



## Diffraction by a plane angular sector, a new derivation

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Thus, other than the null series result, the current in the shadow region does not have an asymptotic series representation. This is an important result because it is the first time (to the author's knowledge) that the failure of an L-K series in the shadow region has been both demonstrated and explained.

While the L-K series does not lead to an exact result for the surface current density, it still holds the potential for providing a tractable improvement to a pure geometrical optics solution. This fact is demonstrated by Ansoorge's [5] calculations for scattering by a dielectric sphere using the ray optics field approach. The attractiveness of the current approach as developed here is due in large part to the fact that the complete L-K representation can be developed entirely from an integral of known functions, i.e.,

$$\int \nabla_0 G(|\Delta \vec{r}|) \exp(j\vec{k}_i \cdot \Delta \vec{r}) d\vec{r}_{i0}.$$

While a complete L-K series development for this integral is prohibitive, it may be possible to obtain the terms up to and including  $k_0^{-2}$  [7]. One of the primary advantages of obtaining the  $k_0^{-1}$  and  $k_0^{-2}$  corrections to the  $k_0^0$  asymptotic expansion of this integral is the recovery of some of the cross-polarizing properties of a rough surface in the high frequency (but not optical) limit.

The results obtained in this communication raise an interesting point. Usually, the first frequency dependent correction to physical optics comes from the nonzero width of the transition zone between the illuminated and shadowed regions on the scatterer and the propagation of creeping waves into the shadow zone. If the L-K series produces a result that is identically zero in the shadow zone, what is the physical source of the  $k_0^{-n}$ ,  $n = 1, 2, \dots$  terms in the illuminated zone? It would appear that this may be due to the transition zone encroaching into the illuminated zone as the frequency is decreased. This is the kind of question that needs to be answered if there is to be a full understanding of the Luneburg-Kline asymptotic representation.

It should be noted that  $\vec{s}_1(\vec{r})$  in (21b) has essentially been computed by Chaloupka and Meckelburg [8] in a very clever application of integration by parts. Their results should provide the basic ingredients to study the question raised in the previous paragraph.

#### ACKNOWLEDGMENT

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## Diffraction by a Plane Angular Sector, A New Derivation

THORKILD B. HANSEN

**Abstract**—The purpose of the present communication is to present an alternative derivation of the exact solution to the scattering problem in which a Hertz dipole illuminates a perfectly conducting plane angular sector.

#### I. INTRODUCTION

The geometrical theory of diffraction (GTD) is one of the most widely used methods for calculating scattered electromagnetic fields. In this theory, the scattered field is approximated by a series in which each term is a contribution from a certain point on the scatterer. These contributing points on the scatterer are called critical points by Van Kampen [1]. To calculate such contributions, diffraction coefficients belonging to each critical point are needed. Diffraction coefficients are found from exact solutions to so-called canonical scattering problems. As an example, the edge diffraction coefficient for the perfectly conducting edge is found from the exact solution to the infinite perfectly conducting wedge. Similarly, the corner diffraction coefficient may be found in principle from the exact solution to a canonical scattering problem that contains a corner. Today there exists only one such solution, that of diffraction from a perfectly conducting plane angular sector, although a corner diffraction coefficient has not been found from it. However, progress has recently been made by Smyshlyayev [2], [3] in deriving a corner diffraction coefficient from this solution.

A plane angular sector is a section of a plane bounded by two half-lines which have common endpoints. As an example, the plane angular sector is a quarter-plane if the half-lines are orthogonal to each other. The vector solution to this scattering problem was first found by Satterwhite [4]. This solution has been used for numerical calculations in several papers e.g. [5]-[7]. Corrections to some misprints in the vector wave function  $\vec{N}$  appearing in these references are given in Section III. Because Satterwhite's solution is the only one existing for a scattering problem where the scatterer contains a corner, it would be valuable to verify it. The purpose of this communication is to present a new derivation of Satterwhite's solution. No attempt is made to derive a corner diffraction coefficient from it.

The method used by Satterwhite [4] can be described as follows. The total electric field is written as a series expansion of vector wave functions. The expansion coefficients are found by integration over a surface which is a sphere, with center at the tip of the plane angular sector, except for an infinitesimal cut around the plane angular sector. During this integration, use is made of Green's second identity for vectors, the divergence theorem, and orthogonal-

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for the expansion coefficients  $\bar{A}_{sm}$  and  $\bar{B}_{sm}$  are obtained

$$\bar{A}_{sm}(\bar{r}', \alpha) = -\frac{2}{\pi} \frac{\alpha^2}{\alpha^2 - \kappa^2} \frac{1}{\Lambda_{sm2}} \bar{M}_{sm2}^I(\bar{r}', \alpha) \quad (6)$$

and

$$\bar{B}_{sm}(\bar{r}', \alpha) = -\frac{2}{\pi} \frac{\alpha^2}{\alpha^2 - \kappa^2} \frac{1}{\Lambda_{sm1}} \bar{N}_{sm1}^I(\bar{r}', \alpha). \quad (7)$$

Insertion of (6) and (7) into (4) and integration with respect to  $\alpha$ , [9] gives our final expressions for the electric dyadic Green's function:

$$\begin{aligned} \bar{G}(\bar{r} | \bar{r}') = j\kappa \sum_{s,m} \left[ \bar{M}_{sm2}^{II}(\bar{r}, \kappa) \bar{M}_{sm2}^I(\bar{r}', \kappa) \frac{1}{\Lambda_{sm2}} \right. \\ \left. + \bar{N}_{sm1}^{II}(\bar{r}, \kappa) \bar{N}_{sm1}^I(\bar{r}', \kappa) \frac{1}{\Lambda_{sm1}} \right], \quad \text{when } |\bar{r}| > |\bar{r}'| \end{aligned} \quad (8)$$

and the same expression with  $\bar{r}$  and  $\bar{r}'$  interchanged when  $|\bar{r}| < |\bar{r}'|$ .

This solution, which agrees with that in [4], is shown in [4] and [9] to satisfy the radiation and the edge condition. Explicit expressions for  $\bar{M}_{smi}$  and  $\bar{N}_{smi}$  may be found in [4] and [9].

Note that when fields are to be calculated within the source region an additional term must be added to the expression in (8) [12], [13].

In conclusion, the solution obtained by Satterwhite [4] is critically confirmed by using the Ohm-Rayleigh method of solution to determine the electric dyadic Green's function for the perfectly conducting plane angular sector.

#### APPENDIX ORTHOGONALITY RELATIONS

The formulas shown in this Appendix are proved in [9] and, to the author's knowledge, have not previously been published.

$$\int_{R^3} \bar{M}_{sm2}^I(\bar{r}, \alpha) \cdot \bar{N}_{s'm'1}^I(\bar{r}, \alpha') dV = 0 \quad (9)$$

$$\begin{aligned} \int_{R^3} \bar{M}_{sm2}^I(\bar{r}, \alpha) \cdot \bar{M}_{s'm'2}^I(\bar{r}, \alpha') dV \\ = \Lambda_{sm2} \frac{\pi}{2} \frac{\delta(\alpha - \alpha')}{\alpha^2} \delta_{ss'} \delta_{mm'} \end{aligned} \quad (10)$$

$$\begin{aligned} \int_{R^3} \bar{N}_{sm1}^I(\bar{r}, \alpha) \cdot \bar{N}_{s'm'1}^I(\bar{r}, \alpha') dV \\ = \Lambda_{sm1} \frac{\pi}{2} \frac{\delta(\alpha - \alpha')}{\alpha^2} \delta_{ss'} \delta_{mm'} \end{aligned} \quad (11)$$

where  $\Lambda_{smi}$  are normalization constants and  $\delta_{pq} = 0$  when  $p \neq q$  and  $\delta_{pp} = 1$ .

#### PROOF OF (10)

The integral in (10) can be written as

$$\begin{aligned} \int_0^{+\infty} j_{\nu_{sm2}}(\alpha r) j_{\nu_{s'm'2}}(\alpha' r) r^2 dr \int_0^\pi \int_{-\pi}^\pi \bar{m}_{sm2}(\theta, \phi) \\ \cdot \bar{m}_{s'm'2}(\theta, \phi) \sigma(\theta, \phi) d\phi d\theta \end{aligned} \quad (12)$$

where  $j_\nu(z)$  is the spherical Bessel function and  $\bar{m}_{sm2}(\theta, \phi)$  is the angular dependent part of the vector wave function  $\bar{M}_{sm2}^I(\bar{r}, \alpha)$ .

Furthermore,  $h_1 h_2 h_3 = r^2 \sigma(\theta, \phi)$  where  $h_1$ ,  $h_2$ , and  $h_3$  are the metrical coefficients in the spheroconal coordinate system [11]. Note that the double integral is independent of  $\alpha$ ,  $\alpha'$ , and  $r$ . In order to evaluate the double integral in (12) we note that

$$\begin{aligned} \nabla \cdot (\Psi_{sm2}(\bar{r}, \alpha) \nabla \Psi_{s'm'2}(\bar{r}, \alpha)) = -\alpha^2 \Psi_{sm2}(\bar{r}, \alpha) \Psi_{s'm'2}(\bar{r}, \alpha) \\ + \nabla \Psi_{sm2}(\bar{r}, \alpha) \cdot \nabla \Psi_{s'm'2}(\bar{r}, \alpha). \end{aligned}$$

By integrating this identity over a spherical volume with center at  $(0, 0, 0)$ , using the divergence theorem and the orthogonality relations for Lamé functions [11, appendix II], one finds that the double integral in (12) is zero when  $s \neq s'$  or  $m \neq m'$ . By using [8, p. 11, eq. (26)] and denoting the double integral in (12) by  $\Lambda_{sm2}$  when  $s = s'$  and  $m = m'$ , the formula (10) is proved.

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#### The Electromagnetic Field of a Slotted Elliptic Cone

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**Abstract**—The exact solution for the electromagnetic field diffracted by a perfectly conducting slotted elliptic cone is determined. The solution is presented in the form of a dyadic Green's function, which is the most general form of a possible solution. Some numerical results are

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