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Study of the atmospheric wake turbulence of a CFD actuator disc model

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Abstract

Modelling wind turbine wake using the standard $k-\epsilon$ atmospheric turbulence model gives a too fast wake dissipation compared to measurements and more advanced turbulence models. The problem is that the relevant scales of turbulence are different from the point of view of the wind turbines and from the point of view of the terrain. As the atmospheric turbulence model imposes the scale of turbulence, part of the influence of the wind turbine on the airflow is happening at the turbulence level. It becomes therefore necessary to add terms in the turbulence equations to account for the wind turbine activity. In this context two different models are compared together and with wind turbine wake measurements. With the proper parameterization, they are both able to produce similar results, in agreement with the measurements.

1 Introduction

Modelling wind turbine wake has been a productive research area since the early 1970s. The trend in the methods used has closely followed the technological advancement of computer science. With the growing popularity of Computational Fluid Dynamics methods (CFD), different wake models have been proposed. First as actuator disc models (e.g. Crespo et al. [1], Sørensen et al.[2], Ammara et al. [3], Réthoré et al. [4]), actuator lines (e.g. Mikkelsen [5], Troldborg [6]), and also as full rotor computations (e.g. Sørensen et al. [7], Zhale [8]). In parallel, modelling atmospheric flows over terrains has been a growing source of interest (e.g. Sørensen [9], Bechmann [10]). It is a natural evolution to try to link the two approaches in order to model wind turbine wake in atmospheric flows.

As the computational cost goes down, and the large clusters become more affordable for wind turbine companies, it becomes possible for them to run the cheapest kind of CFD atmospheric wake computations, based on Reynolds Averaged Navier-Stokes (RANS) turbulence models, and actuator disc model. Nonetheless, linking these two models without a specific treatment gives an unrealistic wake dissipation. El Kasmi et al. [11] addressed this issue and proposed to use a model of Chen and Kim [12], adding an extra term in the dissipation equation proportional to the production of turbulence, to enhance the transfer of large scale turbulence to small-scale turbulence. Cabezon et al. [13] investigated further this issue by considering several other modification of the $k-\epsilon$ model, as well as using a Reynolds-Stress turbulence model which are more computationally expensive, but which seemed to give better results.

In a different field of atmospheric physics, the area of forest canopy modelling, researchers have also been working on this issue. As the wind turbines can be modelled as actuator disc of porous body forces, the forest trees influence on the local wind is also modelled as body forces. Some modifications of the $k-\epsilon$ model can be found, for example, in Sogachev [14], Sogachev et al. [15] and Sanz [16]. The main difference with the model proposed by El Kasmi et al., is that all the added terms are proposed to be proportional to the canopy drag coefficient, and some extra terms are added in the Turbulence Kinetic Energy (TKE) equation.

In the present work, after a theoretical presentation of the problem, we propose to modify the $k-\epsilon$ equations based on an adaptation of Sanz's model to wind turbine wake. The added terms

are chosen to be proportional to forces acting on the wind turbine blades, as well as the lost mean kinetic energy transfered into turbulence.
After a parameter study, the results obtained are shown to behave closer to the wind turbine wake measurements than the standard $k-\epsilon$ model, and give encouraging prospects concerning the use of actuator discs in atmospheric flow conditions.

2 Presentation of the problem

2.1 Theoretical analysis

In a RANS turbulence model, the turbulence and the mean flow are separated into two parts. On one side, the influence of turbulence over the mean flow is modelled through the addition of the eddy viscosity μ_τ to the molecular viscosity μ.

$$\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_j}(\rho U_i U_j) - \frac{\partial}{\partial x_j}\left[(\mu + \mu_t)\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] + \frac{\partial P}{\partial x_i} = S_{Ui}, \quad (1)$$

where U is the velocity, P is the pressure and S_U is the body force.

On the other side, the influence of the mean flow over k and ϵ is proportional to the production of turbulence P_τ, which is related to the local gradient of the mean flow velocity.

$$P_\tau = \mu_\tau \frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \quad (2)$$

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho U_j k) - \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_\tau}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] = P_\tau - \rho\epsilon \quad (3)$$

$$\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_j}(\rho U_j \epsilon) - \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_\tau}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right] = C_{\epsilon 1}\frac{\epsilon}{k}P_\tau - \rho C_{\epsilon 2}\frac{\epsilon^2}{k} \quad (4)$$

It is interesting to notice that there are no pressure, or pressure fluctuation terms neither in the production of turbulence, nor in the k or ϵ equations. In fact during the derivations of the Reynolds Averaged Navier-Stokes equations, there are some pressure diffusion terms that remain in the turbulence kinetic energy equation, but they are assumed to be neglectable to the transport of turbulence kinetic energy (see Wilcox [17] p.110).

The eddy viscosity concept is accounting for the dissipative effect of the large eddies over the mean flow shear. μ_τ is directly proportional to the square of the turbulence kinetic Energy k divided by the dissipation of turbulence kinetic energy ϵ. As they are both positive values, the eddy viscosity is always positive, and always has a positive dissipative effect.

$$\mu_\tau = \rho C_\mu \frac{k^2}{\epsilon}, \quad (5)$$

where C_μ is a constant.

In neutral atmospheric flows, in order to satisfy the log-law, the eddy viscosity is linearly dependent with the height. So the higher an observer is situated, the larger the eddies are going to be, and the more effective their viscous effect is going to be on the local mean flow.

$$\mu_\tau = \rho\kappa u_\tau y, \quad (6)$$

where κ is the von Karma constant, u_τ is the friction velocity and y is the height.

When a wind turbine is placed in an atmospheric turbulent flow, the large eddies are considered differently by the wind turbine and by the terrain. An observer positioned on a rotating wind turbine blade sees the large atmospheric eddies as a fluctuation of relative velocity. Nonetheless, the length scales and the time scales of this relative velocity fluctuations are not considered as turbulence by the blade. Indeed, the pressure on the blade has the time to adapt to the change of relative velocity, and the resulting forces acting on the flow are also proportionally adjusted. But

from the point of view of the terrain, which sees the large eddies as turbulence, the wind turbine is actually acting on the turbulence as well as the mean air flow. In other words, the length and time scales considered to be turbulence from the point of view of the terrain are of several order of magnitudes larger than those considered to be turbulence from the point of view of the wind turbine blades.
The problem is that a RANS model sets an arbitrary threshold from which scale the wind fluctuations are considered to be turbulence. This value is linked with the observed height, which is imposed the terrain turbulence model. In order to comply with this restriction, there should be turbulence terms proportional to the wind turbine activity in the k and ϵ equations. But because the pressure fluctuations have been neglected in the $k-\epsilon$ model, and the body forces are considered to act only on the mean flow, the action of the turbine on the turbulence is not be taken into account. And this is true not only for a steady state actuator disc, but also for an unsteady actuator line model, and even an unsteady full rotor computation.

Another phenomena that can be overlooked is that the wind turbine by itself generates turbulence. There is, of course, the shear generated turbulence created by the gradient of the wake velocity. This type of velocity is taken care by the $k-\epsilon$ model by the production of turbulence term P_τ. But the turbine generates as well some blade tip and root vortices and some blade shed vortices. Finally, the wake rotation can also be considered as a contribution to the turbulence, after few rotor diameters. After breaking down, both the vortices and the wake rotation also take part to the wake dissipation. This type of turbulence is not automatically accounted in a steady state RANS turbulence model.

These two opposite phenomena are combined in something referred, in the forest canopy research area, as the short-circuiting of turbulence cascade [16]. The concept is that the trees (and here the wind turbines) extract some kinetic energy from the larger scale of the turbulence spectrum and re-inject some at a lower scale, short-circuiting the natural phenomenon of turbulence cascade. The difference between the two amount of kinetic energy is partly lost as heat through viscous effects, and, at least in the case of the wind turbine, partly extracted as mechanical power by the rotation of the rotor.

The relevant questions to this problem are therefore, how much influence has the wind turbine over the turbulence, and how to model it.

2.2 Influence of the effective Reynolds number on wake dissipation

To address these questions we first investigated the eddy viscosity threshold at which the wind turbine wake begins to be significantly affected. In order to do that we switched off the turbulence model and increased the molecular viscosity to simulate roughly the influence of the atmospheric turbulence over a rotor. Figure 1 shows the axial velocity of an actuator disc with axial and tangential non-homogeneous forces.

According to the hub height, the friction velocity and the local density, the eddy viscosity seen by an actuator disc in atmospheric turbulence is of the same order of magnitude than Figure 1 d). What is interesting to notice here, is that the wake seems unaffected by the viscosity for an effective Reynolds number down to 10,000. Lower than that the effect of the viscosity becomes gradually stronger. So in theory, by operating at an eddy viscosity 100 time smaller, one could avoid the problem of unrealistic wake diffusion.

2.3 Comparison with a Large-Eddy Simulation

Large-Eddy Simulation (LES) is another type of turbulence model where there is a distinction made between the large scale eddies which are simulated and the lower scale of turbulence which is modelled. The threshold between the two is typically of the order of the cell size. Because of this, the eddy viscosity is effectively of the order of the cell size. If the cells are small enough, it is therefore possible to operate an unsteady simulation at a eddy viscosity up to 50 times smaller

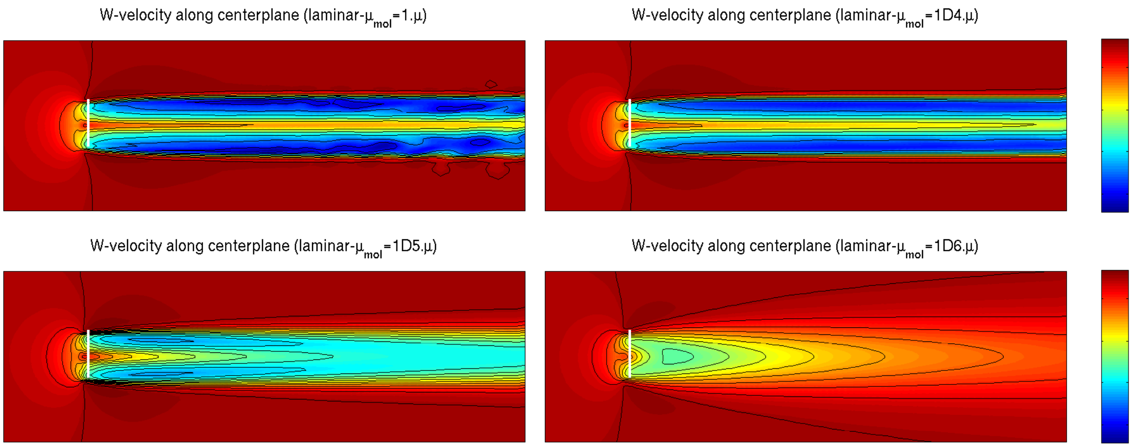

Figure 1: Axial-velocity for different molecular viscosities ($Re = 10^8$, 10^4, 10^3, 10^2)

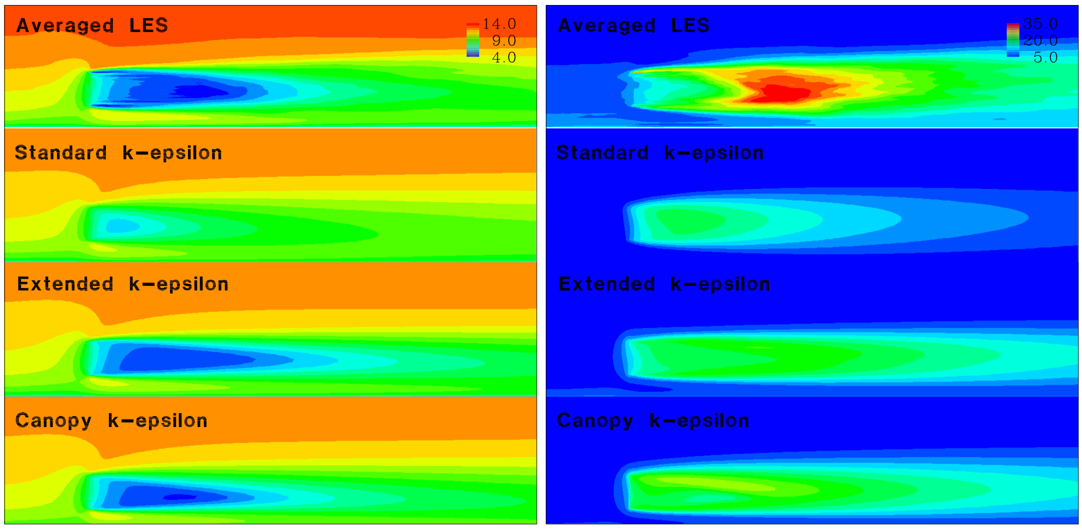

Figure 2: Axial-velocity [m/s] and turbulence intensity [%] in the vertical plane passing through an actuator disc

than the eddy viscosity seen in an equivalent $k-\epsilon$ simulation at hub height. An hybrid RANS-LES model based on the EllipSys CFD flow solver (Sørensen [9] and Michelsen [18]) has been developed by Bechmann [10]. It is able to simulate the atmospheric boundary layer turbulence by building up turbulence in a preprocessing cyclic box. An unsteady LES computation was then run using this preprocessed turbulence box as an input and using an actuator disc model [4]. After obtaining a full development of the wake, the simulation was averaged over 3000 iterations to build an equivalent mean result that could be compared with a steady state RANS simulation. Figure 2 compares the axial velocity and turbulence intensity of the averaged LES with the standard, extended and canopy $k-\epsilon$ computation, which are presented in the following section.

3 Methods

3.1 Extended $k-\epsilon$ model

El Kasmi et al. [11] proposed to use an extended $k-\epsilon$ model of Chen and Kim [12]. The idea is to add a term in the turbulence kinetic energy dissipation ϵ to enhance the creation of dissipation proportionally to the production of turbulence.

$$S_\epsilon = C_{\epsilon 4}\frac{P_\tau^2}{\rho k} \tag{7}$$

Adding this term around the wind turbine is basically compensating the production of turbulence created by the axial velocity shear by a proportional increase in the dissipation. It is justified physically by arguing that it represents the "energy transfer rate from large-scale turbulence to small-scale turbulence controlled by the production range scale and the dissipation rate time scale." Chen and Kim [12]. This term is therefore meant to account for the short-circuiting of turbulence cascade as previously described. The influence of the turbine over the turbulence is therefore assumed to be carried on through the production of shear turbulence.
This model introduces practically 2 free independent parameters to "tune" the turbulence model to measurements, the dissipation constant $C_{\epsilon 4}$ and the area over which it is applied.
The axial velocity development was compared with different sets of measurements by El Kasmi et al [11] and by Cabezon et al. [13]. The model seems to behave in agreement with the measurements in most of the cases.

3.2 Canopy model

In the forest and urban canopy research area, researchers have also proposed some modifications of the turbulence kinetic energy and dissipation. (see Sogachev et al [14] for a full review). Sanz [16] adopted a pragmatic approach and proposed to model the influence of the trees as a source and sink of k and ϵ.

$$S_U = \frac{1}{2}C_x U^2 \tag{8}$$

$$S_k = \frac{1}{2}C_x\left(\beta_p U^3 - \beta_d U k\right) \tag{9}$$

$$S_\epsilon = \frac{1}{2}C_x\left(C_{\epsilon 4}\beta_p\frac{\epsilon}{k}U^3 - C_{\epsilon 5}\beta_d U\epsilon\right) \tag{10}$$

The term proportional to β_p is accounting for the production of wake turbulence, and represents the ratio of mean kinetic energy transfered directly into turbulence (not the shear generated turbulence). While the term proportional to β_d accounts for the short circuiting of turbulence cascade, so the transfer of energy between the large scale turbulence to smaller scales of turbulence. In forest canopies the set of values used are in general around $\beta_p = 1$ and $\beta_d = 4$ [14], [16]. It is interesting to note that by using $\beta_p = 1$ they assume that the totality of the energy lost by the mean air flow is transfered into turbulence and so neglect the viscous effects, which tend to transform mean kinetic energy into heat.

3.3 Adapted canopy model

Estimation of the wake turbulence dissipation parameter β_d

In order to determine the size of β_d we propose to derive the RANS equations assuming that the force is adjusting to the wind fluctuations. In order to simplify the derivations we consider that the turbine is aligned with the x direction, and the thrust force is only dependent to the axial velocity U. As the fluctuations are assumed to be small, the thrust coefficient is assumed constant. The momentum source term is found using the local induced velocity at the disc by expressing the thrust of the turbine $C_T = 4a(1-a)$ locally at the disc.

$$S_U = -\frac{1}{2}\rho A C_x U^2, \quad \text{with} \quad C_x = \frac{4a}{1-a}. \tag{11}$$

where U is the induced velocity at the disc, a is the axial induction factor so that the velocity as the disc is $U = (1-a)U_\infty$. The axial induction factor a can be determined from the thrust coefficient C_T (e.g. Hansen [19])

$$a = \frac{1}{2}\left(1 - \sqrt{1 - C_T}\right). \tag{12}$$

The induced velocity at the disc is separated into two parts, a mean component, and a fluctuation component $U = U + u'$.

$$S_U(U + u') = -\frac{1}{2}\rho A C_x (U + u')^2 \tag{13}$$
$$= -\frac{1}{2}\rho A C_x (U^2 + 2Uu' + u'u'). \tag{14}$$

Taking the time averaging removes the $2Uu'$ term.

$$\overline{S_U(U + u')} = -\frac{1}{2}\rho A C_x (U^2 + \overline{u'u'}). \tag{15}$$

The term $\overline{u'u'}$ remaining is nonetheless neglectable in comparison with the term U^2. For example, for a turbulence intensity lower than 10%, $\overline{u'u'} < 100U^2$.

The same procedure can be applied to derive the k equation. The derivation is based on the following time average (Wilcox [17]).

$$\overline{u'_i \aleph(u_j) + u'_j \aleph(u_i)} = 0 \tag{16}$$

The source term for k can be derived in a similar fashion.

$$S_{k,d}) = -\frac{1}{2}\rho A C_x \overline{(u' + v' + w')(U^2 + 2u'U + u'u')} \tag{17}$$
$$= -\frac{1}{2}\rho A C_x \left((\overline{u'u'} + \overline{u'v'} + \overline{u'w'})U + \overline{u'u'u'} + \overline{u'u'v'} + \overline{u'u'w'}\right) \tag{18}$$

The $\overline{u'_i u'_j}$ terms are defined as the Reynolds-Stress Tensor and are modelled as

$$\overline{u'_i u'_j} = \frac{2}{3}k\delta_{ij} - \frac{\mu_\tau}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{19}$$

The $\overline{u'_i u'_i u'_j}$ terms are accounting for the turbulent transport, and are usually modelled as

$$\overline{u'_i u'_i u'_j} = -2\frac{\mu_\tau}{\rho\sigma_k}\frac{\partial k}{\partial x_j} \tag{20}$$

The final expression of $S_{k,d}$ is therefore

$$S_{k,d} = -\frac{1}{2}\rho A C_x \left[\frac{2}{3}kU - \frac{\mu_\tau}{\rho}\left(\frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} + \frac{\partial V}{\partial x} + \frac{\partial W}{\partial x}\right)U - 2\frac{\mu_\tau}{\rho\sigma_k}\left(\frac{\partial k}{\partial x} + \frac{\partial k}{\partial y} + \frac{\partial k}{\partial z}\right)\right] \tag{21}$$

The canopy model proposed by Sanz [16] neglects the velocity and turbulence derivative terms which leaves only the term proportional to kU.

$$S_{k,d} = -\frac{1}{2}\rho\beta_d A C_x kU, \tag{22}$$

with $\beta_d = 2/3$.

Nonetheless, it is difficult to argue that the terms neglected do not play a significant influence on the turbulence. And without a proper estimation of the error done while neglecting these terms it is safer to keep a certain flexibility with this parameter. It is quite interesting to notice that the value generally used in forest modelling ($\beta_d \approx 4$) is significantly larger than the one derived here.

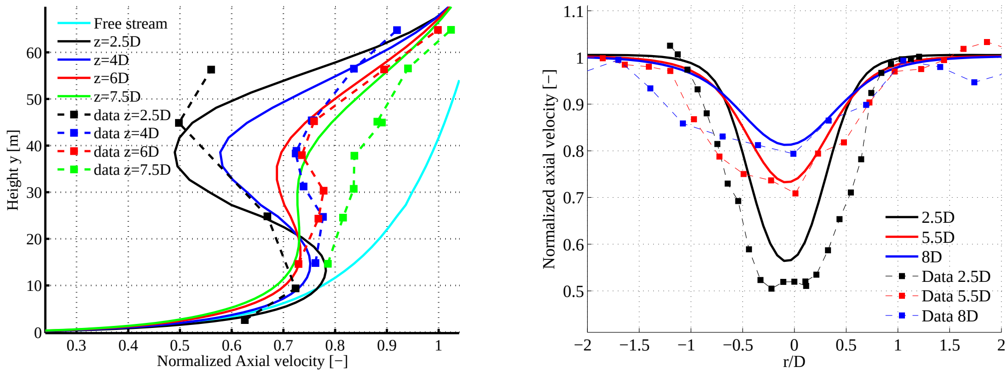

Figure 3: Adapted canopy model vs measurements: Left: Nibe [20] and Right: Sexbierum [20]

Estimation of the wake turbulence production parameter β_p

β_p accounts for the ratio of kinetic energy lost by the mean airflow which is converted into turbulence. In the case of wind turbines, the viscous losses are not negligible, and there is in addition the energy extracted from the system through the mechanical work of the rotor.
We propose to estimate β_p based on the wind turbine energy budget using the thrust and power coefficients. From basic actuator disc theory (e.g. Hansen [19]), we have

$$P = \frac{1}{2}\rho A C_x U^3 \tag{23}$$

The power coefficient C_P of the wind turbine gives an estimate of how much energy has been extracted as electricity. This neither includes the transfer losses (e.g. mechanical, electrical), and neither the viscous effect on the blade which dissipate kinetic energy as heat. These losses can be assumed to be proportional to the cube of the induced velocity at the wind turbine rotor. The wind turbine energy budget can then be applied to estimate the kinetic energy transfered into turbulence.

$$P = P_{\text{turb}} + P_{\text{losses}} + P_{\text{elec}} \tag{24}$$

$$P_{\text{turb}} = \frac{1}{2}\rho A C_x \left(1 - \frac{C_{\text{H}} + C_P}{4a(1-a)^2}\right) U^3 \tag{25}$$

where C_H is the heat losses coefficient. In the following simulations, we chose to use the set of values $C_H = 0.15$ and $C_P = 0.4$ $C_T = 0.8$ which gives $\beta_p = 0.05$.

4 Results

The adapted canopy $k-\epsilon$ model was tested on two sets of measurements. The Nibe measurements done by Taylor [20], and the Sexbierum wind farm measurements by Cleijne [20]. In both cases, the actuator disc model developed by Réthoré and Sørensen [4] was used. The forces on the disc where estimated using the thrust coefficient of the wind tubines, scaled over the free stream wind profile. The added terms in the k and ϵ equations were estimated using the local velocity and turbulence values at the disc. The sets of parameters used for both Nibe and Sexbierum was: $C_{\epsilon 4} = 1.6$, $C_{\epsilon 5} = 0.2$, $\beta_p = 0.05$, $\beta_d = 3/2$.

5 Discussion

In Figure 2, the wake recovers faster in the standard $k-\epsilon$ model. The main reason is that the eddy viscosity is higher than in the other models, so the mean axial velocity dissipated faster. The

two modified models have been tuned to give a similar wake development to the averaged LES. The aim of this exercise was to study the behavior of the turbulence intensity.
The two modified models presents a similar turbulence intensity, at the exception of the close wake region for the canopy model which has a decrease. The averaged LES seems to have a slower increase of turbulence but eventually becomes of much higher intensity than the other models. the wind turbine, by applying a force, acts on the mean air flow velocity through a gradient of pressure. And this gradient of pressure is acting in a spherical area up to 1 rotor diameter around the wind turbine. Any fluctuation of the force should be propagated as a fluctuation of pressure in the turbulence. This can explain why the turbulence is not increasing as much upstream the disc in the averaged LES computation while there is a significant gradient of velocity. Moreover, while watching the LES animation of the turbulent flow going through the rotor, one can clearly see that the actuator disc acts as a filter of turbulence, and damps the incoming fluctuations in the close region, both upstream and downstream the rotor. The canopy model, as it acts only in the region where the body forces are applied, is not able to damp the fluctuations created by the axial velocity shear upstream region of the rotor. To reflect this, the source term S_k should be transformed into a pressure fluctuation acting on the turbulence in that spherical region. The increase of turbulence in the upstream region of the rotor could explain why the canopy model is not able to reach the full wake deficit observed in the close wake region of the Sexbierum measurements on Figure 3.
The adapted canopy model and the extended model compare similarly to the data. One problem that was seen is that they are both quite sensitive to the parameters and a wrong tuning could have a strong influence on the wake recovery distance. Moreover, increase of turbulence in the upstream region of the rotor as well as the initial decrease of turbulence intensity in the close wake region (only for the adapted canopy model) are not looking very physical. But there is hope that this could be prevented by adapting the forces fluctuations into pressure fluctuations based on the local pressure gradient in the k and ϵ equations,

6 Conclusion and future work

The canopy models originally developed for forest and urban areas can be adapted to the study of the wind turbine wakes. One challenge was to derive the equivalents parameters for the wind turbine. The model can be tuned to compare satisfyingly with measurements.
Nonetheless more work needs to be done to adapt the canopy model, which is originally designed for distributed force over large areas, into a disc of discreet body forces.
The two modified RANS atmospheric turbulence models open the prospect of faster full scale CFD wind farm wake analysis. The next step is therefore to compare them with wind farm measurements.

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