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General $V(\lambda)$ mismatch index: History, current state and new ideas

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The general $V(\lambda)$ mismatch index f_1' specifies the mismatch between the relative spectral responsivity of a photometer, and the spectral luminous efficiency function for photopic vision, $V(\lambda)$. A short review of its historical development explains the reasons for the current definition and which adjustments may be helpful in the future. The properties of the current definition are described in detail. It is very likely that photometers will be calibrated with a white light-emitting diode (LED) light source as the reference in the future. This might involve the need for a more appropriate definition of the general $V(\lambda)$ mismatch index, either by using a different normalisation in f_1' for the relative spectral responsivity of the photometer or by introducing a different type of function for assessing the mismatch. On the other hand, the measurement of coloured LEDs is also becoming increasingly important. So, is a single general mismatch index for white and coloured light sources sufficient?

1. Motivation, specific objective

The general $V(\lambda)$ mismatch index, f_1', quantifies the mismatch between the relative spectral responsivity of a photometer, $s_{\mathrm{rel}}(\lambda)$, and the spectral luminous efficiency function for photopic vision, $V(\lambda)$. It was defined for the first time by the CIE in 1982[1] to describe the photometric performance of photometers under general lighting conditions, and a value close to zero denotes photometers that require minimal corrections under light sources with spectral distributions (SDs) different from the defined reference sources used in photometry.

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1.1 Original f_1' definition

Photometer response is based on a combination of the relative spectral responsivity $s_{\mathrm{rel}}(\lambda)$ and the source SD. Since incandescent light played an important role in general illumination, the photometer responsivity was first normalised with the relative SD of CIE standard illuminant A, $S_{\mathrm{A}}(\lambda)$, to consider the mismatch (Equation 1) as defined in ISO/CIE DIS 11664.[2]

$$s_{\mathrm{rel}}^{*}(\lambda)=\frac{\int_{\lambda=380\,\mathrm{nm}}^{780\,\mathrm{nm}} S_{\mathrm{A}}(\lambda)\cdot V(\lambda)\cdot \mathrm{d}\lambda}{\int_{\lambda=380\,\mathrm{nm}}^{780\,\mathrm{nm}} S_{\mathrm{A}}(\lambda)\cdot s_{\mathrm{rel}}(\lambda)\cdot \mathrm{d}\lambda}\, s_{\mathrm{rel}}(\lambda) \quad (1)$$

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According to the ISO/CIE 19476,[3] f_1' is then defined as shown in Equation (2):

$$f_1' = \frac{\int_{\lambda=380\,\text{nm}}^{780\,\text{nm}} \left| s_{\text{rel}}^{*}(\lambda) - V(\lambda) \right| \cdot \mathrm{d}\lambda}{\int_{\lambda=380\,\text{nm}}^{780\,\text{nm}} V(\lambda) \cdot \mathrm{d}\lambda} \tag{2}$$

The original definition of the general $V(\lambda)$ mismatch index f_1' for the function $V(\lambda)$ can be generalised to any other normalised target function $s_{\text{T}}(\lambda)$ required to be spectrally matched. For this purpose, a normalised spectral responsivity $s_{\text{rel,T}}^{*}(\lambda)$ is calculated using the relative SD of CIE standard illuminant A, $S_{\text{A}}(\lambda)$, as a weighting (Equation (3)).

$$s_{\text{rel,T}}^{*}(\lambda) = \frac{\int_{\lambda=\lambda_{\text{T,min}}}^{\lambda_{\text{T,max}}} S_{\text{A}}(\lambda) \cdot s_{\text{T}}(\lambda) \cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda\text{min}}^{\lambda_{\text{max}}} S_{\text{A}}(\lambda) \cdot s_{\text{rel}}(\lambda) \cdot \mathrm{d}\lambda} s_{\text{rel}}(\lambda) \tag{3}$$

The general mismatch index $f'_{1,\text{T}}$ for a specific target function $s_{\text{T}}(\lambda)$ can then be defined as (Equation (4)):

$$f'_{1,\text{T}} = \frac{\int_{\lambda=\lambda_{\text{T,min}}}^{\lambda_{\text{T,max}}} \left| s_{\text{rel,T}}^{*}(\lambda) - s_{\text{T}}(\lambda) \right| \cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\text{T,min}}}^{\lambda_{\text{T,max}}} s_{\text{T}}(\lambda) \cdot \mathrm{d}\lambda} \tag{4}$$

The following rules for the wavelength interval of the integrals have been used in this paper:

- For all integrals including a multiplication by $V(\lambda)$, the lower and upper integration limits are 360 nm and 830 nm, which is the region where $V(\lambda)$ is defined. Exceptions are Equations (1) and (2) (definition of f_1') because the integration range was defined from 380 nm to 780 nm due to historical reasons.
- For all integrals including a multiplication by another target function $s_{\text{T}}(\lambda)$, the lower and upper integration limits are $\lambda_{\text{T,min}}$ and $\lambda_{\text{T,max}}$, limiting the spectral range to that where $s_{\text{T}}(\lambda)$ is defined.
- For all integrals including a multiplication by the detector responsivity $s_{\text{rel}}(\lambda)$, the lower and upper integration limits λ_{min} and λ_{max} should refer to the entire wavelength interval where the detector responsivity $s_{\text{rel}}(\lambda)$ has non-zero values.

Nowadays, almost all sources to be measured in photometry are light-emitting diode (LED)-based. Therefore, in the near future, it is very likely that another standard illuminant will replace the current one (CIE standard illuminant A, $S_{\text{A}}(\lambda)$) in the calibration of photometers. It is planned to use the SD of an LED illuminant, $S_{\text{L}}(\lambda)$, which might involve the need for a more appropriate redefinition of the general $V(\lambda)$ mismatch index, either by using a different normalisation in f_1' for the relative spectral responsivity of the photometer or by introducing a different type of function for assessing this mismatch. Furthermore, the present general mismatch index might not predict the expected range of spectral mismatch errors when measuring coloured LED-based light sources.

1.2 Spectral mismatch correction factors

The luminous responsivity of a photometer depends on the SD of the calibration light source, $S_{\text{C}}(\lambda)$. When used for measuring a light source with a different SD, $S_{\text{Z}}(\lambda)$, this luminous responsivity must be corrected using the spectral mismatch correction factor (SMCF) $F\left(S_{\text{C}}(\lambda), S_{\text{Z}}(\lambda)\right)$. This factor can be expressed by Equation (5):

$$F\left(S_{\mathrm{C}}(\lambda), S_{\mathrm{Z}}(\lambda)\right)=\frac{\int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\mathrm{C}}(\lambda) \cdot s_{\mathrm{rel}}(\lambda) \cdot \mathrm{d} \lambda}{\int_{\lambda=\lambda_{\mathrm{T}, \min}}^{\lambda_{\mathrm{T}, \max}} S_{\mathrm{C}}(\lambda) \cdot s_{\mathrm{T}}(\lambda) \cdot \mathrm{d} \lambda} \frac{\int_{\lambda=\lambda_{\mathrm{T}, \min}}^{\lambda_{\mathrm{T}, \max}} S_{\mathrm{Z}}(\lambda) \cdot s_{\mathrm{T}}(\lambda) \cdot \mathrm{d} \lambda}{\int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\mathrm{Z}}(\lambda) \cdot s_{\mathrm{rel}}(\lambda) \cdot \mathrm{d} \lambda} \tag{5}$$

$$s_{\mathrm{rel}, \mathrm{T}, \mathrm{C}}(\lambda)=\frac{\int_{\lambda=\lambda_{\mathrm{T}, \min}}^{\lambda_{\mathrm{T}, \max}} S_{\mathrm{C}}(\lambda) \cdot s_{\mathrm{T}}(\lambda) \cdot \mathrm{d} \lambda}{\int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\mathrm{C}}(\lambda) \cdot s_{\mathrm{rel}}(\lambda) \cdot \mathrm{d} \lambda} s_{\mathrm{rel}}(\lambda)=a_{\mathrm{C}} s_{\mathrm{rel}}(\lambda) \tag{6}$$

$$F\left(S_{\mathrm{C}}(\lambda), S_{\mathrm{Z}}(\lambda)\right)=\frac{1}{a_{\mathrm{C}}} \frac{\int_{\lambda=\lambda_{\mathrm{T}, \min}}^{\lambda_{\mathrm{T}, \max}} S_{\mathrm{Z}}(\lambda) \cdot s_{\mathrm{T}}(\lambda) \cdot \mathrm{d} \lambda}{\int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\mathrm{Z}}(\lambda) \cdot s_{\mathrm{rel}}(\lambda) \cdot \mathrm{d} \lambda}=\frac{\int_{\lambda_{\mathrm{T}, \min}}^{\lambda_{\mathrm{T}, \max}} S_{\mathrm{Z}}(\lambda) \cdot s_{\mathrm{T}}(\lambda) \cdot \mathrm{d} \lambda}{\int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\mathrm{Z}}(\lambda) \cdot s_{\mathrm{rel}, \mathrm{T}, \mathrm{C}}(\lambda) \cdot \mathrm{d} \lambda} \tag{7}$$

This expression can be simplified by introducing $s_{\mathrm{rel,T,C}}(\lambda)$, a modified expression of Equation (3), using the SD of the calibration illuminant $S_{\mathrm{C}}(\lambda)$ as the weighting function for the normalisation instead of the SD of CIE standard illuminant A, $S_{\mathrm{A}}(\lambda)$, as shown in Equation (6).

The integral ratio in Equation (6) results in a wavelength-independent normalisation factor a_{c}. Therefore, the SMCF in Equation (5) can be rearranged while using Equation (6) with $s_{\mathrm{rel,T,C}}(\lambda)=a_{\mathrm{C}}s_{\mathrm{rel}}(\lambda)$ to Equation (7).

Equation (7) shows the connection of the SMCF in Equation (5) with the general mismatch index through the normalisation $s_{\mathrm{rel,T,C}}(\lambda)$ of the relative spectral responsivity $s_{\mathrm{rel}}(\lambda)$ and shows its practical importance.

2. History

The first notation of f_1' can be found in CIE 53[1] as an informative note. The preferred measure in CIE 53[1] for quantifying the mismatch, $f_{1,\mathrm{CIE}}$, was the maximum deviation due to spectral mismatch for five defined light sources with SDs $S_{\mathrm{Z},i}(\lambda)$ as shown in Equation (8).

$$f_{1,\mathrm{CIE}}=\max_{i=1\ldots5}\left|F\left(S_{\mathrm{C}}(\lambda), S_{\mathrm{Z},i}(\lambda)\right)-1\right| \tag{8}$$

Later in CIE 69,[4] the general $V(\lambda)$ mismatch index f_1' was introduced as a CIE recommendation. In preparation for these early CIE publications, numerous articles (e.g., Krochmann and Reissmann,[5] Krystek and Erb[6] and Krochmann and Rattunde[7]) were published, especially in German. These articles systematically showed that the selection of f_1' was reasonable under the technical conditions of that time and proved the advantages over other methods under discussion at that time.

In particular, the derivation and justification for the normalisation of the relative spectral responsivity $s^{*}_{\mathrm{rel}}(\lambda)$ using the relative SD of the calibration light source (denoted here as $s_{\mathrm{rel,T,C}}(\lambda)$ in Equation (6) and described as

$s_{\mathrm{rel},3}(\lambda)$ in the bibliographical source) can be found in Krochmann and Reissmann.[5]

The discussions can be summarised as follows: The difference between the realised and targeted functions can be defined by Equation (9):

$$\Delta s(\lambda) = s_{\mathrm{rel}}(\lambda) - s_{\mathrm{T}}(\lambda) \tag{9}$$

However, this type of calculation generally leads to overestimating the goodness of the fit. For the sake of comparison, it is more appropriate to show the relative error $f(\lambda)$ instead of the difference $\Delta s(\lambda)$, as shown in Equation (10).

$$f(\lambda) = \frac{s_{\mathrm{rel}}(\lambda)}{s_{\mathrm{T}}(\lambda)} - 1 \tag{10}$$

This is the measurement deviation for monochromatic light of wavelength λ. This can be modified to express the measurement deviation at wavelength λ when a photometric detector is calibrated using the usual calibration source, CIE standard illuminant A, by introducing $s^{*}_{\mathrm{rel}}(\lambda)$ from Equation (1) into Equation (10) instead of $s_{\mathrm{rel}}(\lambda)$. Finally, this approach was extended in Geutler *et al.*[8] and CIE 53[1] to a more general relative error metric, denoted as f_1' in CIE 53,[1] given by Equation (11).

$$f_1 = \frac{\int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} \left| s^{*}_{\mathrm{rel,T}}(\lambda) - s_{\mathrm{T}}(\lambda) \right| \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} s_{\mathrm{T}}(\lambda)\, \mathrm{d}\lambda} \text{ later defined as } f_1' \tag{11}$$

$$f_{\mathrm{Erb}}\left(s_{\mathrm{rel}}(\lambda)\right) = \sqrt{\frac{\int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} \left(s_{\mathrm{rel,T}}(\lambda) - s_{\mathrm{T}}(\lambda)\right)^2 \cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} s_{\mathrm{T}}{}^2(\lambda) \cdot \mathrm{d}\lambda}} \tag{12}$$

$$F\left(S_{\mathrm{Z}}(\lambda)\right) = \sqrt{\frac{\int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} S_{\mathrm{Z}}^2(\lambda) \cdot \mathrm{d}\lambda \int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} s_{\mathrm{T}}{}^2(\lambda) \cdot \mathrm{d}\lambda}{\left(\int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} S_{\mathrm{Z}}(\lambda) \cdot s_{\mathrm{T}}(\lambda) \cdot \mathrm{d}\lambda\right)^2}} \tag{13}$$

A good summary of all the previous literature can be found in Krochmann and Rattunde.[7] It contains many calculations with different quality metrics and correlations, using a set of 223 detectors and 46 light sources (41 different others besides the five CIE light sources from CIE 53[1]), showing the correlation between quality metrics and SMCFs.

A more theoretical approach was published in Krystek and Erb.[6] This involved a separation of the spectral mismatch error due to the photometer spectral response from that due to the spectral characteristics of the calibration source. In this approach, the detector-related dependence was described through Equation (12) and the light source-related dependence was given in Equation (13).

The definition of f_{Erb} in Equation (12) renders the index independent of features of the calibration light source, by only including features of the detector. It was suggested as the general $V(\lambda)$ mismatch index.

3. Current state

The current definition of the general $V(\lambda)$ mismatch index can be found in ISO/CIE 19476,[3] and the use of f_1' for the estimation of the maximum SMCF for phosphor-based white LEDs and RGB-type white LEDs can be found in CIE S 025:2015.[9] Here, a good correlation between f_1' and the minimum and maximum SMCF of phosphor-based white LEDs was found. However, the minimum and maximum SMCF for white light produced by the mixed emission from RGB LEDs cannot be convincingly estimated based on f_1'. Furthermore, the spectral mismatch error of coloured LEDs is not predictable based on f_1'. This means a new or modified approach is needed for current lighting situations.

4. Other approaches

Czibula and Makai[10] showed that f_1' is not sufficient to describe or even predict the spectral mismatch correction for LEDs, especially for coloured LEDs. The authors introduced two additional measurements using auxiliary detectors to estimate the spectral mismatch correction factor precisely.

Young *et al.*[11] later introduced a new quality metric f_{LED} as the average absolute spectral mismatch error over a wavelength region relative to the error for the central wavelength of that region. Here, the influence of bandwidth changes and central wavelength changes for the spectral mismatch correction factor was discussed. The result was not a single value but a characteristic field or matrix. Csuti *et al.*[12] took this one step further and introduced a partial index $f'_{1,\mathrm{PART}}$ calculating a form of f_1' for specific wavelength regions (blue, green, yellow and red) based on a calibration/normalisation with a coloured LED in that region.

A new approach for the definition of an adjusted $V(\lambda)$ mismatch index f_1'' was introduced by Ferrero *et al.*,[13,14] which provides a better correlation to the average absolute spectral mismatch error than f_1', when this error is evaluated for broadband sources (phosphor-based LEDs and blackbody sources). The following sections will describe these ideas in a more detailed manner.

4.1 Czibula and Makai

Czibula and Makai[10] stated that f_1' is insufficient to describe or even predict the necessary mean spectral mismatch correction for LEDs (especially coloured LEDs). They suggested an improvement of the spectral correction based on the measurement of auxiliary detectors sensitive for blue (B1) and red (R1) light, which was developed from simulations and measurements of about 1000 detectors and real/simulated spectra of about 150 LEDs.

The prediction of the SMCF using the auxiliary information was very good. There was an additional investigation to calculate the ideal pair of auxiliary detectors, but the correction results were not improved compared to the real auxiliary detectors B1 and R1 used for the first part of the work.

This is the first study that suggests that even minimal spectral information is sufficient to obtain very precise SMCF information.

4.2 Young *et al.*

Young *et al.*[11] introduced a new mismatch index called f_{LED} (usable for coloured LEDs only), defined as the 'average spectral mismatch error over a wavelength region relative to the central wavelength of that region'. They used a model-based specific SMCF as shown in Equation (14):

$$\varepsilon_{\lambda_c,\lambda_m,\Delta\lambda_c} = 1 - \frac{\int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\text{mLED}}\left(\lambda,\lambda_p=\lambda_c,\Delta\lambda_c\right)\cdot s_{\text{rel}}(\lambda)\cdot \mathrm{d}\lambda \int_{\lambda=\lambda_{\text{T},\min}}^{\lambda_{\text{T},\max}} S_{\text{mLED}}\left(\lambda,\lambda_p=\lambda_m,\Delta\lambda_c\right)\cdot s_{\text{T}}(\lambda)\cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\text{T},\min}}^{\lambda_{\text{T},\max}} S_{\text{mLED}}\left(\lambda,\lambda_p=\lambda_c,\Delta\lambda_c\right)\cdot s_{\text{T}}(\lambda)\cdot \mathrm{d}\lambda \int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\text{mLED}}(\lambda,\lambda_p=\lambda_m,\Delta\lambda_c)\cdot s_{\text{rel}}(\lambda)\cdot \mathrm{d}\lambda} \quad (14)$$

where $S_{\text{mLED}}(\lambda, \lambda_p, \Delta\lambda)$ represents a model function for the SD of an LED with peak wavelength λ_p and a full-width at half maximum (FWHM) $\Delta\lambda_c$. This is similar to the SMCF in Equation (5) and accounts for the spectral mismatch error of a photometer under the following conditions:

- calibrated with an LED source with a peak wavelength $\lambda_p = \lambda_c$, and a FWHM $\Delta\lambda_c$,
- measuring an LED source with a peak wavelength $\lambda_p = \lambda_m$ and the same FWHM $\Delta\lambda_c$.

The variable given by Equation (15):

$$w_{\text{LED}}\left(\lambda_c,\Delta\lambda_c\right) = \frac{\int_{\lambda_m=p_1}^{p_2} \left|\varepsilon_{\lambda_c,\lambda_m,\Delta\lambda_c}\right|\cdot \mathrm{d}\lambda}{p_2 - p_1} \quad (15)$$

describes the average spectral mismatch error across the wavelength range $\lambda_m \in (p_1, p_2)$, where p_1 and p_2 are the wavelength limits of the wavelength range considered.

If we modify the FWHM parameter (calibration at FWHM $\Delta\lambda_c$ and measuring at FWHM $\Delta\lambda_m$) instead of varying the peak wavelength parameter, we obtain Equation (16).

$$\eta_{\lambda_c,\Delta\lambda_c,\Delta\lambda_m} = 1 - \frac{\int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\text{mLED}}\left(\lambda,\lambda_p=\lambda_c,\Delta\lambda_c\right)\cdot s_{\text{rel}}(\lambda)\cdot \mathrm{d}\lambda \int_{\lambda=\lambda_{\text{T},\min}}^{\lambda_{\text{T},\max}} S_{\text{mLED}}\left(\lambda,\lambda_p=\lambda_c,\Delta\lambda_m\right)\cdot s_{\text{T}}(\lambda)\cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\text{T},\min}}^{\lambda_{\text{T},\max}} S_{\text{mLED}}\left(\lambda,\lambda_p=\lambda_c,\Delta\lambda_c\right)\cdot s_{\text{T}}(\lambda)\cdot \mathrm{d}\lambda \int_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\text{mLED}}(\lambda,\lambda_p=\lambda_c,\Delta\lambda_m)\cdot s_{\text{rel}}(\lambda)\cdot \mathrm{d}\lambda} \quad (16)$$

$$h_{\text{LED}}\left(\lambda_c,\Delta\lambda_c\right) = \frac{\int_{\Delta\lambda_m=h_1}^{h_2} \left|\eta_{\lambda_c,\Delta\lambda_c,\Delta\lambda_m}\right|\cdot \mathrm{d}h}{h_2 - h_1} \quad (17)$$

$$f_{\text{LED}}\left(\lambda_c,\Delta\lambda_c\right) = \sqrt{w_{\text{LED}}^2\left(\lambda_c,\Delta\lambda_c\right) + h_{\text{LED}}^2\left(\lambda_c,\Delta\lambda_c\right)} \quad (18)$$

We can define now the average spectral mismatch error while varying the bandwidth using Equation (17).

Equation (17) describes the average spectral mismatch error while varying the LED model bandwidth in the range $\Delta\lambda_m \in (h_1, h_2)$, where h_1 and h_2 are the minimum and maximum FWHM bandwidths considered. Combining the Equations (15) and (17), Young *et al.*[11] defined an LED mismatch index as shown in Equation (18).

According to Young *et al.*,[11] a reasonable region for p_1 to p_2 and h_1 to h_2 would be ± 5 nm around the central values of λ_c and $\Delta\lambda_c$.

This proposal by Young *et al.*[11] was the first to use LED models with different central wavelength and bandwidth characteristics to calculate

SMCFs and summarise a specific spectral mismatch index from all this information.

4.3 Csuti *et al.*

Csuti and Kránicz[15] and the additional contributions from Csuti *et al.*[12,16] summarised discussions of CIE TC2-45 to prepare the CIE report CIE 127[17] at the end of 2003. The discussion in technical committee TC 2-45 included two key proposals. One proposal is shown in Equations (19) and (20). Note that compared with Equation (5), the weighting of the relative spectral responsivities with the relative SD of the calibration source in the first integral ratio was left out here.

$$\begin{aligned} f_{1,\mathrm{LED,YO}} &= \max\left|F\left(S_{\mathrm{mLED}}\left(\lambda,\lambda_0,\Delta\lambda\right)\right)-1\right| \\ \text{with } \lambda_0 &= 450\ \mathrm{nm}, 460\ \mathrm{nm},\ldots,650\ \mathrm{nm} \\ \Delta\lambda &= 20\ \mathrm{nm} \end{aligned} \tag{19}$$

$$F\left(S_{\mathrm{mLED}}\left(\lambda,\lambda_0,\Delta\lambda\right)\right)=\frac{\int\limits_{\lambda=\lambda_{\min}}^{\lambda_{\max}} s_{\mathrm{rel}}(\lambda)\cdot \mathrm{d}\lambda \int\limits_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} S_{\mathrm{mLED}}\left(\lambda,\lambda_0,\Delta\lambda\right)\cdot s_{\mathrm{T}}(\lambda)\cdot \mathrm{d}\lambda}{\int\limits_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} s_{\mathrm{T}}(\lambda)\cdot \mathrm{d}\lambda \int\limits_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\mathrm{mLED}}\left(\lambda,\lambda_0,\Delta\lambda\right)\cdot s_{\mathrm{rel}}(\lambda)\cdot \mathrm{d}\lambda} \tag{20}$$

The second proposal is shown in Equation (21), but this is not very helpful because problems with the spectral matching can be averaged out. However, neither of these suggestions were published in the final report CIE 127,[17] meaning that the references in Csuti *et al.*[12] are the only remaining information.

$$\begin{aligned} f_{1,\mathrm{LED,KM}} &= \mathrm{average}\left|F\left(S_{\mathrm{mLED}}\left(\lambda,\lambda_0,\Delta\lambda\right)\right)-1\right| \\ \text{with } \lambda_0 &= 400\ \mathrm{nm}, 460\ \mathrm{nm},\ldots,700\ \mathrm{nm} \\ \Delta\lambda &= 20\ \mathrm{nm} \end{aligned} \tag{21}$$

Equation (5) can be modified to Equation (22):

$$F_{\mathrm{mLED}}\left(S_{\mathrm{C}}(\lambda),S_{\mathrm{mLED}}\left(\lambda,\lambda_i\right)\right)=\frac{\int\limits_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\mathrm{C}}(\lambda)\cdot s_{\mathrm{rel}}(\lambda)\cdot \mathrm{d}\lambda \int\limits_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} S_{\mathrm{mLED}}\left(\lambda,\lambda_p=\lambda_i,\Delta\lambda\right)\cdot s_{\mathrm{T}}(\lambda)\cdot \mathrm{d}\lambda}{\int\limits_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} S_{\mathrm{C}}(\lambda)\cdot s_{\mathrm{T}}(\lambda)\cdot \mathrm{d}\lambda \int\limits_{\lambda=\lambda_{\min}}^{\lambda_{\max}} S_{\mathrm{mLED}}(\lambda,\lambda_p=\lambda_i,\Delta\lambda)\cdot s_{\mathrm{rel}}(\lambda)\cdot \mathrm{d}\lambda} \tag{22}$$

where $S_{\mathrm{mLED}}\left(\lambda,\lambda_p=\lambda_i,\Delta\lambda\right)$ is an LED model (see Section 5.1 for details) with a peak wavelength of λ_i and a bandwidth (FWHM) of $\Delta\lambda = 20\,\mathrm{nm}$. Based on the approach used in Equation (19), an f_1' value for LEDs can be defined as follows in Equation (23):

$$f_{1,\mathrm{LED,YO}}' = \max_{\lambda_i}\left\{\left|F_{\mathrm{mLED}}\left(S_{\mathrm{C}}(\lambda),S_{\mathrm{mLED}}\left(\lambda,\lambda_i,\Delta\lambda\right)\right)-1\right|\right\} \tag{23}$$

Table 1 Wavelength ranges suggested in Csuti *et al*[15]

Index *i*	Name	Integration range		λ_p Peak wavelength for reference LED (nm)
		$\lambda_{S,i}$ Start wavelength (nm)	$\lambda_{E,i}$ End wavelength (nm)	
0	Blue: BL	465	540	460
1	Green: GN	490	560	503
2	Yellow: YE	550	630	611
3	Red: RD	620	660	659

For this definition, it is necessary to define the wavelength range (400–650 nm) and the steps (10 nm) to get consistent results. However, a definition of this form leads to problems when calculating the measurement uncertainty.

The suggestion from Csuti *et al.*[15] was to use four different wavelength ranges, instead of the maximum over the complete wavelength range, which was used in Equation (23). A new index called f'_{1,PART_i} was proposed, which used a special version of the normalised spectral responsivity of the photometer and different calibration LEDs, designated LED_i, in each of the chosen wavelength ranges as shown in Equation (24):

$$s_{\mathrm{rel,LED}_i}(\lambda) = \frac{\int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} S_{\mathrm{mLED}}(\lambda,\lambda_i,\Delta\lambda)\cdot s_{\mathrm{T}}(\lambda)\cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\mathrm{min}}}^{\lambda_{\mathrm{max}}} S_{\mathrm{mLED}}(\lambda,\lambda_i,\Delta\lambda)\cdot s_{\mathrm{rel}}(\lambda)\cdot \mathrm{d}\lambda} s_{\mathrm{rel}}(\lambda) \tag{24}$$

f'_{1,PART_i} is defined in Equation (25):

$$f'_{1,\mathrm{PART}_i} = \frac{\int_{\lambda=\lambda_{\mathrm{S},i}}^{\lambda_{\mathrm{E},i}} \left| s_{\mathrm{rel,LED}_i}(\lambda) - s_{\mathrm{T}}(\lambda)\right| \cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\mathrm{T,min}}}^{\lambda_{\mathrm{T,max}}} s_{\mathrm{T}}(\lambda)\cdot \mathrm{d}\lambda} \tag{25}$$

For the integration limits in the nominator see Table 1.

However, the problem of Equation (25) is the lack of a reference source for each required calibration condition.

The complete calculation process is demonstrated in the following figures. Figure 1 shows the relative spectral responsivity of sample photometer #2 and $V(\lambda)$. The normalised SDs of the reference LEDs (normalised to an integral of 1, see Table 1 for the definition of the wavelength ranges and reference LEDs) are shown in Figure 2. Using these data, Figure 3 shows the calculated differences between the normalised spectral responsivities of photometer #2 (Equation (24)) and $V(\lambda)$.

Further explanations for handling SMCFs, especially with monochromatic light and LED models, can be found in Sections 5.2 and 5.3.

4.4 Ferrero *et al.*

Ferrero *et al.*[13] proposed an alternative index f''_1, based on the idea that the spectral shape of the relative deviation between the relative spectral responsivity and $V(\lambda)$ impacts the resulting

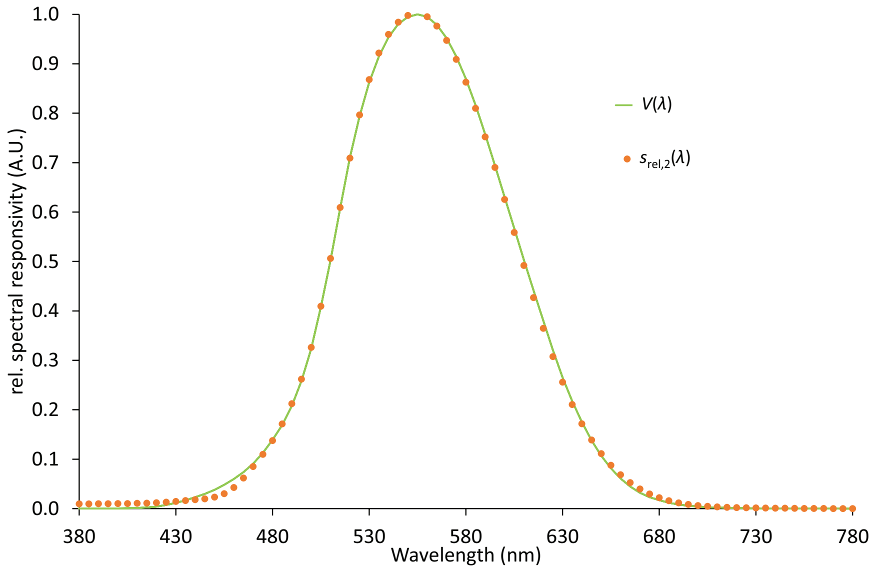

Figure 1 Relative spectral responsivity of a sample photometer #2 compared to $V(\lambda)$

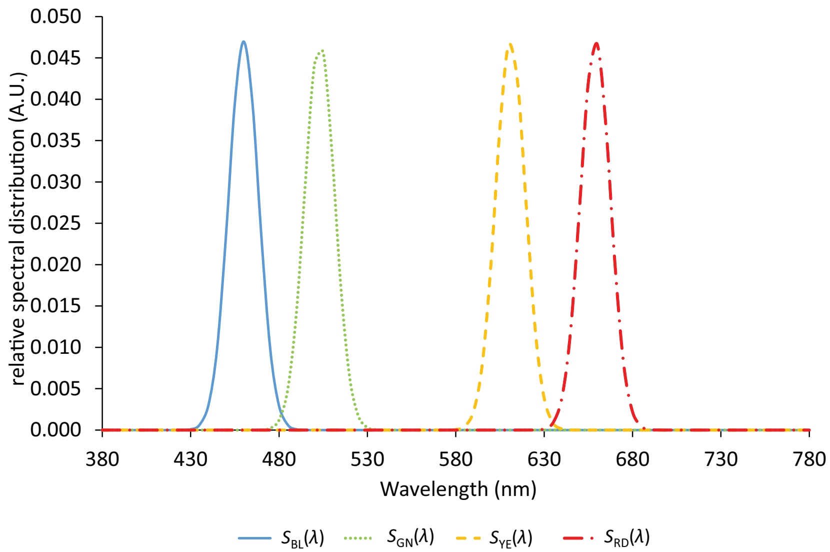

Figure 2 Relative spectral distributions of the reference LEDs used in f'_{1,PART_i}

mean spectral mismatch error. For instance, any deviations that vary rapidly with wavelength have a lower impact when broadband light sources are to be measured with a photometer because those variations are smoothed out by integration when calculating the luminous responsivity. Therefore, a formalism based on the Fourier transform of the relative error

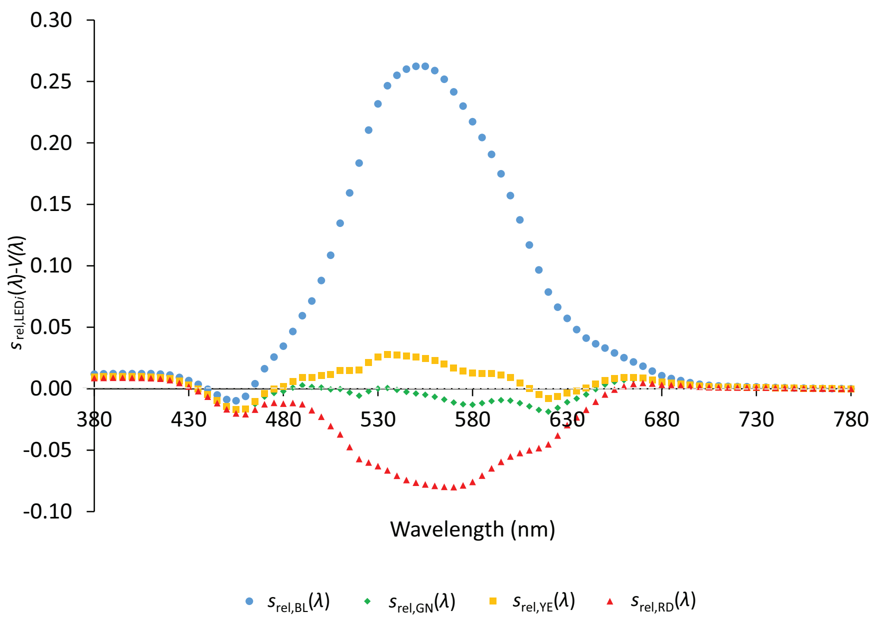

Figure 3 Difference between the normalised spectral responsivities of sample photometer #2, calculated using Equation (24) for each LED, and $V(\lambda)$

function was used for defining the alternative index f_1''.

Introducing a relative spectral responsivity $\bar{s}_{rel}(\lambda)$ without any further weighting in Equation (26):

$$\bar{s}_{rel}(\lambda) = \frac{\int_{\lambda=\lambda_{T,min}}^{\lambda_{T,max}} s_T(\lambda)\cdot d\lambda}{\int_{\lambda=\lambda_{min}}^{\lambda_{max}} s_{rel}(\lambda)\cdot d\lambda} s_{rel}(\lambda) \quad (26)$$

and using the spectral frequency ν_λ, a cut-off spectral frequency $\nu_{\lambda,c}$ and δ_s as the Fourier transform of the normalised difference function δ_s as shown in Equation (27):

$$\delta_s(\lambda) = \frac{\bar{s}_{rel}(\lambda) - s_T(\lambda)}{\int_{\lambda=\lambda_{T,min}}^{\lambda_{T,max}} s_T(\lambda)\, d\lambda} \quad (27)$$

f_1'' is defined in Equation (28):

$$f_1'' = \sqrt{2\int_{\nu_\lambda=0}^{\nu_{\lambda,c}} \left|\widehat{\delta_s}\right|^2 d\nu_\lambda} \quad (28)$$

f_1'' equals the standard deviation of δ_s when the cut-off frequency $\nu_{\lambda,c}$ is infinite, but it is lower for a finite $\nu_{\lambda,c}$. A general value of $\nu_{\lambda,c}$ can be defined for general lighting purposes, but others can be recommended for more specific lighting scenarios, such as those that consist exclusively of phosphor-based LED sources. Ferrero *et al.*[13] showed that this alternative index results in a better correlation than f_1' with the mean mismatch errors in broadband or mixed scenarios, but not for narrowband colour LEDs, where the correlation is lower.

5. Further considerations

5.1 Coloured LED modelling

A simple model $S_{mLED}(\lambda, \lambda_0, \Delta\lambda)$ (Equations (29) and (30)) of the SD of an LED with central wavelength λ_0 and bandwidth (FWHM) $\Delta\lambda$ is introduced for further calculation:

$$S_{\text{mLED}}(\lambda,\lambda_0,\Delta\lambda) = e^{-4\ln(2)\cdot((\lambda-\lambda_0)/\Delta\lambda)^2} / \Delta\lambda\sqrt{\pi/\ln(16)} \tag{29}$$

$$\int_{\lambda=0}^{\infty} S_{\text{mLED}}(\lambda)\cdot \mathrm{d}\lambda = 1 \tag{30}$$

$$S_{\text{Ohno}}(\lambda,\lambda_0,\Delta\lambda_{0,5}) = \frac{1}{3}\left(g\left(\lambda,\lambda_0,\Delta\lambda_{0,5}\right) + 2\cdot g^5\left(\lambda,\lambda_0,\Delta\lambda_{0,5}\right)\right) \tag{31}$$

$$g\left(\lambda,\lambda_0,\Delta\lambda_{0,5}\right) = e^{-\left((\lambda-\lambda_0)/\Delta\lambda_{0,5}\right)^2} \tag{32}$$

The numerator in Equation (29) was also used in Young *et al.*[11] for modelling coloured LEDs. More complex models could also be used, but there is no significant improvement for general purposes. However, for the specific determination of an actual SMCF, the measured SD of the LED should be taken into account and not the modelled one.

As an example of a more complex model, Ohno[18] published the first realistic model as shown in Equations (31) and (32).
Note: In this model, the FWHM is about $0.9\cdot\Delta\lambda_{0,5}$.

5.2 Spectral mismatch correction factors for monochromatic light sources

For monochromatic light sources emitting light only at a specific wavelength λ, the SMCF in Equation (7) becomes Equation (33).

5.3 Spectral mismatch correction factors for LEDs

Similarly, for LED sources, the SMCF can be expressed, from Equation (7), as shown in Equation (34):

$$F(\lambda) = \frac{s_{\text{T}}(\lambda)}{s_{\text{rel,T,C}}(\lambda)} \tag{33}$$

$$F_{\text{LED}}\left(S_{\text{C}}(\lambda), S_{\text{mLED}}(\lambda,\lambda_0,\Delta\lambda)\right) = \frac{\int_{\lambda=\lambda_{\text{T,min}}}^{\lambda_{\text{T,max}}} S_{\text{mLED}}(\lambda,\lambda_0,\Delta\lambda)\cdot s_{\text{T}}(\lambda)\cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\text{T,min}}}^{\lambda_{\text{T,max}}} S_{\text{mLED}}(\lambda,\lambda_0,\Delta\lambda)\cdot s_{\text{rel,T,C}}(\lambda)\cdot \mathrm{d}\lambda} \tag{34}$$

Inserting the coloured LED model from Equation (29), this can be arranged as shown in Equation (35):

$$F_{\text{LED}}\left(S_{\text{C}}(\lambda), S_{\text{mLED}}(\lambda,\lambda_0,\Delta\lambda)\right) = \frac{\int_{\lambda=\lambda_{\text{T,min}}}^{\lambda_{\text{T,max}}} \exp\left(k\cdot\left((\lambda-\lambda_0)/\Delta\lambda\right)^2\right)\cdot s_{\text{T}}(\lambda)\cdot \mathrm{d}\lambda}{\int_{\lambda=\lambda_{\text{T,min}}}^{\lambda_{\text{T,max}}} \exp\left(k\cdot\left((\lambda-\lambda_0)/\Delta\lambda\right)^2\right)\cdot s_{\text{rel,T,C}}(\lambda)\cdot \mathrm{d}\lambda} \tag{35}$$

with the factor k shown in Equation (36):

$$k = -4\ln(2) \quad (36)$$

The expression $\int \exp\left(k \cdot \left((\lambda - \lambda_0)/\Delta\lambda\right)^2\right)$ represents a convolution (operator *) generally described as shown in Equation (37).

$$f(\lambda) * g(\lambda) = \int_{\lambda'} g(\lambda - \lambda') f(\lambda') \mathrm{d}\lambda' \quad (37)$$

This allows the spectral mismatch correction factor to be expressed in Equation (38).

$$F_{\mathrm{LED}}\left(S_{\mathrm{C}}(\lambda), S_{\mathrm{mLED}}(\lambda, \lambda_0, \Delta\lambda)\right) = \frac{S_{\mathrm{mLED}}(\lambda, \lambda_0, \Delta\lambda) * s_{\mathrm{T}}(\lambda)}{S_{\mathrm{mLED}}(\lambda, \lambda_0, \Delta\lambda) * s_{\mathrm{rel,T,C}}(\lambda)} \quad (38)$$

6. Conclusion

This article explains the basics of the general $V(\lambda)$ mismatch index f_1' and its background. Furthermore, based on the published literature, it is shown that several approaches lead to better predictions of the general $V(\lambda)$ mismatch of photometers for general lighting situations using LED sources.

The following objectives can be used as a guide for the next steps towards a new or extended definition for the quality metric used to describe the spectral mismatch of photometers:

- Define a $V(\lambda)$ mismatch index such that its values provide better correlation than the current f_1' values with the average or maximum spectral mismatch error introduced when measuring light sources that are spectrally different to the calibration light source.
- The definition should be independent of any specific set of light sources.
- The definition should be independent of the wavelength resolution of the relative spectral responsivity $s_{\mathrm{rel}}(\lambda)$ (some minimal requirements for the sampling and bandwidth must be defined).
- Take into account the requirements for simple measurement uncertainty calculation (be careful with maximum values, absolute values, etc.).
- The mismatch index should be easy to implement by manufacturers and easily reproducible by users.

As an initial suggestion, we propose the following:

- Change the current definition of f_1' using Equations (3) and (4) to a similar one using a normalisation function for the relative spectral responsivity $s_{\mathrm{rel}}(\lambda)$ of the detector, which is connected to the relative SD of the light source actually used for calibration rather than standard illuminant A, that is, replace $S_{\mathrm{A}}(\lambda)$ with $S_{\mathrm{C}}(\lambda)$.
- Add a different approach to f_1'. Since f_1' was defined only for general lighting, it might be necessary to provide a new index to evaluate the mismatch of photometers for lighting conditions under coloured LED sources. A graph should be provided showing the SMCFs for a set of simulated coloured LEDs of 20 nm FWHM and a simple model as shown in Equation (29).

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