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Low-field extremum in the surface impedance of metals at MHz frequencies

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The first detailed measurements of the low-field extremum for a cylindrical sample at MHz frequencies are reported. It is shown that the model suggested by Sibbald *et al.* is the only interpretation of the low-field extremum which can account either for the measurements reported here or those carried out for similar extrema observed in flat samples. The theoretical basis for this interpretation is investigated, and the importance of Fermi surface geometry is given special emphasis.

Measurements of the surface impedance of metals at liquid-helium temperatures frequently reveal a large extremum in the vicinity of zero external magnetic field at MHz frequencies.[1-7] The temperature and frequency dependences of this low-field extremum have been measured in detail in the case of flat copper samples and, on the basis of such measurements, the surface impedance extremum has been attributed to quantized skipping electrons whose depth of penetration into the metal is much less than the skin depth.[1] This interpretation of the low-field extremum has been recently contested, however, by other authors[2] who argue that the measurements in copper can be interpreted equally well in terms of skipping electron trajectories which span the entire MHz skin depth. Other interpretations of the low-field extremum have also been suggested over the years[3,4] and, in view of the fact that at least four qualitatively different interpretations have now been proposed to interpret the same or similar low-field extrema, some of them explicit to the sample geometry itself, it would be desirable to have additional detailed measurements of the low-field extremum in different samples and sample geometries which could be used to determine which of the various interpretations best describes the experimental extremum actually observed in the immediate vicinity of zero magnetic field. It is the purpose of this letter to report the results of such measurements for cylindrical samples and to give a critical discussion of how well the different interpretations can be reconciled with experiment. In particular, it is pointed out that the quantized skipping-electron interpretation is the only interpretation which has been proposed that fully agrees with detailed measurements of the low-field extremum either in flat or cylindrical samples and that the same interpretation is equally consistent with less detailed measurements of the low-field extremum in other metals.

The choice of a molybdenum cylinder represented a compromise between metals with simpler Fermi surfaces, such as the alkali and noble metals, for which the low-field extremum can be observed only with extreme difficulty, if at all, in cylindrical samples[2] and metals such as tin and indium[3,6] which exhibit measurable low-field extrema in cylindrical samples, but whose significantly less well-known Fermi surfaces would greatly hinder an unambiguous theoretical analysis of the phenomenon. The diameter of the sample (4.5 mm) and the mean free path of the skipping electrons ($\lesssim$0.1 mm) were such that the effects of sample curvature on the low-field extremum would be clearly observable while, at the same time, the motion of the skipping electrons around the circumference of the cylinder would be limited to a small angular region of the Fermi surface ($\lesssim 5°$ in arc length).[7] The molybdenum cylinder was mounted inside a coaxial cylindrical coil of an rf oscillator and an external dc magnetic field applied along the common coil-sample axis. Typical experimental low-field extrema are shown in Fig. 1. Both the line shape and position of the low-field extremum are characteristic of similar low-field extrema observed in cylindrical and flat samples of other metals.

According to the quantized-skipping-electron interpretation proposed by Sibbald *et al.*, the experimentally observed low-field extremum in dZ/dH will occur at a magnetic field H_0 such that the mean free path of the skipping electrons, l, is comparable to the arc length of the shortest-allowed (i.e., lowest-quantum-number) quantized-skipping-electron trajectories whose depth of penetration into the metal is much less than the skin depth at MHz frequencies [Fig. 2(a)]; i.e.,[1]

$$H_0 \cong (hK^{1/2})/(el^{3/2}) . \qquad (1)$$

Here Z is the surface impedance for rf currents perpendicular to an applied dc magnetic field H, oriented along the sample surface, K is the radius of curvature of the Fermi surface measured in the

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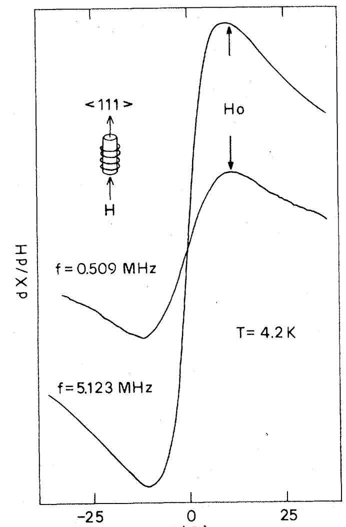

FIG. 1. Experimental derivative curves of the imaginary part of the surface impedance dX/dH for a molybdenum cylinder, obtained in the same run by changing only the frequency of the external rf generator. The frequency independence of the position of the low-field extremum H_0 (at fixed temperature) illustrated above was also found to be valid up to the highest frequencies employed (37 MHz) to within an experimental uncertainty of ± 5–10% in H_0.

plane of the electron motion, h is Planck's constant, and e is the electronic charge.[8] Equation (1) predicts that the position of the low-field extremum will be frequency independent but will depend on the temperature (through the factor l) in contrast to other interpretations proposed for the low-field extremum (illustrated in Fig. 2) which predict

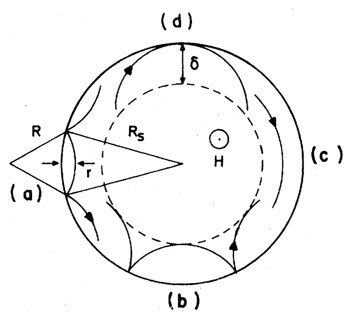

FIG. 2. Different interpretations of the low-field extremum as they would apply to a cylindrical sample: (a) the quantized skipping-electron interpretation of Sibbald *et al.* (Ref. 1); (b) the semiclassical skipping-electron interpretation of Pal and Falk and of Gantmakher *et al.* (Ref. 2); (c) the cylindrical-size-effect interpretation of Tsoi and Meirovich (Ref. 3); (d) the semiclassical skimming-electron interpretation of Drew (Ref. 4). R is the cyclotron radius, R_s is the sample radius, δ is the rf skin depth, r is the depth of penetration of low-quantum-number skipping-electron trajectories ($r \ll \delta$), and H is the dc magnetic field.

either a frequency-dependent value for H_0 [Figs. 2(b) and 2(d)][2,4] or a value which is temperature independent [Fig. 2(c)].[3] Detailed measurements of the low-field extremum reported here for molybdenum (Figs. 1 and 3; see also Ref. 9), as well as those previously reported for copper,[1] clearly show that the position of the low-field extremum is, in fact, both independent of frequency (for changes in frequency by a factor of 10–20 or more) and very strongly temperature dependent thus ruling out all of the interpretations for the low-field extremum other than the quantized skipping-electron interpretation.

An important point which has not been explicitly emphasized before is that it is not possible to distinguish experimentally between the various skipping- and skimming-electron interpretations [Figs. 2(a), 2(b), and 2(d)] of the low-field extremum on the basis of the temperature dependence of the low-field extremum alone[9] since the range of temperatures over which the low-field extremum can be accurately measured experimentally is too small to make such a distinction possible and, more fundamentally, because the position (and amplitude) of the low-field extremum given by the various interpretations assume that l (and K) are the same over all portions of the Fermi surface which contribute to the low-field extremum. In general, however, both K and l will vary considerably over the Fermi surface, leading to a broadening of the observed dZ/dH extremum in magnetic

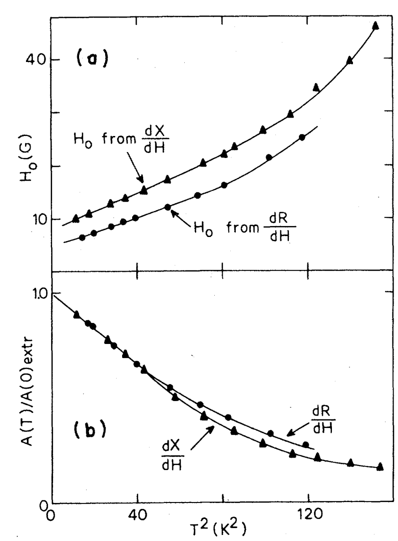

FIG. 3. The position H_0 and amplitude $A(T)$ of the low-field extremum, shown in Fig. 1, as a function of the square of the temperature T. $A(T) \equiv |(dR/dH)_{H_0}|$ and $A(T) \equiv |(dX/dH)_{H_0}|$ for the extrema observed in the real and the imaginary parts of the surface impedance, respectively. $A(0)_{\mathrm{extr}}$ is the value of $A(T)$ extrapolated to zero temperature.

field as well as a line shape and amplitude which will be complicated functions of temperature.[10] Such broadening effects could be expected to be especially large for cylindrical samples where essentially the entire Fermi surface will contribute to the surface impedance,[11] and it can therefore be expected that an equation such as Eq. (1) will be valid only for small changes in l, i.e., for small changes in the temperature. This is illustrated by the fact that a simple temperature dependence for the position of the low-field extremum is, in fact, observed only at low temperatures in molybdenum [$H_0 \simeq T^2$, Fig. 3(a)] where the mean free path exhibits only a weak temperature dependence resulting from residual electron-electron scattering proportional to T^2 for all major cross sections of the molybdenum Fermi surface.[12] A similar argument holds for an experimental determination of the temperature dependence of the amplitude of the low-field extremum [Fig. 3(b)].[13]

In spite of its agreement with all experimental measurements of the low-field extremum, the quantized-skipping-electron interpretation has never received a detailed theoretical treatment in the literature, and it is therefore of considerable interest[2] to investigate to what extent Eq. (1) can be understood in terms of the well-known Prange-Nee theory for the contribution of skipping electrons to the surface impedance derivative dZ/dH[14]:

$$\frac{dZ}{dH} = (\text{const}) \int v\gamma \frac{d}{dh} \sum_{m,n} \frac{(i-\sqrt{3})\alpha_{mn}^2(\beta, h)}{1 - h^{2/3}(\zeta_m - \zeta_n) + (i/\omega\tau)} dA \,, \tag{2}$$

$$h \equiv \gamma H = (e/\hbar)(v^3/2K\omega^3)^{1/2}H \,, \; \beta \equiv (v/2K\omega)^{1/2}(1/\delta) \,. \tag{3}$$

Here $v = v(A)$ is the component of the Fermi velocity parallel to the rf electric field, $\alpha_{mn}(\beta, h)$ is the matrix element describing the transitions induced between the various quantized-skipping-electron states by the rf electric field, ζ_m and ζ_n are the roots of the corresponding skipping-electron wave functions, dA is the area element of the Fermi surface corresponding to the skipping-electron motion in real space,[7] and τ is the relaxation time between effective scattering events for the skipping electrons.

At GHz frequencies, Eq. (1) rapidly converges yielding a good quantitative description of most of the features experimentally observed in dZ/dH at low magnetic fields. This rapid convergence at GHz frequencies results from the fact that the relatively small GHz skin depth effectively limits the summation in Eq. (2) to a small number of terms ($\lesssim 10^2 - 10^3$) representing transitions between low-quantum-number skipping-electron trajectories whose depth of penetration into the metal is less than or comparable to the skin depth where the rf electric fields and hence the matrix elements appearing in Eq. (2) are appreciable. At MHz frequencies the skin depth is much larger and the number of terms required for a similar convergence is too large ($\sim 10^6 - 10^8$) for direct numerical calculations of all sources of magnetic-field variation in dZ/dH to be practical using Eq. (2). In contrast to GHz frequencies, however, there will be a large contribution to dZ/dH from transitions between low-quantum-number skipping-electron trajectories which penetrate only a fraction of the MHz skin depth into the sample. A calculation of this low-quantum-number skipping-electron contribution to the surface impedance has never been carried out using the Prange-Nee theory at MHz frequencies but such a calculation is of particular interest here since it is precisely the low-quantum-number skipping-electron trajectories which would be expected to play a major role in determining the position of the low-field extremum according to the quantized-skipping-electron interpretation of Sibbald *et al.*[1] We have therefore carried out such calculations at MHz frequencies using the same number of low-quantum-number skipping-electron transitions ($\sim 10^2 - 10^3$) which exhibited excellent convergence at GHz frequencies.[15] A broad extremum was found for the theoretically calculated value of dZ/dH at low magnetic fields. However, the convergence of the numerical calculations, especially at higher magnetic fields, is extremely poor, and it must therefore be concluded that the contribution of low-quantum-number skipping-electron trajectories, calculated on the basis of the Prange-Nee theory, cannot explain the low-field extremum observed experimentally.

In summary, the quantized-skipping-electron interpretation of Sibbald *et al.* is the only interpretation of the low-field extremum at MHz frequencies which can account for detailed measurements of the phenomenon in both flat and cylindrical metallic samples and which also agrees with less detailed measurements in other samples. Numerical calculations suggest that a large number of quantized-skipping-electron trajectories contribute to the extremum but a detailed quantitative understanding will require that the Fermi-surface geometry and possibly the nature of electron scattering at the surface be taken into account. In view of the experimental success of the quantized-skipping-electron interpretation and the failure of the Prange-Nee theory to provide a practical quantitative description of the low-field extremum, it is hoped that his work will encourage a more useful and rigorous theoretical formulation of the low-field surface impedance of metals at MHz frequencies.

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[7]The various interpretations of the low-field extremum discussed in this work implicitly assume that the electronic motion is confined to a small portion of the Fermi surface. Qualitatively different phenomena will arise when this condition is not satisfied; e.g., see T. G. Blaney, Philos. Mag. 20, 23 (1969).

[8]The position of the dZ/dH extremum for a cylinder will be displaced in magnetic field by an amount $\hbar K/eR_s$, where R_s is the cylinder radius; e.g., see R. E. Doezema, J. F. Koch, and U. Strom, Phys. Rev. 182, 717 (1969).

[9]For example, the same experimental data in copper have been fit equally well to both $H_0 \sim l^{-3/2}$ and $H_0 \sim l^{-2}$ dependencies, viz., Refs. 1 and 2. In addition, the semiclassical skipping- and skimming-electron interpretations predict *identical* frequency and temperature dependences for H_0, viz., Refs. 2 and 4.

[10]Such so-called k_H broadening effects would also be expected to be more important at low frequencies ($\omega\tau \lesssim 1$ and $\omega\tau \ll 1$) where variations in K (and l) can affect both the position and the width of the extremum; e.g., see R. A. Gordon and J. B. Frandsen, J. Phys. (Paris) 39 Coll. C6, Supp. No. 8, p. C6-1135 (1978).

[11]A detailed analysis of the form of the molybdenum Fermi surface as well as additional measurements in truncated or slightly etched cylinders suggest that the signal in molybdenum originates primarily from small closed regions of the Fermi surface [R. A. Gordon (unpublished)]. The standard assumption that a single localized portion of the Fermi surface is solely responsible for the signal will, however, lead to a more rapid temperature variation in H_0 over large ranges of temperature; R. A. Gordon, in *Proceedings of the Fourteenth International Conference on Low Temperature Physics*, edited by M. Krusius and M. Vuorio (North-Holland, Amsterdam, 1975), Vol. 4, p. 333.

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[13]A similar analysis of the experimental temperature dependence of the amplitude of the low-field extremum has also been carried out for indium; D. G. deGroot, Ph. D. thesis, Vrije University, Amsterdam, 1974 (unpublished).

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[15]The numerical calculations were carried out in collaboration with J. B. Frandsen using the standard cylindrical Fermi-surface approximation.