_{et} as

$$G^{\mathrm{H}}_{et} = \mathrm{COP}_{et} L^{\mathrm{E}}_{et} \qquad \forall e, t\,, \tag{21}$$

where the coefficient of performance COP_{et} is a time-varying parameter in our model. This allows us to include its approximate ambient temperature dependence. The heat output of the heat pump is subject to upper and lower bounds:

$$0 \leq G^{\mathrm{H}}_{et} \leq \overline{G}^{\mathrm{H}}_e \qquad \forall e, t\,. \tag{22}$$

The refrigeration cabinets are the heat source of the heat pump. This implies that the available heat depends on the temperature dynamics in these cabinets. We model the refrigerator temperature T^{F}_{et} using a linear difference equation as

$$T^{\mathrm{F}}_{et+1} - T^{\mathrm{F}}_{et} = A_e\left(T^{\mathrm{I}}_{et} - T^{\mathrm{F}}_{et}\right) - B_e\left(G^{\mathrm{H}}_{et} - L^{\mathrm{E}}_{et}\right) \quad \forall e, t\,, \tag{23}$$

where T^{I} is the indoor temperature in the supermarket. The parameters A_e and B_e may differ per excess heat provider, depending on the physical characteristics of the refrigerators. The refrigerator temperature is subject to bounds:

$$\underline{T}^{\mathrm{F}}_e \leq T^{\mathrm{F}}_{et} \leq \overline{T}^{\mathrm{F}} \qquad \forall i, t\,. \tag{24}$$

The average temperature of the refrigerator over chosen time periods $\mathcal{P}$ must also stay within pre-set limits:

$$T^{\mathrm{F}-}_e \leq \frac{1}{|P|}\sum_{t\in P} T^{\mathrm{F}}_{et} \leq T^{\mathrm{F}+}_e \qquad \forall e, P \in \mathcal{P}\,, \tag{25}$$

which ensures that the refrigerator temperature will not be on lower or upper bounds for longer periods of time. The periods must be defined such that $\bigcup_{P\in\mathcal{P}} P = \mathcal{T}$, so that all time steps are part of at least one period. Finally, the heat output from the heat pump is subject to ramping limits:

$$\underline{R}_e \leq G^{\mathrm{H}}_{et+1} - G^{\mathrm{H}}_{et} \leq \overline{R}_e \qquad \forall e, t\,. \tag{26}$$

Recall that we already provided the feasible space for excess heat producers resulting from the previous constraints in Section II-D in Equation (10).

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[Pub. B] A network-aware market mechanism for decentralized district heating systems

A network-aware market mechanism for decentralized district heating systems

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Abstract

District heating systems become more distributed with the integration of prosumers, including excess heat producers and active consumers. This calls for suitable heat market mechanisms that optimally integrate these actors, while minimizing and allocating operational costs. We argue for the inclusion of network constraints to ensure network feasibility and incentivize loss reductions. We propose a network-aware heat market as a Quadratic Program (QP), which determines the optimal dispatch and a set of nodal marginal prices. While heat network dynamics are generally represented by non-convex constraints, we convexify this formulation by fixing temperature variables and neglecting pumping power. The resulting variable flow heating network model leaves the sign and size of the nodal heat injections flexible, which is important for the integration of prosumers. The market is based on peer-to-peer trades to which we add explicit loss terms. This allows us to trace network losses back to the producer and consumer of these losses. Through a dual analysis we reveal loss components of nodal prices, as well as relations between nodal prices and between seller and buyer prices. A case study illustrates the advantages of the network-aware market by comparison to our proposed loss-agnostic benchmark. We show that the network-aware market mechanism effectively promotes local heat consumption and thereby reduces losses and total cost. We conclude that the proposed loss-aware market mechanism can help reduce operating costs in district heating networks while integrating prosumers.

Keywords— District heating; peer-to-peer market; loss allocation; prosumers; convex optimization.

Nomenclature

Super- and subscripts

DHW	Domestic Hot Water
E	Electricity
g	Index for the grid agent
H	Heat
L	Loss
N	Nodal
R	Return side
SH	Space Heating

S	Supply side
i, j	Indices for prosumers
n	Index for a heat node
n_i	Index for heat node of prosumer i
p	Index for a DHN pipe
$p(n_1, n_2)$	Index for a DHN pipe from node n_1 to n_2
t	Time index
Parameters	
α	Binary, 1 for DLG, 0 for CLG
COP_i	Coefficient of Performance of heat pump of agent i [-]
$\hat{L}$	Forecasted load [W]
$\tilde{u}$	Utility function scaling factor [-]
$\tilde{w}_{ij}$	Loss factor from i to j [-]
c	Cost per energy unit [EUR/Wh]
c_f	Heat carrier specific heat capacity [$\mathrm{J\,kg^{-1}\,K^{-1}}$]
f	Space heating flexibility factor [-]
T_n	Temperature at node n [°C]
Sets	
Γ	Set of optimization variables
$\mathcal{I}$	Set of prosumers
$\mathcal{I}_n$	Set of prosumers at heat node n
$\mathcal{N}$	Set of heat nodes
$\mathcal{P}$	Set of DHN pipelines
$\mathcal{T}$	Set of time indices
$S_n^{+/-}$	Set of pipelines starting/ending at node n
Variables	
γ, μ, τ	Symbols used for dual variables
$\dot{m}$	Mass flow rate [kg/s]
π	Price [EUR / MWh]
τ_{ij}	Trade from i to j [W]
b_{ij}	Heat bought by i from j [W]
C	Total cost [EUR]
G	Power generated [W]
L	Power consumed [W]
P	Net heat power injection [W]
R	Revenue [EUR]
s_{ij}	Heat sold by i to j [W]
u	Prosumer utility function
w_{ij}	Loss caused by sale s_{ij} [W]

1 Introduction

1.1 Context

District heating is expected to play an important role in future carbon neutral energy systems, especially in urban areas [1]. Through a district heating network, excess heat from industrial processes can be distributed to households, thereby facilitating the decarbonisation of heat generation. Example sources of excess heat include supermarkets and data centers that produce heat as a by-product of their refrigeration or cooling system. Excess heat generation is usually less flexible than conventional generation, as only limited deviation from a reference production profile is possible. In order to compensate for a less flexible supply, flexibility on the demand side is needed. Studies show that households can provide such flexibility, among others using the virtual heat storage of buildings [2]. The presence of excess heat producers and active consumers marks the rise of the prosumer in heating systems, a new market participant that has already gained interest in power systems. Prosumers are defined as proactive consumers that may possess assets for local energy generation, conversion and/or storage [3]. Heat prosumer assets include, for instance, heat accumulators, heat pumps, and solar collectors.

The structure and operation of district heating systems will change significantly with the rise of distributed (excess) heat sources and heat prosumers. It remains an open question how heat markets should adapt to this new paradigm. Existing heat pricing methods do not succeed in providing consumers and generators with the right incentives to exploit their flexibility [4]–[6], nor do they deal with challenges related to the operation of a more distributed system. There is thus a need for the design of heat markets that exploit the benefits prosumers can bring to the heating system, while facilitating the integration of more distributed generation from a network operator's perspective. For the latter, markets should help ensuring network feasibility and minimizing operational costs, including the cost of heat loss.

To this end, we aim to design a network-aware market mechanism suitable for district heating systems with distributed generators and prosumers. In a more distributed system, it becomes more challenging to ensure network feasibility and operate the system efficiently. In this context, studies have pointed to the advantage of *network-aware* markets, which include explicit network constraints [7], [8]. Such markets guarantee network feasibility and economic efficiency of operation in a system with high penetration of distributed generators. If managed and integrated properly, it has been shown that prosumers can facilitate network operation and reduce system costs [9]. In order to optimally coordinate prosumers while exploiting their value, the concept of *consumer-centric* markets has attracted attention. In the next Section, we review the literature on network-aware market design for heating as well as electricity systems, including works involving consumer-centric network-aware market design.

1.2 Status quo of network-aware operations and markets

The need for network-aware optimal dispatch has long been recognized for electrical power systems, in order to minimize operating costs while meeting system and security constraints [10], [11]. Optimal energy flow for the heating and gas sector has also been a topic of interest, see e.g. the literature review in [12]. Possibly due to the liberalisation of electricity markets, as well as increased decentralization, optimal flow problems for electricity have been researched most extensively. Solving Optimal Power Flow (OPF) problems in systems with many agents (e.g., producers and consumers) has become more complex [13]. A standard OPF-based electricity market minimizes generation costs subject to power flow equations and operational constraints. The Alternating Current (AC) OPF considers the full non-linear power flow equations in its constraints, and thus represents the power flows most accurately. However, the non-convexity of this formulation has many drawbacks, such as a general lack of optimality guarantees on solver solutions and intractability of larger problems. Much research has therefore focused on approximation and convexification of the AC OPF.

A similar problem arises when designing network-aware heat market mechanisms, due to the highly complex, non-linear nature of district heating network dynamics. In the most general and most accurate variable-flow-variable-temperature (VFVT) formulation, the flow, pressure, and temperature of the heat carrier are variable. In the control literature, the resulting Mixed Integer Non-Linear Problem (MINLP) is solved using iterative methods, such as in [14]–[16]. However, convex formulations are preferred in many applications, including market design. One may apply convex relaxations and retrieve a solution to the original problem after solving, as done in e.g. [17] and [18]. However, there are no guarantees on the magnitude of the optimality gap, and the nonlinear problems can quickly become intractable for larger number of nodes. We therefore consider such methods unsuitable for our purpose. A second convexification method is to fix flow variables to arrive at the constant-flow-variable-temperature (CFVT) formulation, e.g. applied in [19]–[22]. The fluid temperature is variable at injection points and throughout the network, so that the network can be used as a heat storage. As a drawback, due to fixed nodal flows, a node must be marked as a net producer or net consumer before market clearing. Nodal temperature constraints even enforce minimal injections and extractions of these pre-appointed producers and consumers, which limits the exploitation of prosumer flexibility considerably. Finally, the variable-flow-constant-temperature (VFCT) formulation convexifies the problem by fixing nodal temperatures, leaving the flow of the heat carrier variable. The heat loss in each pipeline is now a fixed share of the transported heat, i.e. loss is multiplicative. VFCT is applied in an optimal dispatch setting in [23]. The authors of [24] apply the VFCT to prevent congestion in a distribution network with a single point of heat injection and several flexible consumers.

Over recent years, consumer-centric electricity markets have been proposed in order to accommodate prosumers. The authors of [7] review approaches for integration of distributed energy sources into power systems, and in this context discuss peer-to-peer mechanisms, as well as network considerations. Network-agnostic peer-to-peer markets based on bilateral trades are formulated in e.g. [25], [26]. More recently, several works have considered network effects within decentralized market frameworks. In [27], the cost for infrastructure usage is allocated to agents using several types of exogenous network charges, without considering explicit grid constraints. The authors of [28] propose a peer-centric market where distribution locational marginal prices reflect network usage charges that peers must pay to the utility. A peer-to-peer market with distribution and transmission grid constraints is formulated in [8], where the authors study the effect of different loss allocation policies as well. The prosumer has also gained interest in the context of district heating, not least because many types of excess heat providers classify as prosumers. The authors of e.g. [29] foresee the presence of prosumers in future heating systems, and highlight the importance of integrating them optimally. It has been shown that prosumers can be a cost-efficient solution to bottleneck problems in heating networks [30]. The literature on consumer-centric heat markets is however limited. From a market perspective, the authors of [31] develop a community-based combined heat and electricity market for a group of prosumers, consisting of an optimal dispatch and different allocation mechanisms, while disregarding network constraints. The aforementioned work [24] proposes a mechanism for exploiting flexible prosumer demands while considering the network. In this work however, there is a single point of heat injection, and prosumers cannot export heat.

1.3 Contributions

The state-of-the-art literature lacks a convex, network-aware heat market that integrates distributed generators and prosumers, while minimizing operational costs. In this work, we propose a VFCT based network-aware market mechanism to fill this gap. The market is network- and loss-aware, minimizing total production cost including the cost of generated losses. The choice for VFCT representation of the heating network is motivated by the need for a convex model as well as variable sign of nodal injections. This comes at the cost of fixing nodal temperatures, so that the storage capacity of the heating network itself is not exploited, and market participants must inject at a fixed temperature. We furthermore neglect pressure and pumping power constraints, so that we can include distributed heat injections. The market mechanism is suitable for any radial network with unidirectional pipeline flows. Nodal flows may be bidirectional, so that prosumer flexibility can be harnessed. Our choice for unidirectional network flow matches current practice in the operation of district heating networks. Bidirectional network flow is envisioned to be realized in fifth generation district heating, but this concept is in an early stages of research and development [32]. Due to the unidirectionality in the pipelines, prosumers cannot sell heat to agents located upstream. In other words, prosumers can only sell heat to agents at their own node, or downstream nodes. For prosumers at the end node, this implies that they can only sell heat to other prosumers at the same node.

We explore several questions related to network heat losses. Who *causes* them? Which generator *compensates* for the losses? And finally, who *pays* for losses? The answers to these questions help use to account for heat loss in both dispatch and pricing. Our market formulation includes peer-to-peer trades. In this work, the main purpose of introducing peer-to-peer trades is to reveal the agents that cause a certain network loss. We link network losses to a particular trade, which allows us to identify which seller and buyer caused a certain amount of heat loss in certain parts of the network. We present two variations of the dispatch mechanism, in which either the distributed generators or the grid connection compensate for heat losses. Furthermore, we propose two allocation mechanisms for the costs of energy and losses. We provide insights in our proposed formulation through detailed analysis, including a derivation of the loss components of nodal prices, and a dual analysis revealing relations between nodal prices and between seller and buyer prices. For fair evaluation of the proposed market mechanism we formulate a network-aware but *loss-agnostic* benchmark. By comparison to this benchmark, we can show the effects of loss-aware dispatch in our case study. The comparison shows that the proposed network- and loss-aware mechanism effectively promotes a more local heat consumption and thereby reduces losses and total costs. Finally, we compare the effect of individual and socialized loss allocation on consumer payments.

To the best of our knowledge, we are the first to engage in a detailed analysis of VFCT-based heat markets, and thereby to provide deeper insights in this formulation. In fact, explicit market considerations apart from optimal dispatch are rarely addressed in the literature. In addition, we consider explicit allocation of loss generation costs, which has not been done for the heat case. Our network and peer-to-peer formulations are partly inspired by the work in [24], but differ from it in several ways. Firstly, we add constraints to prevent arbitrage, ensuring a unique solution. We furthermore allow for multiple points of heat injection. As a result we need to omit the pumping power constraint.

The remainder of this article is organized as follows. We describe the components of the system under consideration in Section 2, including the district heating network model and agent representation. Section 3 presents the proposed market mechanisms, as well as the benchmark. The price of energy and loss are discussed in Section 4, including two different loss allocation policies. Next, the properties of the proposed market are illustrated in a case study in Section 5. We draw conclusions and discuss future work in Section 6.

2 System description

This Section describes the dynamics of the considered heating system, as well as the representation of agents present in this system. First, the general district heating system setup is introduced in Section 2.1. Sections 2.2 and 2.3 respectively present the heating system model and the agent model that are used in our market formulation. Temporal coupling is introduced through the load flexibility model in the latter Section. Therefore, we need to use a time index $t \in \mathcal{T}$.

2.1 District heating system representation

Figure 1 provides a graphical representation of the district heating system setup. The district heating network consists of a supply and a return side. Heat generators extract cold fluid from the return side and inject hot fluid on the supply side. Supply-side pipelines then transport the hot fluid to heat consumers, which extract the hot fluid from the supply side and inject cold fluid in the return side.

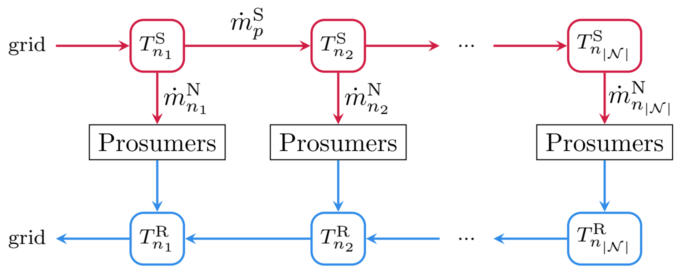

Figure 1: Representation of district heating network with prosumers and connection to greater heating grid. The supply side is colored red, the return side blue. Nodal temperatures and flows, as well as pipeline flows are indicated.

The network is a directed graph $(\mathcal{N}, \mathcal{P})$, where $\mathcal{N}$ is a set of nodes connected by pipes $\mathcal{P} \subseteq \mathcal{N} \times \mathcal{N}$. Each node and pipe consists of a supply and a return side, which are indicated by superscripts S and R. Any pipe $p \in \mathcal{P}$ is defined by its supply-side start and end node, i.e., if $p = (n_1, n_2)$, then the flow in the supply side of pipe p goes from node n_1 to node n_2. The system state is described by nodal supply T^{S}_{n} and return temperatures T^{R}_{n}, nodal mass flow rates $\dot{m}^{\mathrm{N}}_{nt}$ (here positive towards the return side), and unidirectional pipeline mass flow rates $\dot{m}^{\mathrm{S}}_{pt}$. Due to mass conservation, the mass flow rates in the supply and return side of a pipe are equal, so that it suffices to consider only the supply-side flow.

We consider a unidirectional district heating network on the distribution level. More specifically, pipeline flow is unidirectional, while nodal flow may be from the supply to the return side or vice versa, so that nodes are free to be net generators or net loads. This allows for prosumer nodes, which do not have to fix the sign of their injection before market clearing. The local system is connected to a larger grid,that may supply heat energy at import price c^{H}_{t}. Due to the unidirectionality, no heat *export* to the larger network is possible. As this work focuses on the heating system, we simplify the connection to the electricity system. It is assumed that all agents are subject to the same known electricity import price c^{E}_{t} for each period. The imported heat and electricity from the grid are denoted $G^{\mathrm{H}}_{\mathrm{g}}$ and $G^{\mathrm{E}}_{\mathrm{g}}$. These quantities are not upper bounded, so that the grid agent can in principle supply unlimited amounts of heat and electricity. However, due to grid constraints introduced in the next Section, these quantities are limited indirectly.

2.2 Model of district heating network dynamics

We will now continue to present our VFCT heating network model. The network-related variables are the nodal flows $\dot{m}^{\mathrm{N}}_{nt}$ and the pipeline flows $\dot{m}^{\mathrm{S}}_{pt}$. The temperature at the supply and return side of each node are model parameters, and they are constant over time. This way, time delays can be neglected. The temperature loss in each pipeline is determined by those fixed values. The supply-side temperature at a node must necessarily be greater than its return-side temperature for the node to be able to extract heat from the system. In addition, temperatures must be non-increasing along pipelines in the direction of the flow. It is important to emphasize that losses are multiplicative in this formulation. That is, a fixed share of the heat injected in a certain pipeline is lost while the heat carrier flows to the next node. This share does not depend on flow or temperatures. However, total system losses are not fixed, as the amount of heat injected in each pipeline is variable. For the VFCT model, fixed nodal temperature values are needed. These may be obtained from measurements, as in [24]. Another option is to estimate temperature losses as in [33] (Equations 7 and 8), or using an average mass flow and temperature loss equations as in [14] (Equation 6).

The set of heating system variables is given by $\Gamma^{\mathrm{DHN}} = \{\dot{m}^{\mathrm{N}}_{nt}, \dot{m}^{\mathrm{S}}_{pt}\}$. Mass is preserved at each node, which means that the difference between incoming and outflowing mass at a node must equal the nodal flow. This translates to

$$\sum_{p \in S_n^-} \dot{m}^{\mathrm{S}}_{pt} - \sum_{p \in S_n^+} \dot{m}^{\mathrm{S}}_{pt} = \dot{m}^{\mathrm{N}}_{nt} \,. \tag{1}$$

Each node contains a heat exchanger that ensures locally produced heat is injected into the network. The nodal power injection and nodal flow are related as

$$\sum_{i \in \mathcal{I}_n} P^{\mathrm{H}}_{it} = -c_{\mathrm{f}}\, \dot{m}^{\mathrm{N}}_{nt} \,(T^{\mathrm{S}}_n - T^{\mathrm{R}}_n) \,, \tag{2}$$

where c_{f} is the heat capacity of water. The nodal flow $\dot{m}^{\mathrm{N}}_{nt}$ is considered positive when the direction is from supply to return side. Note that the temperature of the injected fluid on the supply and return side of node n must equal the fixed temperatures T^{S}_n and T^{R}_n, respectively. Finally, the flow variables are subject to limits. The flow in pipelines is unidirectional, and upper bounded:

$$0 \leq \dot{m}^{\mathrm{S}}_{pt} \leq \overline{\dot{m}}^{\mathrm{S}}_p \,. \tag{3}$$

The nodal flows are bidirectional, but bounded in size:

$$-\underline{\dot{m}}^{\mathrm{N}}_n \leq \dot{m}^{\mathrm{N}}_{nt} \leq \overline{\dot{m}}^{\mathrm{N}}_n \,. \tag{4}$$

2.3 Consumption model

A set of agents $\mathcal{I}$ is present in the system. Agent i is located at a node n_i in the district heating network. The set of agents at node n is denoted $\mathcal{I}_n$; multiple agents may thus be located at a single node. The agents have an hourly heat load L^{H}_{it} and generation G^{H}_{it}, resulting in a total heat injection P^{H}_{it} given by

$$P^{\mathrm{H}}_{it} = G^{\mathrm{H}}_{it} - L^{\mathrm{H}}_{it} \,. \tag{5}$$

The heat load consists of an inflexible domestic hot water load $\hat{L}^{\mathrm{DHW}}$ and a partially flexible space heating load L^{SH},

$$L^{\mathrm{H}}_{it} = L^{\mathrm{SH}}_{it} + \hat{L}^{\mathrm{DHW}}_{it} \,. \tag{6}$$

It is assumed that the heat is generated using heat pumps (HPs), which have an electric load $L^{\mathrm{E,hp}}_{it}$. The electric load and heat production are related by the heat pump Coefficient of Performance (COP) as

$$G^{\mathrm{H}}_{it} = \mathrm{COP}_i L^{\mathrm{E,hp}}_{it} \,. \tag{7}$$

The COP can in principle vary in time, but is considered fixed in this work. All agent variables are collected in the set $\Gamma^{\mathrm{agent}} = \{P^{\mathrm{H}}, L^{\mathrm{H}}, L^{\mathrm{SH}}, G^{\mathrm{H}}, L^{\mathrm{E,hp}}\}$. The generation is upper and lower bounded as follows

$$0 \leq G^{\mathrm{H}}_{it} \leq \overline{G}^{\mathrm{H}}_{it} \,. \tag{8}$$

Flexibility in prosumer consumption is represented as follows. First, the space heating profile of agent i may at most deviate from this agent's reference profile $\hat{L}^{\mathrm{SH}}_i$ by a maximum flexibility $\overline{f}_i$ at each time:

$$\max\{\hat{L}^{\mathrm{SH}}_{it} - \overline{f}_i, 0\} \leq L^{\mathrm{SH}}_{it} \leq \hat{L}^{\mathrm{SH}}_{it} + \overline{f}_i \,. \tag{9}$$

Note that if $\hat{L}^{\mathrm{SH}}_{it} - \overline{f}_i < 0$, the lower bound on the space heating consumption is zero. In addition, the total heat consumption for space heating has to equal the total in the profile, i.e.

$$\sum_{t \in \mathcal{T}} L^{\mathrm{SH}}_{it} = \sum_{t \in \mathcal{T}} \hat{L}^{\mathrm{SH}}_{it} \,, \tag{10}$$

which means that space heating load may be shifted but has to be consumed eventually.

The utility is inversely proportional to the squared deviation from this profile. This relation is scaled by a time and agent dependent factor $\tilde{u}_{it}$ representing the importance of following the space heating profile. The resulting utility as a function of the space heating demand is

$$u_{it}(L^{\mathrm{SH}}_{it}) = -\tilde{u}_{it}\,(L^{\mathrm{SH}}_{it} - \hat{L}^{\mathrm{SH}}_{it})^2 \,. \tag{11}$$

Heat generation costs for agent i are given by

$$c^{\mathrm{H}}_{it} = c^{\mathrm{E}}_{\mathrm{gt}} L^{\mathrm{E,hp}}_{it} = c^{\mathrm{E}}_{\mathrm{gt}} \frac{G^{\mathrm{H}}_{it}}{\mathrm{COP}_i} \,. \tag{12}$$

3 Optimal dispatch strategies

We will now define the three different dispatch strategies we consider in our market designs. An overview of the proposed dispatch strategies is first provided in Section 3.1. Next, we present our formulation of peer-to-peer trades in Section 3.2. This formulation allows us to derive the losses each agent is causing, which we do in the following Section 3.3. The objective functions used for loss-aware and for loss-agnostic dispatch are provided in Section 3.4. Finally, we summarize the full optimization problem in Section 3.5.

3.1 Overview

We design three optimal dispatch strategies, that differ from eachother in one or two aspects. The first option that can be switched is loss-awareness versus loss-agnosticism. This setting is discussed in Section 3.4. Our aim in this work is to show the benefits of loss-aware dispatch compared to the status quo in heat markets. We use the loss-agnostic dispatch as our benchmark for showing these benefits. This benchmark is intended to resemble current practices in heat markets, which usually do not consider the network nor operational costs. However, for a fair comparison to our network-aware and loss-aware market, the benchmark needs to respect network constraints.

The second setting is centralized loss generation (CLG) versus decentralized loss generation (DLG), which determines the generator that compensates for losses, as we will discuss in Section 3.2. Combining these options gives four different dispatch strategies, illustrated in Table 1. However, as indicated, we exclude the loss-agnostic DLG variant, as it is unlikely to be applicable in reality, nor does it mimic any existing market setups. Moreover, it results in counter-intuitive dispatch and prices, as the loss generation is competing with energy generation for the limited capacity local generators. In all cases, we assume non-strategic and regulation-agnostic agents. By the latter we mean that agents are not able to anticipate hindsight payments, which implies they do not change behavior in the dispatch because of the hindsight payments.

Reasons for including network constraints in the optimal dispatch are twofold. Firstly, for all three proposed dispatch strategies, the resulting dispatch will be *feasible*. This means that the physical limitations posed by the network are respected by any resulting dispatch. Second, the network model implicitly models heat loss, so that the cost of loss related to a certain unit of consumed energy is directly included in the market. This cost will influence the choices of heat consumers when buying

	DLG	**CLG**
loss-aware	loss-aware DLG	loss-aware CLG
loss-agnostic	loss-agnostic DLG	loss-agnostic CLG

Table 1: Overview of dispatch strategies considered.

heat: it may happen that a generator close to a certain consumer is preferred over a cheaper but far away generator, when the latter trade becomes more expensive due to loss costs. This effect can be observed in our loss-aware dispatch. In the loss-agnostic dispatch, we artificially remove the loss costs from the objective function, as we will formalize in Section 3.4. The loss-agnostic dispatch is agnostic to the cost of loss only: the losses are still produced and transported, and the dispatch remains feasible in the network.

In our proposed markets, generator bids are of price-quantity format. That is, they bid a maximum quantity $\overline{G}_t^{\mathrm{H}}$ for each time step, as well as their generation costs per unit for each time. The consumer bids are more complex: they bid a fixed load $\hat{L}^{\mathrm{DHW}}$, a minimum and maximum quantity for flexible load L^{SH}, and the total to consume flexible load over the entire day. In addition, the price component of the consumer bid comes as a quadratic utility function $u_{it}(\hat{L}^{\mathrm{SH}}, \hat{L}^{\mathrm{DHW}})$. If the market were to be decomposed, and agents would engage in bilateral trades in a decentralized system, the consumers would not need to hand all this information to the market operator. Instead, the information would be used in their local optimization problem.

3.2 Peer-to-peer trades

The market includes peer-to-peer trades, which enable agents to negotiate directly with one another and agree on bilateral heat trades. Our formulation is an extension of a common peer-to-peer setup, as described in for example [34]. The extension consists of constraints that prevent arbitrage, as well as an explicit loss representation. We define a trade τ_{ij} between agent i and j, which is positive if i sells and negative if i buys heat. The grid agent is denoted using the index g. Trades define an amount of energy that is received by the buyer, not including any losses on the way. This means that, besides the traded heat τ_{ij}, the losses w_{ij} associated with the trade need to be generated by some agent. We introduce a binary parameter $\alpha_{ij} \in \{0, 1\}$ to indicate whether the seller of trade τ_{ij} will be responsible for producing the losses caused by this trade ($\alpha_{ij} = 1$) or whether grid import will be used to compensate for losses ($\alpha_{ij} = 0$). We will refer to these respective cases as *distributed loss generation* (DLG) and *centralized loss generation* (CLG). An illustration of peer-to-peer trading variables is given in Figure 2.

A trade τ_{ij} can be decomposed into sales $s_{ij} \geq 0$ and buys $b_{ij} \geq 0$ as

$$\tau_{ijt} = s_{ijt} - b_{ijt} \,. \tag{13}$$

Trade reciprocity is ensured by the constraint

$$s_{ijt} = b_{jit} \,. \tag{14}$$

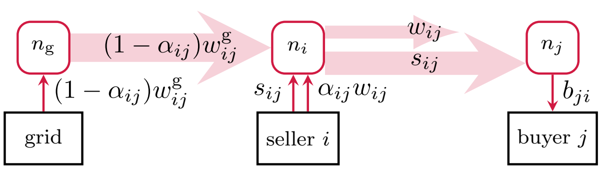

Figure 2: Visualisation of peer-to-peer variables related to the trade $\tau_{ij} = s_{ij} = b_{ji} = -\tau_{ji}$. If $\alpha = 1$, the seller generates the losses, while this is done by the grid in case $\alpha = 0$

The variable $b_{ii} = s_{ii}$ represents energy bought from own production, used for self-consumption. This self-consumption variable b_{ii} is used to prevent arbitrage and ensure a unique solution, using the following constraints. Agents can buy no more and no less than the energy they need for consumption,

$$\sum_{j\in\mathcal{I}} b_{ijt} = L^{\mathrm{H}}_{it}\,, \tag{15}$$

which prevents reselling of bought energy and thus prevents arbitrage. Note that the sum in this Equation includes the self-consumption b_{ii}. Among others, this constraint ensures that for any i, j, no agent is both buying from and selling to a single other agent, i.e. $b_{ij} = 0 \vee b_{ji} = 0$.

An agent must generate an amount equal to the total sale of heat plus the associated loss generation allocated to this agent,

$$\sum_{j} s_{ijt} + \alpha_{ij} w_{ijt} = G^{\mathrm{H}}_{it}\,. \tag{16}$$

The losses w_{ij} will be quantified in the next Section. The grid agent must produce

$$\sum_{ij} (1-\alpha_{ij}) w^{\mathrm{g}}_{ijt} + \sum_{j} (s_{\mathrm{g}jt} + \alpha_{ij}\, w_{\mathrm{g}jt}) = G^{\mathrm{H}}_{\mathrm{g}t}\,, \tag{17}$$

where w^{g}_{ij} is the amount the grid agent must inject to compensate for the losses in the trade τ_{ij}, which is quantified in the next Section.

The primal variables related to the peer-to-peer trading are $\Gamma^{\mathrm{p2p}} = \{t, b, s, w, w^{\mathrm{g}}\}$.

3.3 Explicit loss formulation

Pipeline losses occur both on the supply and return side in the network. These losses are implicit in (1) and (5). In order to provide more insight in the losses, and to be able to allocate losses to market participants, we turn to a more explicit loss formulation.

Suppose without loss of generality that trade $\tau_{ijt} > 0$, so agent i is the seller of the nonzero trade with agent j. In the VFCT formulation, the loss associated to trade τ_{ij} is a fixed share of this trade, depending on the nodal temperatures at n_i and n_j. To derive this share, suppose $s_{ij} = b_{ji} > 0$. Then the change in power injection (and thus flow) at the receiving node n_j equals

$$\Delta P^{\mathrm{H}}_{n_j} = -\Delta L^{\mathrm{H}}_{n_j} = -b_{ji} = -c_f \Delta \dot{m}_{n_j}\,(T^{\mathrm{S}}_{n_j} - T^{\mathrm{R}}_{n_j}) \tag{18}$$

by (2). By continuity of flow, $\Delta \dot{m}_{n_j} = -\Delta \dot{m}_{n_i}$. Assuming the losses are supplied by the seller i, the change in power injection at node n_i is

$$\begin{aligned} \Delta P^{\mathrm{H}}_{n_i} = \Delta G^{\mathrm{H}}_{n_i} &= -c_f \Delta \dot{m}_{n_i}\,(T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}) \\ &= c_f \Delta \dot{m}_{n_j}\,(T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i})\,. \end{aligned} \tag{19}$$

The lost energy is equal to the difference between generated and consumed energy in this trade, as given by (19) and (18) respectively. The total loss associated to the trade τ_{ij} thus equals

$$\begin{aligned} w_{ij} &= c_{\mathrm{f}} \Delta \dot{m}_{n_j} (T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}) - c_{\mathrm{f}} \Delta \dot{m}_{n_j} (T^{\mathrm{S}}_{n_j} - T^{\mathrm{R}}_{n_j}) \\ &= c_{\mathrm{f}} \Delta \dot{m}_{n_j} (T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}) - b_{ji}\,. \end{aligned} \tag{20}$$

Thus, we derive the constant relationship $\tilde{w}_{ij}$ between w_{ij} and b_{ji} as

$$\tilde{w}_{ij} = \frac{w_{ij}}{b_{ji}} = \frac{\Delta G^{\mathrm{H}}_{n_i} - b_{ji}}{b_{ji}} = \frac{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}{T^{\mathrm{S}}_{n_j} - T^{\mathrm{R}}_{n_j}} - 1\,. \tag{21}$$

An important observation is that, as a result of assuming constant nodal temperatures, the losses become *multiplicative*, i.e. a fixed *share* of the energy transported through a pipeline. Note furthermore that the factor $\tilde{w}_{ij}$ is only non-negative if the temperature gradient between supply and return side at n_i is at least as large as the gradient at n_j. This is the case as long as n_j is downstream of n_i, or $n_i = n_j$.

Loss component for Decentralized Loss Generation ($\alpha_{ij} = 1$)

We define w_{ij} as the loss associated with the trade τ_{ij} in case that the seller of this trade is also producing the loss, i.e. $\alpha_{ij} = 1$. If $\tau_{ij} < 0$, so i is the buyer of this trade, then $w_{ij} = 0$ while now $w_{ji} \geq 0$ represents the loss associated with this trade. In other words, $s_{ij} \geq 0 \implies w_{ij} \geq 0, w_{ji} = 0$, whereas $b_{ij} \geq 0 \implies w_{ij} = 0, w_{ji} \geq 0$. The explicit computation of the losses caused by a certain trade is

$$w_{ijt} = \tilde{w}_{ij}\, s_{ijt}\,,$$

where it is important to note this holds under the assumption that the seller produces these losses. Furthermore, losses must be positive, i.e. $w_{ijt} \geq 0$. Combining this with the nonnegativity of s, we enforce that $s_{ijt} = 0$ if $\tilde{w}_{ij} < 0$. The interpretation of this is that j cannot buy from i if j is upstream of i, so these constraints exclude physically impossible trades.

Loss component for Centralized Loss Generation ($\alpha_{ij} = 0$)

In this case, the grid agent has to inject an amount of energy that results in an amount of w_{ijt} arriving at the node of the seller of trade τ_{ijt}. If $w_{ijt} > 0$, i is the seller of the trade τ_{ijt}. Therefore, an amount of w_{ijt} must arrive at node i. This means that the grid agent must produce

$$w^{\mathrm{g}}_{ijt} = \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, w_{ijt} = \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij}\, s_{ijt}\,.$$

Again, the loss must be positive, i.e. $w^{\mathrm{g}}_{ijt} \geq 0$.

3.4 Loss-aware and loss-agnostic objective functions

The difference between our loss-aware and loss-agnostic dispatch lies in the objective function f^{obj} of the respective optimization problems. The objective of both markets is to maximize some form of social welfare (or equivalently, minimize negative social welfare). In the loss-aware dispatch, total production cost is included, which consists of the costs of energy sold to a consumer and the cost of producing losses:

$$f^{\mathrm{awa}}(G^{\mathrm{H}}_{it}) = \sum_{t\in\mathcal{T}} \left(c^{\mathrm{H}}_t\, G^{\mathrm{H}}_{\mathrm{g}t} + \sum_{i\in\mathcal{I}} \left(-u_{it} + \frac{c^{\mathrm{E}}_t}{\mathrm{COP}_i} G^{\mathrm{H}}_{it} \right) \right)\,. \tag{22}$$

In the loss-agnostic benchmark, the objective function of the market is adapted to minimize only the production cost of consumed load, while disregarding the cost of losses:

$$\begin{aligned} f^{\mathrm{agn}}(G^{\mathrm{H}}_{it}, w_{ijt}) = &\sum_{t\in\mathcal{T}} c^{\mathrm{H}}_t \left(G^{\mathrm{H}}_{\mathrm{g}t} - \sum_{j\in\mathcal{I}} w_{\mathrm{g}jt} - \sum_{i,j\in\mathcal{I}} (1-\alpha_{ij}) w^{\mathrm{g}}_{ijt} \right) + \\ &\sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{I}} \left(-u_{it} + \frac{c^{\mathrm{E}}_t}{\mathrm{COP}_i} \left(G^{\mathrm{H}}_{it} - \sum_{j\in\mathcal{I}} \alpha_{ij} w_{ijt} \right) \right)\,. \end{aligned} \tag{23}$$

After a loss-agnostic dispatch, the cost of losses may be computed and allocated a posteriori as discussed in Section 4.2. The total cost of losses is therefore in general different when using these two objective functions. Note that without appointment of an agent responsible for loss generation, there will in general not be a unique solution in the loss-agnostic case.

3.5 Resulting overall market optimization problem

To arrive at a complete market formulation, we combine the constraints defined over Sections 2 and 3 in a single optimization problem in (24). Dual variables are indicated for each constraint, where we use μ for equality constraints and γ for lower and upper bounds. These dual variables are used in our derivation of energy and loss prices in Section 4.1 and Appendices A.1-A.4.

The objective function f^{obj} is either f^{awa} for loss-aware dispatch, or f^{agn} for loss-agnostic dispatch. The optimization variables are $\Gamma = \{P^{\text{H}}_{\text{g}}, G^{\text{H}}_{\text{g}}\} \cup \Gamma^{\text{DHN}} \cup \Gamma^{\text{agents}} \cup \Gamma^{\text{p2p}}$. We list network constraints in (24b)-(24e), and the agents' own constraints in (24g)-(24l). To synchronize notations between the grid agent and prosumers, we add a power injection constraint for the grid agent in (24f). Trade related constraints are in (24m)-(24q), and loss related constraints in (24r)-(24u). In the absence of preferences with respect to trading partners, as in this work, the same dispatch as the one resulting from our peer-to-peer market can be obtained from an equivalent pool formulation as well.

$$
\begin{aligned}
\min_{\Gamma} \quad & f^{\text{obj}} && && \text{(24a)}\\
& \sum_{p\in S_n^-} \dot{m}^{\text{S}}_{pt} - \sum_{p\in S_n^+} \dot{m}^{\text{S}}_{pt} = \dot{m}^{\text{N}}_{nt} && : \mu^{mc}_{nt} && \text{(24b)}\\
& \sum_{i\in\mathcal{I}_n} P^{\text{H}}_{it} = -c_{\text{f}}\, \dot{m}^{\text{N}}_{nt}\, (T^{\text{S}}_n - T^{\text{R}}_n) && : \mu^{HE}_{nt} && \text{(24c)}\\
& 0 \le \dot{m}^{\text{S}}_{pt} \le \overline{\dot{m}}^{\text{S}}_p && : \underline{\gamma}^{\text{mp}}, \overline{\gamma}^{\text{mp}} && \text{(24d)}\\
& -\underline{\dot{m}}^{\text{N}}_n \le \dot{m}^{\text{N}}_{nt} \le \overline{\dot{m}}^{\text{N}}_n && : \underline{\gamma}^{\text{mn}}, \overline{\gamma}^{\text{mn}} && \text{(24e)}\\
& P^{\text{H}}_{\text{g}t} = G^{\text{H}}_{\text{g}t} - L^{\text{H}}_{\text{g}t} && : \mu^{\text{inj}}_{\text{g}t} && \text{(24f)}\\
& P^{\text{H}}_{it} = G^{\text{H}}_{it} - L^{\text{H}}_{it} && : \mu^{inj}_{it} && \text{(24g)}\\
& L^{\text{H}}_{it} = L^{\text{SH}}_{it} + \hat{L}^{\text{DHW}}_{it} && : \mu^{\text{Ltot}}_{it} && \text{(24h)}\\
& G^{\text{H}}_{it} = \text{COP}_i L^{\text{E,hp}}_{it} && : \mu^{\text{cop}}_{it} && \text{(24i)}\\
& 0 \le G^{\text{H}}_{it} \le \overline{G}^{\text{H}}_{it} && : \underline{\gamma}^{\text{G}}_{it}, \overline{\gamma}^{\text{G}}_{it} && \text{(24j)}\\
& \max\{\hat{L}^{\text{SH}}_{it} - \overline{f}_i, 0\} \le L^{\text{SH}}_{it} \le \hat{L}^{\text{SH}}_{it} + \overline{f}_i && : \underline{\gamma}^{\text{SH}}_{it}, \overline{\gamma}^{\text{SH}}_{it} && \text{(24k)}\\
& \sum_{t\in\mathcal{T}} L^{\text{SH}}_{it} = \sum_{t\in\mathcal{T}} \hat{L}^{\text{SH}}_{it} && : \mu^{\text{eb}}_i && \text{(24l)}\\
& \tau_{ijt} = s_{ijt} - b_{ijt} && : \mu^{\text{T}}_{ijt} && \text{(24m)}\\
& s_{ijt} = b_{jit} && : \mu^{\text{R}}_{ijt} && \text{(24n)}\\
& \sum_j b_{ijt} = L^{\text{H}}_{it} && : \mu^{\text{B}}_{it} && \text{(24o)}\\
& \sum_j s_{ijt} + \alpha_{ij} w_{ijt} = G^{\text{H}}_{it} && : \mu^{\text{S}}_{it} && \text{(24p)}\\
& \sum_{ij} (1-\alpha_{ij}) w^{\text{g}}_{ijt} + \sum_j (s_{\text{g}jt} + w_{\text{g}jt}) = G^{\text{H}}_{\text{g}t} && : \mu^{\text{S}}_{\text{g}t} && \text{(24q)}
\end{aligned}
$$

$$w_{ijt} = \tilde{w}_{ij}\, s_{ijt} \qquad : \mu^{\mathrm{w}}_{ijt} \tag{24r}$$
$$w_{ijt} \geq 0 \qquad : \underline{\gamma}^{\mathrm{w}}_{ijt} \tag{24s}$$
$$w^{\mathrm{g}}_{ijt} = \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij}\, s_{ijt} \qquad : \mu^{\mathrm{wg}}_{ijt} \tag{24t}$$
$$w^{\mathrm{g}}_{ijt} \geq 0 \qquad : \underline{\gamma}^{\mathrm{wg}}_{ijt} \tag{24u}$$

4 Pricing and loss allocation mechanisms

After the optimal schedule is determined using one of the three dispatch strategies presented in Section 3.1, payments and revenues have to be determined for all market participants using an allocation mechanism. In this Section we propose different allocation mechanisms. First, the allocation of energy and loss cost is discussed in Section 4.1. Then, Section 4.2 presents different loss allocation mechanisms.

4.1 Marginal price of energy and loss

Different choices of consumed energy and loss prices are possible. In this work, we consider nodal marginal pricing. In the Appendix, we derive dual relations for the different markets, and determine the nodal prices. It is derived that the price per unit of loss and price per unit of energy are equal. However, the amount of loss per unit of consumed energy depends on where the energy is imported from, and therefore the loss cost per unit of energy consumed increases with distance from the generator. We denote nodal marginal prices as π^{N}_{nt}, and seller i and buyer j marginal price as π^{s}_{it} and π^{b}_{jt} respectively.

In Appendix A.2, we show that the unit price received by seller i is given by $\mu^{\mathrm{S}}_{it} + \mu^{\mathrm{inj}}_{it}$, whereas the price per unit consumed for buyer j is $-\mu^{\mathrm{B}}_{jt} + \mu^{\mathrm{inj}}_{jt}$. We further derive that for loss-aware DLG, these prices relate as

$$\pi^{\mathrm{b}}_{jt} = -\mu^{\mathrm{B}}_{jt} + \mu^{\mathrm{inj}}_{jt} = (1 + \tilde{w}_{ij}) \left(\mu^{\mathrm{S}}_{it} + \mu^{\mathrm{inj}}_{it}\right) = (1 + \tilde{w}_{ij})\pi^{\mathrm{s}}_{it}\,. \tag{25}$$

As a result, the cost of loss connected to energy sale s_{ijt} on the DLG loss-aware market is given by

$$C^{\mathrm{L,DLG}}_{ijt} = w_{ijt}\, \pi^{\mathrm{s}}_{it}\,. \tag{26}$$

In the loss-aware CLG market the prices relate as

$$\begin{aligned} \pi^{\mathrm{b}}_{jt} = -\mu^{\mathrm{B}}_{jt} + \mu^{\mathrm{inj}}_{jt} &= \mu^{\mathrm{S}}_{it} + \mu^{\mathrm{inj}}_{it} + \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij}(\mu^{\mathrm{S}}_{\mathrm{g}t} + \mu^{\mathrm{inj}}_{\mathrm{g}t}) \\ &= \pi^{\mathrm{s}}_{it} + \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij}\pi^{\mathrm{s}}_{\mathrm{g}t} \end{aligned} \tag{27}$$

as derived in Appendix A.3. So, losses are paid for at the nodal price of the node of grid connection. Therefore, in the CLG loss-aware market the cost of loss connected to sale s_{ijt} is given by

$$C^{\mathrm{L,CLG}}_{ijt} = w^{\mathrm{g}}_{ijt}\pi^{\mathrm{s}}_{\mathrm{g}t}\,. \tag{28}$$

Finally, for the loss-agnostic CLG, we show in Appendix A.4 that the buyer and seller marginal price are always equal, i.e. $\pi^{\mathrm{s}}_{it} = \pi^{\mathrm{b}}_{jt}$, regardless of whether the agents are at different nodes (as long as there is no congestion). This implies that the losses are not paid by the buyer directly, which is what was intended with this formulation. The costs of losses are not optimized and therefore the cost of loss cannot be obtained from any dual variable. Instead, the cost of loss has to be computed in hindsight. As in the loss-aware markets, we choose to price the loss in hindsight at the nodal price of the generator of the loss, in this case the grid node. Thus, the cost of loss connected to sale s_{ijt} is computed in hindsight in the same way as in CLG loss-aware market as given in (28), .

Note that we price the loss w_{ij} at the nodal price of generator i in all three market formulations.

4.2 Allocation mechanisms for loss costs

Once the generation cost of losses are known, these costs have to be allocated to market participants. A *loss allocation policy* is a system for distributing loss costs over generators and loads. It is desirable that such a policy is budget balanced, i.e. the loss payments add up to the cost of loss, so that the network operator does not suffer a loss or earn a profit.

In this work, two budget balanced loss allocation policies are investigated:

1. *individual*: the buyer of heat pays for the generation of the losses associated with this trade.
2. *socialized*: losses are paid for collectively a posteriori. Each consumed unit is charged the average cost of loss per unit consumed, so that the loss costs are distributed proportionally to prosumer total consumption.

In the individual loss allocation policy, losses are paid by the individual that is causing them, while the cost of losses are shared evenly using the socialized loss allocation policy. In particular, the socialized loss cost per unit consumed is taken as an average over the entire time period considered, so that the unit price of loss is equal for any time t. This is intended to mimic current network and loss charges, which are usually a fixed price per unit, where the network and loss costs for the whole year are socialized in a grid tariff. The socialized loss allocation is a pro rata method, as described in [35] for the allocation of electrical loss costs. In our case, we allocate 100% of the loss costs to the consumers. Pro rata procedures are network-agnostic, i.e. loads near generating nodes pay the same loss price per unit consumed as loads far away from generating nodes [35].

It should be noted that the individual loss allocation policy suits our loss-aware markets naturally. In other words, in the loss-aware markets the agents are dispatched optimally, given that the losses are allocated according to the individual loss allocation policy. For these loss-aware markets, the dispatch would be different if the agents would be able to anticipate the socialized loss allocation. For the loss-agnostic market, this is the case for both the individual and the socialized loss allocation. In this work, we however assume that the agents are not able to anticipate the loss allocation post-processing, i.e. they are regulation-agnostic. To summarize, the socialized loss allocation is an ex post step for all market types, while the individual loss allocation is integrated in the market clearing for the loss-aware markets.

For the loss-aware DLG, the revenue R_{it} of generator i at time t is computed as

$$R_{it} = \sum_{j\in\mathcal{I}} (s_{ijt} + w_{ijt})\, \pi^{\mathrm{N}}_{n_i t} = \sum_{j\in\mathcal{I}} s_{ijt}\pi^{\mathrm{N}}_{n_i t} + \sum_{j\in\mathcal{I}} w_{ijt}\pi^{\mathrm{N}}_{n_i t} \tag{29}$$

so that every generated unit costs the nodal price at n_i. In the rightmost expression the cost is split into cost of consumed energy and cost of loss. When using the individual loss allocation policy, consumer j is paying $w_{ijt}\pi^{\mathrm{N}}_{n_i t}$ to make up for the cost of loss. In the socialized loss allocation policy, the costs of loss are shared over all consumed units, so that the cost of loss per unit consumed π^{L} in DLG becomes

$$\pi^{\mathrm{L,DLG}} = \frac{\sum_{i,j,t} w_{ijt}\pi^{\mathrm{N}}_{n_i t}}{\sum_{i,t} L^{\mathrm{H}}_{it}}\,, \tag{30}$$

and in CLG becomes

$$\pi^{\mathrm{L,CLG}} = \frac{\sum_{i,j,t} w^{\mathrm{g}}_{ijt}\pi^{\mathrm{N}}_{n_{\mathrm{g}} t}}{\sum_{i,t} L^{\mathrm{H}}_{it}}\,. \tag{31}$$

5 Numerical results

Through an illustrative case study we show the benefits of loss-aware dispatch, and the drawbacks of not considering the cost of loss. We look at total scheduled generation volumes, total amount of losses, total cost and cost of loss, and agent payments and revenues. In addition, we visualize the effect of loss- and network-awareness on nodal (locational) marginal prices. The aim of this case study is to illustrate the properties of the proposed market mechanisms, rather than to mimic a specific real system as closely as possible. All case study inputs, as well as an implementation of the three market variations in Julia, and example analysis of the outcomes can be found in our GitHub repository[1].

5.1 Case study setup

We simulate an hourly day-ahead market for 24 hours. We consider a simple network topology without branches, see Figure 3, even though the market mechanisms are also suitable for other types of radial systems. The system consists of 11 nodes. The supply temperature at the most upstream node n_1 is 90°C, and the return side temperature of the most downstream node n_{11} is set to 40°C. Precise ways of determining constant nodal temperatures are beyond the scope of this work. For illustrative purposes, we will assume a temperature loss of $0.1\mathrm{K\,m^{-1}}$ on the supply side, and a loss of $0.05\mathrm{K\,m^{-1}}$ in return side pipelines. A total of 28 prosumers are present in the system, of which 6 generate excess heat using heat pumps, while the remaining 22 cannot generate heat. The used reference DHW and space heating load profiles are measurements from the Nordhavn neighbourhood in Copenhagen, collected in the EnergyLab Nordhavn project [36]. The excess heat generators are located at all even nodes and node 1, i.e. node $1, 2, 4, 6, 8, 10$, which are marked as HP in Figure 3. The remaining agents without production are distributed over the remaining nodes, so that nodes $3, 5$, and 7 contain 4 agents each, and node 9 and 11 contain 5 agents each. The excess heat generators have identical maximum generation capacity, while their heat pump COPs differ. The six heat pump COP values range linearly from 3.27 to 3.46.

The agents are subject to a variable electricity price, whereas the heat import price is constant. This is a situation that currently can be the case for consumers in for example Denmark: consumers can opt-in on variable electricity prices, whereas heat prices are always constant (apart from possible seasonal differences). The price curves used in this case study are plotted in Figure 4. The electricity price profile is a series of day-ahead prices from Nord Pool Elspot on January 8$^{\mathrm{th}}$ 2021. This price only includes the energy price, so the real price paid by consumer is higher, especially in Denmark where the price of energy is around 20% of the total price [2]. To be more realistic, we multiply the price signal by

[1] github.com/linde-fr/network-aware-heat-market
[2] www.forsyningstilsynet.dk/tal-fakta/priser/elpriser/prisstatistik-1-kv-2021

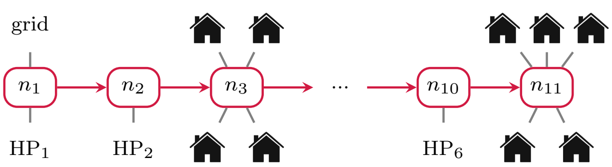

Figure 3: Overview of case study district heating system. Each of the nodes $1, 2, 4, 6, 8, 10$ contains a single prosumer with a heat pump (marked with HP). Nodes $3, 5, 7$ each contain 4 flexible consumers, while nodes $9, 11$ contain 5 each. The grid agent injects heat at node 1. The flow is unidirectional from the supply side of node 1 to 11. The return side is equal to the displayed supply network, with reversed flow directions.

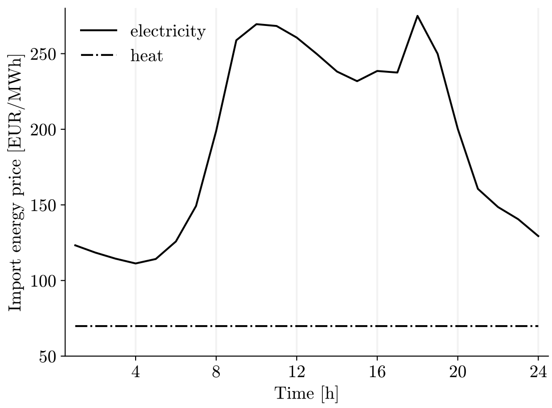

Figure 4: Heat and electricity import prices used in case study.

2.5. The constant heat import price signal is set to 524 DKK/MWh or 69.87 EUR/MWh, which equals the consumer heat price set by the Danish heat provider HOFOR [3].

We consider two variants of this case study: case I and case II. The cases differ only in the marginal costs of the local heat pumps, which allows us to illustrate different properties of our mechanism, and to distinguish between general outcomes and case-dependent outcomes. In case I, the heat pumps are placed so that marginal production costs decrease (i.e. COPs increase) with distance from the heating grid connection. This means that the excess heat producer at node 1 will have the highest marginal cost, followed by the producer at node 2, then node 4, etc. In case II, the heat pump order is reversed, so that the marginal production cost increases and COP decreases with distance from the grid connection. Now the marginal costs of the producer at node 1 will be lowest. In the next Section, we use case studies I and II to compare the different market mechanisms, and illustrate relevant properties.

5.2 Results

First, it should be noted that a network-agnostic dispatch in a unidirectional network with multiple producers may be infeasible. The main reason for this is the lack of directional awareness, i.e. downstream producers can be scheduled for an amount greater than the loads they are able to reach. An important benefit of the considered market mechanisms (including the loss-agnostic benchmark) is that the resulting dispatch is guaranteed to be feasible, as opposed to network-agnostic markets. In addition, the loss-aware dispatch is optimal under the assumption that grid temperature cannot be varied. When projecting network-agnostic market outcomes in the feasible space, as is common practice in current district heat dispatch, such optimality guarantees do not exist.

Scheduled generators

We compare the dispatch of local generators in the loss-aware markets, both DLG and CLG, to our loss-agnostic CLG benchmark. In the loss-aware markets, the cost of loss is taken into account in the dispatch, while the loss-agnostic benchmark minimizes production cost of consumed heat only. It is expected that distant generators have an advantage in the loss-aware markets, as their loss costs are lower. This effect is indeed seen in the dispatch shown for case study I in Figure 5a and case study II in Figure 5b, where the generation over the entire time horizon is shown per node. In both

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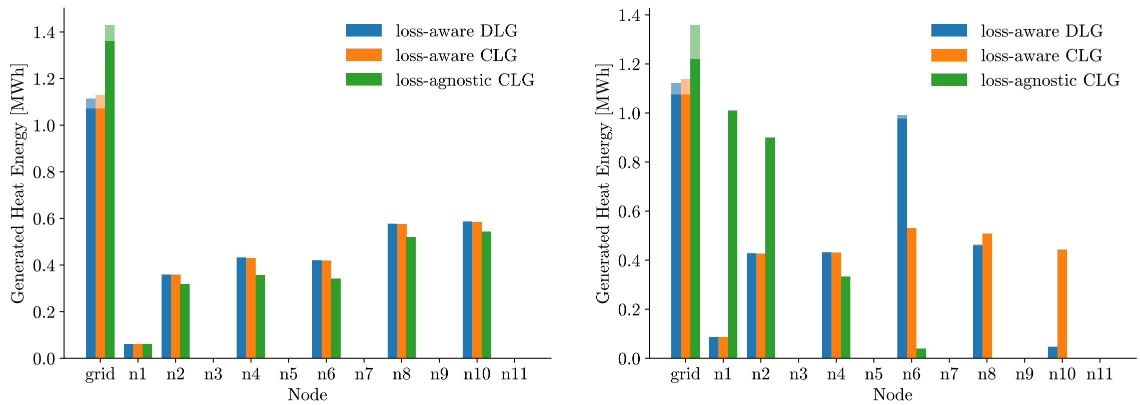


(a) Case I: decreasing marginal costs

(b) Case II: increasing marginal costs

Figure 5: Total scheduled nodal generation. The shaded part represents the generated heat that is lost on the way to the consumer.

loss-aware markets (blue and orange bars), distant heat producers are scheduled to generate a greater amount of energy than in the loss-agnostic benchmark (green bars). This shows that loss-aware dispatch promotes more local heat consumption. We note that for case II, the DLG has a far more widely varying generation between nodes 6-10. This observation is most certainly case-study specific, and would not be there if focusing on a different case-study application.

In case I, the loss-aware markets' generation schedules are equal except for the loss generation, which is shifted from the local generators to the grid agent in the CLG dispatch. This similarity of schedules is not a general result, as we see for case II in Figure 5b. In case II, the loss-aware DLG and CLG dispatches are not equal for node 6, 8, and 10. Compared to DLG, loss-aware CLG increases the generated heat at the most distant generation nodes 8 and 10, while reducing the generated heat at node 6. This is due to the higher cost of loss for these generators in the CLG market compared to the DLG market, as the losses have to be imported from the grid node that is far from node $6-10$. In other words, distant nodes have a greater incentive to minimize losses in loss-aware CLG than in loss-aware DLG.

Total heat loss and total generation costs

Table 2 gives an overview of the total heat loss over 24 hours for each market mechanism and each case. Note that the heat loss in all three markets is considerably higher in case II than in case I, because in case II the cheaper generators are located far from the distant consumers. As expected, the loss-agnostic market results in the largest heat loss in both case I and II. The table also shows the percentage decrease in heat loss relative to the loss-agnostic case for the other two markets. In case II, this decrease is most dramatic: the heat loss is almost halved in the two loss-aware markets, compared to the loss-agnostic case. These results show that loss-aware dispatch leads to a decrease in network heat loss.

One may expect that the losses of loss-aware DLG must always be lower than loss-aware CLG. Interestingly however, the results of case II in Table 2 show loss-aware CLG may lead to lower total losses than loss-aware DLG. The explanation for this is that in loss-aware CLG, losses are more expensive as they have to be imported from the grid, so that local production is stimulated even more than in loss-aware DLG. This may result in more frequent scheduling of the distant generators, and thereby a

		loss-agnostic CLG	loss-aware CLG	loss-aware DLG
case I	Total heat loss [kW]	68.63	57.32	56.71
	% decrease	-	16.48 %	17.37 %
case II	Total heat loss [kW]	137.34	64.48	72.99
	% decrease	-	53.05 %	46.85 %

Table 2: Total heat loss for the three different market mechanisms in case I and II. Percentage decrease with respect to the loss-agnostic CLG market is provided for the other two market mechanisms.

reduction in total heat loss.

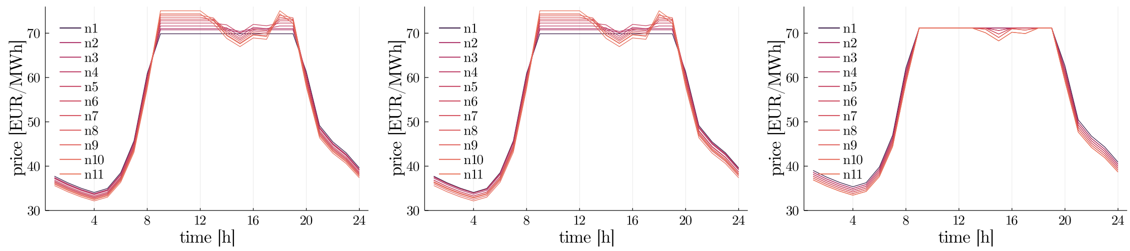
Table 3 summarizes total generation costs for all three market mechanisms, for both case I and II. In addition, the cost reductions in percentages compared to loss-agnostic CLG are given. Loss-agnostic dispatch is expected to increase the cost of loss, and thereby the total cost. Furthermore, it is expected that CLG leads to higher generation costs than DLG, because the feasible space is restricted by forcing the grid agent to produce all losses, rather than leaving it to the cheapest generator. These effects are indeed observed for both case I and case II, as seen in Table 3. In case II, the total costs are lower for loss-aware DLG than loss-aware CLG, despite the observed higher heat loss. The cost differences between the different market types are largest in case II, whereas they are minimal in case I. The reason for this is that in case I, the different market mechanisms lead to rather similar generator schedules. In all, we show here that loss-aware dispatch decreases operational costs resulting from heat loss.

		loss-agnostic CLG	loss-aware CLG	loss-aware DLG
case I	Total generation cost [M€]	198.24	197.85	197.54
	% decrease	-	0.19 %	0.35 %
case II	Total generation cost [M€]	201.76	199.54	199.2
	% decrease	-	1.10 %	1.27 %

Table 3: Total generation costs for the three different market mechanisms in case I and II. Percentage decrease with respect to the loss-agnostic CLG market is provided for the other two market mechanisms.

Locational marginal prices

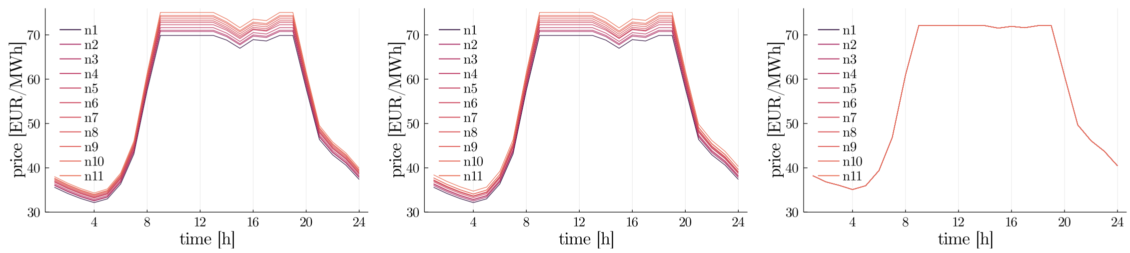
The Locational Marginal Prices (LMPs) as a function of time for case I are shown in Figure 6, and for case II in Figure 7. These prices depend on the choice of loss allocation policy as well. For the loss-aware markets, we use the individual loss allocation that suits these markets naturally. For the loss-agnostic benchmark, we use the socialized loss allocation that is most suitable to this market. Two effects on the nodal prices can be distinguished: the effect of losses and the effect of (unidirectional) flow constraints. The effect of losses is only present in the loss-aware markets, most clearly in case II in Figure 7a and 7b, but also in, for example, hour 10 of the loss-aware markets in case I. The LMP at node 1 equals the import price, and as one moves away from the grid node the LMP increases by a (known) factor. The effect of unidirectional flow can be seen in both loss-aware and loss-agnostic markets of case I in hour 14 to 17. It occurs when a cheap marginal generator is located downstream, so that it cannot supply some upstream nodes. As a result, marginal prices may be lower at distant nodes. The effect is not seen in case II, as the cheaper marginal generator is always upstream. Concluding, in the loss-aware markets the LMPs may differ per node for two reasons, namely due to losses and/or due to heat flow restrictions. In the loss-agnostic case, the LMPs only differ between nodes due to heat flow restrictions, and downstream LMPs will always be lower than or equal to upstream LMPs.



(a) loss-aware DLG (b) loss-aware CLG (c) loss-agnostic CLG

Figure 6: Locational marginal prices as a function of time in Case I. We consider individual loss allocation for the loss-aware markets, and socialized loss allocation for the loss-agnostic benchmark.

On the shape of the LMP curves, it is also visible at which hours nodal heat is generated and when heat is imported from the grid. The latter is the case in those hours where the LMP curve is at high level, and horizontal. Here, we see the effect of having a constant heat import price while having a variable electricity price. Local generators are selling heat at times of low electricity price, i.e until 9 AM and after 9 PM, no matter if grid heat is actually cheap at that time. Heat is imported between 10 AM and 20 PM, which includes the evening demand peak. This puts a pressure on the heat transmission grid, which may be prevented if a variable cost of importing heat is communicated to the consumer. This illustrates the need for variable heat import prices, or even combined heat and electricity markets.



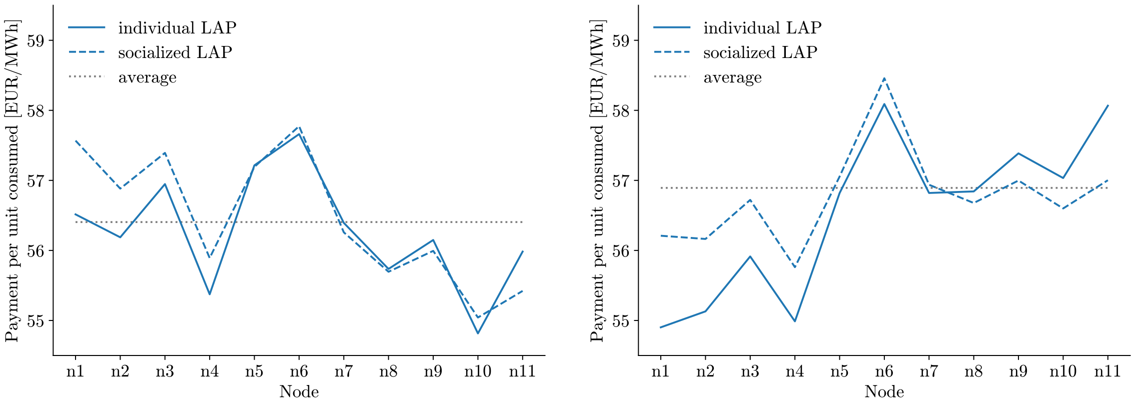
(a) loss-aware DLG (b) loss-aware CLG (c) loss-agnostic CLG

Figure 7: Locational marginal prices as a function of time in Case II. We consider individual loss allocation for the loss-aware markets, and socialized loss allocation for the loss-agnostic benchmark.

Payments: individual VS socialized loss allocation

In this Section we illustrate how consumers may be affected by loss-aware dispatch under different loss allocation policies, by looking at the average consumer price per unit consumed. This average price includes the costs of consumed energy and of loss, where the latter is either socialized or individual, while the former is individual in all cases. The proposed loss allocation policies affect the payments made by consumers of heat, whereas the revenues received by generators are equal for the different loss allocation policies.

The socialized loss allocation policy redistributes the cost of loss, so that an equal loss price is paid for all consumed units. As a result, nodes with an above average loss costs per unit consumed will pay a lower price in the socialized case than in the individual case, and vice versa for nodes with below average loss costs. The redistribution of loss costs is most intuitive in case II, as illustrated in Figure 8b for the loss-aware DLG market. In case II, where marginal generation costs increase with distance



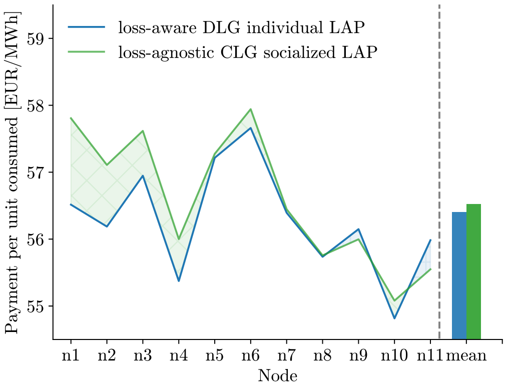
(a) Case I

(b) Case II

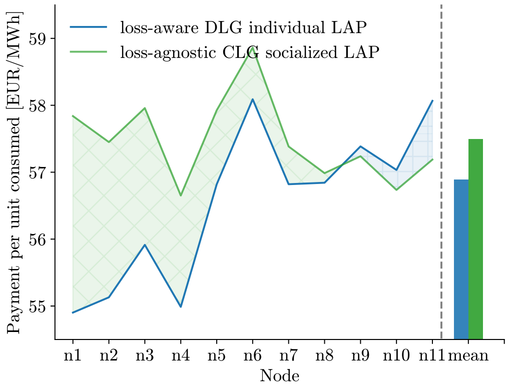
Figure 8: Loss-aware DLG: average consumer price per unit consumed as a function of node for the different loss allocation policies.

from the grid node, the cost of loss increases with distance from the grid node. As a result, it can be seen that the socialized loss allocation generally reduces the unit price for distant nodes, whereas it increases the price for nodes closer to the grid connection. The socialization of loss cost also pulls most nodal prices towards the average price, for all nodes except node 6 and 8. This is not a general result, but it is a result of the fact that most nodes with higher loss costs also have higher energy costs. For case I in Figure 8a, there is also a trend that the socialized loss allocation redistributes loss costs from distant nodes to proximal nodes, but it does not hold for all nodes. For instance, node 10 pays a higher unit price under the socialized loss allocation. For case I, as opposed to case II, most nodes experience an average unit price *further* from the average under the socialized loss allocation. In other words, if energy costs are individualized, the socialization of loss costs may lead to larger differences in unit prices, which is the opposite of what may be expected from socialization.

Finally, we compare the average unit price of loss-aware DLG market with individual loss allocation policy, and of loss-agnostic CLG market with socialized loss allocation policy. The loss-aware DLG market with individual loss allocation is the one giving optimal incentives to market participants for reducing losses, but leads to the highest loss price differentiation between them. The loss-agnostic CLG market with socialized loss allocation is closest to existing practices. The comparison is shown in Figure 9, which shows both the average unit price as a function of node, and the overall average. In both case I and II, the overall average unit price is higher for the loss-agnostic market. As a result, most nodes pay a lower unit price in loss-aware DLG with individual loss allocation, even though the loss is paid by the individual. Only those nodes with the very highest loss costs benefit in the loss-agnostic CLG with socialized loss costs.



(a) Case I



(b) Case II

Figure 9: Average price per unit consumed as a function of node. Comparison between loss-aware DLG with individual loss allocation policy and loss-agnostic CLG with socialized loss allocation policy

6 Conclusion

District heating systems become more distributed with the integration of prosumers, including excess heat producers and active consumers. This calls for suitable heat market mechanisms that optimally integrate these actors while ensuring network feasibility and considering operational costs. To this end, we have proposed two variants of a network- and loss-aware heat market mechanism, as well as a network-aware but loss-agnostic benchmark. The markets are formulated as Quadratic Programs. In our distributed loss generation (DLG) market variant, losses caused by a certain trade are produced by the seller of that trade, while in the centralized loss generation (CLG) formulation this is done by a grid agent. In the benchmark, the loss costs are excluded from the objective function. We used peer-to-peer trades to explicitly link losses to certain market participants. The mechanisms are suitable for radial, unidirectional district heating systems with bidirectional nodal flow, , and allow for multiple heat injection points. We have derived dual relations for the proposed markets and the benchmark, and determined the nodal marginal prices. Based on these nodal prices we formulated allocation mechanisms for energy and loss production costs. We considered two allocation mechanisms for the cost of loss: an *individual* and a *socialized* policy. In the former, loss costs are allocated to the buyer of the trade causing the losses, whereas in the latter they are socialized. Through a case study we have illustrated several properties of the proposed market mechanisms by comparing to our benchmark. Most importantly, we have shown that the designed loss-aware dispatch may schedule distant generators despite their higher production costs, in case this reduces the total cost *including cost of loss*. We have shown that the total heat loss and cost of heat loss are reduced in a loss-aware dispatch compared to our loss-agnostic benchmark. In conclusion, we have shown that the loss-aware market mechanisms can help promote local consumption and reduce operating costs in district heating networks, while integrating distributed generators and prosumers.

6.1 Discussion

Compared to network-aware mechanisms in the literature, our formulation has the advantage that it can include multiple distributed generators and flexible loads, while leaving the size and sign of nodal power injection variable. The latter means that no unit commitment decisions are fixed before dispatch, and prosumer nodes are free to either consumer or produce, as opposed to constant-flow variable-temperature (CFVT) formulations. These advantages come at the price of other desirable properties. First of all, generators (consumers) must inject water at the respective nodal supply (return) temperature, while CFVT or full variable-flow variable-temperature (VFVT) formulations allow for variable injection temperatures. This may be a problem for some agents in practice. A way to deal with this would be to have punish deviations from the set temperature. For example, generators would not be paid for the additional energy in case they inject at a too high temperature, and would pay a fee in case they inject at a lower temperature than expected. Another challenge is that the fixed nodal temperatures in our formulation have a large impact on dispatch and prices, and should therefore be selected carefully. New, fair methods of determining these temperatures are needed.

The main purpose of adding peer-to-peer trades in this work was to trace losses back to certain producers and consumers. There are other reasons these peer-to-peer trades may appeal in practice. For one, the formulation allows for including consumer preferences, both based on the source of the heat as well as on the amount of loss. This would be done by adding weighting factors to the objective function, as for example in [26]. Moreover, for implementation of a consumer-centric heat market in practice, the peer-to-peer formulation is appealing because it forms the basis for deriving a decentralized negotiation mechanism, in which all market participants solve a local optimization problem. The peer-to-peer market is then cleared using a distributed optimization method, such as the Alternating Direction Method of Multipliers (ADMM), in which agents would negotiate directly with one another. ADMM is an iterative method. In every iteration, the market participants all solve a local optimization problem, and sends trade proposals to each trading partner. For more details on reformulation to distributed setup, as well as a discussion of the implications of a decentralized peer-to-peer market in practice, refer to for example [27]. Our individual peer-to-peer dispatch satisfies conditions to be solved by ADMM as summarized in [37]. Therefore it is possible to solve the dispatch in a distributed, fully decentralized manner. Note that the socialized loss allocation policy is not readily suitable for use in a distributed setup.

6.2 Future perspectives

Future research could investigate the inclusion of operating costs and constraints related to water pumping in the network. This is done in [24] for a system with a single point of heat injection, using a first order approximation of bilinear expression for pumping energy. This approximation cannot be used in the presence of multiple injection points, but can perhaps be generalized to the multiple injection setting. Otherwise, one could respond to approximate pumping costs as a recourse action, after market clearing. In such a setup, the output of the simplified market clearing without pumping equations may be seen as the operating point for the system. Then, by adding McCormick relaxations of pumping equations to the simplified setup, one could see how much it would be beneficial to deviate from these operating points. Those deviations may eventually be seen as potential recourse actions.

The proposed market design is only suitable for systems with unidirectional pipeline flow. It could be adapted to accommodate systems with bidirectional flow. For instance, pipelines losses could remain a fixed share of transported energy, i.e. the losses would still be multiplicative. A loss factor would have to be determined for each pipeline. It is however not trivial how nodal temperatures and pipeline flows could remain part of such model, and this should be investigated further.

In this work, we have focused on the heat market, and therefore simplified connections to the electricity system. Already in this simplified case, it became clear that consumers that are subject to variable electricity prices but fixed heat import prices may shift their load in a way unfavourable for the heating system. This highlights the need for variable heat prices in the presence of variable electricity prices. In future work, the proposed heat market mechanisms can be extended to a combined heat and electricity market.

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A Appendix

In this Appendix we derive dual variable relations for the three market setups considered in this work.

A.1 Dual variable relations between nodes

The relations derived in this Section hold for all market formulations in this work. To determine the relations between the nodal dual variables μ^{HE}_{nt}, we derive the first order conditions for flow in node n and pipe $p = (n_1, n_2)$

$$\dot{m}^{\mathrm{N}}_{nt} : -\mu^{\mathrm{mc}}_{nt} + c_{\mathrm{f}}(T^{\mathrm{S}}_{n} - T^{\mathrm{R}}_{n})\mu^{\mathrm{HE}}_{nt} - \underline{\gamma}^{\mathrm{mn}} + \overline{\gamma}^{\mathrm{mn}} = 0 \tag{32a}$$
$$\dot{m}^{\mathrm{S}}_{pt} : \mu^{\mathrm{mc}}_{n_2t} - \mu^{\mathrm{mc}}_{n_1t} - \underline{\gamma}^{\mathrm{mp}} + \overline{\gamma}^{\mathrm{mp}} = 0\,. \tag{32b}$$

Now consider two nodes n_1 and n_2 connected by pipe $p = (n_1, n_2)$. If none of the pipeline flow bounds are active, it holds that $\underline{\gamma}^{\mathrm{mp}} = 0 = \overline{\gamma}^{\mathrm{mp}}$, so that $\mu^{\mathrm{mc}}_{n_1t} = \mu^{\mathrm{mc}}_{n_2t}$. Combined with (32b) and assuming that none of the nodal flow bounds are active, the following relation between $\mu^{\mathrm{HE}}_{n_1t}$ and $\mu^{\mathrm{HE}}_{n_2t}$ can be derived

$$c_{\mathrm{f}}(T^{\mathrm{S}}_{n_1} - T^{\mathrm{R}}_{n_1})\mu^{\mathrm{HE}}_{n_1t} = c_{\mathrm{f}}(T^{\mathrm{S}}_{n_2} - T^{\mathrm{R}}_{n_2})\mu^{\mathrm{HE}}_{n_2t}$$
$$\mu^{\mathrm{HE}}_{n_2t} = \frac{T^{\mathrm{S}}_{n_1} - T^{\mathrm{R}}_{n_1}}{T^{\mathrm{S}}_{n_2} - T^{\mathrm{R}}_{n_2}}\mu^{\mathrm{HE}}_{n_1t}\,. \tag{33}$$

A.2 Loss-aware DLG peer-to-peer market

In this Section we derive the dual relations for the DLG loss-aware peer-to-peer market. Consider the following first order conditions for this case. Those are

$$G^{\mathrm{H}}_{it} : \frac{df^{\mathrm{loss}}}{dG^{\mathrm{H}}_{it}} - \mu^{\mathrm{inj}}_{it} + \cancel{\mu^{\mathrm{cop}}_{it}} + \overline{\gamma}^{\mathrm{G}}_{it} - \underline{\gamma}^{\mathrm{G}}_{it} - \mu^{\mathrm{S}}_{it} = 0 \tag{34a}$$
$$L^{\mathrm{H}}_{it} : \mu^{\mathrm{inj}}_{it} + \mu^{\mathrm{Ltot}}_{it} - \underline{\gamma}^{\mathrm{L}}_{it} - \mu^{\mathrm{B}}_{it} = 0 \tag{34b}$$
$$L^{\mathrm{SH}}_{it} : \frac{df^{\mathrm{loss}}}{dL^{\mathrm{SH}}_{it}} + \mu^{\mathrm{eb}}_{i} - \mu^{\mathrm{Ltot}}_{it} + \overline{\gamma}^{\mathrm{SH}}_{it} - \underline{\gamma}^{\mathrm{SH}}_{it} = 0 \tag{34c}$$
$$P^{\mathrm{H}}_{it} : \mu^{\mathrm{HE}}_{n_it} + \mu^{\mathrm{inj}}_{it} = 0 \tag{34d}$$
$$t_{ijt} : \mu^{\mathrm{T}}_{ijt} = 0 \tag{34e}$$
$$w_{ijt} : \mu^{\mathrm{S}}_{it} + \mu^{\mathrm{w}}_{ijt} - \underline{\gamma}^{\mathrm{w}}_{ijt} = 0 \tag{34f}$$
$$s_{ijt} : \cancel{-\mu^{\mathrm{T}}_{ijt}} + \mu^{\mathrm{R}}_{ijt} + \mu^{\mathrm{S}}_{it} - \tilde{w}_{ij}\mu^{\mathrm{w}}_{ijt} - \underline{\gamma}^{\mathrm{s}}_{ijt} = 0 \tag{34g}$$
$$b_{ijt} : \cancel{\mu^{\mathrm{T}}_{ijt}} - \mu^{\mathrm{R}}_{jit} + \mu^{\mathrm{B}}_{it} - \underline{\gamma}^{\mathrm{b}}_{ijt} = 0 \tag{34h}$$

We can identify that the price each agent perceives for generation is $\mu^{\mathrm{S}}_{it} + \mu^{\mathrm{inj}}_{it}$. For the perceived price for loads, we combine (34b) and (34c) to

$$\mu^{\mathrm{inj}}_{it} + \frac{df^{\mathrm{loss}}}{dL^{\mathrm{SH}}_{it}} + \mu^{\mathrm{eb}}_{i} + \overline{\gamma}^{\mathrm{SH}}_{it} - \underline{\gamma}^{\mathrm{SH}}_{it} - \underline{\gamma}^{\mathrm{L}}_{it} - \mu^{\mathrm{B}}_{it} = 0\,. \tag{35}$$

which shows that the perceived price for load i is $-(\mu^{\mathrm{B}}_{it} - \mu^{\mathrm{inj}}_{it}) = -\mu^{\mathrm{B}}_{it} + \mu^{\mathrm{inj}}_{it}$. Note that μ^{eb}_{i} is seen as a bound on the load.

Next, we derive relations between seller and buyer price of a nonzero trade t_{ijt}. Assume the sale from agent i to j is nonzero, i.e. $s_{ijt} = b_{jit} > 0$. For a marginal generator i,

$$\mu^{\mathrm{inj}}_{it} + \mu^{\mathrm{S}}_{it} = \frac{df^{\mathrm{loss}}}{dG_{it}}\,. \tag{36}$$

By (34g), it holds that

$$\mu^{\mathrm{S}}_{it} = -\mu^{\mathrm{R}}_{ijt} + \tilde{w}_{ij}\mu^{\mathrm{w}}_{ijt} \tag{37}$$

as $\underline{\gamma}^{\mathrm{s}}_{ijt} = 0$. Similarly, by (34h) for $b_{jit} > 0$

$$\mu^{\mathrm{B}}_{jt} = \mu^{\mathrm{R}}_{ijt}\,. \tag{38}$$

Substituting (38) in (37) gives

$$\mu^{\mathrm{S}}_{it} = -\mu^{\mathrm{B}}_{jt} + \tilde{w}_{ij}\mu^{\mathrm{w}}_{ijt}\,. \tag{39}$$

Case 1: i and j are at different nodes, so $n_i \neq n_j$. Still assuming $s_{ijt} > 0$, the loss must be positive, so $w_{ijt} > 0$ and $\underline{\gamma}^{\mathrm{w}}_{ijt} = 0$. Then it follows from (34f) that

$$\mu^{\mathrm{S}}_{it} = -\mu^{\mathrm{w}}_{ijt}\,. \tag{40}$$

Substituting this in (39), we derive that

$$\begin{aligned} \mu^{\mathrm{S}}_{it} &= -\mu^{\mathrm{B}}_{jt} - \tilde{w}_{ij}\mu^{\mathrm{S}}_{it} \\ (1+\tilde{w}_{ij})\mu^{\mathrm{S}}_{it} &= -\mu^{\mathrm{B}}_{jt}\,. \end{aligned} \tag{41}$$

Combining this with nodal price relations in (33), we derive that

$$\begin{aligned} \text{buyer price } &= -\mu^{\mathrm{B}}_{jt} + \mu^{\mathrm{inj}}_{jt} = -\mu^{\mathrm{B}}_{jt} - \mu^{\mathrm{HE}}_{n_j t} \\ &= -\mu^{\mathrm{B}}_{jt} - \frac{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}{T^{\mathrm{S}}_{n_j} - T^{\mathrm{R}}_{n_j}}\mu^{\mathrm{HE}}_{n_i t} \\ &= (1+\tilde{w}_{ij})\mu^{\mathrm{S}}_{it} - \frac{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}{T^{\mathrm{S}}_{n_j} - T^{\mathrm{R}}_{n_j}}\mu^{\mathrm{HE}}_{n_i t} \\ &= (1+\tilde{w}_{ij})\left(\mu^{\mathrm{S}}_{it} + \mu^{\mathrm{inj}}_{it}\right) = (1+\tilde{w}_{ij}) \cdot \text{seller price }\;, \end{aligned} \tag{42}$$

where the equality on the third line comes from (41). This relation shows that the buyer price is a fixed factor greater than the seller price, which implies that the buyer pays more per unit of consumed heat than the seller is paid per unit of produced heat. This way, the buyer is experiencing the cost of loss. The price of energy and loss is equal.

Case 2: i and j are at the same node, so $n_i = n_j$. Then by (34d) it holds that $\mu^{\mathrm{inj}}_{it} = \mu^{\mathrm{inj}}_{jt}$. The loss must be zero, i.e. $w_{ijt} = 0$ which implies $\underline{\gamma}^{\mathrm{w}}_{ijt} \geq 0$. From (39) and the fact that $\tilde{w}_{ij} = 0$ if $n_i = n_i$, we derive

$$\mu^{\mathrm{S}}_{it} = -\mu^{\mathrm{B}}_{jt}\,. \tag{43}$$

This relation is as expected, as there are no losses when i and j are at the same node.

A.3 Loss-aware CLG market

So in this case, $\alpha_{ij} = 0$ for all agents. This results in several changes in the first order conditions. First of all, in the derivative with respect to w_{ijt}, now we have $\mu^{\mathrm{S}}_{\mathrm{g}t}$ instead of μ^{S}_{it}. This yields

$$G^{\mathrm{H}}_{it} : \frac{df^{\mathrm{loss}}}{dG^{\mathrm{H}}_{it}} - \mu^{\mathrm{inj}}_{it} + \cancel{\mu^{\mathrm{cop}}_{it}} + \overline{\gamma}^{\mathrm{G}}_{it} - \underline{\gamma}^{\mathrm{G}}_{it} - \mu^{\mathrm{S}}_{it} = 0 \tag{44a}$$

$$G^{\mathrm{H}}_{\mathrm{g}t} : \frac{df^{\mathrm{loss}}}{dG^{\mathrm{H}}_{\mathrm{g}t}} - \mu^{\mathrm{inj}}_{\mathrm{g}t} - \underline{\gamma}^{\mathrm{G}}_{\mathrm{g}t} - \mu^{\mathrm{S}}_{\mathrm{g}t} = 0 \tag{44b}$$

$$L^{\mathrm{H}}_{it} : \mu^{\mathrm{inj}}_{it} + \mu^{\mathrm{Ltot}}_{it} - \underline{\gamma}^{\mathrm{L}}_{it} - \mu^{\mathrm{B}}_{it} = 0 \tag{44c}$$

$$
\begin{aligned}
&L_{it}^{\mathrm{SH}} : \frac{df^{\mathrm{loss}}}{dL_{it}^{\mathrm{SH}}} - \mu_{it}^{\mathrm{Ltot}} + \overline{\gamma}_{it}^{\mathrm{SH}} - \underline{\gamma}_{it}^{\mathrm{SH}} = 0 && \text{(44d)}\\
&P_{it}^{\mathrm{H}} : \mu_{n_i t}^{\mathrm{HE}} + \mu_{it}^{\mathrm{inj}} = 0 && \text{(44e)}\\
&t_{ijt} : \mu_{ijt}^{\mathrm{T}} = 0 && \text{(44f)}\\
&w_{ijt}^{\mathrm{g}} : \mu_{\mathrm{g}t}^{\mathrm{S}} + \mu_{ijt}^{\mathrm{wg}} - \underline{\gamma}_{ijt}^{\mathrm{wg}} = 0 && \text{(44g)}\\
&s_{ijt} : \cancel{-\mu_{ijt}^{\mathrm{T}}} + \mu_{ijt}^{\mathrm{R}} + \mu_{it}^{\mathrm{S}} - \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}\, \tilde{w}_{ij} \mu_{ijt}^{\mathrm{wg}} - \underline{\gamma}_{ijt}^{\mathrm{s}} = 0 && \text{(44h)}\\
&b_{ijt} : \cancel{\mu_{ijt}^{\mathrm{T}}} - \mu_{jit}^{\mathrm{R}} + \mu_{it}^{\mathrm{B}} - \underline{\gamma}_{ijt}^{\mathrm{b}} = 0 && \text{(44i)}
\end{aligned}
$$

The price of loss for the trade between agents i and j is a fixed share of the nodal price at the grid node. We can see that for any time t, it holds that $\mu_{ijt}^{\mathrm{wg}} = \mu_{\mathrm{g}t}^{\mathrm{S}}$ for all nonzero losses w_{ijt}. Most of the analysis of previous Section still holds, but we derive a new expression for price of loss in the case that i and j are at different nodes (Case 1). Assuming that the sale from i to j is positive, i.e. $s_{ijt} > 0$, it follows from (44h) and (44i) that

$$
\begin{aligned}
\mu_{it}^{\mathrm{S}} &= -\mu_{ijt}^{\mathrm{R}} + \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}\, \tilde{w}_{ij} \mu_{ijt}^{\mathrm{wg}}\\
&= -\mu_{jt}^{\mathrm{B}} + \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}\, \tilde{w}_{ij} \mu_{ijt}^{\mathrm{wg}}\,, \qquad (45)
\end{aligned}
$$

which can be rewritten using (44g) to

$$
\mu_{it}^{\mathrm{S}} = -\mu_{jt}^{\mathrm{B}} - \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}\, \tilde{w}_{ij} \mu_{\mathrm{g}t}^{\mathrm{S}} \qquad (46)
$$

$$
\mu_{jt}^{\mathrm{B}} = -\mu_{it}^{\mathrm{S}} - \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}\, \tilde{w}_{ij} \mu_{\mathrm{g}t}^{\mathrm{S}}\,. \qquad (47)
$$

In addition, we derive the following relation between nodal prices at n_i, n_j, and n_g using the nodal price relation in (33) and the definition of $\tilde{w}_{ij}$ in (21):

$$
\begin{aligned}
\mu_{n_j t}^{\mathrm{HE}} &= \frac{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}{T_{n_j}^{\mathrm{S}} - T_{n_j}^{\mathrm{R}}} \mu_{n_i t}^{\mathrm{HE}} = \left(\tilde{w}_{ij} + 1\right) \mu_{n_i t}^{\mathrm{HE}} = \mu_{n_i t}^{\mathrm{HE}} + \tilde{w}_{ij} \mu_{n_i t}^{\mathrm{HE}}\\
&= \mu_{n_i t}^{\mathrm{HE}} + \tilde{w}_{ij} \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}} \mu_{n_\mathrm{g} t}^{\mathrm{HE}}\,. \qquad (48)
\end{aligned}
$$

Using (46) and (48), we can finally establish the following relation between the buyer and seller price in the loss-aware CLG market:

$$
\begin{aligned}
\text{buyer price } &= -\mu_{jt}^{\mathrm{B}} + \mu_{jt}^{\mathrm{inj}} = -\mu_{jt}^{\mathrm{B}} - \mu_{n_j t}^{\mathrm{HE}} = -\mu_{jt}^{\mathrm{B}} - \left(\mu_{n_i t}^{\mathrm{HE}} + \tilde{w}_{ij} \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}} \mu_{n_\mathrm{g} t}^{\mathrm{HE}} \right)\\
&= \mu_{it}^{\mathrm{S}} + \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}\, \tilde{w}_{ij} \mu_{\mathrm{g}t}^{\mathrm{S}} - \mu_{n_i t}^{\mathrm{HE}} - \tilde{w}_{ij} \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}} \mu_{n_\mathrm{g} t}^{\mathrm{HE}}\\
&= \mu_{it}^{\mathrm{S}} + \mu_{it}^{\mathrm{inj}} + \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}\, \tilde{w}_{ij} (\mu_{\mathrm{g}t}^{\mathrm{S}} + \mu_{\mathrm{g}t}^{\mathrm{inj}})\\
&= \text{seller price } + \frac{T_{n_\mathrm{g}}^{\mathrm{S}} - T_{n_\mathrm{g}}^{\mathrm{R}}}{T_{n_i}^{\mathrm{S}} - T_{n_i}^{\mathrm{R}}}\, \tilde{w}_{ij} \cdot \text{ grid node price }\,. \qquad (49)
\end{aligned}
$$

A.4 Loss-ignorant CLG market

In this case, $\alpha_{ij} = 0$ for all trades ij, and the objective function does not include the cost of loss, as in (23). The only difference in the first order conditions compared to loss-aware CLG is in the condition for w^{g}_{ijt}, which now includes a derivative of the objective function:

$$w^{\mathrm{g}}_{ijt} : \frac{df^{\mathrm{noloss}}}{dw^{\mathrm{g}}_{ijt}} + \mu^{\mathrm{S}}_{\mathrm{g}t} + \mu^{\mathrm{wg}}_{ijt} - \underline{\gamma}^{\mathrm{wg}}_{ijt} = 0\,. \tag{50}$$

Substituting this relation in (45) while assuming the loss is nonzero gives

$$\begin{aligned} \mu^{\mathrm{S}}_{it} &= -\mu^{\mathrm{B}}_{jt} + \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij}\mu^{\mathrm{wg}}_{ijt} \\ &= -\mu^{\mathrm{B}}_{jt} - \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij} \left(\frac{df^{\mathrm{noloss}}}{dw^{\mathrm{g}}_{ijt}} + \mu^{\mathrm{S}}_{\mathrm{g}t} \right) \end{aligned} \tag{51}$$

so that the buyer price can be decomposed into

$$\mu^{\mathrm{B}}_{jt} = -\mu^{\mathrm{S}}_{it} - \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij} \left(\frac{df^{\mathrm{noloss}}}{dw^{\mathrm{g}}_{ijt}} + \mu^{\mathrm{S}}_{\mathrm{g}t} \right) . \tag{52}$$

By the first order conditions of grid generation in (44a)

$$\mu^{\mathrm{S}}_{\mathrm{g}t} = \frac{df^{\mathrm{loss}}}{dG^{\mathrm{H}}_{\mathrm{g}t}} - \mu^{\mathrm{inj}}_{\mathrm{g}t} - \underline{\gamma}^{\mathrm{G}}_{\mathrm{g}t}\,. \tag{53}$$

Assuming nonzero loss, we know that $\underline{\gamma}^{\mathrm{G}}_{\mathrm{g}t} = 0$, and the buyer price decomposition becomes

$$\begin{aligned} \mu^{\mathrm{B}}_{jt} &= -\mu^{\mathrm{S}}_{it} - \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij} \left(\frac{df^{\mathrm{noloss}}}{dw^{\mathrm{g}}_{ijt}} + \frac{df^{\mathrm{loss}}}{dG^{\mathrm{H}}_{\mathrm{g}t}} - \mu^{\mathrm{inj}}_{\mathrm{g}t} \right) \\ &= -\mu^{\mathrm{S}}_{it} + \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij}\mu^{\mathrm{inj}}_{\mathrm{g}t} \\ &= -\mu^{\mathrm{S}}_{it} + \frac{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}}{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}\, \tilde{w}_{ij} \frac{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}{T^{\mathrm{S}}_{n_{\mathrm{g}}} - T^{\mathrm{R}}_{n_{\mathrm{g}}}} \mu^{\mathrm{HE}}_{n_i t} \\ &= -\mu^{\mathrm{S}}_{it} + \tilde{w}_{ij}\mu^{\mathrm{HE}}_{n_i t}\,. \end{aligned} \tag{54}$$

In the second equality we use that $\frac{df^{\mathrm{noloss}}}{dw^{\mathrm{g}}_{ijt}} + \frac{df^{\mathrm{loss}}}{dG^{\mathrm{H}}_{\mathrm{g}t}} = -c^{\mathrm{H,imp}}_t + c^{\mathrm{H,imp}}_t = 0$, and in the third equality we rewrite $\mu^{\mathrm{inj}}_{\mathrm{g}t}$ using the nodal relations in (33).

Combining this result with the nodal price relation in (33) gives the following relation between seller and buyer price:

$$\begin{aligned} \text{buyer price } &= \mu^{\mathrm{B}}_{jt} + \mu^{\mathrm{inj}}_{jt} = \mu^{\mathrm{B}}_{jt} - \mu^{\mathrm{HE}}_{n_j t} \\ &= \mu^{\mathrm{B}}_{jt} - \frac{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}{T^{\mathrm{S}}_{n_j} - T^{\mathrm{R}}_{n_j}} \mu^{\mathrm{HE}}_{n_i t} \\ &= -\mu^{\mathrm{S}}_{it} + \tilde{w}_{ij}\mu^{\mathrm{HE}}_{n_i t} - \frac{T^{\mathrm{S}}_{n_i} - T^{\mathrm{R}}_{n_i}}{T^{\mathrm{S}}_{n_j} - T^{\mathrm{R}}_{n_j}} \mu^{\mathrm{HE}}_{n_i t} \\ &= -(\mu^{\mathrm{S}}_{it} + \mu^{\mathrm{HE}}_{n_i t}) = -(\mu^{\mathrm{inj}}_{it} + \mu^{\mathrm{S}}_{it}) = \text{seller payment}\,, \end{aligned} \tag{55}$$

where the left equality on the last line comes from the definition of $\tilde{w}_{ij}$. This equality of buyer and seller price shows that no one is paying for the losses generated by the grid agent.

[Pub. C] On the efficiency of energy markets with non-merchant storage

On the Efficiency of Energy Markets with Non-Merchant Storage

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***Abstract*—Energy market designs with non-merchant storage have been proposed in recent years, with the aim of achieving optimal integration of storage. In order to handle the time-linking constraints that are introduced in such markets, existing works commonly make simplifying assumptions about the end-of-horizon storage level. This work analyzes market properties under such assumptions, as well as in their absence. We find that, although they ensure cost recovery for all market participants, these assumptions generally lead to market inefficiencies. Therefore we consider the design of markets with non-merchant storage without such simplifying assumptions. Using an illustrative example, as well as detailed proofs, we provide conditions under which market prices in subsequent market horizons fail to reflect the value of stored energy. We show that this problem is essential to address in order to preserve market efficiency and cost recovery. Finally, we propose a method for restoring these market properties in a perfect-foresight setting.**

***Index Terms*—non-merchant storage, energy market design, passive storage, market efficiency.**

I. Introduction

A. Context and Motivations

The need for large-scale energy storage to balance intermittent and stochastic renewables in future energy systems has become apparent. The IEA Energy Outlook predicts that utility-scale battery storage will increase from less than 20 GW in 2020 to over 3 000 GW by 2050 [1]. It remains an open question how to best integrate storage in energy markets. The urgency of this question is illustrated by a recent FERC order that requires system operators to facilitate market participation of electric storage, and to provide fair compensation for the provided services, given the physical and operational characteristics of these assets [2].

Currently, most energy markets treat storage like any other load or generator, which means that the storage operator submits price-quantity bids. Storage submitting bids in the market is also called *merchant storage*. One major disadvantage of merchant storage is that it can have negative effects on social welfare [3], [4], depending on its market power. To mitigate these effects, the concept of *non-merchant* or *passive* storage in energy markets has been investigated in recent years [5]–[8]. A non-merchant storage does not submit bids, but its operation is co-optimized with generation and loads to achieve the highest social welfare. An appealing property of this set-up is that it is comparable to network-aware markets, which are prevalent in practice, e.g. in the U.S. markets. Similar to the way that power lines are included in the market to perform spatial arbitrage, a non-merchant storage performs temporal arbitrage. Thus, the storage is scheduled to achieve more efficient use of energy systems. Non-merchant storage would typically be a public asset owned by the system or network operator, but it could also be privately owned.

Time-linking constraints are inevitably part of optimal dispatch problems for storage. With merchant storage, these time-linking constraints can be considered in the individual optimal bidding problem of the storage operator. The challenges associated with time-linking constraints in this case have been studied extensively for both price-taker [9], [10] and price-maker storage [11]–[14]. These challenges are transferred to the market operator in the case of non-merchant storage. Energy markets are cleared sequentially for subsequent finite time horizons, but due to intertemporal constraints, the different market horizons do influence one another. This raises the question of how to account for the subsequent market clearing in the current clearing, and vice versa. For an overview of market design approaches for markets with intertemporal constraints we refer to [15].

B. Status Quo and Limitations

Existing works on market clearing with non-merchant storage often neglect or simplify relations between subsequent market horizons, leading to 'myopic' decision-making regarding the state of energy of the storage at the end of horizon. This is pointed out as a research gap by the authors of [16]. A non-myopic, optimal final state of energy is challenging to determine, because it depends on both current and future market horizons. The non-merchant storage literature largely bypasses this problem using the following common simplifying assumptions. Most works do not impose any constraints on the final state of energy in the storage [5]–[8]. Another common simplification is to enforce that the storage state of energy is equal at the start and end of a horizon [17]–[19]. A possible state-of-energy profile resulting from either of these myopic approaches is plotted in blue in Figure 1, whereas an example optimal profile given perfect information about future market intervals is depicted in black. Such simplifying assumptions lead to suboptimal use of the storage, loss of social welfare, and market inefficiencies. These effects are sometimes overlooked in existing works that consider market properties within a single horizon [8]. As an alternative, a 'future-aware' method, in red in Figure 1, could approach the perfect profile if good end-of-horizon decisions were made.

Several previous works do consider setting a future-aware end-of-horizon level for the non-merchant storage, but this has only been done using rolling-horizon methods [15], [20], [21]. However, problems can even occur when the final state of energy is set perfectly, both in rolling-horizon or other kinds of look-ahead markets. They result from the fact that

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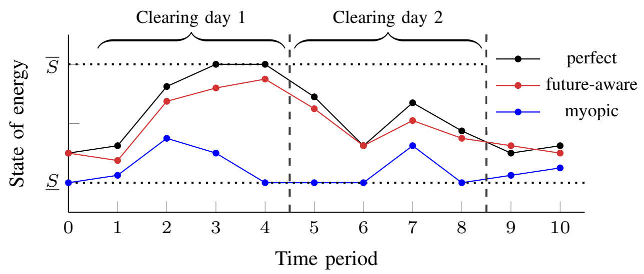


Fig. 1. Illustrative example: the effect of (simplifying) assumptions on the state of energy of the storage over several market clearing horizons.

subsequent optimization horizons are not aware of losses and gains incurred by market participants in past horizons [20]. As a result, it can happen that the storage improves social welfare, while not recovering its cost. Solutions to this problem have been proposed in a rolling-horizon setting, and with a focus on ramping constraints [20], [22].

C. *Contributions and Structure of the Paper*

The end-of-horizon issue for non-merchant storage is systematically overlooked, as illustrated by many studies that make simplifying assumptions on the final state of energy. To draw attention to this, we aim to convey two key messages in a rigorous manner. Our first message is that it is important to set a future-aware end-of-horizon storage level, instead of making simplifying assumptions. Second, we show that a new problem arises in the absence of the simplifying assumptions, namely, that market prices in subsequent market horizons may fail to reflect the value of storage. This problem is essential to address, as the market can fail to provide dispatch-following incentives and cost recovery for the storage. Previous works have touched upon these issues, but only in a rolling-horizon setting, and with a focus on ramping constraints. Instead of using a rolling-horizon approach, we consider a more general formulation where the final storage level is constrained to take a certain value, to be determined for each market clearing.

To provide intuitive understanding of these two situations, we first provide illustrative examples. In addition, we use a more formal approach to give a general and rigorous explanation of why these problems may occur, and are likely to occur in practice. For the first message, we prove that common simplifying assumptions ensure cost recovery for the storage, but lead to market inefficiencies. Related to the second message, we provide sufficient conditions in Theorem 1 under which the market prices in subsequent market horizons will fail to reflect the value of storage, and thus fail to provide dispatch-following incentives and possibly cost recovery for the storage. A final contribution of this work is a proposed market-clearing procedure with non-merchant storage which is efficient and ensures cost recovery, given perfect foresight about future market-clearing parameters.

The remainder of this paper is organized as follows. In Section II, we introduce the set-up of this work, including the market-clearing model and pricing scheme. Section III analyzes the impact of common assumptions on market properties. In Section IV, we show why and under which conditions dispatch-following incentives and cost recovery for the storage are not ensured, even when the final state of energy is set to its optimal value. In Section V, we propose a method for ensuring that the desirable market properties hold, in the perfect foresight setting. Finally, Section VI concludes. , followed by several appendices.

II. Market Model and Definitions

We formulate the market-clearing problem with a non-merchant storage and detail our assumptions. Hereafter, *market horizon* refers to a set of time periods covered by a given market clearing. For example, a day-ahead market would have a market horizon of 24 hours. The next market horizon would include the following 24 hours. We disregard what happens in other markets trading for the same day, e.g., intraday and balancing. We first introduce the storage model in Section II-A before proceeding to the market formulation in Section II-B. Next, we formulate the dual problem and discuss the pricing mechanism in Section II-C. Finally, we define market properties in Section II-D. Nomenclature and the rules followed for notations are available in Appendix A.

A. *Storage Model*

We assume there is a single storage system. This is equivalent to including multiple storage systems, as we do not consider grid constraints. The storage system is described by

$$0 \le e_t \le \overline{S}, \quad \forall t \qquad : (\underline{\nu}_t, \overline{\nu}_t) \tag{1a}$$
$$e_t = e_{t-1} + b_t, \quad \forall t \neq 1 \qquad : (\rho_t) \tag{1b}$$
$$e_1 = E^{\text{init}} + b_1 \qquad : (\rho_1) \tag{1c}$$
$$e_{\text{H}} = E^{\text{end}} \qquad : (\xi)\,. \tag{1d}$$

Here and in the following, dual variables are indicated within parentheses on the right of the corresponding constraint. For time step t in the market-clearing horizon $\mathcal{T}$, the state of energy of the storage system is a decision variable denoted by e_t. It is bounded between 0 and the storage capacity $\overline{S}$ in (1a). The variable b_t represents the energy charged ($b_t > 0$) or discharged ($b_t < 0$). The storage energy balance is described by (1b) and (1c), where E^{init} is the initial state of energy. We omit charging and discharging losses, so that a single variable for charging and discharging suffices. The inclusion of charging and discharging losses would change the dual problem slightly, but as will be argued later on, it would not alter our main message. We disregard charging and discharging limits, since they make the formulations and derivations heavier by introducing additional dual variables, while they do not affect conclusions. Our storage-system model is stylized as we focus on the time-linking aspect in (1b). The representation of the storage system would be more detailed in a practical market-clearing model. For a discussion on the storage model assumptions, we refer to [23], [24]. The final state of energy for $t = H = |\mathcal{T}|$ can be set to a predefined value E^{end} with (1d). Constraints (1c) and (1d) generalize what is found in the literature, where the initial level is often assumed to be equal to zero and the final level is unconstrained, or where $E^{\text{end}} = E^{\text{init}}$.

The best value of E^{end} could be determined in many different ways, e.g., rolling horizon, online learning, reinforcement learning, perfect foresight, etc. It is a topic in itself and we will not address it here. In Section IV, we consider the situation where E^{end} is chosen optimally, i.e., with perfect foresight of future market horizons. Our formulation is general in the way that it includes various approaches proposed in the literature as special cases. Alternatives to imposing the final level with (1d) could be to have a rolling-horizon set-up or to include an estimated value of stored energy in the objective function of the market-clearing [13]. Similar end-of-horizon problems also arise in those settings.

B. Market-Clearing Formulation

In the market-clearing problem with non-merchant storage, the storage does not submit price-quantity bids. However, storage operational constraints (1a)-(1c), and in some cases (1d), are included in the clearing. The storage is used as an asset to move cheap energy between time periods, similar to the way that power lines can be included in the market clearing to move energy between nodes or zones. As we focus on the time-linking effects of storage, network constraints are excluded for simplicity. The addition of network constraints would, however, not alter our main results*.

We consider two versions of the market clearing with non-merchant storage, namely, a free and a constrained market. In the *constrained* market, (1d) is included in the optimization problem, which gives the market operator the option to set the end-of-horizon storage level with future market horizons in mind. In the *free* market on the other hand, this constraint is omitted. The constrained market-clearing problem for the time periods $t \in \mathcal{T}$, denoted by $\mathbf{C}(\mathcal{T})$, is

$$\max_{\mathbf{x}} \quad \sum_{t\in\mathcal{T}} \left(\sum_{l\in\mathcal{L}} U_{lt} d_{lt} - \sum_{g\in\mathcal{G}} C_{gt} p_{gt} \right) \tag{2a}$$

$$\text{s.t.} \quad \sum_{l\in\mathcal{L}} d_{lt} + b_t - \sum_{g\in\mathcal{G}} p_{gt} = 0, \quad \forall t \in \mathcal{T} \quad : (\lambda_t) \tag{2b}$$

$$0 \le p_{gt} \le \overline{P}_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \quad : (\underline{\mu}_{gt}, \overline{\mu}_{gt}) \tag{2c}$$

$$0 \le d_{lt} \le \overline{D}_{lt}, \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \quad : (\underline{\chi}_{lt}, \overline{\chi}_{lt}) \tag{2d}$$

$$(1a) - (1d)\,. \tag{2e}$$

Here, $\mathbf{x}$ is the vector grouping all primal variables, which are indicated with Roman lowercase letters. The generators are indexed by $g \in \mathcal{G}$, and loads by $l \in \mathcal{L}$. The production of g at t is a decision variable p_{gt}, and similarly d_{lt} gives the demand of l at t. The individual bid and offer prices are U_{lt} and C_{gt}. These correspond to the demand utility and the generation cost under the assumption that participants bid truthfully. The objective function (2a) is to maximize the social welfare, calculated as the difference between total demand utility and total generation cost over the given time horizon. Constraint (2b) enforces balance between production and demand at each time t, including the charged or discharged energy. Production and demand limits are enforced in (2c) and (2d), with maximum $\overline{P}_{gt}$ and $\overline{D}_{lt}$, respectively.

The free market-clearing problem for the time periods $t \in \mathcal{T}$, denoted by $\mathbf{F}(\mathcal{T})$, is obtained from the constrained problem by removing (1d).

C. Dual Problem and Pricing

The dual problem $\mathbf{CD}(\mathcal{T})$ of (2) is

$$\min_{\boldsymbol{\zeta}} \quad \sum_{t\in\mathcal{T}} \left(\sum_{g\in\mathcal{G}} \overline{P}_{gt}\overline{\mu}_{gt} + \sum_{l\in\mathcal{L}} \overline{D}_{lt}\overline{\chi}_{lt} + \overline{S}\overline{\nu}_t \right) + E^{\text{init}}\rho_1 - E^{\text{end}}\xi \tag{3a}$$

$$\text{s.t.} \quad C_{gt} - \underline{\mu}_{gt} + \overline{\mu}_{gt} - \lambda_t = 0, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{3b}$$

$$-U_{lt} - \underline{\chi}_{lt} + \overline{\chi}_{lt} + \lambda_t = 0, \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{3c}$$

$$-\rho_t + \lambda_t = 0, \quad \forall t \in \mathcal{T} \tag{3d}$$

$$-\underline{\nu}_t + \overline{\nu}_t + \rho_t - \rho_{t+1} = 0, \quad \forall t \in \mathcal{T} \setminus \{H\} \tag{3e}$$

$$-\underline{\nu}_H + \overline{\nu}_H + \rho_H - \xi = 0 \tag{3f}$$

$$\underline{\mu}_{gt}, \overline{\mu}_{gt}, \underline{\chi}_{lt}, \overline{\chi}_{lt}, \underline{\nu}_t, \overline{\nu}_t \ge 0\,. \tag{3g}$$

The vector $\boldsymbol{\zeta}$ groups all dual variables. We include the dual variables associated with the non-negativity constraints of the primal variables as they are used in our subsequent proofs. We choose the signs of the free variables ρ_1 and ξ in a way that will facilitate our derivations. The dual problem $\mathbf{FD}(\mathcal{T})$ of the free market clearing is obtained from (3) by removing ξ and the terms related to it.

A thorough analysis of the dual problem is available in [8]. One useful interpretation of ρ_t is that it represents the marginal value of having an additional unit of energy stored at the end of hour t. Constraint (3d) implies $\rho_t = \lambda_t$ for all t. Therefore, we use ρ and λ interchangeably in the remainder of this article. The inclusion of charging and discharging losses would change relation (3d) by a factor.

We consider a pricing system where all market participants, including the storage system, buy and sell energy at the hourly market price λ_t. Alternatively, the participants could be paid rents evaluated from other dual variables, as is done in [5], [6], [8]. The two payment systems are equivalent, as proven in [5], [8] for the case that $E^{\text{init}} = E^{\text{end}} = 0$.

Constraints (3e) and (3f) establish a relation between the value of ρ_t in subsequent time periods. These constraints show that the market prices λ_t in two subsequent hours only differ if the storage state of energy is at a bound. This can be seen as intertemporal congestion, similar to the way line congestion in network-aware markets may lead to differences in nodal or zonal prices.

We define the concept of a *time zone* similar to a spatial zone in network-aware markets. A time zone is the longest possible set of consecutive time steps with the same market price. We illustrate the concept of a time zone in Figure 2.

Definition 1 (Time zone). *A set of time steps $\mathcal{Z}$ with $\min_{t\in\mathcal{Z}} t = z_0$ and $\max_{t\in\mathcal{Z}} t = Z$ is a time zone if and only if*

1) *$\mathcal{Z}$ only includes consecutive time steps*
2) *for all $t \in \mathcal{Z}$ it holds that $\lambda_t = c$ for some constant c*

*Storage can also modify the dispatch between buses of the system and have an impact on line congestions, which is studied in [7].

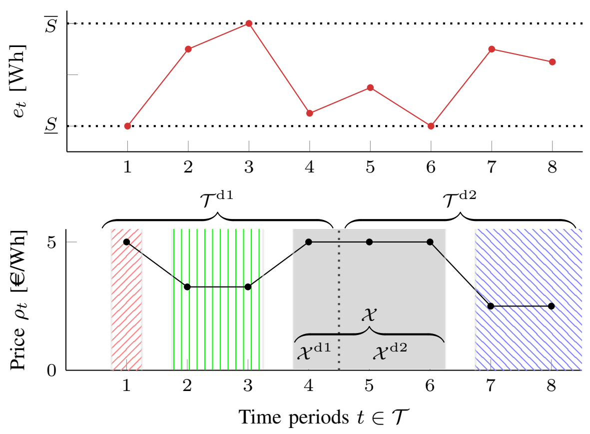


Fig. 2. Bottom: Example of market-clearing price dividing the market horizon into four time zones. Notation for sets of time indices used in later sections is introduced. Top: Storage state-of-energy (e_t) profile related to the below market price signal.

3) $\rho_{z_0-1} \neq c$ and $\rho_{Z+1} \neq c$.

Denote the time zone of time step t by $\mathcal{Z}_t$.

The bottom plot in Figure 2 shows four time zones. Within each time zone, the market price $\lambda_t = \rho_t$ is constant. The upper plot in the figure shows an example state-of-energy profile for the storage that can accompany the price profile below. As ρ decreases from $t = 1$ to $t = 2$, the storage must be at a lower bound at $t = 1$, due to constraint (3e). The same constraint implies that when the price increases, e.g., from $t = 3$ to $t = 4$, the storage must be at an upper bound at $t = 3$.

D. Market Properties

In the next sections, we evaluate the following desirable properties of the market clearing.

Market prices are *dispatch supporting* when no market player desires to unilaterally deviate from the outcomes of the market clearing. Alternatively, one could say that the market provides *dispatch-following incentives*. The market is *efficient* if social welfare is maximized and the market provides dispatch-following incentives. There is *cost recovery* if the profit of every participant is guaranteed to be non-negative. Related works might use other terms to describe similar market properties, e.g., individual rationality. Another critical market property is *revenue adequacy*, which holds when the market operator has no deficit. We do not mention it in the rest of the paper because it is ensured regardless of the assumptions on the final storage level.

In the remainder, we assume that there is perfect competition, in the sense that demands and generators participating in the market bid their true utility and cost. Indeed, we focus on the effects of the storage system on the market properties and disregard the possibility of strategic bidding for the other participants, which would also exist without the storage system.

III. Market Properties under Common Assumptions on the Final Storage Level

In this section, we analyze the influence of common simplifying assumptions on market properties, in particular, cost recovery for the storage system and market efficiency. We evaluate two assumptions. The first is to not have a constraint on the final level. The market clearing will then schedule the storage in order to maximize the social welfare in the current horizon only, while neglecting future social welfare. This means that the storage state of energy will be at its lower bound by the end of each market horizon, unless negative prices occur. The second assumption is to assume that storage state of energy is equal at the start and end of a horizon. Often this level is set to zero, which is not necessarily a good choice, as the storage operator could prefer to start the day with some energy available in case the prices are high. We explore how these assumptions ensure cost recovery over each individual market horizon, but may lead to market inefficiency.

A. Illustrative Example I

In this example, we clear the market for two sequential days of two hours each, identified by the sets $\mathcal{T}^{\mathrm{d1}}$ and $\mathcal{T}^{\mathrm{d2}}$. The storage capacity is $\overline{S} = 2.5\mathrm{Wh}$. We consider a single load and two generators, a cheaper one and a more expensive one. The time-dependent inputs are summarized in Appendix B. The corresponding code is available online [25].

We look at the profit of the storage system with different assumptions on the initial state of energy E^{init} and final state of energy E^{end} on the first day. These results are shown in Table I. In the first case, the initial level is set to 0 and the final level is left free, or equivalently set to zero (if no negative prices occur), which is the most common assumption. In the second case, the initial and the final level on each day are equal, which is also a common assumption. The value is set to $1.25\mathrm{Wh}$ (half of the capacity). In the third case, the preference of the storage, given perfect information about the second day, is taken into account. The storage starts empty, and the final level on the first day is set to $2.5\mathrm{Wh}$. The final level on the second day is left free, under the assumption that there is no subsequent market clearing, which is sufficient to show the potential issues. Indeed, in this last case, the storage earns negative profit on the first day, but this is compensated for by a higher profit on the second day. Note that the storage never obtains a negative daily profit in the other two cases.

The total social welfare obtained is equal to 46€ in the first case and 55.5€ in the last case. So not only the profit of the storage is higher in the last case, but the social welfare is too. The second case's social welfare cannot be compared to the other cases since the storage is initially not empty. This difference in social welfare is due to the fact that in the last case, the storage system is used in the first hour of the second day instead of the most expensive generator. In order to charge the storage, the less expensive generator produces more on the first day.

In the following, we generalize these results and show that if the final state of energy is forced to be equal to the initial state of energy or not fixed at all, the daily profit of the storage

TABLE I
PROFIT (€) OF THE STORAGE SYSTEM FOR VARIOUS INITIAL AND FINAL STATE OF ENERGY (Wh) FOR EXAMPLE I

	t	$E^0 = E^{\text{end}} = 0$	$E^0 = E^{\text{end}} = 1.25$	$E^0 = 0$ and $E^{\text{end}} = 2.5$
$\mathcal{T}^{\text{d1}}$	1	-4	-4	-10
	2	4	4	-2.5
$\mathcal{T}^{\text{d2}}$	3	-11	-11	6
	4	11	11	9
Total $\mathcal{T}^{\text{d1}}$		0	0	-12.5
Total $\mathcal{T}^{\text{d2}}$		0	0	15
Total		0	0	2.5

is never negative. However, these assumptions on the final storage level will most often give a solution that is not optimal for the storage, nor the social welfare.

B. Cost Recovery for the Storage System

In both market-clearing formulations $\mathbf{F}(\mathcal{T})$ and $\mathbf{C}(\mathcal{T})$, cost recovery holds for the generators and loads. For the storage system however, cost recovery is not automatically ensured within a single market interval. For example, if the final level is set to be higher than the initial level and the prices are positive and decreasing during the market interval, the storage system pays for charging at the last hour and does not recover that cost in this market interval. However, we show in this section that under certain conditions, cost recovery for the storage within each market interval is ensured.

1) Cost recovery in $\mathbf{C}(\mathcal{T})$ with equal initial and final state of energy: In order to evaluate cost recovery for $\mathbf{C}(\mathcal{T})$, we express the profit of the storage as a function of the initial and final state of energy. We use the profit-maximization problem of the storage and strong duality to find this relationship. Related to the market-clearing model (2), the profit maximization for the storage is

$$\begin{aligned}
\max_{\mathbf{b},\mathbf{e}} \quad & \sum_{t\in\mathcal{T}} -\lambda_t b_t && \text{(4a)}\\
\text{s.t.} \quad & 0 \le e_t \le \overline{S}, \quad \forall t\in\mathcal{T} && :(\underline{\nu}_t, \overline{\nu}_t) \quad \text{(4b)}\\
& e_t = e_{t-1} + b_t, \quad \forall t\in\mathcal{T}, t\neq 1 && :(\rho_t) \quad \text{(4c)}\\
& e_1 = E^{\text{init}} + b_1 && :(\rho_1) \quad \text{(4d)}\\
& e_H = E^{\text{end}} && :(\xi)\,. \quad \text{(4e)}
\end{aligned}$$

The objective of the storage is to maximize its profit (4a), considering that it is paid the market price λ_t when discharging ($b \le 0$), and it pays the market price λ_t when charging ($b \ge 0$). The market price is taken as an input. The dual problem of (4) is

$$\begin{aligned}
\min_{\boldsymbol{\rho},\boldsymbol{\nu}} \quad & \sum_{t\in\mathcal{T}} \overline{S}\overline{\nu}_t + E^{\text{init}}\rho_1 - E^{\text{end}}\xi && \text{(5a)}\\
\text{s.t.} \quad & \lambda_t - \rho_t = 0, \quad \forall t\in\mathcal{T} && \text{(5b)}\\
& -\underline{\nu}_t + \overline{\nu}_t + \rho_t - \rho_{t+1} = 0, \quad \forall t\in\mathcal{T}, t\neq H && \text{(5c)}\\
& -\underline{\nu}_H + \overline{\nu}_H + \rho_H - \xi = 0 && \text{(5d)}\\
& \underline{\nu}_t, \overline{\nu}_t \ge 0\,. && \text{(5e)}
\end{aligned}$$

At an optimal solution, which is identified with the superscript x^*, strong duality holds, and the values obtained for the primal and dual objective functions are equal. The profit at an optimal solution is thus

$$\sum_{t\in\mathcal{T}} -\lambda_t b_t^* = \sum_{t\in\mathcal{T}} \overline{S}\overline{\nu}_t^* + E^{\text{init}}\rho_1^* - E^{\text{end}}\xi^*\,. \tag{6}$$

Adding (5c) for all t to (5d) for the optimal solution, and multiplying by E^{end}, we get

$$E^{\text{end}}\xi^* = -\sum_{t\in\mathcal{T}} E^{\text{end}}\underline{\nu}_t^* + \sum_{t\in\mathcal{T}} E^{\text{end}}\overline{\nu}_t^* + \rho_1^* E^{\text{end}}\,. \tag{7}$$

Substituting (7) in the right side of (6), we obtain

$$\sum_{t\in\mathcal{T}} -\lambda_t b_t^* = \sum_{t\in\mathcal{T}} (\overline{S} - E^{\text{end}})\overline{\nu}_t^* + \sum_{t\in\mathcal{T}} E^{\text{end}}\underline{\nu}_t^* + \rho_1^*(E^{\text{init}} - E^{\text{end}})\,. \tag{8}$$

This equality shows that if the final level is set equal to the initial level, the profit of the storage system is non-negative, due to the non-negativity of $\overline{\nu}_t$ and $\underline{\nu}_t$, and the fact that $\overline{S} \ge E^{\text{end}}$. However, $E^{\text{init}} = E^{\text{end}}$ is a special case. In general, we do not have guarantees that the profit of the storage system is non-negative, and in fact it is easy to find counter-examples, as was provided in Section III-A.

2) Cost recovery in $\mathbf{F}(\mathcal{T})$: We show here that not having a constraint on the final state of energy is also a special case for which the profit of the storage system is non-negative. For $\mathbf{F}(\mathcal{T})$, constraint (5d) is modified to

$$-\underline{\nu}_H + \overline{\nu}_H + \rho_H = 0\,. \tag{9}$$

Strong duality from (6) simplifies to

$$\sum_{t\in\mathcal{T}} -\lambda_t b_t^* = \sum_{t\in\mathcal{T}} \overline{S}\overline{\nu}_t^* + E^{\text{init}}\rho_1^*\,. \tag{10}$$

Summing all constraints in (5c) and (9) for the optimal solution, we get

$$\sum_{t\in\mathcal{T}} \overline{\nu}_t^* = \sum_{t\in\mathcal{T}} \underline{\nu}_t^* - \rho_1^*\,. \tag{11}$$

Since $\underline{\nu}_t^*$ is non-negative,

$$\sum_{t\in\mathcal{T}} \overline{\nu}_t^* \ge -\rho_1^*\,. \tag{12}$$

Multiplying both sides by E^0, and using that $\overline{S} \ge E^0$, we derive

$$\overline{S}\sum_{t\in\mathcal{T}} \overline{\nu}_t^* \ge E^0 \sum_{t\in\mathcal{T}} \overline{\nu}_t^* \ge -E^{\text{init}}\rho_1^*\,. \tag{13}$$

We conclude

$$\sum_{t\in\mathcal{T}} -\lambda_t b_t^* = \overline{S}\sum_{t\in\mathcal{T}} \overline{\nu}_t^* + E^{\text{init}}\rho_1^* \ge 0\,. \tag{14}$$

In other words, the profit of the storage system is certainly non-negative when the final state of energy is unconstrained.

The absence of cost recovery over a single market interval is not necessarily an issue: the storage may still recover its cost in the subsequent market intervals. In that sense, it would be more relevant to evaluate cost recovery for the storage over an infinitely long time horizon (or equivalent). However, if market intervals are considered completely separate, cost recovery cannot be ensured. Setting the initial and final levels to the same value is a way to overcome this, but it has an impact on other market properties, as we discuss next.

C. Market Efficiency

We now show that enforcing the initial level to be equal to the final level can pose problems in terms of market efficiency. We argue that under this simplification, the storage system could have an incentive to unilaterally deviate from the market-clearing outcomes. Indeed, the storage system profit-maximization problem would actually take into account a longer horizon than just one market interval, to best utilize the potential of temporal arbitrage. The storage profit maximization should be evaluated over a longer horizon $\mathcal{T}^{\text{long}}$, where $H^{\text{long}} > H$, with $H = |\mathcal{T}|$ and $H^{\text{long}} = |\mathcal{T}^{\text{long}}|$:

$$\max_{\mathbf{b},\mathbf{e}} \quad \sum_{t \in \mathcal{T}^{\text{long}}} -\lambda_t b_t \tag{15a}$$

$$\text{s.t.} \quad 0 \leq e_t \leq \overline{S}, \quad \forall t \in \mathcal{T}^{\text{long}} \tag{15b}$$

$$e_t = e_{t-1} + b_t, \quad \forall t \in \mathcal{T}^{\text{long}}, t \neq 1 \tag{15c}$$

$$e_1 = E^{\text{init}} + b_1 \,. \tag{15d}$$

Thus, if the final level at the end of the market interval is not set to a value found in the storage profit-maximization problem, i.e. if $E^{\text{end}} \neq e^*_H$, where e^*_H is an optimal solution for (15), the market output will not be optimal for the storage system and the storage will have an incentive to deviate.

The social welfare is also impacted, as shown in illustrative example I. Clearing a set of market intervals together will always give the best value of the social welfare over the entire set of time periods included, since the social welfare is maximized. When we instead clear these market intervals sequentially without setting the final value of the storage properly for every single clearing, the social welfare may be reduced.

In conclusion, it is a good idea to allow system operators to set the final storage level to a sensible level, based on information about future market intervals. However, doing so brings new challenges. In the following , we analyze this type of market clearing and discuss the duality issues that arise.

IV. Market Properties of the Multi-Interval Market Clearing

While the previous section focused on market properties within a single market interval, this section establishes several results for the multi-interval case. The main aim is to determine the effect of splitting the market clearing in multiple intervals on the optimal primal and dual solutions. Ideally, the splitting should not affect the solutions. We show that even though the primal solution may be unaffected, this is not necessarily the case for the dual solution.

First, we specify the formulations used in the remainder of this work in Section IV-A, including the definition of the *full-horizon* and *split-horizon* problems. In Section IV-B we use an illustrative example to show how solutions to these optimization problems may differ, and why this deserves attention. The following sections formalize and generalize the results of this illustrative example. In Section IV-C, we show that the primal solutions to these two problems are equivalent. However, as will be shown in Section IV-D, market efficiency is not ensured in the split-horizon problem, while it does hold for the full-horizon problem. We provide conditions under which market inefficiencies will occur in the split-horizon problem.

A. Definition of the Full-Horizon and Split-Horizon Problems

We consider a horizon $\mathcal{T}$ that consists of two market intervals, assumed to be days. The time periods of first and second days are collected in the respective sets $\mathcal{T}^{\text{d1}}$ and $\mathcal{T}^{\text{d2}}$, where $H = |\mathcal{T}^{\text{d1}}|$ and $H+1$ is the first hour of $\mathcal{T}^{\text{d2}}$. The end of the entire horizon is $T = |\mathcal{T}|$. We define the *full-horizon* problem as the market clearing for the entire horizon $\mathcal{T}$. The full-horizon optimization model is given by $\mathbf{F}(\mathcal{T})$, as already defined in Section II-B.

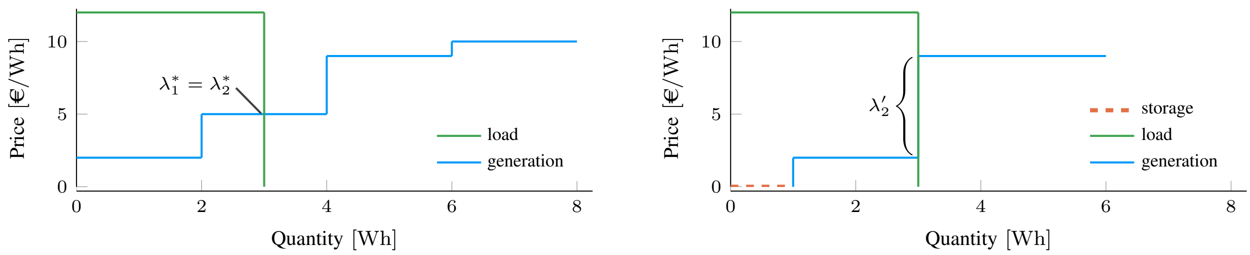
We use the term *split-horizon* problem for the case where we clear the market sequentially for $\mathcal{T}^{\text{d1}}$ and then for $\mathcal{T}^{\text{d2}}$. The split-horizon optimization model is given by $\mathbf{C}(\mathcal{T}^{\text{d1}})$ for the first and $\mathbf{F}(\mathcal{T}^{\text{d2}})$ for the second day. In the split-horizon market with *perfect foresight*, the final storage level on $\mathcal{T}^{\text{d1}}$ is set optimally. That is, E^{end} is equal to the final optimal storage level e^*_H when solving the full-horizon problem. For fair comparison, the storage level at the end of the second day is unconstrained in both the full-horizon and the split-horizon problem. The two optimization problems for the split-horizon problem are independent, besides the fact that one must choose the parameters right, which means that E^{end} on the first day should equal to the initial storage level E^{init} on the second day. As the optimization problems are independent, they can be solved in an equivalent combined optimization without changing their solution. The term *split-horizon problem* refers to this combined optimization. The objective functions of the full-horizon and split-horizon problems are equal.

We assume that we can disregard what happens after time $t = |\mathcal{T}| = T$, i.e. we take the simplifying assumption that there is a finite horizon for which the market with non-merchant storage needs to be cleared. This simplification is justified by the need for a perfect baseline to which we can compare the split-horizon solution. Over a finite horizon, the results from Section III show that the market clearing with non-merchant storage achieves the optimal solution, and cost recovery is ensured for the storage. Therefore, we can consider this full-horizon market clearing as a perfect benchmark.

Optimal values in the full- and split- horizon problem are denoted with $*$ and $'$, respectively. Our proofs use the Karush-Kuhn-Tucker (KKT) conditions of the full- and split-horizon problems, in particular those corresponding to dual constraints from (3). For completeness, we provide the full sets of KKT conditions in Appendix C.

B. Illustrative Example II

Suppose the market is cleared for two sequential hours. The market participants include a single load, two generators, and a non-merchant storage with capacity $\overline{S} = 2.5\text{Wh}$. The time-dependent parameters, demand and production limits, utility, and cost, are summarized in Appendix B, and will also be visible in market-clearing diagrams. The corresponding code is available online [25].



(a) Full-horizon

(b) Split-horizon for $t = 2$

Fig. 3. Market-clearing diagram for illustrative example II

In order to evaluate the effect of splitting the market clearing on primal and dual solutions, we solve both the split-horizon problem with perfect foresight and the full-horizon problem. We thus show that the lack of dispatch support is not due to imperfect information, but an inherent effect of the market splitting. The optimal values for relevant variables are summarized in Table II. The primal variables of the full- and split-horizon problems have equal optimal values and therefore appear in the table only once.

In the full-horizon problem, the two hours are cleared together. As the storage is not at a bound after the first hour, the two hours comprise a single time zone, so that the market-clearing diagram for the two hours can be depicted as in Figure 3a. The unique optimal solution is to charge the storage to $e_1^* = 1\text{Wh}$, and discharge it fully at $t = 2$. The corresponding optimal values for the market price are $\boldsymbol{\lambda}^* = [5, 5]€/\text{Wh}$. The storage improves the optimal value of the objective function by $1\text{W h} \cdot (9-5)€/\text{Wh} = 4€$ compared to the case without storage, as one unit of generation by the more expensive generator with cost of $9€/\text{Wh}$ at $t = 2$ is replaced by the cheaper generator with cost of $5€/\text{Wh}$ at $t = 1$.

The split-horizon problem with perfect foresight clears the two hours separately, while enforcing in the first market interval that $E^{\text{end}} = e_1^* = 1\text{Wh}$. The primal solution is equal to that of the full-horizon market – this result we formalize in Section IV-C. In the first hour, $\lambda'_1 = 5€/\text{Wh}$. In the market for the second hour, none of the loads and generators is marginal. This is depicted in the market-clearing diagram in Figure 3b. Therefore, there is no longer a unique dual solution, as all values λ'_2 in the range $[2, 9]€/\text{Wh}$ are optimal. This interval includes the full-horizon optimum for λ_2, so the full-horizon dual solution is still optimal for the split-horizon dual problem. This is a general result, as we will prove in Section IV-C.

The dual multiplicity that can arise in the split-horizon problem deserves attention for several reasons. If $\lambda'_2 \in [2, 5)€/\text{Wh}$ is selected, the market does not provide dispatch-following incentives to the storage, which can increase its profit by decreasing e_1. In this example, the storage even earns a negative profit, illustrating that cost recovery is not ensured. At the same time, this dispatch of the storage does achieve the optimal social welfare. Furthermore, as shown in this example, there exists a price that supports the dispatch.

One could argue that just selecting the 'correct', dispatch-supporting dual value for $\lambda'_2 = 5€/\text{Wh}$ would solve this problem. However, in order to do so the market operator first of all needs to be aware of the possible existence of multiple dual solutions. Solvers usually return only one solution, which is not necessarily the property-preserving one. Second, if dual multiplicity occurs in the first market interval rather than the second, then the market operator cannot yet know which dual solution will preserve market properties. Finally, selecting a dispatch-supporting dual becomes more complicated in an imperfect-foresight setting, where the chosen level E^{end} may be suboptimal in hindsight.

TABLE II
SELECTED OPTIMAL PRIMAL AND DUAL VALUES IN ILLUSTRATIVE EXAMPLE II

t	e_t(Wh)	d_{1t}(Wh)	p_{1t}(Wh)	p_{2t}(Wh)	λ_t^*(€)	λ'_t(€)
1	1	0	1	0	5	5
2	0	3	2	0	5	[2, 9]

C. Shared Solutions of the Full- and Split-Horizon Problems

In this section, we investigate whether the solutions of the full-horizon are also solutions to the split-horizon problem. Lemma 1 shows that primal solutions to the split-horizon problem with perfect foresight and those to the full-horizon problem are identical.

Lemma 1. *Let e_H^* be part of an optimal solution $\mathbf{x}^*$ to the full-horizon problem. Then $\mathbf{x}^*$ is an optimal primal solution to the full-horizon problem if and only if $\mathbf{x}^*$ is an optimal primal solution to the split-horizon problem with $E^{\text{end}} = e_H^*$.*

Proof. In Appendix D-A. □

Next, Lemma 2 shows that any optimal primal and dual solution to the original problem can be converted to an equivalent solution for the split-horizon problem. This correspondence is sometimes overlooked in the literature. For example, [20] discusses the existence of dual solutions to the split-horizon problem that are not dispatch supporting, but does not mention that the original dual solution remains a solution to the split-horizon problem too.

Lemma 2. *Any optimal primal and dual solution pair $\{\mathbf{x}^*, \boldsymbol{\zeta}^*\}$ to the full-horizon problem is also an optimal solution to the split-horizon problem, with additional split-horizon variable ξ' taking the value $\xi' = \rho_{H+1}^*$.*

Proof. In Appendix D-B. □

D. Weak Solutions to the Split-Horizon Problem

While every optimal dual solution to the full-horizon problem is also an optimal dual solution to the split-horizon problem, the reverse generally does not hold. In this section, we discuss the existence of a weak optimal dual solution admitted by the split-horizon problem, but not by the full-horizon problem. First, we define this type of dual solution, and show why it is a problem. Next, we provide sufficient conditions for the existence of weak dual solutions in the split-horizon problem in our main Theorem 1.

Definition 2 (Weak dual)**.** *A dual solution to the split-horizon problem is **weak** if the resulting price is not dispatch supporting for the non-merchant storage. Such a weak dual exists if and only if one of the following situations occurs.*

1) $e_H \in (\underline{S}, \overline{S})$ *and* $\rho'_H \neq \rho'_{H+1}$
2) $e_H = \underline{S}$ *and* $\rho'_H < \rho'_{H+1}$
3) $e_H = \overline{S}$ *and* $\rho'_H > \rho'_{H+1}$.

For a weak dual solution, the storage operator has incentive to unilaterally deviate from the schedule determined in the market. This incentive exists if the solution to the storage's profit maximization problem (4) given market prices $\{\lambda_t\}$ is different from the schedule determined in the market. We distinguish two cases:

1) If $\rho'_H = \xi' > \rho'_{H+1}$, the storage can improve its profit by decreasing e'_H. This is possible in case $e_H \neq \underline{S}$, i.e. both in situations 1) and 3) of Definition 2.
2) If $\rho'_H = \xi' < \rho'_{H+1}$, the storage can improve its profit by increasing e'_H. This is possible in case $e_H \neq \overline{S}$, i.e. both in situations 1) and 2) of Definition 2.

In extreme cases, a weak dual solution can cause the loss of cost-recovery for the storage operator. For example, this happens if the storage bought all of the energy e_H at the price ρ'_H and is scheduled to sell all of it for a lower ρ'_{H+1}.

Theorem 1. *Assume all cost and utility bids are unique*[*]*. If the optimal solution to the full-horizon problem is such that $H+1$ and T are in different time zones, and*

$$\rho^*_H = \rho^*_{H+1}\,, \tag{16}$$

then the split-horizon problem with perfect foresight admits a weak dual solution.

Proof. We construct a weak dual solution to the split-horizon problem, based on the given dual solution to the full-horizon problem. More precisely, we derive values of $\delta > 0$ for which

$$\rho'_{H+1} = \rho^*_{H+1} + \delta \tag{17}$$

is part of an optimal dual solution $\boldsymbol{\zeta}'$ that satisfies the KKT conditions of the non-myopic split-horizon problem with perfect foresight. We construct δ in such a way that ρ^*_H can remain unchanged, i.e. $\rho'_H = \rho^*_H$. In this proof, we refer to KKT conditions coming from the dual and primal problems, which have been presented in previous sections.

By (16), H and $H+1$ belong to the same time zone $\mathcal{X}$. Denote the time periods in $\mathcal{X}$ that are part of day 1 by the set $\mathcal{X}^{\mathrm{d1}}$, and those that are part of day 2 by $\mathcal{X}^{\mathrm{d2}}$. These sets are illustrated in Figure 2.

Due to uniqueness of cost and utility bids, there can at most be one marginal load or generator in $\mathcal{X}$ for a single time period $t \in \mathcal{X}$, which is part of either $\mathcal{X}^{\mathrm{d1}}$ or $\mathcal{X}^{\mathrm{d2}}$. Therefore, at least one of $\mathcal{X}^{\mathrm{d1}}$ and $\mathcal{X}^{\mathrm{d2}}$ does not have a marginal load or generator. We assume that this is the case for $\mathcal{X}^{\mathrm{d2}}$. The proof is similar for the other case.

By uniqueness of cost and utility bids, ρ^*_H can at most be equal to a single cost or utility bid within $\mathcal{X}$. If there is no marginal load or generator in $\mathcal{X}$, it can happen that ρ^*_H is not equal to any cost or utility bid.

From the KKT conditions of the full-horizon problem corresponding to dual constraints (3b) and (3c), for scheduled loads and generators in $\mathcal{X}^{\mathrm{d2}}$ it holds that

$$\overline{\chi}^*_{lt} = U_{lt} - \rho^*_t \geq 0 \qquad \text{for } \forall t \in \mathcal{X}^{\mathrm{d2}},\, l \in \mathcal{L}^+_t \tag{18}$$
$$\overline{\mu}^*_{gt} = \rho^*_t - C_{gt} \geq 0 \qquad \text{for } \forall t \in \mathcal{X}^{\mathrm{d2}},\, g \in \mathcal{G}^+_t\,. \tag{19}$$

where $\mathcal{L}^+_t = \{l \in \mathcal{L}|\, d^*_{lt} > 0\}$ and $\mathcal{G}^+_t = \{g \in \mathcal{G}|\, p^*_{gt} > 0\}$. The inequalities are actually strict for all but a single scheduled load or generator, except possibly the load or generator that has a cost bid equal to ρ^*_H. Similarly, for unscheduled loads and generators in $\mathcal{X}^{\mathrm{d2}}$,

$$\underline{\chi}^*_{lt} = \rho^*_t - U_{lt} > 0 \qquad \text{for } \forall t \in \mathcal{X}^{\mathrm{d2}},\, l \in \mathcal{L}^0_t \tag{20}$$
$$\underline{\mu}^*_{gt} = C_{gt} - \rho^*_t > 0 \qquad \text{for } \forall t \in \mathcal{X}^{\mathrm{d2}},\, g \in \mathcal{G}^0_t\,. \tag{21}$$

where $\mathcal{L}^0_t = \{l \in \mathcal{L}|\, d^*_{lt} = 0\}$ and $\mathcal{G}^0_t = \{g \in \mathcal{G}|\, p^*_{gt} = 0\}$.

We define the maximum positive change in ρ_t for $t \in \mathcal{X}^{\mathrm{d2}}$ as

$$\overline{\Delta}^+ = \min_{t \in \mathcal{X}^{\mathrm{d2}}} \left\{\overline{\chi}^*_{lt} \,|\, l \in \mathcal{L}^+_t\right\} \cup \left\{\underline{\mu}^*_{gt} \,|\, g \in \mathcal{G}^0_t\right\}. \tag{22}$$

This is defined so that an increase of ρ_{H+1} by $\overline{\Delta}^+$ can be counterbalanced by changing $\overline{\mu}$, $\underline{\mu}$, $\overline{\chi}$, $\underline{\chi}$, so that KKT conditions corresponding to (3b) and (3c) still hold. Similarly, the maximum negative change in ρ_t for $t \in \mathcal{X}^{\mathrm{d2}}$ is

$$\overline{\Delta}^- = \min_{t \in \mathcal{X}^{\mathrm{d2}}} \left\{\underline{\chi}^*_{lt} \,|\, l \in \mathcal{L}^0_t\right\} \cup \left\{\overline{\mu}^*_{gt} \,|\, g \in \mathcal{G}^+_t\right\}. \tag{23}$$

Non-negativity of $\overline{\Delta}^+$ and $\overline{\Delta}^-$ follows from (18)-(21) and (19)-(20), respectively. In fact, at least one of $\overline{\Delta}^+$ and $\overline{\Delta}^-$ is strictly positive

$$\overline{\Delta}^+ > 0 \vee \overline{\Delta}^- > 0\,, \tag{24}$$

since the inequalities in (18) and (19) are strict for all but a single load or generator.

Other than KKT conditions corresponding to (3b) and (3c), the dual variable ρ_t for $t \in \mathcal{X}^{\mathrm{d2}}$ is bound by an end of time-zone relation due to the KKT condition corresponding to dual constraint (3e):

$$\rho^*_X = \rho^*_{X+1} + \underline{\nu}^*_X - \overline{\nu}^*_X\,, \tag{25}$$

[*]More specifically, utility bids are unique $U_{lt} = U_{l't'} \iff l = l' \wedge t = t'$, cost bids are uniqueand $C_{gt} = C_{g't'} \iff g = g' \wedge t = t'$, and $U_{lt} \neq C_{gt'}\ \forall t, t', l, g$.

where X denotes the final time period in $\mathcal{X}$. As $X < T$, it holds that $\underline{\nu}_X^* > 0 \vee \overline{\nu}_X^* > 0$ by the definition of a time zone. From (25) we can derive the following additional constraint on δ:

$$\delta \in [-\infty, \overline{\nu}_X^*] \qquad \text{if } \overline{\nu}_X^* > 0 \tag{26}$$
$$\delta \in [-\underline{\nu}_X^*, \infty] \qquad \text{if } \underline{\nu}_X^* > 0\,. \tag{27}$$

Combining (24), (26), and (27),

$$\delta \in \left[-\overline{\Delta}^-, \min\left\{\overline{\nu}_X^*, \overline{\Delta}^+\right\}\right] \qquad \text{if } \overline{\nu}_X^* > 0 \tag{28}$$
$$\delta \in \left[-\min\left\{\underline{\nu}_X^*, \overline{\Delta}^-\right\}, \overline{\Delta}^+\right] \qquad \text{if } \underline{\nu}_X^* > 0\,. \tag{29}$$

Condition (24) ensures that either the upper or lower bound on δ is nonzero, in both these cases.

Finally, we can construct the weak dual solution accordingly:

$$\rho_t' = \rho_t^* + \delta \qquad \forall t \in \mathcal{X}^{\mathrm{d2}} \tag{30}$$
$$\overline{\chi}_{lt}' = \overline{\chi}_{lt}^* - \delta \qquad \forall t \in \mathcal{X}^{\mathrm{d2}}, l \in \mathcal{L}_t^+ \tag{31}$$
$$\underline{\chi}_{lt}' = \underline{\chi}_{lt}^* + \delta \qquad \forall t \in \mathcal{X}^{\mathrm{d2}}, l \in \mathcal{L}_t^0 \tag{32}$$
$$\overline{\mu}_{gt}' = \overline{\mu}_{gt}^* + \delta \qquad \forall t \in \mathcal{X}^{\mathrm{d2}}, g \in \mathcal{G}_t^+ \tag{33}$$
$$\underline{\mu}_{gt}' = \underline{\mu}_{gt}^* - \delta \qquad \forall t \in \mathcal{X}^{\mathrm{d2}}, g \in \mathcal{G}_t^0 \tag{34}$$
$$\underline{\nu}_X' - \overline{\nu}_X' = \underline{\nu}_X^* - \overline{\nu}_X^* + \delta\,. \tag{35}$$

□

Intuitively, the conditions in Theorem 1 describe the setting in which the full-horizon optimal solution has a time zone which extends over the two different days, but does not cover the entire second day. When this time zone is split over two market-clearing intervals, at least one of the parts will lack a marginal load or generator, and the optimal market-clearing price for the affected time periods can take multiple values. These conditions are *sufficient* but not necessary, i.e. there are other cases in which a weak dual solution to the split-horizon problem exists.

Regarding the assumptions in Theorem 1, the uniqueness of cost and utility bids excludes certain miscellaneous exceptional cases, such as the case that all market participants are either on an upper or lower bound, but there is still no freedom in the related dual variables. We further assume that the time zone $\mathcal{X}$ that spans the two days does not include $t = T$, the final time period of day two. This assumption is used to exclude the following very specific and unlikely case. It could happen that $T \in \mathcal{X}$, while at the same time $e_T^* \in (\underline{S}, \overline{S})$. By KKT condition given by dual constraint (3f), this implies that $\rho_t^* = 0$ for all $t \in \mathcal{X}$. If there is a marginal generator bidding 0 in the part of $\mathcal{X}$ that is on day one, this fixes $\rho_H' = 0$ for the split-horizon problem too. Furthermore, due to dual constraint (3f), it must hold that $\rho_{H+1}' = 0$.

V. Restoring Market Efficiency

In this section we propose a method to ensure equivalence between dual solutions to the split-horizon problem and the full-horizon problem. We explain how our method solves the problem in case of perfect foresight, and discuss its use in an imperfect foresight setting.

A. Proposed method

In the perfect foresight setting, the optimal dual variables to the full-horizon problem, including ρ_{H+1}^*, are assumed known or predicted perfectly when clearing the first-day market. We denote solutions to the restored split-horizon problem using $''$. When clearing the market for the second day, ρ_H'', $\underline{\nu}_H''$, and $\overline{\nu}_H''$ are known.

Our method aims to modify the split-horizon problem in order to restore the dual constraint that is missing in the split-horizon problem:

$$\rho_{H+1} = \rho_H - \underline{\nu}_H + \overline{\nu}_H\,, \tag{36}$$

while it is present in the full-horizon problem.

First, we modify problem $\mathbf{C}(\mathcal{T}^{\mathrm{d1}})$. Instead of fixing the final storage level e_H using a constraint, we use the value ρ_{H+1}^* to steer the optimal dispatch of the storage. This is done by adding $e_H\,\rho_{H+1}^*$ to the objective, where ρ_{H+1}^* is the perfect prediction of this dual variable. As a result, the first-day dual problem includes the dual constraint

$$-\underline{\nu}_H + \overline{\nu}_H + \rho_H - \rho_{H+1}^* = 0 \tag{37}$$

instead of (3f), where it should be noted that ρ_{H+1}^* is a parameter in the day 1 problem.

Next, we modify problem $\mathbf{F}(\mathcal{T}^{\mathrm{d2}})$, using ρ_H'', $\underline{\nu}_H''$, and $\overline{\nu}_H''$ determined by the modified day 1 problem as parameters. The initial storage level e_H is now included in the second day problem as a variable instead of a parameter. We add $e_H\,(\rho_H'' - \underline{\nu}_H'' + \overline{\nu}_H)$ to the objective function. The dual problem for day 2 will then include the following constraint related to variable e_H:

$$-\rho_{H+1} + \rho_H'' - \underline{\nu}_H'' + \overline{\nu}_H'' = 0\,, \tag{38}$$

which equals the missing dual constraint, with dual variables related to day 1 as parameters.

B. Illustrative Example

We apply the proposed method to illustrative example II from Section IV-B. In the perfect foresight case, we assume that the optimal value $\rho_2^* = 5€/\mathrm{Wh}$ is known at the time of market clearing for day 1. Using this value, we obtain $e_H'' = 1\mathrm{W\,h}$, just as was obtained in the full-horizon problem, as well as the dual variable $\rho_H'' = 5€/\mathrm{Wh}$. The second day problem no longer admits multiple dual solutions, because the new dual constraint (38) enforces that $\rho_{H+1}'' = \rho_H'' = 5€/\mathrm{Wh}$. In summary, both λ_H'' and λ_{H+1}'' are equal to their original values in the full-horizon problem. As a result, market efficiency (including dispatch-following incentives) and cost recovery for the storage are again ensured.

C. Imperfect Foresight

In an imperfect foresight setting, the market operator would make an error in estimating ρ_{H+1} on the first day, and the storage level e_H may be set to a suboptimal level compared to the perfect foresight case. If that happens, the proposed method is not guaranteed to retrieve to the optimal full-horizon solution, nor is market efficiency ensured. Furthermore, the values of e_H determined in the first- and second-day problems are likely to differ. As a result, the determined storage dispatch can be infeasible.

VI. Discussion & Conclusion

The inclusion of non-merchant storage in energy market clearing has received attention, among others for its potential to increase social welfare. In this work, we have argued against several simplifying assumptions that are commonly made in the literature regarding final state of energy of the non-merchant storage, in particular, to set the final state of energy equal to the initial state of energy, or to disregard the final level altogether. We have shown that under these assumptions, market efficiency is not ensured, as the market may fail to provide dispatch-following incentives for the storage system. In addition, the storage can only perform time arbitrage within a single market interval, but not between market horizons, resulting in a loss of overall social welfare. However, allowing the final state of energy to take any value can also have negative consequences on the market properties if not handled carefully. First, one must determine the value of the final state of energy that is optimal for the storage system. Second, the market prices may not reflect the relation between different market horizons.

Regarding the latter, we have shown that the split-horizon market may fail to provide dispatch-following incentives for the storage, even when the final state of energy is set perfectly. However, we have shown that any solution to the full-horizon problem is also a solution to the split-horizon problem. This changes the nature of the problem, compared to what is discussed the literature. It shows that there always *exists* a property-preserving dual solution to the split-horizon problem, namely, the dual solution to the full-horizon problem. However, there may in addition exist optimal dual solutions to the split-horizon problem, which are infeasible to the full-horizon dual problem and lead to market inefficiencies. Therefore, the challenge *in the perfect foresight setting* becomes that of selecting the correct dual solution. We have provided sufficient conditions for the existence of weak dual solutions. Finally, we have proposed a method that restores market properties in the split-horizon problem, in the perfect-foresight case.

However, it becomes more complicated in the imperfect-foresight case, where the value of E^{end} may turn out to be suboptimal in hindsight. The proposed solution cannot be applied in case e_H is suboptimal in hindsight. Future work should develop solutions for the imperfect-foresight case. Here, it should be considered that the suboptimal final state of energy leads to a social welfare loss compared to the perfect-foresight case. It is a nontrivial question how this loss should be fairly divided over the market participants, especially since the storage may improve social welfare in expectation.

Furthermore, we note that illustrative examples and proofs in this work are valid for a market clearing with linear cost and utility functions. We have restricted ourselves to this setting for simplicity, and because this type of market clearing is common in practice. However, the market efficiency problem can also arise in the nonlinear convex case, under similar conditions. Future works should analyze this case in further detail.

Finally, we have neglected the problem of determining the value for the final state of energy. In future work, we will focus on how to determine this value, both under perfect and imperfect information.

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Appendix A
Nomenclature and Notations

Nomenclature

Sets and indices

$g \in \mathcal{G}$	Generators
$l \in \mathcal{L}$	Loads
$t \in \mathcal{T}$	Time periods

Parameters

$\overline{D}_{lt}$	Maximum consumption of l at t
$\overline{P}_{gt}$	Maximum output of g at t
$\overline{S}$	Storage capacity
C_{gt}	Cost of g at t
E^{end}	Final state of energy
E^{init}	Initial state of energy
U_{lt}	Utility of l at t

Variables

$\mathbf{x}$	Vector grouping all the primal variables
b_t	Energy charged or discharged for t
d_{lt}	Demand of l at t
e_t	State of energy at the end of t
p_{gt}	Production of g at t

Dual variables

$\boldsymbol{\zeta}$	Vector grouping all the dual variables
λ_t	Dual variable associated with the energy balance at time t. Market price at t.

A. Notation rules

Here we give the rules that are followed for the notations.

- Sets are represented with calligraphic capital letters: $\mathcal{T}$
- Primal variables are represented with lower-case Roman letters: p_{gt}
- Dual variables are represented with Greek letters: ξ
- Indices are given as subscript: e_t
- Characters in bold font indicate vectors grouping all the variables: $\mathbf{x}$
- Parameters are given with capital letters: C_{gt}
- The overline is used for maximum: $\overline{P}$
- The underline is used for minimum: $\underline{P}$
- Superscripts are used to further describe the variables or parameters: E^{end}
- The superscript $*$ is used to describe an optimal value of primal or dual variables: b_t^*
- The superscript $'$ is used to describe an optimal value of primal or dual variables of the split-horizon problem (in Section IV): λ_t'
- The superscript $''$ is used to describe an optimal value of primal or dual variables for the restored split-horizon problem (in Section V): ρ_H''

Appendix B
Time-dependent data for the illustrative examples

TABLE III
TIME-DEPENDENT INPUTS FOR THE ILLUSTRATIVE EXAMPLE OF SECTION III

	t	$\overline{D}_{1t}$(Wh)	$\overline{P}_{1t}$(Wh)	$\overline{P}_{2t}$(Wh)	U_{1t} (€)	C_{1t} (€)	C_{2t} (€)
$\mathcal{T}^{\text{d1}}$	1	0	2	2	12	4	8
	2	1	2	2	12	5	10
$\mathcal{T}^{\text{d2}}$	3	3	2	2	12	2	9
	4	3	2	2	12	6	11

TABLE IV
TIME-DEPENDENT INPUTS FOR THE ILLUSTRATIVE EXAMPLE OF SECTION IV

t	$\overline{D}_{1t}$(Wh)	$\overline{P}_{1t}$(Wh)	$\overline{P}_{2t}$(Wh)	U_{1t}(€)	C_{1t}(€)	C_{2t}(€)
1	0	2	2	12	5	10
2	3	2	2	12	2	9

Appendix C
KKT conditions

A. KKT conditions full-horizon problem

The KKT conditions of $\mathbf{N}(\mathcal{T})$ are

$$C_{gt} - \underline{\mu}_{gt} + \overline{\mu}_{gt} - \lambda_t = 0, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{39a}$$
$$- U_{lt} - \underline{\chi}_{lt} + \overline{\chi}_{lt} + \lambda_t = 0, \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{39b}$$
$$- \rho_t + \lambda_t = 0, \quad \forall t \in \mathcal{T} \tag{39c}$$
$$- \underline{\nu}_t + \overline{\nu}_t + \rho_t - \rho_{t+1} = 0, \quad \forall t \in \mathcal{T} \setminus \{T\} \tag{39d}$$
$$- \underline{\nu}_T + \overline{\nu}_T + \rho_T = 0 \tag{39e}$$
$$0 \leq p_{gt} \perp \underline{\mu}_{gt} \geq 0, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{39f}$$
$$0 \leq \overline{P}_{gt} - p_{gt} \perp \overline{\mu}_{gt} \geq 0, \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{39g}$$
$$0 \leq d_{lt} \perp \underline{\chi}_{lt} \geq 0, \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{39h}$$
$$0 \leq \overline{D}_{lt} - d_{lt} \perp \overline{\chi}_{lt} \geq 0, \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{39i}$$
$$0 \leq e_t \perp \underline{\nu}_t \geq 0, \quad \forall t \in \mathcal{T} \tag{39j}$$
$$0 \leq \overline{S} - e_t \perp \overline{\nu}_t \geq 0, \quad \forall t \in \mathcal{T} \tag{39k}$$
$$(2b), (1b), (1c) \tag{39l}$$

B. KKT conditions split-horizon problem

The KKT conditions of $\mathbf{L}(\mathcal{T}^{\text{d1}})$ are

$$C_{gt} - \underline{\mu}_{gt} + \overline{\mu}_{gt} - \lambda_t = 0, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}^{\text{d1}} \tag{40a}$$
$$- U_{lt} - \underline{\chi}_{lt} + \overline{\chi}_{lt} + \lambda_t = 0, \quad \forall l \in \mathcal{L}, t \in \mathcal{T}^{\text{d1}} \tag{40b}$$
$$- \rho_t + \lambda_t = 0, \quad \forall t \in \mathcal{T}^{\text{d1}} \tag{40c}$$
$$- \underline{\nu}_t + \overline{\nu}_t + \rho_t - \rho_{t+1} = 0, \quad \forall t \in \mathcal{T}^{\text{d1}} \setminus \{H\} \tag{40d}$$
$$- \underline{\nu}_H + \overline{\nu}_H + \rho_H - \xi = 0 \tag{40e}$$
$$0 \leq p_{gt} \perp \underline{\mu}_{gt} \geq 0, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}^{\text{d1}} \tag{40f}$$
$$0 \leq \overline{P}_{gt} - p_{gt} \perp \overline{\mu}_{gt} \geq 0, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}^{\text{d1}} \tag{40g}$$
$$0 \leq d_{lt} \perp \underline{\chi}_{lt} \geq 0, \quad \forall l \in \mathcal{L}, t \in \mathcal{T}^{\text{d1}} \tag{40h}$$
$$0 \leq \overline{D}_{lt} - d_{lt} \perp \overline{\chi}_{lt} \geq 0, \quad \forall l \in \mathcal{L}, t \in \mathcal{T}^{\text{d1}} \tag{40i}$$
$$0 \leq e_t \perp \underline{\nu}_t \geq 0, \quad \forall t \in \mathcal{T}^{\text{d1}} \tag{40j}$$

$$0 \le \overline{S} - e_t \perp \overline{\nu}_t \ge 0, \qquad \forall t \in \mathcal{T}^{\mathrm{d1}} \tag{40k}$$
$$e_H = E^{\mathrm{end}} \tag{40l}$$
$$e_1 = E^{\mathrm{init}} + b_1 \tag{40m}$$
$$e_t = e_{t-1} + b_t, \qquad \forall t \in \mathcal{T}^{\mathrm{d1}} \setminus \{1\} \tag{40n}$$
$$\sum_{l\in\mathcal{L}} d_{lt} + b_t - \sum_{g\in\mathcal{G}} p_{gt} = 0, \qquad \forall t \in \mathcal{T}^{\mathrm{d1}} \tag{40o}$$

The KKT conditions of $\mathbf{N}(\mathcal{T}^{\mathrm{d2}})$ are

$$C_{gt} - \underline{\mu}_{gt} + \overline{\mu}_{gt} - \lambda_t = 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}^{\mathrm{d2}} \tag{41a}$$
$$-U_{lt} - \underline{\chi}_{lt} + \overline{\chi}_{lt} + \lambda_t = 0, \qquad \forall l \in \mathcal{L}, t \in \mathcal{T}^{\mathrm{d2}} \tag{41b}$$
$$-\rho_t + \lambda_t = 0, \qquad \forall t \in \mathcal{T}^{\mathrm{d2}} \tag{41c}$$
$$-\underline{\nu}_t + \overline{\nu}_t + \rho_t - \rho_{t+1} = 0, \qquad \forall t \in \mathcal{T}^{\mathrm{d2}} \setminus \{T\} \tag{41d}$$
$$-\underline{\nu}_T + \overline{\nu}_T + \rho_T = 0 \tag{41e}$$
$$0 \le p_{gt} \perp \underline{\mu}_{gt} \ge 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}^{\mathrm{d2}} \tag{41f}$$
$$0 \le \overline{P}_{gt} - p_{gt} \perp \overline{\mu}_{gt} \ge 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}^{\mathrm{d2}} \tag{41g}$$
$$0 \le d_{lt} \perp \underline{\chi}_{lt} \ge 0, \qquad \forall l \in \mathcal{L}, t \in \mathcal{T}^{\mathrm{d2}} \tag{41h}$$
$$0 \le \overline{D}_{lt} - d_{lt} \perp \overline{\chi}_{lt} \ge 0, \qquad \forall l \in \mathcal{L}, t \in \mathcal{T}^{\mathrm{d2}} \tag{41i}$$
$$0 \le e_t \perp \underline{\nu}_t \ge 0, \qquad \forall t \in \mathcal{T}^{\mathrm{d2}} \tag{41j}$$
$$0 \le \overline{S} - e_t \perp \overline{\nu}_t \ge 0, \qquad \forall t \in \mathcal{T}^{\mathrm{d2}} \tag{41k}$$
$$e_{H+1} = E^{\mathrm{init}} + b_{H+1} \tag{41l}$$
$$e_t = e_{t-1} + b_t, \qquad \forall t \in \mathcal{T}^{\mathrm{d2}} \setminus \{H+1\} \tag{41m}$$
$$\sum_{l\in\mathcal{L}} d_{lt} + b_t - \sum_{g\in\mathcal{G}} p_{gt} = 0, \qquad \forall t \in \mathcal{T}^{\mathrm{d2}} \tag{41n}$$

Appendix D
Proofs

A. Proof of Lemma 1

Proof. The full-horizon and split-horizon problems have the same objective function. The optimal solution to the full-horizon problem $\mathbf{x}^*$ lies in the feasible space $\mathcal{F}'$ of the split-horizon problem, i.e., $\mathbf{x}^* \in \mathcal{F}'$, because it satisfies the additional constraint $e_H = E^{\mathrm{end}} = e^*_H$. Therefore, the optimal objective of the full-horizon problem is a lower bound for the split-horizon problem. The feasible space $\mathcal{F}'$ of the split-horizon problem with the added constraint is a subspace of the feasible space $\mathcal{F}$ of the full-horizon problem, i.e., $\mathcal{F}' \subset \mathcal{F}$. Thus, the optimal objective of the split-horizon problem is a lower bound for the full-horizon problem. This implies that the split- and full-horizon problems attain the same maximum.

As any optimum $\mathbf{x}' \in \mathcal{F}$ and any optimum $\mathbf{x}^* \in \mathcal{F}'$, the sets of optimal primal solutions must be equal.

□

B. Proof of Lemma 2

Proof. As the problem is convex, the KKT conditions are necessary and sufficient. Therefore, any feasible point in the KKT conditions of the split problem is an optimal solution to it. The KKT conditions of the split-horizon problem only differ from the KKTs of the full-horizon problem in the equations

$$-\underline{\nu}_H + \overline{\nu}_H + \rho_H - \xi = 0 \tag{42}$$
$$e_H = E^{\mathrm{end}}\,, \tag{43}$$

which replace the full-horizon KKT condition

$$-\underline{\nu}_H + \overline{\nu}_H + \rho_H - \rho_{H+1} = 0\,. \tag{44}$$

As we in the perfect foresight case set $E^{\mathrm{end}} = e^*_H$, KKT condition (43) holds for any optimal primal solution to the full-horizon problem. As for condition (42), we can see that it is satisfied by the optimal solution to the full-horizon problem in case $\xi' = \rho^*_{H+1}$.

Therefore, any optimal solution to the full-horizon problem, augmented with $\xi' = \rho^*_{H+1}$, also satisfies the KKT conditions for the split problem. As the KKT conditions are sufficient for convex problems, this means that this solution is also an optimal solution to the split-horizon problem. □

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