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The ΔV-Method: An Intuitive Method for Analyzing Soft-Charging Capabilities of Hybrid Switched-Capacitor DC-DC Converters

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***Abstract*—This paper presents an inspection method for analyzing the soft-charging capabilities of switched-capacitor converters (SCC). The method utilizes a multi-step visual approach to derive the voltage changes (ΔV) across each flying capacitor, leading to an intuitive way of understanding the behavior of hybrid-SCC topologies on circuit level. The method is presented by examples and is used to obtain capacitance ratios and split-phase timings for Hybrid Dickson topologies.**

***Index Terms*—hybrid switched-capacitor converter, soft-charging, energy efficient**

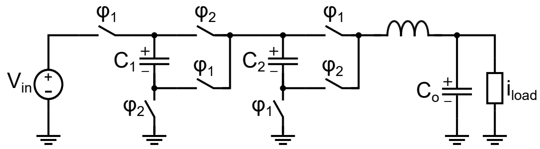
I. Introduction

The increased requirements for large conversion ratio, high efficient power converters in applications such as data centers, automotive and USB-C power delivery DC-DC systems is leading to the popularity of hybrid switched-capacitor power converters [1]–[7]. They utilize the high voltage step-down ability from switched-capacitor converters (SCC) together with one or more inductors to achieve voltage regulation and sometimes soft-charging of the flying capacitors. The soft-charging capabilities of SCC topologies with a current source in series with the output have been analyzed in [8]. This current source is usually an inductor forming a hybrid DC-DC converter. Nevertheless, hybrid converters are most efficient if all capacitors experience soft-charging. Soft-charging takes place when all capacitors are being charged/discharged without the occurrence of current spikes. This removes part of the equivalent output resistance called the slow switching limit impedance [9]. Most often soft-charging is achieved by constraining the capacitor currents with an inductor. In [8] the soft-charging capability of a topology is determined by deriving the reduced loop matrix in all phases and combining it with the constraint that the voltage changes of each single capacitor have to sum up to zero across all switching phases. Solving these systems of linear equations for the two constraints then results in the required normalized capacitor values to achieve soft-charging for a given topology. However, the presented method in [8] requires linear algebra which lacks the intuition that visual approaches like charge flow analysis provide. Therefore it is difficult to learn and does not give the designer an intuitive understanding on the limiting factors for a given SCC topology. To provide more insight, this work presents a method to determine soft-charging capabilities of SCC topologies by inspection. The method can also be used for multi-phase systems and for multiple inductors placed locally in a topology. Thereby it attributes to better understanding of the limitations and criteria for achieving soft-charging, which can lead to hybrid SCCs with an improved efficiency and power density.

II. Determination of Soft-Charging Capability by Inspection

The proposed method uses two constraints for evaluating the soft-charging capability of a topology. (1) the capacitor voltage change in all phases due to charge being delivered to the load must sum to zero for each individual flying capacitor ($\Delta V_{c,i}^1 + \Delta V_{c,i}^2 + \ldots + \Delta V_{c,i}^j = 0$). (2) Kirchhoff's voltage law (KVL) must hold for the voltage changes in each loop of the topology in all phases. Utilizing both constraints ensures that there is no voltage mismatch between the capacitors during phase transitions and therefore no charge redistribution losses for the capacitors. The method uses these constraints in a multi-step approach to find the normalized voltage changes across each capacitor for all phases. This is similar in structure to how the charge flow analysis by [9] is performed. Together with the charge flow analysis the capacitance scalings required for soft-charging can then be determined. Since the method analyzes the voltage changes and ΔV is used to symbolize these changes the method is referred to as voltage change method (ΔV-method).

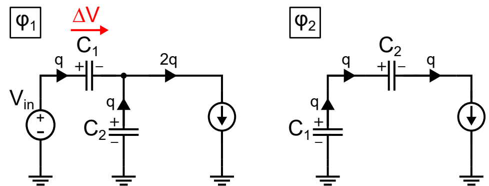
To expand on the method consider the 3:1 Fibonacci SCC in Fig. 1a. Here the charge flow analysis for the SCC was already performed and the result can be seen in Fig. 1b. Large capacitors like the input and output capacitor are approximated as voltage sources and large inductors are approximated as current sources. Similar to the charge flow analysis a good starting point should be chosen for the ΔV-method. In this example, since C_1 is connected to a source it is chosen as a starting point. Using that the input voltage source does not have any voltage change when delivering charge we can find



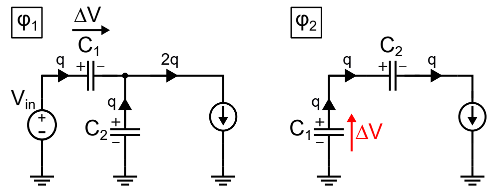
(a) 3:1 Fibonacci SCC with an inductor in series with the output.

$$\begin{bmatrix} q_{c,1}^1 \\ q_{c,2}^1 \end{bmatrix} = \begin{bmatrix} +1 \\ -1 \end{bmatrix}, \begin{bmatrix} q_{c,1}^2 \\ q_{c,2}^2 \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

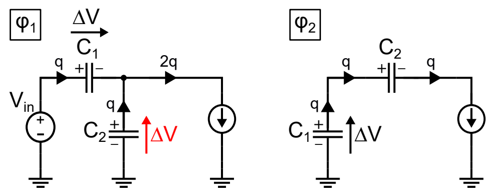
(b) 3:1 Fibonacci SCC capacitor charge flows for each phase.



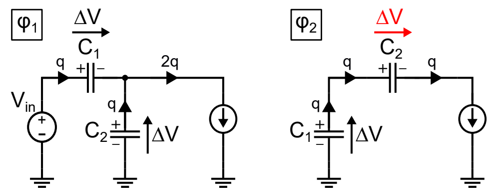
(c) ΔV-method: step 1 phase networks.



(d) ΔV-method: step 2 phase networks.



(e) ΔV-method: step 3 phase networks.



(f) ΔV-method: step 4 phase networks.

Fig. 1: Step-by-step example of using the ΔV-method to determine soft-charging capabilities for a 3:1 Fibonacci SCC with an inductor at the output.

all the capacitor voltage changes by iteration in Fig. 1 as follows:

- Phase 1: Charge flowing into the positive terminal of C_1 yields: $\Delta V_{c,1}^1 = +1\Delta\text{V}$ indicated across C_1 in Fig. 1c.
- Phase 2: Because of constraint (1): $\Delta V_{c,1}^2 = -1\Delta\text{V}$, i.e., the polarity of the voltage across C_1 changes (but not its absolute value).
- Phase 1: KVL yields: $\Delta V_{c,2}^1 = \Delta V_{in}^1 - \Delta V_{c,1}^1 = -1\Delta\text{V}$.
- Phase 2: Because of constraint (1) the polarity of $\Delta V_{c,2}$ changes with respect to phase 1 leading to: $\Delta V_{c,2}^2 = +1\Delta\text{V}$.

All capacitor voltage changes have now been found and can be summarized as:

$$\begin{bmatrix} \Delta V_{c,1}^1 \\ \Delta V_{c,2}^1 \end{bmatrix} = \begin{bmatrix} +1 \\ -1 \end{bmatrix}, \begin{bmatrix} \Delta V_{c,1}^2 \\ \Delta V_{c,2}^2 \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \end{bmatrix} \tag{1}$$

Using that $C_i = q_{c,i}/\Delta V_{c,i}$ we also directly find the capacitances using the charge flows from Fig. 1b:

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{2}$$

The 3:1 Fibonacci topology is therefore capable of achieving soft-charging with an inductor in series with the output if both flying capacitors are the same size. This fits with the results of the previously presented method in [8]. Note that the actual value of ΔV is not required to determine the normalized capacitances and, in turn, to assess if the topology is soft-charging capable.

More complex topologies than the 3:1 Fibonacci converter can be assessed using the same inspection method. In Fig. 2a the 4:1 Dickson topology with an inductor in series with the output can be seen. The solution after using the ΔV-method is shown in Fig. 2b with the voltage changes for each phase summarized in Fig. 2c together with the result of the charge flow analysis.

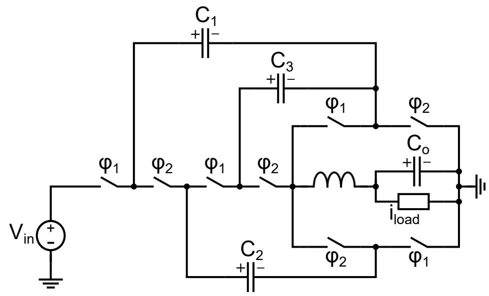
The method is performed in multi-steps as follows:

- Phase 1: Charge flowing into positive terminal of C_1 yields: $\Delta V_{c,1}^1 = +1\Delta\text{V}$.
- Phase 2: $\Delta V_{c,i}^1 + \Delta V_{c,i}^2 = 0$ yields: $\Delta V_{c,1}^2 = -1\Delta\text{V}$.
- KVL:
  - Phase 1: $\Delta V_{c,1}^1 = \Delta V_{c,3}^1 - \Delta V_{c,2}^1$.
  - Phase 2: $\Delta V_{c,1}^2 = \Delta V_{c,3}^2 + \Delta V_{c,2}^2$.
  - Using this with $\Delta V_{Cj}^1 + \Delta V_{Cj}^2 = 0$ yields: $\Delta V_{c,2} = 0$.
- Phase 2: Utilizing that $\Delta V_{c,2}^2 = +0$ yields for $\Delta V_{c,3}^2$: $\Delta V_{c,3}^2 = \Delta V_{c,1}^2 = -1\Delta\text{V}$.
- Phase 1: $\Delta V_{Cj}^1 + \Delta V_{Cj}^2 = 0$ yields $\Delta V_{c,3}^1$: $\Delta V_{c,3}^1 = +1\Delta\text{V}$.

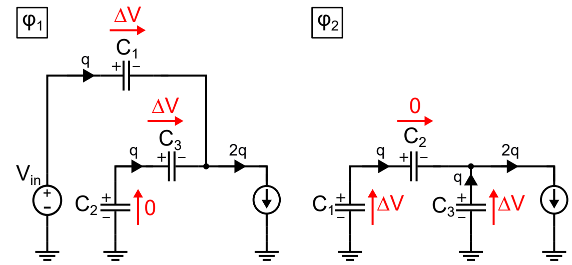
The capacitances can again be found based on the ΔV-method and the results of the charge flow analysis:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \infty \\ 1 \end{bmatrix} \tag{3}$$

For the 4:1 Dickson topology an infinitely large capacitor would be required for C_2. Therefore, the topology can not realize soft-charging. Signed zero is used for $\Delta V_{c,2}$ to clarify,



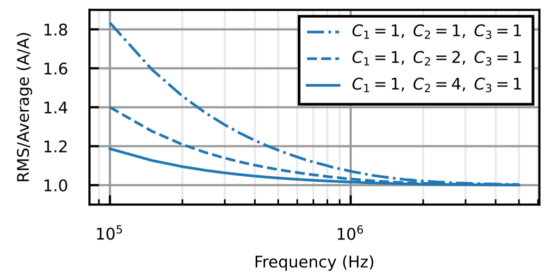
(a) 4:1 Dickson SCC with an inductor in series with the output.



(b) 4:1 Dickson SCC phase networks with notated voltage changes.

$$\begin{bmatrix} q^1_{c,1}\\ q^1_{c,2}\\ q^1_{c,3}\end{bmatrix} = \begin{bmatrix}+1\\-1\\+1\end{bmatrix}, \quad \begin{bmatrix} q^2_{c,1}\\ q^2_{c,2}\\ q^2_{c,3}\end{bmatrix} = \begin{bmatrix}-1\\+1\\-1\end{bmatrix}$$
$$\begin{bmatrix} \Delta V^1_{c,1}\\ \Delta V^1_{c,2}\\ \Delta V^1_{c,3}\end{bmatrix} = \begin{bmatrix}+1\\-0\\+1\end{bmatrix}, \quad \begin{bmatrix} \Delta V^2_{c,1}\\ \Delta V^2_{c,2}\\ \Delta V^2_{c,3}\end{bmatrix} = \begin{bmatrix}-1\\+0\\-1\end{bmatrix}$$

(c) 4:1 Dickson SCC capacitor charge flows and voltage changes for each phase.



(d) Simulated 4:1 Dickson SCC current ratio between RMS current and average of the absolute current of C_1 for three different designs with same total capacitance.

Fig. 2: Using the ΔV-method to determine soft-charging capabilities for a 4:1 Dickson SCC with an inductor at the output.

that the required C_2 capacitance to ensure soft-charging is positive. This is not always the case for other topologies such as the Ladder and Cockcroft-Walton topologies, where a negative voltage change is required for some capacitors, when the capacitors are charged resulting in a negative capacitance requirement to achieve soft-charging. In Fig. 2d the simulated ratio between the RMS current and the average of the absolute current of C_1 is shown swept over frequency for three different choices of C_2 with the total capacitance kept the same and assuming no inductor current ripple for simplicity. Figure 2d shows that as we increase C_2, the capacitors approaches soft-charging. While the capacitors still experience a voltage mismatch, it is lower than for the hard-charging case, which leads to a lower output resistance at the slow-switching limit. Therefore, in practice the total losses due to the charge redistribution are decreased compared to the hard-charging case for the same total capacitance sizes and switching frequency.

The above examples demonstrate that the ΔV-method is a powerful analysis tool that obtains results after a few simple steps. In the case of the 4:1 Dickson topology it was possible to find a KVL constraint for solving the voltage mismatch. This is also the case for Dickson topologies with higher conversion ratios and for any topology encountered by the authors such as the Series-Parallel, Ladder, Doubler and Cockcroft-Walton topologies. The solution obtained by the ΔV-method can always be verified afterwards by checking both KVL and that the voltage change for each capacitor sums to zero for a full switching period. The inspection method only requiring simple KVL equations to analyze the soft-charging capabilities makes it easy to solve the soft-charging requirements even for complicated hybrid converter topologies.

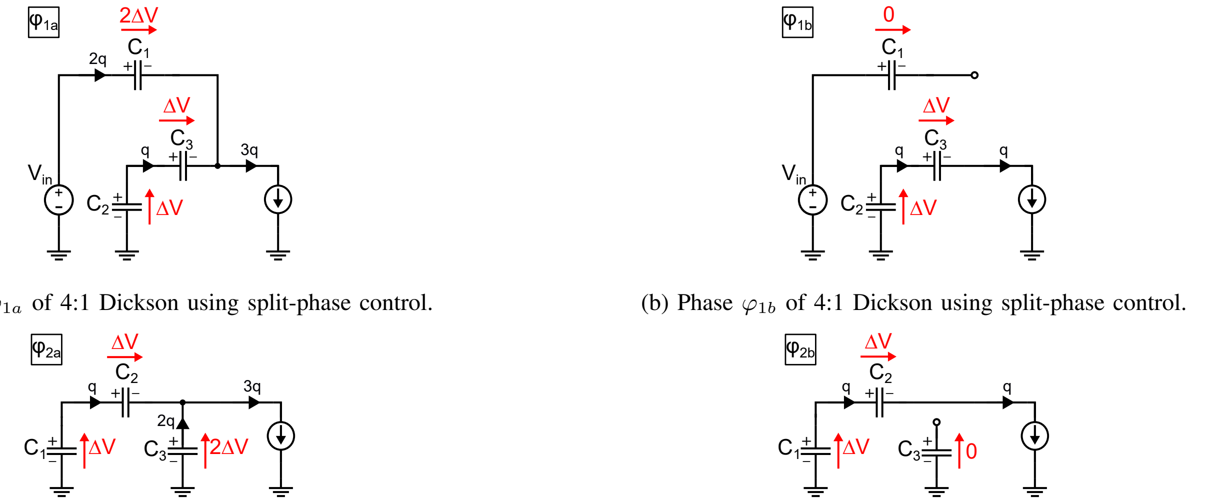
III. Applying the ΔV-Method

To verify the applicability of the ΔV-method in the analysis of hybrid converters, the inspection method is used to analyze the split-phase control method presented in [10] and the soft-charging capabilities of the Hybrid Dickson topologies for higher conversion ratios.

A. Analyzing Split-Phase Control of 4:1 Dickson Using The ΔV-Method

In [10] it is shown that the 4:1 Dickson topology can be made soft-charging by splitting both phases φ_1 and φ_2 in two phases each, as shown in Fig. 3. The goal is to find the duty cycles of each phase that permit soft-charging. Applying charge flow vector analysis we find that the system of equations for the 4 phase networks in Fig. 3 is underdetermined. This is because the lengths of the split-phases constitute two degrees of freedom. Each combination of capacitor sizes leads to different timings that achieve soft-charging. For this example we choose $C_1 = C_2 = C_3 = C$ and find the suitable split-phase duty cycles as follows. Denoting $q^{1a}_{c,1} = 2q$ the charge flow analysis yields equation (4).

$$\begin{aligned} 2q + q^{2a}_{c,1} + q^{2b}_{c,1} &= 0 \\ 2q + q^{1a}_{c,2} + q^{1b}_{c,2} &= 0 \end{aligned} \tag{4}$$



(a) Phase φ_{1a} of 4:1 Dickson using split-phase control.

(b) Phase φ_{1b} of 4:1 Dickson using split-phase control.

(c) Phase φ_{2a} of 4:1 Dickson using split-phase control.

(d) Phase φ_{2b} of 4:1 Dickson using split-phase control.

Fig. 3: Phase networks showing charge flow and voltage changes for soft-charging operation using split-phase and assuming equal capacitor sizing.

The two missing constraints in (4) are found with the ΔV-method by using KVL on phases φ_{1a} and φ_{2a}, as shown in equation (5). Phase φ_{1b} and φ_{2b} do not offer more KVL constraints.

$$\begin{aligned} 2\Delta V &= \Delta V_{c,3}^{1a} + \Delta V_{c,2}^{1a} \\ \Delta V_{c,3}^{2a} &= \Delta V_{c,1}^{2a} + \Delta V_{c,2}^{2a} \end{aligned} \tag{5}$$

Using that $\Delta V = q/C$, (5) can also be formulated in terms of the charge vectors (6).

$$\begin{aligned} q_{c,1}^{1a}/C_1 &= q_{c,3}^{1a}/C_3 + q_{c,2}^{1a}/C_2 \\ q_{c,3}^{2a}/C_3 &= q_{c,1}^{2a}/C_1 + q_{c,2}^{2a}/C_2 \end{aligned} \tag{6}$$

Using that $q_{c,1}^{1a} = 2q$ and $C_1 = C_2 = C_3$ together with (4) and (6) we find that $q_{c,1}^{2a} = q_{c,1}^{2b} = q_{c,2}^{1a} = q_{c,2}^{1b} = -q$. The switching phase networks with their voltage changes and charge flows are depicted in Fig. 3. The voltage change for all capacitors in all phases is:

$$\begin{aligned} &\begin{bmatrix} \Delta V_{c,1}^{1a} \\ \Delta V_{c,2}^{1a} \\ \Delta V_{c,3}^{1a} \end{bmatrix} = \begin{bmatrix} +2 \\ -1 \\ +1 \end{bmatrix}, \begin{bmatrix} \Delta V_{c,1}^{1b} \\ \Delta V_{c,2}^{1b} \\ \Delta V_{c,3}^{1b} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ +1 \end{bmatrix}, \\ &\begin{bmatrix} \Delta V_{c,1}^{2a} \\ \Delta V_{c,2}^{2a} \\ \Delta V_{c,3}^{2a} \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \\ -2 \end{bmatrix}, \begin{bmatrix} \Delta V_{c,1}^{2b} \\ \Delta V_{c,2}^{2b} \\ \Delta V_{c,3}^{2b} \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \\ 0 \end{bmatrix} \end{aligned} \tag{7}$$

Additionally, the charge flow for the input voltage, the flying capacitors and the output in matrix form is:

$$\begin{aligned} &\begin{bmatrix} q_{in}^{1a} \\ q_{c,1}^{1a} \\ q_{c,2}^{1a} \\ q_{c,3}^{1a} \\ q_{out}^{1a} \end{bmatrix} = \begin{bmatrix} +2 \\ +2 \\ -1 \\ +1 \\ +3 \end{bmatrix}, \begin{bmatrix} q_{in}^{1b} \\ q_{c,1}^{1b} \\ q_{c,2}^{1b} \\ q_{c,3}^{1b} \\ q_{out}^{1b} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ +1 \\ +1 \end{bmatrix}, \\ &\begin{bmatrix} q_{in}^{2a} \\ q_{c,1}^{2a} \\ q_{c,2}^{2a} \\ q_{c,3}^{2a} \\ q_{out}^{2a} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ +1 \\ -2 \\ +3 \end{bmatrix}, \begin{bmatrix} q_{in}^{2b} \\ q_{c,1}^{2b} \\ q_{c,2}^{2b} \\ q_{c,3}^{2b} \\ q_{out}^{2b} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ +1 \\ 0 \\ +1 \end{bmatrix} \end{aligned} \tag{8}$$

From (7) and (8) it can be seen that it is possible to achieve soft-charging by using split-phase control and having all capacitors the same size. Finally the required duty cycle of any switching phase to ensure soft-charging can be calculated by considering the output charge (q_{out}) compared to the output charge summed across all phases:

$$\begin{aligned} D_{1a} &= t_{1a}/T_{sw} = q_{out}^{1a}/q_{out,tot} = 3/8 \\ D_{1b} &= t_{1b}/T_{sw} = q_{out}^{1b}/q_{out,tot} = 1/8 \\ D_{2a} &= t_{2a}/T_{sw} = q_{out}^{2a}/q_{out,tot} = 3/8 \\ D_{2b} &= t_{2b}/T_{sw} = q_{out}^{2b}/q_{out,tot} = 1/8 \end{aligned} \tag{9}$$

It should be noted that this calculation of duty cycle assumes a constant output current and no inductor current ripple. Inferring the duty cycle from the charge flow will become more complex for large current ripple operating modes [11], [12].

$$\mathbf{\Delta V^1} = \begin{bmatrix} +1 \\ -0.5 \\ +0.5 \\ -1 \end{bmatrix}, \ \mathbf{\Delta V^2} = \begin{bmatrix} -1 \\ +0.5 \\ -0.5 \\ +1 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

(a) Voltage changes for each phase and required capacitor values for 5:1 Dickson SCC.

$$\mathbf{\Delta V^1} = \begin{bmatrix} +1 \\ -0 \\ +1 \\ -0 \\ +1 \end{bmatrix}, \ \mathbf{\Delta V^2} = \begin{bmatrix} -1 \\ +0 \\ -1 \\ +0 \\ -1 \end{bmatrix},$$

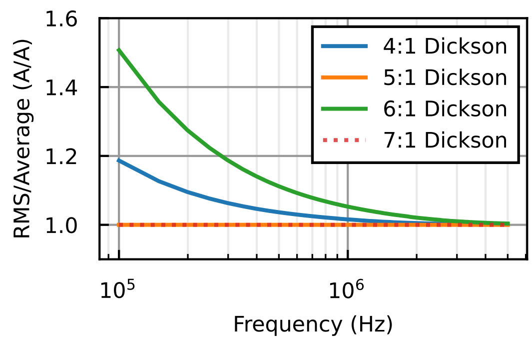
$$\mathbf{C} = \begin{bmatrix} 1 \\ \infty \\ 1 \\ \infty \\ 1 \end{bmatrix}$$

(b) Voltage changes for each phase and required capacitor values for 6:1 Dickson SCC.

$$\mathbf{\Delta V^1} = \begin{bmatrix} +1 \\ -1/3 \\ +2/3 \\ -2/3 \\ +1/3 \\ -1 \end{bmatrix}, \ \mathbf{\Delta V^2} = \begin{bmatrix} -1 \\ +1/3 \\ -2/3 \\ +2/3 \\ -1/3 \\ +1 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 \\ 3 \\ 3/2 \\ 3/2 \\ 3 \\ 1 \end{bmatrix}$$

(c) Voltage changes for each phase and required capacitor values for 7:1 Dickson SCC.



(d) Simulated Dickson SCC current ratio between RMS and average of the absolute current of C_1 for 4 different conversion ratios.

Fig. 4: The voltage changes and capacitor scalings required for soft-charging for the 5:1, 6:1 and 7:1 Dickson topology using only two phases together with the simulated current ratio between RMS and average of the absolute currents for C_1 for all conversion ratios.

B. Soft-Charging Capabilities of Dickson Topology for Different Conversion Ratios

While the split-phase control can ensure soft-charging of the flying capacitors for the 4:1 Dickson topology we can also use the ΔV-method to analyze the soft charging capabilities of the Dickson topology at other conversion ratios. The analysis approach is the same as presented in Section II and has been done for the 5:1, 6:1 and 7:1 Dickson SCC. The resulting voltage changes and required capacitor sizes are shown in Fig. 4. The simulated current ratio between RMS and average of the absolute current in C_1 for the different conversion ratios can be seen in Fig. 4d. For the simulation the total capacitance has been kept constant between the ratios. The plot reveals that all odd conversion ratios can achieve soft-charging without any additional split-phase control and using only a single inductor at the output. Therefore, the designer should consider if an odd conversion ratio Hybrid Dickson could be used instead in the desired application. The 5:1 and 7:1 Dickson converters do require that the two phases have different duty cycles to achieve soft-charging. The duty cycles can again be obtained by observing the output charge in each phase compared to the summed output charge for both phases. For the 5:1 Dickson this means that the duty cycle required for soft-charging is:

$$\begin{aligned} D_1 &= t_1/T = q^1_{out}/q_{out,tot} = 3/5 \\ D_2 &= t_2/T = q^2_{out}/q_{out,tot} = 2/5 \end{aligned} \tag{10}$$

For the 7:1 Dickson the duty cycle required for soft-charging is:

$$\begin{aligned} D_1 &= t_1/T = q^1_{out}/q_{out,tot} = 4/7 \\ D_2 &= t_2/T = q^2_{out}/q_{out,tot} = 3/7 \end{aligned} \tag{11}$$

C. Improving Converter Design by Topology Insight from the ΔV*-Method*

The fact that the soft-charging capabilities of a specific topology can also be conversion ratio dependent is mentioned in [12]. In 2018 [1] presented a new 7:1 Dual-Inductor Hybrid converter (DIHC) topology, which can achieve soft-charging using two interleaved inductors and achieve output voltage regulation using PWM control. The work in [1] states that the Hybrid Dickson converter can not achieve soft-charging without split-phase control such as in [10]. While this is true for even order Hybrid Dickson converters, the ΔV-method, presented here, shows that the 7:1 DIHC in [1] could also have been implemented with a 7:1 Hybrid Dickson, thereby using only a single inductor at the output. The output voltage regulation of the 7:1 Hybrid Dickson can then be achieved with similar PWM control.

IV. Conclusion

An inspection method for determining the soft-charging capabilities of hybrid DC-DC SCCs is introduced. It uses simple steps to visually analyze the voltage changes ΔV across each capacitor. This process leads to an intuitive understanding on how to achieve soft-charging for a given SCC topology, which can then be used to improve efficiency and power density of the converter. The method is applied to determine split-phase timings for a 4:1 Hybrid Dickson converter and to show the inherent soft-charging capability of odd ratio Hybrid Dickson converters. The ΔV-method is easy to use since it does not require the designer to describe the system equations. Instead the phase equivalent circuits are analyzed to help gathering faster and easier insight into specific hybrid converter topologies.

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