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Levring, O. A.; Pedersen, Niels Falsig; Samuelsen, Mogens Rugholm

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Hf alloys used as a component of the composite.

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[1]K. Inoue and K. Tachikawa, Proc. 12th International Conf. Low. Temp. Phys., Kyoto, 1970, p. 482.
[2]K. Inoue and K. Tachikawa, Appl. Phys. Lett. **25**, 94 (1974).
[3]K. Inoue and K. Tachikawa, Appl. Phys. Lett. **29**, 386 (1976).
[4]K. Inoue and K. Tachikawa, IEEE Trans. Magn. **MAG-13**, 840 (1977).
[5]First reported in this article.
[6]K. Inoue, H. Wada, T. Kuroda, and K. Tachikawa, Appl. Phys. Lett. **38**, 939 (1981).
[7]B. S. Brown, J. W. Hafstrom, and T. A. Klipert, J. Appl. Phys. **48**, 1759 (1977).
[8]T. Luhman and M. Suenaga, IEEE Trans. Magn. **MAG-13**, 800 (1977).
[9]A. Pebler and E. A. Gulbransen, Trans. Met. Soc. AIME **239**, 1593 (1967).
[10]E. A. Pushkarev, N. S. Petrenko, and V. A. Finkel, Phys. Status Solidi A **47**, K145 (1978).
[11]J. W. Ekin, Adv. Cryo. Eng. **24**, 306 (1978).
[12]J. W. Ekin, Cryogenics **20**, 611 (1980).
[13]J. W. Ekin, IEEE Trans. Magn. **MAG-15**, 197 (1979).
[14]G. Rupp, IEEE Trans. Magn. **MAG-15**, 189 (1979).
[15]See, for example, K. Togano, T. Asano, and K. Tachikawa, J. Less Comm. Metals **68**, 15 (1979); K. Tachikawa, Y. Tanaka, Y. Yoshida, T. Asano, and Y. Iwasa, IEEE Trans. Magn. **MAG-15**, 391 (1979).

Fluxon motion in long overlap and inline Josephson junctions

O. A. Levring, N. F. Pedersen, and M. R. Samuelsen
Physics Laboratory I, The Technical University of Denmark, DK-2800 Lyngby, Denmark

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The motion of a single fluxon in long Josephson junctions of the overlap and inline geometries is investigated. It is concluded that if the junction is long and the damping is not too large then zero-field steps exist also in the inline junction. These zero-field steps are found to be mathematically identical to those of the overlap junctions in spite of the fact that the fluxon dynamics are quite different in the two cases.

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Recently large interest has been focused on fluxon motion in long Josephson junctions. Two different geometries are usually considered. In the overlap type of junctions the so-called zero-field steps are observed in the *IV* curve. They are due to resonant motion of (a few) fluxons being reflected at the boundaries.[1,2] In the inline geometry a so-called resistive branch may be observed in the *IV* characteristic.[3] This is due to a (continuous) generation of fluxons in one end and antifluxons in the other end of the junction and a subsequent annihilation in the center.[4] In this letter we discuss the differences and similarities between fluxon motion in the two geometries. One of the important similarities is that zero-field steps may also exist in inline junctions if certain conditions are satisfied.

The motion of fluxon in a long and narrow Josephson junction is governed by the perturbed sine-Gordon equation[5]

$$\phi_{xx} - \phi_{tt} = \sin\phi + \alpha\phi_t + \eta, \tag{1}$$

where ϕ is phase difference between the two superconducing films. The spatial variable in the long direction, x, is measured in units of the Josephson penetration depth $\lambda_j = (\hbar/2ed\mu_0 J)^{1/2}$, and time t in units of the reciprocal plasma frequency ω_0^{-1}, where $\omega_0 = (2eJ/\hbar C)^{1/2}$. Here J is the Josephson current density, d is the magnetic thickness of the barrier, and C is the capacitance per unit area. $\alpha = 1/\sqrt{\beta_c}$, where β_c is the McCumber parameter $\beta_c = 2eJC/\hbar G^2$, where G is the conductance per unit area due to quasiparticles. The unnormalized length of the junction is L (defines the x direction) and the unnormalized width is W. These dimensions are subject to the conditions $L \gg \lambda_J \gg W$. The surface current density in the London penetration layer of one of the films is given by

$$i_x = \lambda_J J\phi_x \tag{2}$$

and thus the magnetic field in the oxide layer is

$$H_y = i_x = \lambda_J J\phi_x. \tag{3}$$

The above considerations are common to the overlap and inline geometries; we now introduce the bias current I_{dc} and boundary conditions separately for the two cases.

In the overlap junction [Fig. 1(a)] the bias current is uniformly distributed across the long direction of the junction and enters through η in Eq. (1), which is then given by

$$\eta^{ov} = I_{dc}/JWL. \tag{4}$$

Defining κ as the magnitude of the (normalized) magnetic field [Eq. (3)] at the junction ends the boundary conditions are

$$\kappa^{ov} = \phi_x(0,t) = \phi_x(l,t) = 0, \tag{5}$$

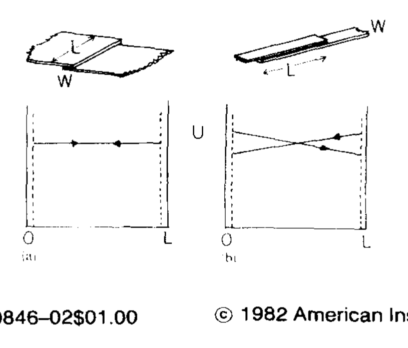

FIG. 1. (a) Overlap geometry. The fluxon propagates with a constant velocity defined by loss and bias. (b) Inline geometry. The fluxon receives energy when being reflected at the ends and loses energy when propagating along the line due to losses.

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where l is the normalized length, $l = L/\lambda_J$ ($l \gg 1$). Here we note that because of the uniformly distributed bias current the critical current of the junction I_c^{ov} is given by $I_c^{ov} = WLJ$.

In the inline junction [Fig. 1(b)] we have

$$\eta^{in} = 0 \quad (6)$$

in Eq. (1), and the bias current is introduced through the magnetic field it creates at the ends of the junction through the boundary conditions

$$\phi_x(0,t) = -\phi_x(l,t) = I_{dc}/2\lambda_J WJ = \kappa^{in}. \quad (7)$$

For this case the critical current I_c^{in} is given by $I_c^{in} = 4\lambda_J WJ$, which is smaller than in the overlap case.

If (for both geometries) one fluxon moves with average (normalized) velocity u, a dc voltage V_{dc} (normalized to $\hbar\omega_0/2e$) is developed across the junction. Here $V_{dc} = 4\pi u/2l = 4\pi/T$, where T is the (normalized) period. In order to determine this average velocity we write the total (normalized) energy on the Josephson transmission line[5]

$$H = \int_0^l \left(\frac{1}{2}\phi_x^2 + \frac{1}{2}\phi_t^2 + (1 - \cos\phi) \right) dx. \quad (8)$$

Differentiating Eq. (8) with respect to time and using η and κ (for the particular geometry) we obtain the perturbation result[5]

$$\frac{dH}{dt} = -\kappa[\phi_t(l,t) + \phi_t(0,t)] - 8u^2\gamma(u)\alpha + 2\pi\eta u. \quad (9)$$

To obtain the last two terms we have used the single-soliton solution to the pure sine-Gordon equation. $\gamma(u)$ is the Lorentz factor $\gamma(u) = (1-u^2)^{-1/2}$. In a stationary condition dH/dt integrated over one period should be zero. Since the phase change over one period is -4π we obtain from Eq. (9)

$$\alpha 8u^2\gamma(u) = 2\pi\eta u + \kappa 8\pi/T. \quad (10)$$

Since $\kappa = 0$ in the overlap geometry, the velocity u is constant. Physically the fluxon is maintained at this constant velocity through the Lorentz force from the uniformly distributed bias current [the term η in Eq. (10)]. This is shown in Fig. 1(a).

For the inline geometry $\eta = 0$, and the soliton velocity decreases along the Josephson line [Fig. 1(b)]. When it reaches the boundary it gets an energy input of $4\pi\kappa$ from the boundary condition and continues its steady-state motion. It should be noted that for these events to occur as described, it is required that the soliton reaches the other end of the junction with only a small velocity change, i.e. $\alpha l \ll 1$ or $l \ll \sqrt{\beta_c}$.

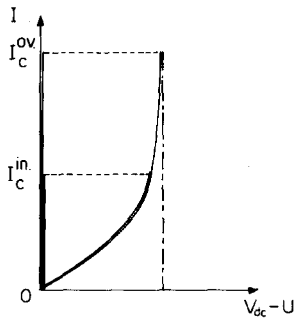

FIG. 2. Supercurrent and first zero-field step for both overlap and inline geometry. The only difference is that due to self-field effects, the critical supercurrent for an inline junction is smaller than the critical supercurrent for the overlap junction.

The (normalized) voltage across the junction, V_{dc}, is given by $V_{dc} = 2\pi u/l$, where u is determined from Eq. (10).

$$u\gamma(u) = \begin{cases} \dfrac{\pi\eta}{4\alpha} = \dfrac{\pi I_{dc}}{4\alpha JWL} & \text{(overlap)} \quad (11a) \\[2ex] \dfrac{\pi\kappa}{2\alpha l} = \dfrac{\pi I_{dc}}{4\alpha JWL} & \text{(inline)} \quad (11b) \end{cases}$$

Hence, the results for the two geometries are identical and the so-called zero-field steps from overlap junction also exist in inline junction as long as $l \ll \sqrt{\beta_c}$.

The common result for the two geometries is shown in Fig. 2. The main difference is that the critical current for the inline junction I_c^{in} is smaller than that for the overlap junction I_c^{ov}, as discussed earlier. Higher-order zero-field steps may be obtained by scaling the voltage with the number of fluxons. If, in the inline geometry, $l \gg \sqrt{\beta_c}$ and $I_{dc} > I_c^{in}$ fluxons are continuously created in one end of the junction and antifluxons in the other. They annihilate each other in the center of the junction and give rise to the so-called displaced linear branch as discussed by Scott and Johnson.[4]

In conclusion, we note that for inline junction with $l \ll \sqrt{\beta_c}$ and $I_{dc} < I_c^{in}$ zero-field steps obeying exactly the same equation as those of overlap junctions occur. This happens in spite of the fact that the soliton dynamics are quite different in the two cases. Because of the more restricted current range for the inline junctions, experimentally observed singularities in the IV curve may erroneously be interpreted as different from those found in overlap junctions.

[1]T. A. Fulton and R. C. Dynes, Solid State Commun. **12**, 57 (1973).
[2]R. D. Parmentier, in *Solitons in Action*, edited by K. Lonngren and A. Scott (Academic, New York, 1978), pp. 173–199.
[3]A. Barone, J. Appl. Phys. **42**, 2747 (1971).
[4]A. C. Scott and W. J. Johnson, Appl. Phys. Lett. **14**, 316 (1969).
[5]A. C. Scott and D. W. McLaughlin, Phys. Rev. A **18**, 1652 (1978).