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Cavity-birefringence-dependent vector pure-quartic soliton fiber laser

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Abstract: Pure-quartic soliton (PQS) fiber lasers provide a promising avenue for exploring novel soliton interaction dynamics and generating high-energy pulses. Here, we present the numerical observation of vector PQSs generation and the evolution dynamics in a mode-locked fiber laser, using the coupled Ginzburg-Landau equations. We investigate the buildup dynamics of vector PQSs in a mode-locked laser with birefringent fibers, passing through three stages: energy amplification, energy pulsation owing to the cross-phase modulation (XPM) effect, and finally stabilization. Depending on the strength of the cavity-birefringence, the evolution of PQSs in non-polarization-maintaining fibers reveals that both the elliptical-polarization vector PQSs and near-linear-polarization vector PQSs can be formed by the energy conservation and balance between the two orthogonal directions. Additionally, we observe the transition process from vector PQSs to scalar PQSs with higher cavity-birefringence, resulting from the failure compensation of the walk-off via the soliton trapping effect between the two orthogonal components. These results provide valuable insights into the ultrafast transient process of vector solitons and enhance the understanding of PQS generation in fiber lasers.

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1. Introduction

Pure-quartic solitons (PQSs), as a specific type of solitons in the nonlinear system, are generated through a balance between the fourth-order dispersion and nonlinear effects [1–4]. PQSs in fiber lasers possess outstanding advantages in energy scaling compared to traditional quadratic solitons. Furthermore, they are relatively insensitive to noise and perturbations, which potentially makes them useful for long-distance communication applications. However, the lack of a suitable experimental platform, such as a piece of fiber with both quadratic and cubic dispersion close to zero, makes limited research in this field. Magnus et al. numerically presented an exact localized analytical solution to the pulse propagation in a piece of fiber that includes the negative second-order and fourth-order dispersion [5]. Michel et al. considered the effect of third-order dispersion on the optical soliton propagation in the presence of fourth-order dispersion [6]. Until 2020, the first PQS fiber laser with both net second-order and third-order dispersion being
zero was experimentally demonstrated, using a spatial light modulator to adjust the net cavity dispersion. It implies that the pulse energy of the PQS scales with the third power of the inverse pulse duration [7]. Since then, a burgeoning corpus of research about PQSs in fiber lasers has been conducted such as the pulsating PQS soliton [8,9], the dissipative PQS [10], the creeping and erupting PQS soliton [11], the dissipative soliton resonance [12], etc.

Due to the material imperfection and fabrication imperfection, intrinsic birefringence always exists in optical fibers. The walk-off effect between the orthogonal polarization solitons typically occurs in non-polarization-maintaining fibers with weak birefringence, while the soliton trapping effect can shift their frequency in opposite directions and exchange their energy through four-wave mixing and cross-phase modulation (XPM), resulting in the formation of a vector soliton in this case [13–16]. Numerous types of vector solitons have been numerically predicted and experimentally demonstrated, such as the group-velocity-locked vector solitons [17,18], polarization-locked vector solitons [19,20], polarization rotation vector solitons [21,22], domain wall vector solitons [23,24]. In principle, a vector soliton is a self-sustaining entity with both polarization and spatial degrees of freedom, giving rise to many interesting phenomena that can be observed by adjusting the cavity-birefringence intensity [25–27]. As for the PQS, its long oscillating tail in the time domain can easily promote the XPM between the two orthogonal components in a weak-birefringence fiber, resulting in fascinating and unique nonlinear dynamics. Consequently, there is a great potential for conducting research on the generation and internal motion of the vector PQS in fiber lasers.

In this study, a numerical model of the vector PQS fiber laser is established based on the coupled Ginzburg-Landau equations. Starting from a pair of weak pulses with orthogonal polarization states, the stable vector PQS can be obtained with weak cavity-birefringence. The simulation results show that the gain competition caused by the XPM effect between the two orthogonal components plays a crucial role in the vector PQS buildup process, which is demonstrated by the obvious complementary energy oscillation processes. Enhancing the strength of the cavity-birefringence, both elliptical-polarization vector PQSs and near-linear-polarization PQSs can be generated. Furthermore, the transition from vector PQSs to scalar PQSs has also been observed due to the imbalance between walk-off and the trapping effect between the two orthogonal components. The results indicate that weakly birefringent fibers also support the generation and propagation of the vector PQSs. The gain competition between the two orthogonal components caused by the XPM effect remains the primary reason for the formation of vector solitons, and the strength of cavity-birefringence dominates the buildup dynamics of vector PQSs.

2. Numerical methods

Our simulation model, as shown in Fig. 1, is composed of six parts in a fiber laser, which includes a pump laser source, a wavelength division multiplexer (WDM), a 2-m Erbium-doped fiber (EDF), a saturable absorber (SA), a 2-m passive single-mode fiber (SMF), and an optical coupler (OC). The pump light is injected into the cavity through the WDM for signal amplification in the EDF. The SA is adopted to initialize the mode-locking operation. 10% of the intra-cavity power is emitted from the OC.

The laser pulse propagates in the cavity is governed by the coupled Ginzburg-Landau equations [28–30]:

$$\frac{\partial A_x}{\partial z} = \frac{1}{2} \Delta \beta_1 \frac{\partial A_x}{\partial T} - \frac{i \beta_2}{2} \frac{\partial^2 A_x}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3 A_x}{\partial T^3} + \frac{i \beta_4}{24} \frac{\partial^4 A_x}{\partial T^4} + \frac{g - \alpha}{2} A_x + \frac{g}{2 \Omega_k^2} \frac{\partial^2 A_x}{\partial T^2} + i \gamma$$

$$\left( |A_x|^2 + \frac{2}{3} |A_y|^2 \right) A_x + \frac{i \gamma}{3} A_x^2 A_y^2 \exp(-2i \Delta \beta z)$$
Fig. 1. Configuration of the vector PQS fiber laser. WDM, wavelength division multiplexer; EDF, Erbium-doped fiber; SA, saturable absorber; SMF, single-mode fiber; OC, optical coupler.

\[
\frac{\partial A_y}{\partial z} = -\frac{1}{2} \Delta \beta y \frac{\partial A_y}{\partial T} - \frac{i \beta_2}{2} \frac{\partial^2 A_y}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3 A_y}{\partial T^3} + \frac{i \beta_4}{24} \frac{\partial^4 A_y}{\partial T^4} + \frac{g - \alpha}{2} A_y + \frac{g}{2 \Omega_g^2} \frac{\partial^2 A_y}{\partial T^2} + i \gamma \left( |A_y|^2 + \frac{2}{3} |A_x|^2 \right) A_y + \frac{i \gamma}{3} A_x^* A_x^2 \exp(2i \Delta \beta z)
\]

Here \( A_x \) and \( A_y \) are the slowly varying amplitudes of two orthogonal polarized pulse envelopes along slow axis and fast axis of fibers. \( z \) and \( T \) are the propagation distance and the transformation time for the retarded frame. Our simulation is based on the fiber with linear birefringence and includes the second-order dispersion \( \beta_2 \), the third-order dispersion \( \beta_3 \), and the fourth-order dispersion \( \beta_4 \). \( \Delta \beta \) and \( \Delta \beta_1 \) are related to the linear birefringence which can be written as:

\[
\Delta \beta = \beta_{0x} - \beta_{0y} = \frac{w_0}{c} B_m
\]

\[
\Delta \beta_1 = \beta_{1x} - \beta_{1y} = \frac{B_m}{c}
\]

where the \( w_0 \) is the central optical frequency, \( c \) is the light speed and \( B_m \) is the strength of modal birefringence defined as the difference between the modal refractive indices along the slow axis \( x \) and the fast axis \( y \):

\( B_m = n_x - n_y \)

The coupled Ginzburg-Landau equations are numerically solved using the fourth-order Runge–Kutta method in the interaction picture [31]. Apart from the birefringence related parameters, the two axes share the same values for other parameters, including the fiber loss \( \alpha \), the gain bandwidth \( \Omega_g \), the nonlinear coefficient \( \gamma \), and the saturable gain \( g \) of the fiber, which can be written as [32,33]:

\[
g = g_0 \exp \left( -\frac{\int (|A_x|^2 + |A_y|^2) dt}{E_{sat}} \right)
\]

where \( g_0 \) is the small signal gain and \( E_{sat} \) is the saturation energy of the EDF; \( g_0 \) is set to be 0 for the passive SMF; Besides, the fiber laser is pulse shaped by the SA with the transmission function [34,35]:

\[
T = 1 - \frac{q_0}{1 + \frac{|A_x|^2 + |A_y|^2}{P_0}}
\]

where \( q_0 \) and \( P_0 \) are the modulation depth and the saturation power of the SA. The specific values of all the parameters are listed in Table 1.
Table 1. Parameters used in the simulations.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_2$</td>
<td>$\beta_{2-\text{EDF}} = 0 \text{ ps}^2/\text{km};$</td>
</tr>
<tr>
<td></td>
<td>$\beta_{2-\text{SMF}} = 0 \text{ ps}^2/\text{km}$</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>$\beta_{3-\text{EDF}} = 0 \text{ ps}^3/\text{km};$</td>
</tr>
<tr>
<td></td>
<td>$\beta_{3-\text{SMF}} = 0 \text{ ps}^3/\text{km}$</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>$\beta_{4-\text{EDF}} = 10 \text{ ps}^4/\text{km};$</td>
</tr>
<tr>
<td></td>
<td>$\beta_{4-\text{SMF}} = -10 \text{ ps}^4/\text{km}$</td>
</tr>
<tr>
<td>$\gamma_{\text{EDF}}$</td>
<td>$\gamma_{\text{EDF}} = 3 \text{ W/km}$</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{\text{SMF}} = 3 \text{ W/km}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\alpha = 0 \text{ dB/km}$</td>
</tr>
<tr>
<td>$\Omega_{g}$</td>
<td>$\Omega_{g} = 49 \text{ nm}$</td>
</tr>
<tr>
<td>$w_0$</td>
<td>$w_0 = 2\pi c/\lambda_0$ ($\lambda_0$: 1560 nm)</td>
</tr>
<tr>
<td>$g_0$</td>
<td>$g_0 = 900 \text{ /km}$</td>
</tr>
<tr>
<td>$E_{\text{sat}}$</td>
<td>$E_{\text{sat}} = 0.01 \text{ nJ}$</td>
</tr>
<tr>
<td>$q_0$</td>
<td>$q_0 = 50%$</td>
</tr>
<tr>
<td>$P_0$</td>
<td>$P_0 = 100 \text{ W}$</td>
</tr>
<tr>
<td>$c$</td>
<td>$c = 300000 \text{ km/s}$</td>
</tr>
</tbody>
</table>

3. Results and discussions
3.1. Elliptical-polarization vector PQSs

Adjustment of the cavity-birefringence has a significant impact on the generation and evolution dynamics of vector solitons. To observe the buildup dynamics of the vector PQSs, different values of the modal birefringence $B_m$ have been adopted. In this study, we first investigate the evolution dynamics of the vector soliton when $B_m$ is $0.1 \times 10^{-6}$, as shown in Fig. 2. The simulation starts with the initial weak pulse: $A = \sqrt{P_0} \times \text{sech}(t/\tau_{\text{os}})$, where $P_0$ is 0.0001 W and $\tau_{\text{os}}$ is 2 ps.

We observe that the pulse intensity output from the two orthogonal axes increases rapidly at the beginning. However, the two orthogonal pulses exhibit pulsating dynamics with energy complementarity in the time domain and gradually become stable after the 1500th round trip. The evolution of the spectra corresponding to Figs. 2(a) and 2(b) is shown in Figs. 2(d) and 2(e), which have a similar dynamic as observed in the time domain. Figures 2(c) and 2(f) depict the pulse and spectrum shapes at the 2000th round trip. It is evident that the laser in the two orthogonal directions has nearly the same shape and intensity in the time and frequency domains, indicating that a coherent coupling process of stable PQSs has been established [36].

In order to have a deep insight into the vector PQS generation, the energy and the phase evolution of the vector PQS versus the round trip number is plotted in Fig. 3. The energy of the two solitons increases quickly to 0.3 nJ around the 20th round trip. And then, the total energy of these two solitons nearly remains the same. However, the energy of the pulse in the fast axis starts to decrease, while that in the slow axis increases in a symmetrical trend. When the energy of the pulse in the slow axis increases to the first vertex around 0.58 nJ at the 186th round trip, the pulse in the fast axis also reaches a minimum value (0.04 nJ). Subsequently, the energy of the pulse in the slow axis features multiple minimum-maximum values and becomes stable after the 1500th round trip. Similarly, the pulse energy in the fast axis exhibits multiple maximum-minimum values and tends to be stable after the 1500th round trip. Notably, the energy dynamics of the two axes are complementary, as the total energy of the two solitons remains constant after the 20th round trip. The evolution process reveals that the buildup of the vector PQS is dominated by gain competition induced by the XPM effect, and that a balance within the pulse packet is formed during energy exchange [37]. After the 1500th round trip, the gain competition forces the two PQSs to have the same energy and constant phase difference. Furthermore, the two PQSs keep a synchronous evolution in the time domain because both positions are near 0 ps all the time. This is because the walk-off between the two pulses has been fully compensated by the soliton trapping effect. The evolution of phase difference between pulses in two axes closely corresponds to the changes in energy relationship. As we can see, the turning points of phase evolution correspond to the intersection points of pulse energy, which is because the nonlinear phase shift of the two orthogonal pulses is directly proportional to the pulse energy. Considering
Fig. 2. The pulse evolution dynamics in (a) the slow axis and (b) the fast axis. (c) The pulse shapes output from the 2000th round trip. The spectra evolution dynamics in (d) the slow axis and the fast axis. (f) The spectra output from the 2000th round trip. $B_m$ is $0.1 \times 10^{-6}$. 
the final output intensity of the two axes and the fixed phase difference, we refer to the PQSs as “elliptical-polarization vector PQSs”.

Fig. 3. (a) The energy of PQSs output from the fast/slow axis and their total energy, and (b) the phase difference evolution dynamics with the increase of the round trips when $B_m$ is $0.1 \times 10^{-6}$.

Enhanced $B_m$ of $1.0 \times 10^{-6}$ leads to the generation of another type of “elliptical-polarization vector PQS”, as depicted in Fig. 4. It is obvious that the two PQSs experience the similar amplification and pulsating dynamics before the stable status is achieved. However, their spectrum shapes are different, indicating the incoherently coupled elliptical-polarization vector PQSs have been obtained [38]. The underlying physics is primarily associated with walk-off and soliton trapping between the two pulses. The cavity-birefringence directly results in the walk-off between the two PQSs in the time domain owing to the different group velocity delay. At the beginning of the evolution (before the 23rd round trip), the weak soliton trapping effect resulting from the weak energy of the two pulses cannot bind them together. Consequently, the two pulses evolve with the mismatched time delay and show opposite directions in the time domain, i.e., the pulses in the slow and fast axes evolve in separate negative and positive time directions, as shown in Fig. 5. However, the increasing pulse energy leads to the accumulation of nonlinear interactions, and gradually compensating for the walk-off with the enhanced soliton trapping effect. Hence, the two pulses have nearly identical evolution traces in the time domain with respect to the round trip number. Around the 180th round trip, the energy of the pulse in the slow axis exceeds that in the fast axis, which results in the pulse in the fast axis gradually following the evolving direction with the pulse in the slow axis and moving towards the negative time direction. With the round trip number increased to ~500, the pulse in the fast axis regains control of the evolving direction of the pulse packet as its energy is larger than that in the slow axis. Due to the combined effects of walk-off and soliton trapping, the temporal positions and the spectrum shapes of the two orthogonal PQSs exhibit significant differences.

3.2. Near-linear-polarization vector PQSs

The situation changes significantly when the cavity-birefringence continues to increase. For example, Fig. 6 illustrates another type of evolution dynamics for vector PQSs in the time domain when the $B_m$ is $1.5 \times 10^{-6}$. Both PQSs show a creeping dynamic toward the negative time direction in Figs. 6(a) and 6(b). However, it is interesting that the two PQSs have the completely different evolution dynamics. The PQS in the slow axis initially evolves towards the left with a slow speed, but then turns quicker after the 200th round trip. On the other hand, the PQS in the fast axis undergoes energy amplification before the 200th round trip and then gradually decreases in energy, but the time direction evolves first towards the right and then towards the left. As mentioned in the previous section, the evolution direction in the time domain is determined by the pulse with the stronger energy. Initially, the two pulses evolve in opposite directions in the
Fig. 4. (a) The pulse and (b) spectrum evolution dynamics in the slow axis. (c) The pulse shapes output from the 7000th round trip. (d) The pulse and (e) spectrum evolution dynamics in the fast axis. (f) The spectra output from the 7000th round trip. $B_m$ is $1 \times 10^{-6}$. 
Fig. 5. The detailed evolution dynamics of the two pulses in (a) the slow axis and (b) the fast axis in the time domain within 1200 round trips.

Fig. 6. The pulse evolution versus the round trip number in (a) the slow axis and (b) the fast axis. (c) The pulse shapes and (d) spectra output from the 1000\textsuperscript{th} round trip. $B_m$ is $1.5 \times 10^{-6}$. 
time domain because of birefringence. With the increase of the round trips, the increased energy of the pulse in the slow axis and the energy exchange between the two pulses can enhance the soliton trapping effect between them, which can gradually compensate for the walk-off. However, the energy exchange between the two axes seems irreversible in this situation. Although the creeping speed of the pulses on the two axes is mutually restrained before the 300th round trip, afterwards, the pulse in the fast axis is completely captured by the pulse in the slow axis, which has taken away almost all the energy. Considering the fact that the energy of the pulse in the slow axis is far greater than that of the fast axis, the pulse in the fast axis will follow the evolution direction with the pulse in the slow axis, i.e., both pulses evolve together towards the negative time direction.

Figure 7 shows the pulse energy evolution in the two axes. As mentioned above, the pulse energy in the two axes increases quickly in the first 20 round trips. Then the pulse energy evolves into a complementary pulsating dynamic and tends to stabilize. Thus, in this case, the gain competition also helps to redistribute the energy in the vector PQSs packet. However, the increased cavity-birefringence causes a different gain-competition process with the case of $B_m = 0.1 \times 10^{-6}$. The PQS in the slow axis has an advantage in obtaining energy over the PQS in the fast axis, resulting in higher pulse energy. Since the pulse energy in the slow axis is much higher than that in another axis at the end, we can name it as the “near-linear polarization vector PQS”.

![Figure 7](image)

**Fig. 7.** The energy of the PQSs output from the fast/slow axis and their total energy evolution dynamics with the increase of the round trips when $B_m = 1.5 \times 10^{-6}$.

### 3.3. Transition from vector PQSs into scalar PQSs

The cavity-birefringence can bring a group-velocity difference between the two solitons, while for the vector soliton, the soliton trapping effect can bind them together. The above results show that the walk-off between the two PQSs induced by the group-velocity difference has been compensated by the trapping effect between them. However, the higher the cavity-birefringence, the larger the walk-off. Therefore, it is interesting to observe what will happen with larger cavity-birefringence.

Figure 8 presents the evolution of the two PQSs in the slow/fast axis when $B_m = 3 \times 10^{-6}$. It can be seen that the intensity of the PQSs increases quickly within 22 round trips. And then, the energy difference increases gradually until the pulse intensity in the slow axis is nearly $1.6 \times 10^9$ times larger than that in the fast axis, because the energy in the slow axis continues to increase, while the pulse energy in the fast axis starts to decrease. However, it should be noted that the walk-off between the two solitons starts quickly since the 22nd round trip, indicating that energy exchange decreases with the increase of round trips despite the existence of oscillating tails in the time domain. The energy evolution is similar to that in the case of $B_m = 1.5 \times 10^{-6}$, but the two PQSs consistently maintained their initial creeping directions in the time domain. The more round trips, the less the overlap between the two PQSs. As the two pulses completely move away
from each other, the XPM effect will disappear and the pulse in one axis will fail to capture the pulse in the other axis. Eventually, in this case, the uncompensated walk-off and the huge energy difference between the two solitons demonstrate that the vector PQS have evolved into the scalar PQS in the end.

![Fig. 8. The pulse evolution in (a) the slow axis and (b) the fast axis when $B_m$ is $3 \times 10^{-6}$.](image)

To gain deeper insights into PQS evolution in birefringent fibers and investigate what the difference is between the conventional nonlinear Schrodinger equation soliton (NLSES) and the PQS, we have made another simulation of the NLSES for comparison, as shown in Fig. 9. In the NLSES simulation model, $\beta_{2-EDF} = \beta_{2-SMF} = -10 \text{ ps}^2/\text{km}$ and $\beta_{4-EDF} = \beta_{4-SMF} = 0 \text{ ps}^4/\text{km}$. The other parameters are the same as those in the case for elliptical-polarization vector PQSs with $B_m$ of $0.1 \times 10^{-6}$. It can be observed that even after undergoing the same roundtrip (i.e., 2000 roundtrips), a stable vector soliton state cannot be achieved. The temporal positions along the two axes nearly remain the same, which indicates the two pulses have trapped together. From the perspective of pulse energy on both axes, the energy exchange between the two axes has been successfully established. The evolution dynamic is similar to the PQS evolution with $B_m$ of $1 \times 10^{-6}$ described in Fig. 5 while with a slower speed.

![Fig. 9. The pulse evolution in the (a) slow axis and (b) fast axis for NLSES when $B_m$ is $0.1 \times 10^{-6}$.](image)

To construct the vector soliton, it is crucial to not only concentrate on the formation of the soliton itself, but also consider the interaction between two orthogonal polarization solitons and their ability to develop into a stable bound state. So, vector solitons primarily rely on the
trapping effect via XPM between the orthogonal polarization solitons, while the presence of birefringence can induce the walk-off effect and weaken the trapping. The key point here is the degree of the pulse interaction induced by XPM during the soliton formation. Through our simulations, we have discovered that a significant characteristic distinguishing the PQS from the conventional NLSES is their intensity distribution and the long oscillating tail feature. This distinction becomes more evident when observed on a log scale, as shown in Figs. 10(a) and 10(b). It is important to note that when two orthogonal PQSs interact through XPM, the energy distribution of these oscillations in the tail can mutually influence each other. In other words, through XPM, the two orthogonal PQSs actively participate in each other’s formation process. So, we observe similar characteristics in terms of oscillation period and amplitude (inserted picture in Fig. 10(b)). With the enhanced birefringence for the near-linear polarization PQS, the degree of their interaction still ensures the trapping of the pulses on the two axes with an increased energy difference. The major energy is concentrated in the slow axis, and the pulse in the slow axis exhibits characteristics of a PQS with the oscillating tail. The pulse in the fast axis has lost its symmetry due to XPM, but a portion of the tail can still be observed on its left side in Fig. 10(c). When the soliton trapping fails with higher birefringence, the transition from vector PQSs into scalar PQSs will occur, as shown in Fig. 10(d).

![Fig. 10. The pulse shapes of the last round trip in log scale for (a) NLSES, (b) the elliptical-polarization vector PQS, (c) the near-linear-polarization vector PQS, and (d) the scalar PQS.](image)

Additionally, according to the formation mechanism [3], the maximum energy of the PQS is usually higher than that of the conventional NLSES under similar cavity structure, which will lead to stronger XPM. And the oscillating tail of PQSs can indeed facilitate XPM. Thus, we can conclude that it is much easier to obtain the stable vector PQS state than the NLSES case.
4. Conclusions

In summary, our numerical simulations reveal the formation and evolution dynamics of stable vector PQSs in the mode-locked fiber laser with weak cavity-birefringence. With appropriate birefringence strengths, both elliptical-polarization vector PQSs and near-linear-polarization vector PQSs can be obtained, which is determined by the results of the gain competition between the two orthogonal components. Moreover, the further enhanced cavity-birefringence can result in the vector PQSs evolving into scalar PQSs because the walk-off between the two solitons cannot be fully compensated by the soliton trapping effect. Our findings open new perspectives for vector soliton pulse evolution dynamics and will bring new insights into the design and application of vector PQS fiber lasers.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

References