Semi-active control of magneto-rheological dampers with negative stiffness

S. Bhowmik & J. Høgsberg
Department of Mechanical Engineering, Technical University of Denmark, Lyngby, Denmark

ABSTRACT: Effective damping of large and flexible structures by semi-active dampers relies greatly on the control strategy applied, which should combine the robustness of passive devices and the increased damping performance often available from active control. For structural control the Magneto-rheological (MR) damper is among the most popular and promising devices due to its low power requirement, high dynamic range, high force capacity and robustness. The objective of this paper is to formulate semi -active feedback control methods based on simple linear damper models, which lead to increased damping performance by introduction of apparent negative damper stiffness. The design of the control strategy aims at maximizing the damping ratio of the critical mode of the structure. Explicit solutions for the complex valued natural frequency of the damped structure and the associated damping ratio are obtained by the two-component system reduction technique, which shows that a potential increase in damping is associated with a similar reduction of stiffness, or even with the introduction of negative stiffness. For linear control strategies negative stiffness can only be realized by active control, which is limited by stability. In the present paper it is realized by having the MR damper force track the simple pure viscous damper model with vanishing stiffness and the Kelvin model with negative stiffness. The applied voltage of the MR damper should be controlled in such a way that the desired control force is followed sufficiently accurate. This is done by letting the desired force be the input to an inverse Bingham model, which provides the corresponding desired voltage level of the MR damper. Numerical simulations are conducted to demonstrate the performance of the proposed semi-active control strategy with apparent negative stiffness.

1 INTRODUCTION

Survival of structures from excessive vibration due to uncontrollable events—environmental and otherwise—is of great importance, where vibration control of large flexible structures, such as building or bridges, is installed primarily to reduce transverse deflections and/or accelerations. Some success toward this goal has been reported in the use of actively controlled devices like for instance active mass dampers. However, relatively large power requirement is a primary drawback for this type of structural control. On the other hand, variable or semi-active damping devices exhibit relatively low power demands and show performance abilities similar to those of active control schemes (Dyke 1996). In the category of semi-active control, simple variable damping devices such as magneto-rheological (MR) damper, have received great attention in the civil engineering community. The MR damper is typically produced in cylindrical form filled with an MR fluid and surrounded by one or more electromagnetic coils. The fluid contains very small magnetically polarizable particles that allow the properties of the fluid to change almost instantaneously according to the strength of the applied magnetic field. A detailed description of MR dampers can be found in Spencer (1997) or Yang et al. (2001). The application of MR dampers to the control of vibration has been applied for mitigation of wind and seismic effects on tall buildings by e.g. Zhang and Roschke (1999) or Jansen and Dyke (1999). An important benefit of an MR damper is its capacity to operate from a remote power source, like a battery or a solar panel, thus increasing its viability during destructive environmental events.

Because of the increasing research in and development of new semi-active devices, the associated formulation of suitable control algorithms is of great importance. Two decades ago Yang et al. (1986) proposed new optimal control algorithms for structural control using standard quadratic performance and Ricccati equation to generate appropriate force. Within the framework related to linear quadratic Gaussian, H_2 or H_∞ control a number of promising approaches have been presented and have lead to the development and implementation of e.g. sliding mode control algorithms. Lately, soft computing techniques, for instance based on fuzzy logic theory, have received some attention. However, the efficiency of these techniques is still to be demonstrated.

In the present paper two main objectives are considered. Initially a framework for design of external dampers is presented. This is based on the maximization of the damping ratio of a critical vibration form and it follows the two-component reduction technique presented by Main and Krenk (2005). This analysis provides expressions for the modal damping ratio, which indicates that effective damping can be obtained by imposing apparent negative stiffness. Expressions for optimal tuning of the simple pure viscous damper model and the Kelvin model with negative stiffness are presented. The second step of the paper considers the realization of the desired control force, where the applied voltage is obtained in terms of the desired control force by a simple inverse Bingham plasticity model. A numerical study is presented where the performance of the introduced semi-active control schemes are evaluated based on the response of a 10 story shear frame structure exposed to the El Centro earthquake.

2 MODELING OF MAGNETO-RHEOLOGICAL DAMPER

The response of MR fluids results from the polarization induced in suspended particles by application of an external magnetic field. The interaction between the resulting induced dipoles causes the particles to form columnar structure, parallel to the applied field. These chain-like structures restrict the motion of the fluid, thereby increasing the viscous characteristic suspension. Spencer et al (1997) presented a phenomenological model to analyze the behavior of a small scale version of a Magneto- rheological (MR) damper. The model is an augmented version of the classic Bouc-Wen hysteresis model, and it appears to be representative over a large frequency and amplitude range. A schematic diagram of the modified Bouc-wen model is presented in Figure 1.

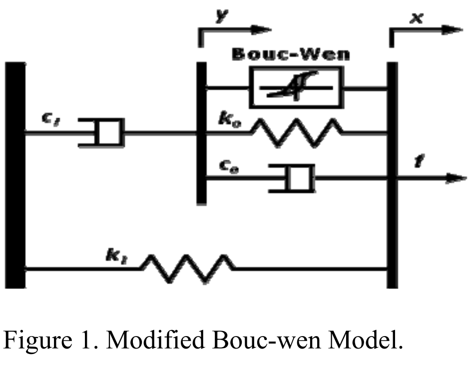

Figure 1. Modified Bouc-wen Model.

The equation governing the force f predicted by the modified Bouc-Wen model is as follows:

$$f = c_1 \dot{y} + k_1 (x - x_0) \quad (1)$$

where the energy dissipation is introduced via the filter

$$\dot{y} = \frac{1}{(c_0 + c_1)}\left[\alpha z + c_0\dot{x} + k_0(x - y)\right] \tag{2}$$

and the hysteresis effect follows from the evolutionary variable z governed by the Bouc-Wen equation

$$\dot{z} = -\gamma|\dot{x} - \dot{y}|z|z|^{n-1} - \beta(\dot{x} - \dot{y})|z|^n + A(\dot{x} - \dot{y}) \tag{3}$$

The following model parameters are typically represented as linear functions of the applied voltage:

$$\alpha = \alpha_a + \alpha_b u \quad , \quad c_1 = c_{1a} + c_{1b} u \quad , \quad c_0 = c_{0a} + c_{0b} u \tag{4}$$

where the applied voltage u is described with a time delay relative to the desired voltage v

$$\dot{u} = -\eta(u - v) \tag{5}$$

Representing the dynamics involved in reaching rheological equilibrium and in driving the electromagnet in the MR damper, a total of fourteen model parameters are to characterize the MR damper using experimental data and e.g. a constrained nonlinear optimization algorithm. Taking displacement, velocity and voltage as input, the model can determine the damper force quite accurately. But due to the nonlinearities of the model its inverse is non-trivial. Therefore, the simple Bingham plasticity model

$$f = \left[f_a + f_b v\right] sign\left[\dot{x}\right] + \left[c_a + c_b v\right]\dot{x} \tag{6}$$

is used to obtain the voltage based on a desired control force, which is derived in the following. Governing parameters for a 1000 kN MR damper for both the Bingham and the modified Bouc-Wen model are given in [2]. These parameters are presented in table 1. It should be noted that the saturation voltage level is around 10 V and that the maximum damper stroke is 0.08 m.

3 DAMPING OF FLEXIBLE STRUCTURES

In dynamic analysis of flexible structures the structural model is typically modeled by finite elements, whereby the equation of motion can be written in the well known form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}_e - \mathbf{w}f(t) \tag{7}$$

where the vector $\mathbf{q}$ contains the generalized displacements and a dot denotes time differentiation, and $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M}$ are the structural stiffness, damping and mass matrix respectively. The external load is represented by the vector process $\mathbf{f}_e$. Finally, a single damper is acting on the structure, where f represents the damper force and $\mathbf{w}$ is the connectivity vector describing the location of the damper. In a collocated setting the displacement of the damper is $x = \mathbf{w}^T\mathbf{q}$. The characteristics of the damper are conveniently formulated in the frequency domain as

$$f(\omega) = H(\omega)x \tag{8}$$

Where H is the frequency dependent transfer function which also contains any damper gain.

The effective damping of the structure is evaluated in terms of its modal damping, where the presence of the external damper generates non-proportional damping with complex valued natural frequencies and mode shape. The two-component system reduction technique by Main and Krenk [3] describes the response as a linear combination of two limiting mode shapes for the structure without damper $\mathbf{u}_0$ and for the structure with the damper fully locked $\mathbf{u}_\infty$, respectively.

$$\mathbf{q} = \mathbf{u}_0 r_0 + \mathbf{u}_\infty r_\infty \tag{9}$$

Table1. MR damper parameters.

Bingham Model							
f_a	100.0 kN	f_b	75.0 kN/V	c_a	81.0 kN s/m	c_b	90.0 kN s/m/V
Modified Bouc-Wen							
α_a	46.2 kN/m	α_b	41.2 kN/m/V	c_{0a}	110.0 kNs/m	c_{0b}	114.3 kNs/m/V
c_{1a}	8359.2kNs/m	c_{1b}	7482.9 kNs/m/V	k_0	0.002 kN/m	k_1	0.0097 kN/m
x_0	0.18 m	η	100 s^{-1}	γ	164.0 m^{-2}	β	164.0 m^{-2}
A	1107.2	n	2				

The first term represents the classical undamped case, while the second term introduces the local effect of the damper described in terms of its ability to lock the structure at its location. The undamped mode shape is governed by the classical eigenvalue problem

$$\left(\mathbf{K}-\omega_0^2\mathbf{M}\right)\mathbf{u}_0=\mathbf{0} \tag{10}$$

A smaller eigenvalue problem exists for the case where the damper link is fully locked defining ω_∞ and $\mathbf{u}_\infty$. The locked mode shape $\mathbf{u}_\infty$ is by construction orthogonal to the connectivity vector which yields the identity $\mathbf{w}^T\mathbf{u}_\infty=0$. Both limiting mode shapes are real-valued and conveniently normalized to unit modal mass,

$$\mathbf{u}_0^T\mathbf{M}\mathbf{u}_0=1 \quad , \quad \mathbf{u}_\infty^T\mathbf{M}\mathbf{u}_\infty=1 \tag{11}$$

Consequently, the limiting natural frequencies are given in terms of the modal stiffness,

$$\mathbf{u}_0^T\mathbf{K}\mathbf{u}_0=\omega_0^2 \quad , \quad \mathbf{u}_\infty^T\mathbf{K}\mathbf{u}_\infty=\omega_\infty^2 \tag{12}$$

The limiting mode shapes and natural frequencies are all real-valued and may be derived directly from the associated generalized eigenvalue problems.

The equations of motion can be described in the reduced two-dimensional subspace by substitution of (9) into (7) followed by pre-multiplication by the limiting mode-shapes, see e.g. Høgsberg and Krenk (2006, 2007). In the frequency domain the reduced system of equations can be written in the following homogeneous form when free vibrations are considered

$$\begin{bmatrix} \omega_0^2-\omega^2+u_0^2H & \kappa(\omega_0^2-\omega^2) \\ \kappa(\omega_0^2-\omega^2) & \omega_\infty^2-\omega^2 \end{bmatrix}\begin{bmatrix} r_0 \\ r_\infty \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{13}$$

The amplitude of the mode shape at damper location is $u_0=\mathbf{w}^T\mathbf{u}_0$ and $\kappa=\mathbf{u}_\infty^T\mathbf{M}\mathbf{u}_0$ represents the coupling between the two limiting mode shapes. Free vibrations require a singular matrix which yields a characteristic equation in ω in e.g. Main and Krenk (2005). This characteristic equation can be re-written in an explicit, given solution when assuming that the coupling parameter $\kappa\approx 1$. This solution is conveniently expressed in terms of the change in natural frequency $\Delta\omega=\omega-\omega_0$ and $\Delta\omega_\infty=\omega_\infty-\omega_0$ relative to the undamped case, whereby it appears in the following compact form

$$\frac{\Delta\omega}{\Delta\omega_\infty}\approx\frac{\mu H}{1+\mu H} \tag{14}$$

In this expression the modal flexibility parameter is given as

$$\mu=\frac{u_0^2}{\omega_\infty^2-\omega_0^2}\approx\frac{u_0^2}{2\omega_0\Delta\omega_\infty} \tag{15}$$

The modal damping ratio is determined as the relative imaginary part of the complex valued natural frequency: $\zeta = \mathrm{Im}\left[\Delta\omega_\infty\right]/|\omega|$. In the case of response dominated by resonance the magnitude of the structural is approximately inverse proportional to the damping ratio of the dominant mode. Thus, a rational and load independent basis for optimal tuning is the maximization of the modal damping ratio ζ with respect to the damper gain. If the approximation $|\omega| \approx \omega_0$ is introduced the damping ratio can be written as

$$\zeta \approx \frac{\Delta\omega_\infty}{\omega_0} \frac{\mu \mathrm{Im}\left[H(\omega_0)\right]}{\left(1+\mu \mathrm{Re}[H(\omega_0)]\right)^2 + \left(\mu \mathrm{Im}\left[H(\omega_0)\right]\right)^2} \tag{16}$$

It is seen from this expression that the stiffness of the damper $\mathrm{Re}[H(\omega_0)]$ appears in the denominator of (16). It is therefore observed that a potential increase in the damping ratio is associated with a corresponding reduction of damper stiffness. And as demonstrated in Krenk and Høgsberg (2006) the application of *negative* stiffness improves the damping efficiency for resonant response conditions. In the present paper simple damper models are considered in the following, with emphasis on the design of models with negative stiffness and the application in a semi-active control setting. The optimal tuning of the damper is based on maximization of the damping ratio, which follows from having the variation $\delta\zeta = 0$.

4 SAMPLE DAMPER MODELS

4.1 *Pure Viscous Damper*

The prototype damper model is the pure viscous model. For this model the damper force is directly proportional to the collocated velocity of the damper motion,

$$f = c\mathbf{w}^T\dot{\mathbf{q}} \tag{17}$$

where c is the viscous parameter.

This means that the damper transfer function is given as

$$H(\omega) = ic\omega \approx ic\omega_0 \tag{18}$$

where the frequency in the transfer function is approximated by the undamped frequency. Substitution into (16) followed maximization of the damping ratio gives the following expression for the optimal viscous parameter

$$c_{opt} = \frac{2\Delta\omega_\infty}{u_0^2} \tag{19}$$

The optimal viscous damper has vanishing stiffness and is therefore a suitable case of reference for damper models with negative stiffness.

4.2 *Kelvin Model*

The general format of a Kelvin type model has both frequency dependent stiffness and viscous components. However, in the present formulation it is assumed that the governing parameters are constant, whereby the damper force can be written as

$$f = \mathbf{w}^T\left[k_d\mathbf{q} + c\dot{\mathbf{q}}\right] \tag{20}$$

It is assumed that the stiffness is given and that c is the gain value of the model. The transfer function can be written as

$$H(\omega) = k_d + ic\omega \tag{21}$$

Substitution into (16) gives the damping ratio and maximization of the damping ratio with respect to *c* leads to the following expression for the optimal viscous parameter of the Kelvin model.

$$c_{opt} = \frac{2\Delta\omega_\infty}{u_0^2} + \frac{k_d}{\omega_0} \tag{22}$$

where the second term introduces the correction due to damper stiffness.

5 CONTROL STRATEGIES

In the present section a control strategy is presented which operates the semi-active MR damper similar to the optimally tuned simple viscous damper and the viscous damper with constant stiffness contribution (Kelvin model). The efficiency of the proposed control strategies depends in the first place on the proper tuning of the simple damper models, which is given in (19) and (22) and subsequently on the ability of the MR damper to track the desired optimal control force.

The following steps outline the design procedure used for the design of the proposed control scheme:

1. Formulate a MR damper model, which should be as detailed and accurate as possible since it represents the actual damper attached to the structure. Therefore, the modified Bouc-Wen model is applied in the present setting.
2. Design of an inverse model of the MR damper, which is used to predict the desired voltage based on the known optimal damper force. This model should be simple so that it can be inverted and therefore the Bingham plasticity model is used.
3. The optimal control force, which is input to the inverse MR model, is computed by (17) for the viscous damper and (20) for the Kelvin model. The corresponding optimal damper gains are given in (19) and (22), respectively.
4. The voltage is determined by the inverse MR damper model. If negative voltage is desired zero volts are chosen instead. Also a maximum voltage level of 10 volts is enforced.

A schematic block diagram of the overall system is shown in Figure 2.

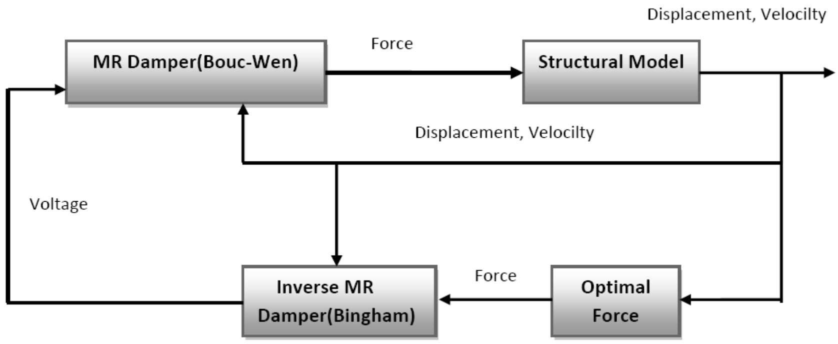

Figure 2. Block diagram of the semi-active control strategy.

6 SIMULATIONS

The efficiency of the semi-active control strategy is assessed for a 10 storied building. The MR damper acts between ground and first floor of the shear frame structure. The governing equation of motion is given in (1), where the mass and stiffness matrix and the connectivity vector are given as

$$\mathbf{M} = m\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & . & & \\ & & & . & \\ & & & & 1 \end{bmatrix} \quad , \quad \mathbf{K} = k\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & & & \\ & & . & & \\ & & & . & -1 \\ & & & -1 & 1 \end{bmatrix} \quad , \quad \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ . \\ . \end{bmatrix}$$

where the concentrated floor mass m and the intersory stiffness k are chosen so that the lowest natural frequency is $\omega_{01} = 2\pi = 6.28$. In the present case m = 5000 and $k = 1.5.10^7$. The applied loading is the El Centro earthquake and simulations are conducted using the Simulink toolbox in Matlab.

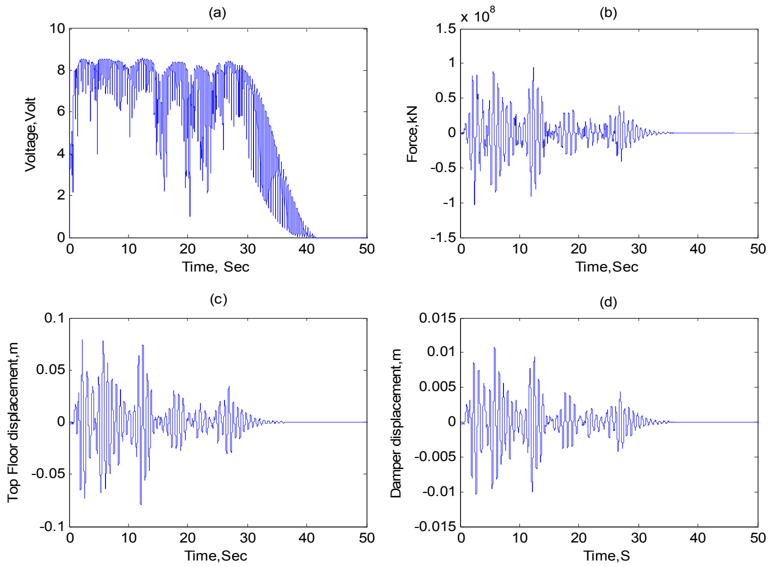

Figure 3. Voltage, force, top floor displacement and damper displacement diagrams for pure viscous

Initially the optimal viscous model is used to generate the desired control force, which is passed on to the inverse MR damper model. Figure 3 shows the applied voltage, the MR damper force, the displacement of the top floor and displacement of the damper. A detailed plot of the response is shown in Figure 4. It is seen that the force-velocity curve in Figure 4a is practically

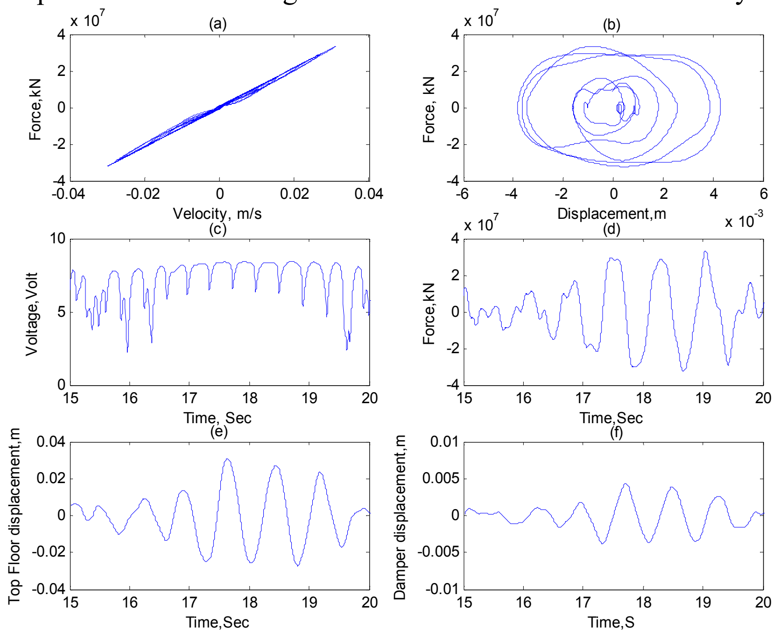

Figure 4. Response diagrams for Pure Viscous.

straight line and that the force-displacement curve in Figure 4b is an ellipse, indicating that the present control is able to capture the characteristics of the pure viscous damper. In Figure 4c the applied voltage is shown, which over a half period is almost constant if the response is sufficiently sinusoidal.

Similar simulations are conducted for the Kelvin model with negative stiffness. For the shear frame structure the limit of negative stiffness is $k_d = -k$, which corresponds to the case where the negative stiffness of the damper has cancelled the structural stiffness between ground and first floor. In the present case two situations are investigated: $k_d = -0.3\,k$ and $k_d = -0.7\,k$. The results of the simulations are plotted in Figures 5 and 6.

When comparing the first two figures in Figures 4, 5 and 6 it is seen that introduction of negative stiffness yields an opening of the force-velocity curves and an inclination of the hysteresis loops in the force-displacement diagram. Thus, the semi-active control scheme is able to capture the desired effect of the viscous component and, in particular, the negative stiffness component. Whereas the voltage in Figure 4c for the pure viscous damper is almost constant over a half vibration period it is seen in Figs. 4c and 5c that the negative stiffness component in the Kelvin model is realized by the semi-active control through a decreasing voltage over a half period. The response magnitude is assessed in terms of performance indices J1, J2, J3 and J4:

$$J1 = \frac{\max_i |x_i(t)|}{\max_i |x_i^{uc}(t)|} \qquad J2 = \frac{\max_i |\ddot{x}_i(t)|}{\max_i |\ddot{x}_i^{uc}(t)|}$$
$$J3 = \frac{\max_i (RMS(x_i))}{\max_i (RMS(x_i^{uc}(t))} \qquad J4 = \frac{\max_i (RMS(\ddot{x}_i))}{\max_i (RMS(\ddot{x}_i^{uc}(t))} \tag{23}$$

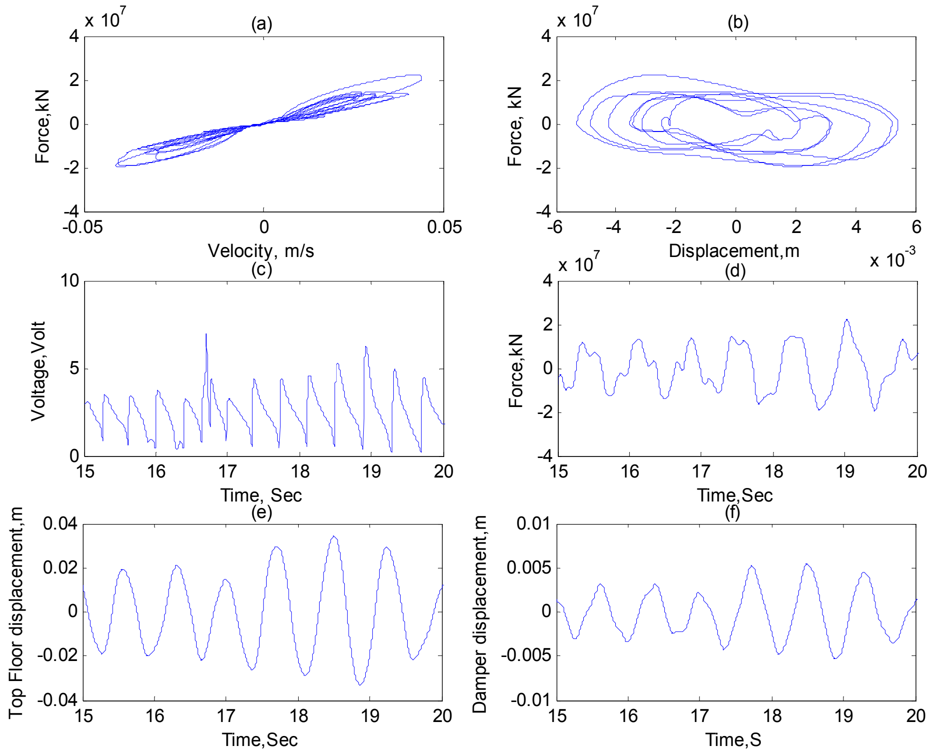

Figure 5. Response diagrams for Kelvin Model ($k_d = -0.3k$).

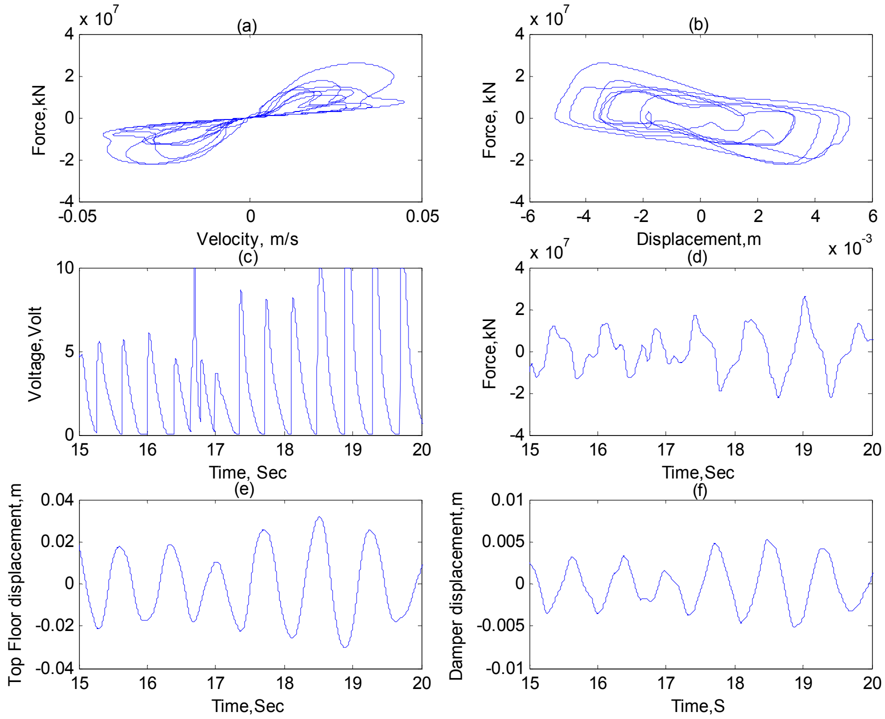

Figure 6. Response diagrams for Kelvin Model ($k_d = -0.7k$).

where superscript *uc* indicates the undamped case. The values of the J1-J4 are given in Table 2 for the three damper models. It is found that all three damper models introduce a significant response reduction compared to the undamped case. The influence of negative stiffness is moderate compared to the viscous case and the increase in negative stiffness from $k_d = -0.3\,k$ to $k_d = -0.7\,k$ is without significant impact. It was found that the voltage generated from inverse damper in some case was quite noisy. Therefore a simple first order filter has been implemented to improve the signal quality of the voltage (Yang 2001)

$$\dot{u}_{fil} + \lambda u_{fil} = \lambda u \tag{24}$$

where u_{des} is the desired force and u is the actual force generated from inverse MR damper model. Here λ is chosen as 31.4 inspired by Yang (2001).

Table2. Performance Analysis.

System Model	J1	J2	J3	J4
Pure Viscous	0.6995	0.9526	0.7154	0.6789
Kelvin with negative stiffness ($k_d = -0.3k$)	0.6745	0.9332	0.6909	0.6645
Kelvin with negative stiffness($k_d = -0.7k$)	0.6574	0.9310	0.6799	0.6615

7 CONCLUSION

The semi-active control strategies for MR damper for getting optimal performance for pure viscous and Kelvin models are investigated above. The optimal force is found for both the cases from Modal analysis. Then this optimal force is used to get desired voltage for MR damper from inverse MR damper. The voltage, force, displacement diagrams are compared for both the case with increasing negative stiffness. From the results it is clearly shown that negative stiffness has significant influence on reduction of displacements and as well as on acceleration. The voltage requirement is reduced with the increase of negative stiffness. Force-displacement diagram is elliptical and changing its inclination angle with the increase of negative stiffness.

For this case earthquake excitation data are used but wind load excitation remains for future work.

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