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A Regulation Method Considering Coupled Voltage Violation and Unbalance in Three-Phase Four-Wire Hybrid AC/DC LVDNs with High Penetration PVs

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Abstract—Low-voltage distribution networks (LVDNs) have witnessed an increasing penetration of distributed photovoltaics (PVs), which exacerbates the existing voltage violation and voltage unbalance issues. PVs, energy storage systems, and voltage source converters can regulate voltages in hybrid AC/DC LVDNs. To solve the above two voltage problems simultaneously, a partition-based coordinated regulation method is proposed in this paper considering the coupling of the two issues and the effect of active power on reactive power in electrical distance calculation. The voltage adjustments in different voltage violation scenarios are obtained based on a graph that describes the coupling of the two issues mathematically. In order to better determine control devices, the equivalent coefficient of active power adjustment on electrical distance calculation is considered in the partition method, in which regulation efficiency, regulation cost, and regulation capability are used as objectives. The coordination of controllable devices is achieved according to three-phase four-wire sensitivities. Case studies are performed to verify the effectiveness and accuracy of the proposed regulation method in hybrid AC/DC LVDNs.

Index Terms—Hybrid AC/DC LVDNs, coupled voltage violation and voltage unbalance, partition-based coordinated control, three-phase four-wire sensitivity

I. INTRODUCTION

Low-voltage distribution networks (LVDNs) are experiencing an increasing penetration of distributed photovoltaic (PV) [1]. However, the capacity-based single-phase or three-phase integration of PV can exacerbate voltage violation and voltage unbalance [2], [3]. For example, serious voltage violations and unbalances frequently and simultaneously occur in the Jinzhai power grid in Anhui Province, China due to the integration of about 40,000 household PV systems. Moreover, voltage violations may cause serious damage to electric equipment. Voltage unbalance may further aggravate power losses and threaten the security of system operation. Therefore, an effective control strategy for addressing the two issues is crucial for PV integration in LVDNs.

Various methods for improving voltage violations and unbalances have been studied in the literature. References [4]-[7] propose voltage control strategies through PV inverters, onload tap changers, and energy storage systems (ESSs). However, they only study either voltage violation or voltage unbalance. It is known that the two issues always occur simultaneously as they are coupled. To address the two issues, [8] develops a priority-based voltage control strategy with voltage source converters (VSCs) in AC/DC LVDNs. [9] and [10] make comprehensive studies for optimizing voltage magnitude and balance profiles. An energy scheduling method is presented in [11] to mitigate voltage violation and voltage unbalance of a microgrid with a small cost. However, none of the above methods considers the coupling of the two issues and the global optimization in [9]-[11] is not justifiable for LVDNs with weak communication.

The coordination between PV inverters and ESS has been considered as a universal approach for voltage management in LVDNs [12]. Moreover, compared with AC systems, DC systems are more suitable for the integration of inverter-based PV systems and ESSs. VSC-based DC systems can provide better power qualities and flexible power control, thus the voltage unbalance can be mitigated through single-phase regulation of VSCs [13]. On the other hand, a hybrid AC/DC LVDN can mitigate overload and voltage violations due to the high-power transfer capacity of DC systems as well as the capability of power flow rescheduling [14]. Therefore, in hybrid AC/DC LVDNs, the coordination of controllable devices such as VSCs, ESSs, and PV inverters are suitable solutions for regulating voltage violations and unbalances. However, their coordinated regulation of active and reactive power still requires proper attention and solutions considering the three-phase four-wire structure on the AC side of a hybrid AC/DC LVDN.

To achieve efficient local control, [15] uses partitioning methods for voltage regulation. Compared to the global control method, the partition-based regulation method offers a simpler model with low computational burdens and allows separate voltage control in each sub-area. Local management of voltage violation and unbalance within the sub-area can prevent the issues from spreading to adjacent sub-areas and causing a wide impact. Therefore, compared to the above methods, the partition-based regulation method is more suitable for LVDNs with complex structures and multiple control devices. When the efficiencies of multiple devices are considered in a network...
partition, it is necessary to define an electrical distance for calculating [16]. However, most existing methods only consider the effects of reactive power since they mainly focus on the reactive power partition of a balanced distribution network and voltage regulation in a transmission network, where the line resistance is much smaller than reactance [17]-[19]. [20] only calculates the electrical distance index of PVs based on active power-voltage sensitivity and reactive power-voltage sensitivity for voltage control in medium-voltage distribution networks (MVDNs). [21] presents a novel performance index based on electrical distance and regulation capability considering both active and reactive power of PV inverters to partition an MVDN. However, their control methods only regulate PV output. In [22], a novel partitioning method considering power balance and electric distance evaluated from active and reactive power sensitivity is proposed for voltage control in MVDNs.

However, unlike balanced MVDNs, the partition indexes calculation and sensitivity-based voltage control method are vulnerable to the three-phase-four-wire structure of LVDNs. Furthermore, optimal control methods in [20]-[22] are unsuitable for LVDNs with weak communication.

Shortcomings of the existing coordinated control methods for addressing voltage violations and unbalances of hybrid AC/DC LVDNs include:

a) Voltage violations and voltage unbalances are solved separately or prioritized in most existing approaches. However, the regulation of phase voltage violation may worsen the voltage unbalance in three phases LVDNs, while voltage violation may also be further deteriorated by mitigating voltage unbalance. The reason is that the existing method of adjusting three-phase voltages to the average value increases the voltage deviation, due to the strong coupling between voltage violation and unbalance. Therefore, to address the two issues simultaneously, it is essential to consider and analyze them in a coordinated manner. Furthermore, a mathematical model is needed to explain and quantify the complex relationship between the two issues.

b) The three-phase structure and regulation capability of active power must be considered in the electrical distance calculation of hybrid AC/DC LVDNs. However, active power will be difficult to be embedded because the electrical distance parameters cannot be obtained. It is because that the comprehensive influence of active and reactive power is considered in the ratio of voltage deviation in the calculation of electrical distances. In this case, the reactive power in the numerator and denominator in the equation cannot be extracted and reduced as a coefficient.

c) Regulation effectiveness can be further improved considering the influence of the neutral line current in LVDNs on the other three phases. Moreover, the coupling between voltage violations and unbalances must be carefully embedded in the regulation method to effectively address the two issues simultaneously.

To tackle the aforementioned technical challenges, this paper proposes a coordinated regulation method for voltage violations and voltage unbalances considering the coupling of the two issues in hybrid AC/DC LVDNs. The specific contributions are summarized as follows:

a) A graph with quantifiable correlation derived from the three-phase analytical model is developed considering the coupling between voltage violations and unbalances. With the graph-based voltage regulation calculation method, voltage violation issues in LVDNs are explicitly classified into different scenarios, based on which the requirements of phase voltage control can be obtained in different scenarios to simultaneously address the two issues.

b) An equivalent coefficient is developed to consider the regulation efficiency of active power in the partition method of three-phase four-wire hybrid AC/DC LVDNs. Therefore, all of the regulation efficiency, cost, and capability of PV, ESSs, and VSCs are accurately considered to effectively determine the control devices in each partition.

c) A sensitivity method of hybrid AC/DC LVDNs is proposed considering the three-phase four-wire topology, based on which a coordinated regulation method for voltage violations and unbalances is proposed based on the developed graph for voltage regulation calculation and the control devices selected by the proposed partition method of hybrid AC/DC LVDNs.

II. PROBLEM DESCRIPTION AND FRAMEWORK

Fig. 1 illustrates the structure of a hybrid AC/DC LVDN. AC side is a three-phase four-wire with a rated voltage of 220 V. DC side is a symmetrical monopole structure with metallic return and the DC rated voltage is ±375 V. Loads and PVs are connected in single-phase between the neutral wire. ESSs and VSCs are connected in three-phase on the AC side. Both active and reactive power outputs of PV inverters, ESSs, and VSCs are utilized for nodal voltage regulation.

Serious voltage violations such as over-voltage and under-voltage and unbalances can frequently and simultaneously occur on the AC side, which are exacerbated due to the increasing unbalanced distribution of distributed generation (e.g. PV). The DC side may face voltage violations because of the integration of single-phase grid-connection PV systems as shown in Fig. 1.

![Fig. 1. Structure of a hybrid AC/DC LVDN.](image)

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is proposed based on the partition method by coordinating multiple devices. The framework of the proposed control method is shown as follows:

**Graph-based voltage regulation calculation method**

- Voltage measurement
  - Three-phase voltage deviations
- Three-phase analytical model
- Divide voltage violation scenarios
- Obtain voltage adjustment ($\Delta U$)

**Partition method**

- Electrical distance with P and Q
- Obj. capacity, cost, and efficiency
- Devices: ESSs, VSCs, and PVs
- Select control devices in each sub-area

**ABCD parameter-based sensitivity calculation**

- Three-phase four-wire topology
- Positions and phases of nodes
- Calculate sensitivities of hybrid AC/DC LVDNs

**Sensitivity-based coordination control method**

- Voltage adjustment
- Sensitivity
- Devices in sub-area
- Determine active and reactive power outputs of devices
- Solve voltage violation and unbalance simultaneously

Fig. 2. The framework of the proposed controlled method.

**a) Graph-Based Voltage Regulation Calculation Method.** The coupling between voltage violation and unbalance is explained in Section III with the three-phase voltage deviations at a node. Then a graph-based regulation calculation method is developed considering voltage violation scenarios and the coupling model, where voltage adjustment is obtained for solving the two issues simultaneously.

**b) Partition Method and ABCD Parameter-Based Sensitivity Calculation.** In Section IV-A, a partitioning method in three-phase four-wire hybrid AC/DC LVDNs is proposed considering the contributions of both active and reactive power on electrical distance calculation. The regulation efficiency, cost, and capability of ESSs, VSCs, and PVs are considered in the multi-objective functions of partition to determine the controllable devices in each sub-area. The optimal partition method is solved through a standard genetic algorithm in Section V. In Section IV-B, a power-voltage sensitivity calculation method is proposed by ABCD parameters for three-phase four-wire hybrid AC/DC LVDNs, considering different positions and phases of nodes. If the network configuration does not change, the sub-areas of LVDNs and the calculated active and reactive power-voltage sensitivity matrices will be constant and stored for use.

e) **Sensitivity-Based Coordination Control Method.** If a voltage issue occurs, the three-phase voltage adjustments will be obtained using a graph-based calculation method based on measured node voltages, and the partition method is utilized to identify the sub-area wherein the node is facing the voltage issue. Then the power outputs of controllable devices within this sub-area will be calculated based on the proposed sensitivity-based coordination control method of hybrid AC/DC LVDNs.

### III. GRAPH-BASED VOLTAGE ADJUSTMENT CALCULATION METHOD

**A. Three-phase Analytical Model of Coupled Voltage Violations and Unbalances**

A three-phase analytical model using the three-phase deviation as variables is proposed to identify the coupling between voltage violation and unbalance in AC LVDNs. The following is the derivation of the proposed coupling model.

The voltage deviations between the measured voltage and the rated voltage in hybrid AC/DC LVDNs as shown in Fig. 1 are:

\[
\Delta U = U_i - U_{\text{rate}}.
\]

(1)

where $U_{\text{rate}}$ is the rated voltage in hybrid AC/DC LVDNs as shown in Fig. 1.

The absolute value of $\Delta U_i$ and the maximum voltage at node $i$ is formulated as follows:

\[
V_{U_i} = (\max [U_{\text{a}}, U_{\text{b}}, U_{\text{c}}]) / U_{\text{a max}} \times 100\% \tag{3}
\]

(3)

where $U_{\text{a max}}$ is the average voltage of three phases at node $i$. The parameter $V_{U_i}$ should not exceed the limit of 2% when the distribution network is in normal operation [24].

It is assumed that the phases of the maximum absolute value of $\Delta U_i$ and the maximum $V_{U_i}$ are both in phase $a$. Thus, (5) can be obtained from (3):

\[
V_{U_i} = \max \left[ \frac{(\Delta U_{i,a} + U_{\text{rate,AC}}) - ((\Delta U_{i,b} + U_{\text{rate,AC}}))}{3 + U_{\text{rate,AC}}} \right] \tag{5}
\]

The absolute value of $\Delta U_{i,a}$ will be maximum when the voltage of phase $a$ is maximum or minimum at node $i$, which means $|\Delta U_{i,a}| = |U_{\text{a max}} - U_{\text{rate,AC}}|$. Therefore, (5) can be converted to

\[
\Delta U_{i,a} = k_1 (\Delta U_{i,b} + \Delta U_{i,c}) + k_2 U_{\text{rate,AC}} \tag{6}
\]

where $k_1$ and $k_2$ are the correlation coefficients. If the voltage of phase $a$ is maximum, $k_1 = (V_{U_i,a} + 1)/(2 - V_{U_i,a})$ and $k_2 = (3 \times V_{U_i,a})/(V_{U_i,a} + 1)$. If the voltage of phase $a$ is minimum, $k_1 = (1 - V_{U_i,a})(V_{U_i,a} + 2)$ and $k_2 = (3 \times V_{U_i,a})(V_{U_i,a} + 2)$.

The coupling between voltage violation and unbalance can be mathematically described through (6).

Furthermore, the $V_{U_i,a}$ is set as the limit value of 2% and is substituted into (6) to further quantify the coupled relation
mentioned above. Therefore, an analytical model of three-phase voltage deviations considering voltage violation and unbalance can be represented as follows (the derivation retains four decimal digits):

1) When $U_{i,a}$ is maximum at node $i$,
   \[ \Delta U_{i,a} \approx 0.5151(\Delta U_{i,b} + \Delta U_{i,c}) + 0.0303. \]  
   (7)

   In this case, the three-phase voltage deviation of node $i$ should meet the constraint
   \[ \Delta U_{i,a} \geq 0.5\left[\Delta U_{i,b} + \Delta U_{i,c}\right]. \]  
   (8)

2) When $U_{i,a}$ is minimum at node $i$,
   \[ \Delta U_{i,a} \approx 0.4851(\Delta U_{i,b} + \Delta U_{i,c}) - 0.0297. \]  
   (9)

   In this case, the three-phase voltage deviation of node $i$ should meet the constraint
   \[ \Delta U_{i,a} < 0.5\left[\Delta U_{i,b} + \Delta U_{i,c}\right]. \]  
   (10)

To facilitate analysis, it is assumed that over-voltage only occurs in phase $a$, thus the sum of the voltage deviations of phases $b$ and $c$ ($\Delta U_{i,b} + \Delta U_{i,c}$) satisfies (11).

\[ \left|\Delta U_{i,b} + \Delta U_{i,c}\right| \leq 0.14. \]  
   (11)

If $U_{i,a}$ is the maximum voltage of the three phases at node $i$, the coupling between single-phase voltage violation and voltage unbalance will be illustrated in Fig. 3(a), according to (7), (8), and (11). If $U_{i,a}$ is the minimum voltage of the three phases at node $i$, the coupled relation will be shown in Fig. 3 (b), according to (9), (10), and (11).

![Fig. 3. A graph of the coupling between the voltage violation and unbalance. (a) The voltage of phase $a$ is maximum at node $i$; (b) the voltage of phase $a$ is minimum at node $i$.](image)

The feasible regions in Fig. 3 are further divided into four subregions, considering the constraints of voltage deviation and unbalance issues:

1) **The blue subregion:**
   AC LVDNs operate normally.

2) **The yellow subregion:**
   Only three-phase voltage unbalance occurs in AC LVDNs.

3) **The green subregion:**
   Only single-phase voltage deviation occurs in AC LVDNs.

4) **The red subregion:**
   Voltage deviation and unbalance occur simultaneously in AC LVDNs.

It should be noted that the phase with the maximum voltage deviation can be different from the phase with the maximum $VU_{i,a}$ at node $i$. However, The method proposed in this paper is still applicable because this approximation assumes that the two phases are the same and therefore has little effect on the regulation results.

Based on the graph shown in Fig. 3, the voltage violation and/or voltage unbalance in LVDNs can be easily known through the three-phase voltage deviations obtained from the voltage measurement at any nodes.

**B. Voltage Adjustment Calculation Method under Different Voltage Violation Scenarios**

In this section, a graph-based calculation method of voltage regulation requirements considering coupled voltage violation and unbalance is developed, according to Fig. 4.

![Fig. 4. A graph for voltage regulation requirement calculation.](image)

Three voltage violation scenarios are divided in LVDNs for the convenience of discussion: single-phase voltage violation, two-phase voltage violation and three-phase voltage violation.

1) **Single-phase voltage violation:**
   If only one phase voltage exceeds the upper limit at node $i$, it will be discussed in the following three cases according to Fig. 4.
   a) Case 1– Over-voltage in phase $a$ occurs as in Yellow regions (BDE): the minimum voltage regulation can be calculated through the red control curve BD.
   b) Case 2– Voltage unbalance occurs as in Yellow regions (ABC): the minimum voltage regulation can be calculated through (8) which is the red control curve BC.
   c) Case 3– Over-voltage and unbalance occur simultaneously in phase $a$ as in Red regions (ABE): the minimum voltage regulation can be calculated through the red control curve CBD.

When phase $a$ voltage exceeds the lower limit, the voltage
2) **Two-phase voltage violation:**
When extreme voltage violations of two phases at node \( i \) occur, the voltage of one phase will be firstly regulated to the voltage limit no matter whether there is voltage unbalance or not. Then the regulation requirement can be determined by the first scenario because the two-phase voltage violation will be turned into a single-phase voltage violation.

3) **Three-phase voltage violation:**
When extreme voltage deviation of the three-phase at node \( i \) exceeds the voltage limit, it can be discussed as the following two cases:

a) Case 1– All the three-phases voltages exceed the upper or lower limit: for example, if they exceed the upper limit, the three-phase voltages will be adjusted to the upper voltage limit 1.07 p.u., as shown at point D in Fig. 4. On the contrary, other situations will be opposite which the three-phase voltages will be adjusted to the lower limit 0.93 p.u., as shown at point D’ in Fig. 4. The \( V_U \) in LVDNs will be 0 % after doing this.

b) Case 2– The three phases experience both over-voltage and under-voltage: there will be a serious unbalance if regulate the three-phases voltages to the limits as in Case 1, (point A or A’). Therefore, if phase \( a \) voltage exceeds the lower limit and the voltages of phases \( b \) and \( c \) exceed the upper limit, the voltages of phases \( b \) and \( c \) will be adjusted to the upper limit 1.07 p.u. and the voltage regulation of phase \( a \) is calculated through the red control in Fig. 4. The \( V_U \) in LVDNs will be 0 % after doing this.

The graph-based voltage regulation calculation method for different voltage violation scenarios in LVDNs is summarized in Fig. 5, based on which the voltage adjustment can be easily calculated. Moreover, the voltage violation and unbalance at one node can be solved simultaneously by adjusting the devices only once or twice.

**Objective Function**
Electrical distance is a measure of the relative variation in voltage magnitudes between two nodes resulting from an alteration in active or reactive power injection at one of the nodes. In this way, the electrical coupling between two nodes can be established by observing the attenuation of voltage variations at nodes \( i \) and \( j \) as follows:

\[
\Delta U_i = \alpha_i A U_j ,
\]  

where \( \alpha_i \) is the interrelationship of voltage variation between nodes \( i \) and \( j \) in AC/DC LVDNs. It should be noted that both active and reactive power affect the voltage in AC LVDNs. Therefore, an equivalent coefficient \( \gamma_{ij} \) is developed for converting the active power to virtual reactive power. The parameters \( \alpha_i \) of AC side and DC side LVDNs can be obtained through (13) and (14).

\[
\alpha_{AC} = \sum_{\phi=a,b,c} \frac{\Delta U_{i,\phi}}{\Delta U_{j,\phi}} = \sum_{\phi=a,b,c} \gamma_{ij} \phi \sum_{\phi=a,b,c} \phi^{AC,\phi} \phi^{AC,\phi} + \phi^{AC,\phi} ,
\]  

\[
\alpha_{DC} = \frac{\Delta U_{i,j}}{\Delta U_{j}} \frac{S_{ij}}{S_{ij}},
\]

where \( S_{ij}^{AC,\phi} \) and \( S_{ij}^{DC,\phi} \) are active power-voltage sensitivity and reactive power-voltage sensitivity of AC LVDNs, and \( \gamma_{ij} \) is the ratio of them [22]. \( S_{ij}^{AC,\phi} \) is the active-power-voltage sensitivity of DC LVDNs.

To obtain symmetrical distances, (15) is taken as the definition of the electrical distances between nodes \( i \) and \( j \) at LVDNs:

\[
D_{ij} = D_{ji} = -\log(\alpha_i \alpha_j). \tag{15}
\]  

Considering the three-phase four-wire configuration, the first objective of the proposed partitioning optimization model is to minimize the product of maximum electrical distances in \( n \) sub-areas. In this case, the differences in the regulation efficiency between sub-areas can be reduced, which can be explained as:

\[
f_i = \min \prod_{n=1}^{N} D_{n,max} ,
\]

where \( D_{n,max} \) is the maximum electrical distance between the nodes with devices and other nodes in the \( n-th \) sub-area. The smaller the electrical distance between nodes, the higher the efficiency of device regulation [27], [28]. \( N \) is the number of partitions.

The second objective is to minimize the total squares of deviations of real and reactive power reserves considering both
regulation capacity and cost.

\[ f_2 = \min \sum_{n=1}^{N} (R_n - \overline{R})^2, \]  

(17)

where \( R_n \) is the power reserve of real and reactive power in partition \( n \). \( \overline{R} \) is the average power reserve of hybrid AC/DC LVDNs.

The power reserves \( R_n \) in AC and DC partitions can be calculated according to (18) and (19), which considers both maximum regulation capacities, cost of regulation devices and voltage requirements of each partition.

\[ R_{nAC}^{SC} = \sum_{l=1}^{w} C_{l, n}^{AC} \sum_{i=1}^{n} S_{l, i}^{AC, \text{wp}} + \sum_{l=1}^{w} C_{l, n}^{SC, CD} \sum_{i=1}^{n} S_{l, i}^{SC, \text{qp}}, \]  

(18)

\[ R_{nDC}^{SC} = \sum_{l=1}^{w} C_{l, n}^{DC, CD} \sum_{i=1}^{n} S_{l, i}^{DC, \text{qp}}, \]  

(19)

where \( v \) and \( w \) are the number of controllable devices and the number of nodes in partition \( n \). \( C_i \) is the regulation cost per unit power of controllable devices. \( P_{\text{wp}}^{AC, CD} \) and \( Q_{\text{qp}}^{SC, CD} \) are active and reactive power capabilities of controllable devices in partition \( n \). If the ratio \( R_n \) is greater than or equal to 1, the total regulation capacity of the devices considering power regulation cost can meet the demand of voltage regulation, and the power in this sub-area will be sufficient. If the ratio \( R_n \) is less than 1, the power regulation capacity in this sub-area will be insufficient.

**Constraints**

The ratings of the controllable devices are considered as the constraints of the partitioning optimization method:

\[(P_n^{\text{CD}})^2 + (Q_n^{\text{CD}})^2 = (S_n^{\text{CD}})^2,\]

(20)

where \( S_{\text{wp}}^{\text{CD}} \) represents the single-phase rated capacity of PVs, ESSs, or VSCs, and the active power of VSCs also needs to meet the constraint:

\[ \sum_{i \in \{a, b, c\}} P_{\text{wp}}^{AC, VSC} = P_{\text{DC, VSC}}^{\text{DC, VSC}} + P_{\text{loss}}^{\text{DC, VSC}}, \]

(21)

where \( P_{\text{loss}}^{\text{DC, VSC}} \) is the power loss of VSCs and is 2% of the rated capacity. \( P_{\text{wp}}^{AC, VSC} \) and \( P_{\text{DC, VSC}}^{\text{DC, VSC}} \) are the active power of VSCs on the AC side and DC side.

**B. Sensitivity-Based Voltage Regulation Method**

A partition-based coordination strategy is developed considering the three-phase four-wire sensitivities in hybrid AC/DC LVDNs. The coordinated voltage control strategy will be triggered if voltage violation and voltage unbalance occurred.

**Sensitivities of hybrid AC/DC LVDNs with three-phase four-wire topology**

The ABCD parameters-based sensitivity calculation method is adopted in this paper considering the three-phase four-wire topology of hybrid AC/DC LVDNs [8]. The voltage sensitivities at node \( j \) due to the injected active and reactive power at node \( i \) are:

\[ S_{ji}^{\text{wp}} = - \sum_{i \in \{a, b, c\}} R_{ji} / U_{\text{rate}}, \]

(22)

where \( C_{l, j} \) is the set of all lines between nodes \( i \) and \( j \). \( R_l \) and \( X_l \) are the equivalent resistance and reactance of the power distribution lines \( l \), which can be calculated through ABCD parameters considering the three-phase four-wire topology on the AC side. The size of the \( S_{ji}^{\text{wp}} \), \( S_{ji}^{\text{qp}} \), \( R_l \), and \( X_l \) matrices is 4x4.

The positions and phases of the nodes will affect the sensitivity calculation as shown in Fig. 6. \( x (x \in \{a, b, c, n\}) \) and \( v (v \in \{a, b, c, n\}) \) (\( x \neq v \)) are used to describe the phases of the two selected nodes.

In a three-phase four-wire line model, the ABCD parameters are the coefficients in the relationship between the voltage and current of two nodes calculated through Kirchhoff’s Current and Voltage Laws [8], [29]. The ABCD parameter-based calculations of \( R_l \) and \( X_l \) are developed in the following two scenarios as shown in Fig. 6.

(1) Same node \((i \rightarrow i)\)

(2) Different node

2.1&2.2) Same line

2.1) Voltage at node \( i \) due to the injected power at node \( j \) \((i \rightarrow j)\).

2.2) Voltage at node \( i \) due to the injected power at node \( j \) \((i \rightarrow j)\).

2.3) Different lines \((i \rightarrow z)\)

Fig. 6. Positions and phases of the selected nodes in a radial LVDN.

The calculations of \( R_l \) and \( X_l \) are developed in the following two scenarios as shown in Fig. 6:

\[
\begin{align*}
\sum R_l &= \begin{bmatrix} \text{Re}(B_{ji}) \end{bmatrix}_{ij}, & \text{if case 1} \\
\sum R_l &= \begin{bmatrix} \text{Re}(A_{ji}) + [B_{ji}]_{ij} \end{bmatrix}_{ij}, & \text{if case 2.1} \\
\sum X_l &= \begin{bmatrix} \text{Re}(A_{ji}) + [B_{ji}]_{ij} \end{bmatrix}_{ij}, & \text{if case 2.2} \\
\sum X_l &= \begin{bmatrix} \text{Re}(B_{ji}) \end{bmatrix}_{ij}, & \text{if case 1} \\
\sum X_l &= \begin{bmatrix} \text{Re}(A_{ji}) + [B_{ji}]_{ij} \end{bmatrix}_{ij}, & \text{if case 2.1} \\
\sum X_l &= \begin{bmatrix} \text{Re}(A_{ji}) + [B_{ji}]_{ij} \end{bmatrix}_{ij}, & \text{if case 2.2} \\
\sum X_l &= \begin{bmatrix} \text{Re}(B_{ji}) \end{bmatrix}_{ij}, & \text{if case 1}
\end{align*}
\]

(24)

(25)

where \([B_{ji}]_{ix}\) is the element in column \( x \) of matrix \( B_j \) and \([B_{ji}]_{nv}\) is the element in row \( v \) of matrix \( B_j \). The size of the \( A, B, a, and b \) matrices is 4x4.

The ABCD parameter calculation method is also applicable to DC distribution networks. The reactive power-voltage sensitivity \( S_{ji}^{\text{qp}} \) of the DC side will be set as zero because there is no reactance of DC lines.

Thus, the active and reactive power adjustment of
controllable devices can be obtained through sensitivity from (26) and (27):
\[
\Delta P = \Delta U / S^{op},
\]
\[
\Delta Q = \Delta U / S^{vq}.
\]

It should be mentioned that the active and reactive power of ESSs, VSCs and PVs can be independently controlled, and the ABCD sensitivity can be utilized for both AC and DC sides.

The changes in active and reactive power of different nodes have a superimposed impact on node voltage, therefore the voltage variation of any node during the coordinated control of active and reactive power of the controllable devices will be satisfied [30]:
\[
\Delta U = \sum S^{op} \Delta P + \sum S^{vq} \Delta Q.
\]

**Coordination strategy**

1) On the AC side

The voltage regulation requirements will be calculated as shown in Fig. 4 if a voltage violation or voltage unbalance occurs. The first priority is to regulate the reactive power of PV inverters in the same partition when PV capacity is still available. The active and reactive power of PV should meet the constraint of DG’s capacity and the adjustment of \( \Delta Q^{PV} \) is obtained through (30).

\[
Q_{PV}^{PV} = \pm \sqrt{(S^{iv})^2 - (P_{PV}^{PV})^2},
\]
\[
\Delta Q_{i,\varnothing}^{PV} = \Delta U_{i,\varnothing} / S^{AC,vq},
\]

where \( S^{iv} \) is the rated capacity of DG, \( P_{PV}^{PV} \) is the active power of DG and \( \Delta Q \) should not exceed \( \Delta P^{PV} \).

Secondly, if the reactive power adjustment cannot be achieved due to the power rating limit of PV, the ESSs or VSCs in the same partition will be considered to regulate the voltage. For VSCs, the reactive power is firstly considered to regulate the voltage because the DC line is not affected by the reactive power regulation on the AC side. The reactive power adjustment \( \Delta Q^{VSC} \) can be calculated by (30). For ESSs, the active power is firstly adjusted to regulate the voltage to avoid the curtailment of PVs, and the active power adjustment \( \Delta P^{ESS} \) can be obtained from:
\[
\Delta P = \Delta U_{i,\varnothing} / S^{AC,op}.
\]

If the VSCs’ reactive capacity or ESS’s active power capacity is insufficient when alleviating AC side voltage violation, both active and reactive power will be used to coordinate and regulate node voltages:
\[
\begin{align*}
\Delta U_{i,\varnothing} &= S^{AC,op} \cdot \Delta P^{AC} + S^{AC,vq} \cdot \Delta Q^{AC} \\
\left( (P^{AC} + \Delta P^{AC})^2 + (Q^{AC} + \Delta Q^{AC})^2 \right)^{1/2} &= S^{CD2},
\end{align*}
\]

DC voltage variation will occur when the active power regulation on the AC side changes. Therefore, the absolute value of ESSs’ or VSCs’ active power will be reduced to release the reactive power capacity, considering reducing the active power regulation effect on the DC side.

Finally, if LVDNs still have voltage violation or voltage unbalance after the adjustment of PVs, ESSs, and VSCs, the curtailment of PVs in the same partition will be proposed to maintain normal voltage on the AC side. \( \Delta P^{PV} \) and \( \Delta Q^{PV} \) can be calculated through (32).

2) On the DC side

If a DC voltage exceeds the limits, the active power of VSCs or ESSs in the same partition will be primarily adjusted to regulate node voltages according to (31). If the active capacity is insufficient, equation (32) will be used to achieve the coordinated control of active power and reactive power so as to solve voltage violation. The absolute value of active power (\( \Delta P^{VSC} \) or \( \Delta P^{ESS} \)) will be released as could as possible by reducing reactive power (\( \Delta Q^{VSC} \) or \( \Delta Q^{ESS} \)). And the curtailment of PV will be adopted when voltage violation is not solved completely through all the above methods.

The flowchart of the proposed sensitivity-based coordination control method for voltage violation and voltage unbalance in hybrid AC/DC LVDNs is shown in Fig. 7.

---

**V. ALGORITHM**

Optimal partition results are obtained through a standard genetic algorithm, and the optimization model is as follows:
\[
\begin{align*}
\min & \quad f_{1} + f_{2} \\
\text{s.t.} & \quad (20) \text{ and } (21)
\end{align*}
\]

In (33), \( f_{1} \) and \( f_{2} \) are the objective functions, including (12)-(16) and (17)-(19) respectively. \( f_{1} \) and \( f_{2} \) divided by \( F_{1} \) and \( F_{2} \) are to normalize the dimension. The \( \Delta U_{i} \) in (18) and (19) is the maximum voltage regulation requirement of each partition, which can be obtained from the worst scenario of the day based on the graph method proposed in Section III.

Using binary encoding and each line \( l \) in hybrid AC/DC LVDNs is represented by a gene \( g_{l} \) to ensure the connectivity of partition. These genes form a chromosome:
\[
g = (g_{1}, g_{2}, g_{3}, \ldots, g_{l})
\]

If the two nodes of line \( l \) are within the same partition, \( g_{l} \) will be 0, otherwise, \( g_{l} \) will be 1.
VI. CASE STUDY

A. Simulation Conditions

A 30-node hybrid AC/DC LVDN embedded with PVs and ESSs, as shown in Fig. 7, is used to verify the proposed method. The DC line denoted by the red line and the AC line, which is represented by the black line, are modified by [24]. The typical profiles of PV output and load for a 24-hour period are given in [24]. Details of the network parameters and load data can be found in [24] as well. The rated power of single-phase PV generation is 3 kW and the rated power of ESS is 5 kWh which is connected to the distribution network in three-phase and can be adjusted in a single phase. The rated capacity of VSC2 is 20 kW. The voltages on the secondary side of distribution transformers are fixed as 1.0 p.u. The penetration of PVs in this study is 127.07%.

Fig. 8. A single-line diagram hybrid AC/DC LVDN integrated with PVs and ESSs.

B. Study-1 Partition Method

In Result 1, the partition method only considers the reactive power capability in electrical distance calculation. In Result 2, the equivalence of real power to reactive power is considered in the proposed partition method.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Partition results</th>
<th>Regulation efficiency</th>
<th>Regulation capacity and cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result 1</td>
<td>[2-7,19-21], [8-10], [11-18], [22-25], [26-30]</td>
<td>0.67</td>
<td>0.45</td>
</tr>
<tr>
<td>Result 2</td>
<td>[2-5,19-21], [6-10], [11-18], [22-26], [27-30]</td>
<td>0.35</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Fig. 8 illustrates the partitioning results of the proposed method in Result 2. For the AC side, nodes 6 and 7 are assigned to Area 2 in Result 2 and Area 1 in Result 1 due to the lower cost of PVs compared with VSC2. Moreover, VSC2 has sufficient active power regulation capacity to satisfy the high voltage regulation requirement in Area 2.

For the DC side, node 26 is in Area 4 in Result 2 and in Area 5 in Result 1, as shown in Table 1. Due to the considerable rated capacity of VSC2, the active power regulation capacity of VSC2 is higher than the ESS at node 30. Therefore, the regulation effect of Result 2 is better than Result 1 because the electrical distance from node 26 to Area 4 equals the distance to Area 5 and the comparable adjustment costs of VSC2 and ESS.

C. Study-2 Voltage Control Method for Solving Voltage Violation and Unbalance Simultaneously

Cases 0 to 3 represent scenarios employing different control strategies, whereas Case 4 is characterized by the implementation of the sensitivity-based control strategy proposed in this paper.

Case 0: Without control.
Case 1: Based on the priority-based voltage control method, the voltage unbalance will be solved first and then the voltage violation [8].
Case 2: Solve the voltage violation with PVs and ESSs and eliminate voltage unbalance with VSC2 simultaneously.
Case 3: Solve the two issues simultaneously based on partitioning Result 1.
Case 4: Solve the two issues simultaneously based on partitioning Result 2 of the proposed partition method.

In Case 0, the distribution network on the AC side has serious voltage violation and unbalance during the day, as shown in Fig. 9. The two issues occur simultaneously from 10:00 to 16:00. The voltage of phase $a$ at node 8 reaches the peak at 13:00, which is 1.1064 p.u. At this time, the maximum $VU$ is 2.71% at node 9 of phase $c$.

In Case 1, the overvoltages at nodes 5 to 18 become worse and the three-phase voltages all exceed the upper limit after managing voltage unbalance first, as shown in Fig. 10(b). It is because that the average voltage of node 9 is greater than 1.07 p.u., which causes the voltages of both phases $a$ and $b$ to be too high to exceed the limit. Implementing the prioritized control method to regulate the voltages to their average value (1.0879 p.u.) can mitigate the unbalance. As illustrated in Fig. 11, the $VU$ of Case 1 has been regulated as the minimum. However, it results in overvoltage in phase $c$, as indicated in Fig. 10(b). Although subsequent voltage control measures can stabilize the operation of the LVDN, regulating the voltage of phase $c$ brings unnecessary device interventions, which will diminish their lifespan.

In Case 2, PVs absorb reactive power to reduce the voltages of phases $a$ and $b$ at node 8 at 13:00. The ESS at node 4 further enhances the voltages of phases $a$ and $b$ by being charged when the capacity of PV is insufficient. At the same time, to alleviate the three-phase unbalance at node 9, VSC2 outputs the reactive power of phase $c$ and absorbs the reactive power of phases $a$ and $b$, as depicted in Fig. 12. However, the voltages of node 8 are overregulated due to the superposition of the regulation effect of ESS and VSC. Therefore, there still voltage violation and unbalance at nodes 7 to 10. Iteration of “voltage measurement and regulation” in Case 2 is needed to solve the
two issues simultaneously, and the iteration result is shown in Fig. 10(c).

![Fig. 10. Three-phase voltages on the AC side at 13:00. (a) Case 0; (b) Case 1; (c) Case 2; (d) Case 3; (e) Case 4.](image)

Fig. 10. Three-phase voltages on the AC side at 13:00.

![Fig. 11. VUs of different cases at 13:00.](image)

Fig. 11. VUs of different cases at 13:00.

increasing reactive power absorbed from the AC lines in Case 4. Although both active and reactive power is coordinated and regulated, the VSC cannot control the voltage back to 1.07 p.u. due to the limited regulation capacity in Case 3. Therefore, the curtailment of about 1.66 kWh of PVs is triggered to mitigate the overvoltages on the AC side.

![Fig. 13. Power of controllable devices for Cases 3 and 4. (a) Case 3; (b) Case 4.](image)

The regulation costs of ESS, VSC, and PV are shown in Table II. The regulation cost in Case 3 exceeds the cost in Case 4 by $2,469.1 due to the high curtailment cost of PVs in Table III. Moreover, in Case 4, the inclusion of PVs at nodes 6 and 7 within Area 2 leads to a reduction in the output of VSC2. This leads to a further reduction in the total expense in Case 4 due to the cost-effectiveness of PV reactive power regulation.

### Table II

<table>
<thead>
<tr>
<th>Devices</th>
<th>Active power regulation ($/kWh)</th>
<th>Reactive power regulation ($/kVarh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>0.3127 (curtailment)</td>
<td>0.0094</td>
</tr>
<tr>
<td>ESS</td>
<td>0.0819</td>
<td>0.0104</td>
</tr>
<tr>
<td>VSC</td>
<td>0.1941</td>
<td>0.0345</td>
</tr>
</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Adjustment Costs ($) for Cases 3 and 4</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The curtailment of PV</td>
<td>0.5175</td>
<td>0</td>
</tr>
<tr>
<td>Reactive power regulation of PV</td>
<td>0.0863</td>
<td>0.1617</td>
</tr>
<tr>
<td>Active power regulation of VSC</td>
<td>1.5095</td>
<td>0</td>
</tr>
<tr>
<td>Reactive power regulation of VSC</td>
<td>2.4691</td>
<td>1.9731</td>
</tr>
<tr>
<td>Total cost</td>
<td>4,5716</td>
<td>2,1348</td>
</tr>
</tbody>
</table>

D. Study-2 Coordinated Control Method Based on Three-Phase Four-Wire Sensitivity

The “Perturb-and-observe” sensitivity calculation method, the three-phase three-wire sensitivity calculation method...
without considering the effect of the neutral line and the three-phase four-wire sensitivity calculation method presented in this paper are used to calculate the sensitivities of AC LVDNs. Voltage qualified rate (VQR) is used to represent the control efficiency in different cases, and the comparison of different sensitivity calculation methods is shown in Table IV. The first method with the highest accuracy is taken as a comparative example.

**TABLE IV**  
**Comparison of Control Results Based on Different Sensitivity Calculation Methods**

<table>
<thead>
<tr>
<th>Sensitivity calculation methods</th>
<th>VQR(%)</th>
<th>Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perturb-and-observe</td>
<td>100%</td>
<td>0.2536</td>
</tr>
<tr>
<td>Three-phase three-wire</td>
<td>98.68%</td>
<td>0.1725</td>
</tr>
<tr>
<td>Three-phase four-wire</td>
<td>99.80%</td>
<td>0.1976</td>
</tr>
</tbody>
</table>

Results show that the calculation time of the three-phase four-wire sensitivity calculation method is 0.0251s longer than that of the three-phase three-wire sensitivity calculation method. The calculation time of the “Perturb-and-observe” sensitivity calculation method is the longest because it requires cyclic power flow calculation for several times.

The difference between the control strategy based on the sensitivity considering the influence of the neutral line and the control result of the comparison example is the smallest, the VQR difference is 0.20%. The accuracy of the proposed method is higher than that of the control method based on three-phase three-wire sensitivity, and the VQR is 1.12% higher than it.

**VII. CONCLUSION**

This paper proposed a partition-based regulation method for managing voltage violation and voltage unbalance simultaneously in three-phase four-wire hybrid AC/DC LVDNs with high penetration of PVs. Unlike the existing priority-based voltage control strategy, the proposed method considers the coupling between voltage violation and unbalance and describes it using a graph based on the three-phase analytical model. The proposed graph visualizes the coupling of the two issues clearly with different subregions and is used to calculate the voltage adjustment for solving the two issues simultaneously. This method is also superior to the existing partition-based regulation method since the electrical distance calculation considers the regulation capability of active power and the three-phase structure, which is more suitable for hybrid AC/DC LVDNs to determine controllable devices in each part. Moreover, the proposed three-phase four-wire sensitivity-based coordinated control method fulfills the speed requirement and enhances accuracy for voltage management. The performance of the proposed voltage control strategy has been demonstrated using a real 24-hour load profile on an unbalanced test hybrid AC/DC LVDNs.

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