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### Experimental relationship between damping and stability of sine-Gordon solitons in Josephson junctions

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We show some experimental results which suggest that total damping, including surface loss, plays a fundamental role in limiting the stability of high-velocity sine-Gordon solitons in real Josephson tunnel junctions.

One of the unsolved problems in experiments on soliton propagation in Josephson junctions is the following: What limits the stability of high-velocity sine-Gordon solitons and causes a switching to another mode before the speed of light is obtained? Observations on real junctions in various geometries show that the soliton disappears at a normalized bias current  $\eta$  in the range  $0.5 < \eta < 0.8$ . Perturbation theory<sup>1,2</sup> (with periodic boundary conditions) predicts stability until  $\eta = 1$  and  $u = 1$ , where  $u$  is the soliton velocity normalized to the speed of light in the barrier  $\bar{c}$ . Some numerical simulations<sup>3</sup> (periodic boundary conditions) show stability all the way to  $\eta = 1$ , while others<sup>4-6</sup> (open-end boundary conditions) do not. Several empirical explanations have been suggested. Pace<sup>7</sup> suggests that the presence of a soliton reduces the effective length of the junction, thus reducing the obtainable  $\eta$ . Others<sup>8,9</sup> have speculated that a resonant interaction with (plasma) oscillations causes the premature switching. We suggest, based on new observations on a long annular Josephson tunnel junction, that there could be a fundamental limit to the soliton's stability that is determined by the total losses in the junction. Switching then takes place when the minimum width of the soliton is obtained, as was also speculated by Pace.<sup>7</sup>

Previous measurements<sup>10</sup> of a single soliton trapped on the annular junction showed excellent agreement with a perturbation solution of the modified sine-Gordon equation:

$$-\phi_{xx} + \phi_{tt} + \sin\phi = -\alpha\phi_t + \beta\phi_{xxt} + \eta, \quad (1)$$

where  $\phi$  is the quantum phase difference across the junction,  $x$  and  $t$  denote partial differentiation in space and time,  $\alpha$  is the tunneling loss coefficient, and  $\beta$  is the coefficient for surface losses. The perturbation solution for the infinite line is<sup>10,11</sup>

$$\frac{\pi}{4}\eta = \alpha p \left( 1 + \frac{\beta}{3\alpha}(1+p^2) \right), \quad (2)$$

where  $p = u\gamma(u)$  is the normalized relativistic momentum of the soliton. Here  $\gamma(u)$  is the Lorentz factor  $\gamma = 1/\sqrt{1-u^2}$ . We note here that the left-hand side may be

interpreted as the force due to the bias current and the right-hand side as the damping forces. Our earlier work has given us reason to believe that we know the coefficients in Eqs. (1) and (2) as well as  $\bar{c}$  and  $I_0$  (the critical current), all at four different temperatures.

In terms of Eqs. (1) and (2), we are now asking: What are the maximum values of  $\eta$  and  $\gamma$  (or  $p$ ) for which the soliton is stable? The results of our measurements, and those inferred from other measurements and simulations in the literature<sup>4-6,12,13</sup> are shown in Fig. 1, which plots the maximum bias  $\eta_c$  against the maximum Lorentz factor,

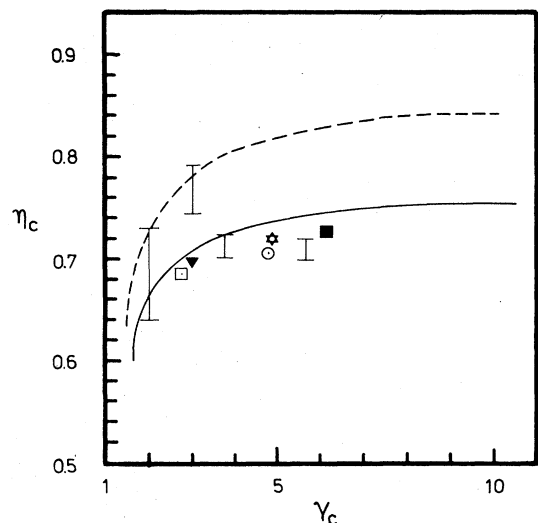


FIG. 1. Plot of the maximum current  $\eta_c$  vs the maximum Lorentz factor  $\gamma_c$  for the lowest-order step on the  $I$ - $V$  curves of various junctions and junction simulations. The four error bars are our results from the annular junction. The open square (Ref. 13) and the star (Ref. 12) are experiments on long junctions. The solid triangle (Ref. 6) and solid square (Ref. 4) are simulations with open ends and  $\beta = 0$ . The open circle (Ref. 5) is a simulation fitted to experimental results with open ends and both  $\alpha$  and  $\beta$  finite.

TABLE I. Parameters for our annular junction at four different temperatures. We have noticed that the last column  $D$  remains essentially constant, while the damping parameters change by an order of magnitude.

$\alpha$	$\beta$	$\gamma_c$	$\alpha\gamma_c$	$\frac{1}{3}\beta\gamma_c^3$	$D = \alpha\gamma_c + (\beta/3)\gamma_c^3$
0.018	0.01	5.59	0.1006	0.582	0.6826
0.055	0.0306	3.64	0.2002	0.4919	0.6921
0.08	0.0445	2.97	0.2376	0.3886	0.6262
0.18	0.10	2.02	0.3636	0.275	0.6386

$\gamma_c = (1 - u_c^2)^{-1/2}$ , where  $u_c$  is the maximum velocity. In this figure the four data points with vertical error bars are our results, for which  $\gamma_c$  has been explicitly determined by fitting to Eq. (2). For all the other points we have used data in the literature and estimated  $\gamma_c$  either by direct calculation from the  $I$ - $V$  curve, or by inference from knowledge of the damping coefficients,  $\eta_c$ , and an empirical relationship described by Eq. (3) below. The error bars on our data result from the fact that for an annular junction the critical current and the step height must be obtained from two independent measurements, which become difficult to reproduce exactly near  $T_c$ . It is interesting to note that the solid-triangle and solid-square points are both simulations with boundaries (long rectangular junctions) and with  $\beta = 0$ . The star and the open square are both experimental measurements on rectangular junctions, and the open circle is a simulation with boundaries and both  $\alpha$  and  $\beta$  damping. Table I lists the parameters for our measurements, and shows that by changing the temperature, we are able to vary  $\alpha$  and  $\beta$  over a full decade. But we have noticed that  $D$ , the last column in the table, remains constant within 5%. That is,

$$\alpha\gamma_c + \frac{\beta}{3}\gamma_c^3 = D \approx 0.66 \pm 5\% \quad (3)$$

Comparing with Eq. (2) we note that  $D$  is approximately the total damping force when switching occurs (the total damping force at switching is  $Du_c$ ; however,  $u_c$  is always close to one<sup>14</sup>). Substituting our measured value of  $D$  back into the perturbation result, Eq. (2), gives  $\eta_c = (4/\pi)D(1 - \gamma_c^{-2})^{1/2}$ . This is the dashed curve in Fig. 1. The solid curve includes the correction<sup>9,15-17</sup> to the perturbation result that has been proposed to account for the finite width of the fluxon as  $u$  goes to 1. We have used<sup>17</sup>  $\eta'_c = \eta_c(1 - \eta_c^2)^{-1/8}$ , where  $\eta'_c$  is the perturbation result and  $\eta_c$  is the solid line of Fig. 1. This curve does a reasonable job of fitting the data.

We propose a qualitative physical argument to explain why our measured quantity  $D$  should be a constant. Since

the damping force varies over the length of the soliton (being largest at the center), we imagine that differential drag forces pull the soliton apart as  $u$  approaches 1.  $D$  is approximately the total drag force on the soliton, based on perturbation theory. We suppose that it is also proportional to the maximum difference in drag over the length of the soliton. When  $D$  exceeds the force binding the soliton together, namely  $\pi/4$  in normalized units, the soliton solution to Eq. (1) disappears and the junction switches out to the quasiparticle branch of the  $I$ - $V$  curve.

We also note that in the limit  $\beta = 0$ , Eq. (3) would become  $\alpha\gamma_c = D$ ; from theoretical and numerical work<sup>2,6</sup> this corresponds to the value of  $\gamma$  where the soliton has a minimum width, with  $D$  approximately equal to 0.7. This supports our picture of the drag forces ultimately pulling the soliton apart.

We have shown experimental indications that the total drag force  $D$  given by Eq. (3) is approximately constant when the soliton solution to Eq. (1) is observed to disappear. That is, Eq. (3) seems to give a good empirical prediction of  $\gamma_c$ . Using the predicted values of  $\gamma_c$  in the perturbation theory yields an upper limit on  $\eta_c$ , the maximum bias that the soliton can sustain. When the finite-width correction is applied to the perturbation result we get a reasonably accurate description of our data and various points taken from the literature. We believe this to be the first indication of real junctions sampling the regime where the Lorentz contraction of the soliton is counterbalanced by drag forces. We realize that some of our arguments are rather qualitative and not fully suited to describe the delicate region near the minimum width of the soliton at  $u \leq 1$ . However, the experimental observation of the constancy of  $D$  in our annular junction is the main result and provides a clue for further experimental and numerical investigations.

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