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NUMERICAL INVESTIGATION OF TURBULENT FORCED CONVECTION IN A DUCT WITH A TRAPEZOIDAL CROSS SECTION

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SUMMARY

The present work concerns development and application of turbulence models for forced convective heat transfer in trapezoidal ducts. The numerical approach is based on the finite volume technique with a non-staggered grid arrangement. The pressure-velocity coupling is handled by using the SIMPLEC-algorithm. Cyclic conditions in the main flow direction are imposed. The standard k-ε model with wall functions is used as a reference. The non-linear k-ε of Speziale is applied to calculate the turbulent shear stresses. The turbulent heat fluxes are calculated by the simple eddy diffusivity (SED) concept, the GGDH method and the WET method. The overall comparison between the methods is presented in terms of the friction factor and average Nusselt number. In particular the secondary flow field is investigated.

1. INTRODUCTION

Ducts with trapezoidal cross sections appear frequently in e.g compact regenerative heat exchangers. Also in recuperative heat exchangers and other cooling systems such ducts may be used.

Fully developed flow and thermal fields for laminar conditions have been investigated extensively while developing hydrodynamic and thermal fields in the entrance region only has been presented in [1]. At turbulent conditions, fully developed velocity distributions and secondary flow pattern were measured by Nikuradse [2] for a specific duct. Another experimental study was conducted by

Rodet [3] for a trapezoidal duct with the corner angle 75°. A numerical investigation was presented in [4], also with the corner angle 75°. These studies suggested that the friction factor could be calculated by the well known Blasius formula if the hydraulic diameter was used.

The present paper concerns fully developed turbulent forced convection in straight ducts having trapezoidal cross sections. The corner angle as well as the ratio of height to base are varied. A numerical calculation procedure including different turbulence models was presented in [5] and is here extended to trapezoidal ducts. A non-linear k-ε turbulence model is applied for calculation of the turbulent shear stresses [6] while the turbulent heat fluxes are determined by the generalized gradient diffusion hypothesis (GGDH) and the wealth equals earnings x time (WET), [7]. The standard k-ε model with a turbulent viscosity and simple eddy diffusivity is also considered.

2. PROBLEM STATEMENT

A straight duct with a trapezoidal cross-section using symmetry conditions is considered as shown in Fig. 1.

![Figure 1. Duct under consideration.](image)

The upper side length of the cross-section is 2b and the duct length is L. Fully developed turbulent flow and periodic conditions at the inlet and outlet are imposed. The ratio of the height to the upper side length is defined as a, (a=h/b).

The overall performance of the duct in terms of the friction factor and Nusselt number are to be determined numerically. The secondary flow motion in the cross-sectional plane is also of major concern. Different turbulence models are used as described in the following sections.

3. BASIC EQUATIONS

The basic equations are the continuity, momentum and energy equations. The flow is considered to be fully developed periodic turbulent flow, in this investigation. The following assumptions are employed: steady state, no-slip at the walls, no natural convection and constant fluid properties. One then has:
\[ \frac{\partial}{\partial x_j} (\rho U_j) = 0 \]  
\[ \frac{\partial}{\partial x_j} (\rho U_i U_j) = -\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] + \frac{\partial}{\partial x_j} (-\rho u_i u_j) \]  
\[ \frac{\partial}{\partial x_j} (\rho U_j T) = \frac{\partial}{\partial x_j} \left[ \frac{\mu}{Pr} \frac{\partial T}{\partial x_j} \right] + (-\rho u_j u_j) \]  

The turbulent shear stresses \(-\rho u_i u_j\) and turbulent heat fluxes \((-\rho u_j u_i)\) are modeled as described in the following sections.

### 3.1 Turbulence models for shear stresses

The \(k\)-\(\varepsilon\) model for steady state is given by

\[ \frac{\partial}{\partial x_j} (\rho U_j k) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \mu_t \right) \frac{\partial k}{\partial x_j} \right] + P_k - \rho \varepsilon \]  
\[ \frac{\partial}{\partial x_j} (\rho U_j \varepsilon) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{\varepsilon} \frac{\varepsilon}{k} P_k - C_{\varepsilon} \rho \frac{\varepsilon^2}{k} \]

where \(P_k\) is the production term and expressed as

\[ P_k = \tau_{ij} \frac{\partial U_i}{\partial x_j} = -\rho u_i u_j \frac{\partial U_i}{\partial x_j} \]

In the linear \(k\)-\(\varepsilon\), \(\tau_{ij}\) is described using Boussinesq eddy viscosity concept as

\[ \tau_{ij} = -\frac{2}{3} \rho k \delta_{ij} + 2\mu_s S_{ij} \]

where

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \]

In the non-linear \(k\)-\(\varepsilon\), the Speziale expression is used. Thus

\[ \tau_{ij} = -\frac{2}{3} \rho k \delta_{ij} + 2\mu_s S_{ij} + 4C_1 C_s \mu_t \frac{k}{\varepsilon} \left( \frac{k}{3} S_{ij} S_{ij} - \frac{1}{3} S_{mn} S_{mn} \delta_{ij} \right) + 4C_1 C_s \mu_t \frac{k}{\varepsilon} \left( \frac{k}{3} S_{ij} S_{ij} - \frac{1}{3} S_{mn} S_{mn} \delta_{ij} \right) \]

where \(\dot{S}_{ij}\) is the frame-indifferent Oldroyd derivative [8] of \(S_{ij}\) in the form of
\[
\dot{S}_{ij} = \frac{\partial S_{ij}}{\partial t} + U_k \frac{\partial S_{ij}}{\partial x_k} - \frac{\partial U_j}{\partial x_k} S_{ij} - \frac{\partial U_i}{\partial x_k} S_{ki} \tag{10}
\]

and
\[
\mu_\tau = \rho C_{\mu} \frac{k^2}{\varepsilon} \tag{11}
\]

\(k\) is the turbulent kinetic energy, \(\varepsilon\) the dissipation rate of the turbulent kinetic energy and \(C_{\mu}\) is a constant. The values of the constants in equations (4), (5), (9) and (11) are given in Appendix 1.

### 3.2 Turbulence Models for Heat Flux

The turbulent heat fluxes are expressed by using three models as follows:

a) Simple Eddy Diffusivity (SED) based on the Boussinesq viscosity model as
\[
\rho u_j \dot{T} = -\frac{\mu_\tau}{\sigma_T} \frac{\partial T}{\partial x_j} \tag{12}
\]

b) Generalized Gradient Diffusion Hypothesis (GGDH) expressed by Daly and Harlow [9] as
\[
\rho u_j \dot{T} = -\rho C_t \frac{k}{\varepsilon} \frac{\partial T}{\partial x_j} \tag{13}
\]

c) Wealth α Earnings×Time (WET) expressed by Launder [10] as
\[
\rho u_j \dot{T} = \rho C_t \frac{k}{\varepsilon} \left( u_j u_k \frac{\partial T}{\partial x_k} + u_k \frac{\partial U_i}{\partial x_k} + \dot{f}_i \right) \tag{14}
\]

where \(\dot{f}_i\) is the buoyancy-driven heat flux which is zero in this case. \(C_t\) is constant and set to 0.3 in both cases.

### 3.3 Periodic Case

The pressure \(P\) is expressed by
\[
P(x, y, z) = -\beta x + P^* (x, y, z) \tag{15}
\]

where \(\beta\) is a constant and represents the non-periodic pressure gradient in the main flow direction. \(P^*\) behaves periodically in the flow direction and is related to the detailed local motions.

In the cyclic case, the dimensionless temperature \(\theta\) is defined as
\[
\theta(x, y, z) = \frac{T(x, y, z) - T_w}{T_b(x) - T_w} \tag{16}
\]

where \(T_w\) is the wall temperature which is constant, and \(T_b\) is the fluid bulk temperature. Using this expression and inserting it into the energy equation (3) one obtains
\[
\frac{\partial}{\partial x_j} (\rho U_j \theta) = \frac{\partial}{\partial x_j} \left[ \frac{\mu}{Pr} \frac{\partial \theta}{\partial x_j} \right] + \gamma + \frac{\partial}{\partial x_j} \left( - \rho u_j \theta \right) \tag{17}
\]

where \( \gamma \) is a periodic parameter and expressed as
\[
\gamma = \lambda \left[ \Gamma \frac{\partial \theta}{\partial x} + \frac{\partial}{\partial x} (\Gamma \theta) - \rho U \theta \right] + \Gamma (\lambda^2 + \frac{\partial \lambda}{\partial x}) \tag{18}
\]

In (18), \( \lambda \) is periodic too and described as
\[
\lambda = \frac{\partial (T_b - T_w)}{\partial x} / (T_b - T_w) \tag{19}
\]

and \( \Gamma = \mu / Pr \).

In the GGDH method, \( \bar{u}_j \theta \) is calculated from
\[
\bar{u}_j \theta = -c_i \frac{k}{\epsilon} (T_b - T_x) \left\{ u_j u (\lambda \theta + \frac{\partial \theta}{\partial x}) + u_j v \frac{\partial \theta}{\partial y} + u_j w \frac{\partial \theta}{\partial z} \right\} \tag{20}
\]

In the WET method, \( \bar{u}_j \theta \) is determined from
\[
\bar{u}_j \theta = -c_i \frac{k}{\epsilon} (T_b - T_x) \left\{ u_j u (\lambda \theta + \frac{\partial \theta}{\partial x}) + u_j v \frac{\partial \theta}{\partial y} + u_j w \frac{\partial \theta}{\partial z} + u_j \frac{\partial U_j}{\partial x_k} \right\} \tag{21}
\]

An additional condition is needed to close the problem since the energy equation contains two unknowns, \( \theta(x, y, z) \) and \( \lambda(x) \). This condition can be obtained from the definition of the bulk temperature. In dimensionless form one has
\[
\int |U| dA_c = \int |U| dA_c \tag{22}
\]

where \( A_c \) is the cross-sectional area in the main flow direction and \( \theta(x, y, z) \) is a non-dimensional temperature profile which repeats itself in the fully developed periodic region.

4. BOUNDARY CONDITIONS

Periodicity conditions at the inlet and outlet are imposed for all variables.
\[
\Phi(x, y, z) = \Phi(x + L, y, z) \quad \Phi = U, V, W, \theta, P^*, k, \epsilon, u_i u_j, u_j \theta, \lambda, S_{ij}
\]

4.1 Wall Boundaries

For the near wall region, the traditional law of the wall is assumed to be valid for both the flow and temperature fields. The procedure adopted here is similar to that presented in e.g. [11] but the details will be omitted in this paper due to lack of space.
The grid distribution employed here is non-uniform. In most cases, the number of grid points or control volumes close to some walls are increased to enhance the resolution and accuracy.

The pressure gradient perpendicular to any wall will be quite small close to the considered wall and is neglected at the wall proximity in the corresponding momentum equation (this has been assumed in the definition of the law of the wall).

The molecular diffusion terms \( \frac{\partial}{\partial x_i} \left( \mu \frac{\partial U_j}{\partial x_j} \right) \), which appear due to the varying viscosity and often are neglected, will also be very small close to any wall. At the grid point adjacent to the wall, these terms are neglected as the equation for the velocity component perpendicular to a wall is solved. The advantage of this procedure is that it enables more efficient convergence in achieving the secondary velocity vector parallel to the wall in the non-staggered grid. However, the accuracy is not lost by this procedure.

5. NUMERICAL SOLUTION PROCEDURE

A boundary fitted coordinate method is applied in order to extend the capabilities of the general finite-volume technique to deal with complex geometries.

The general idea of this method is to map the complex flow domain in the physical space to a rectangular domain in the computational space by using a curvilinear coordinate transformation. Also, the Cartesian coordinate system in the physical space is replaced by a general non-orthogonal coordinate system. This method is implemented in CALC-BFC [12]. The authors developed CALC-BFC slightly in order to deal with the complex turbulent flows by developing some computational tensors which have no physical meaning. These tensors allow convenient handling of such complex turbulent flows near the walls and symmetry planes in the non-staggered grid.

The momentum equations are computationally solved for U, V and W on a non-staggered grid. The Rhie-Chow interpolation method is used for the velocity components at the control volume faces. The pressure velocity coupling is handled by the SIMPLEC-method. TDMA and SIP (SIM) based algorithms are employed for solving the equations. The convective terms are treated by the hybrid, van Leer, QUICK, Upwind and MUSCL schemes while the diffusive terms are treated by the central-difference scheme.

5.1 Sample Calculations

The Prandtl number was set to 0.73 and the Reynolds number was varied from 20000 to 100000 by choosing appropriate values of \( \beta \), the per-cycle pressure gradient. The computations were terminated when the sum of the absolute residuals normalized by the inflow was below \( 10^{-5} \) for all variables except temperature. The
temperature field did not converge to less than about \(5 \times 10^{-4}\) mostly because of combination of additional source terms from non-orthogonal grids and cyclic parameter \(\lambda(x)\), at the base corner. The calculations were carried out on a DEC 3000/400 AXP computer. Different number of grid points were chosen in the \(y\)- and \(z\)-direction for different \(h/b\), 21 to 42. The calculations show that increasing the number of grid points in the \(y\)- and \(z\)-direction does not improve the results (Nu-number and friction factor) significantly.

6. ADDITIONAL EQUATIONS

Some additional formulas are used in this study in order to calculate, \(Re\), \(\Delta p\), \(\mu\), Fanning friction factor, Nu-number both in each cross-sectional area and overall performance (\(Nu_x\) and \(Nu_{ov}\) respectively). These calculations are based on e.g. [5, 11], but are omitted here due to lack of space.

7. RESULTS and DISCUSSION

The secondary flow velocity vectors predicted by Speziale’s non-linear \(k-\varepsilon\) model in a trapezoidal duct in the fully developed region are shown in Fig. 2 (with corresponding main flow contours). These secondary flow patterns are predicted at all the considered Reynolds numbers. The same flow structure is valid for various base-corner angle.

It should be strictly noted that the kinetic energy gradient terms \(\frac{\partial}{\partial x_j}(\rho k)\) and the molecular diffusion terms \(\frac{\partial}{\partial x_j}(\mu \frac{\partial U_j}{\partial x_j})\) have not been neglected in the momentum equations although this is commonly done.

By neglecting the \(\frac{\partial}{\partial x_j}(\rho k)\) one obtains only one secondary vortex instead of three distinguished vortices as in Fig. 2 (the fourth one which appears beside the upper left side one is unstable and becomes weaker as the computations converged and is hard to distinguish here). The reason of this phenomenon might be explained since the mathematical model of such ducts with arbitrary cross-section may have two mathematical solutions. Neglecting the kinetic gradient terms makes one of the solutions to disappear from the computational model and the computations converge to the other one which might be a wrong physical solution. In other words, the computations can not detect the first solution. However, these terms can be neglected in a square cross-sectional duct because the turbulent kinetic energy is symmetric there and the mathematical model has only one solution and the computations converge to this solution.
Neglecting the molecular diffusion terms \( \frac{\partial}{\partial x_j} \left( \mu \frac{\partial U_j}{\partial x_i} \right) \) makes the secondary velocity flow significantly weaker. The secondary velocity flow should be about 2-5% of the main flow, depending on the Re-number.

Having these terms in the momentum equations provides additional wall treatment in a non-staggered grid arrangement which can be handled by the local computational tensors mentioned in chapter 5.

Table 1 shows the calculated Nu\(_x\) (local)-, overall Nu-numbers and Fanning friction factors for height to upper-side length ratio \(a/h=b=2\). Table 2 and 3 show some corresponding calculations for \(a=b=1\) and 0.5 with 60\(^\circ\) and 45\(^\circ\) base angle. The results can be compared with the corresponding Nu-number for circular cross-section which is shown in the tables.

### Table 1. Local (Nu\(_x\)), overall Nu-number and Fanning friction factors for a trapezoidal duct with h/b=2 and 60\(^\circ\) base-angle.

<table>
<thead>
<tr>
<th>Type</th>
<th>Re</th>
<th>Nu(_x)</th>
<th>Nu(_{ov})</th>
<th>Nu(_{circ})</th>
<th>f x 10(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-(\varepsilon)</td>
<td>100128.4</td>
<td>208.1</td>
<td>230.4</td>
<td>209.5</td>
<td>6.308</td>
</tr>
<tr>
<td>SED(^2)</td>
<td>100167.4</td>
<td>194.0</td>
<td>206.3</td>
<td>209.6</td>
<td>5.822</td>
</tr>
<tr>
<td>GGDH(^2)</td>
<td>&quot;</td>
<td>224.2</td>
<td>238.9</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>WET(^2)</td>
<td>&quot;</td>
<td>225.2</td>
<td>240.4</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>k-(\varepsilon)</td>
<td>80245.8</td>
<td>171.6</td>
<td>189.8</td>
<td>175.5</td>
<td>6.601</td>
</tr>
<tr>
<td>SED(^2)</td>
<td>80003.2</td>
<td>160.2</td>
<td>169.8</td>
<td>175.1</td>
<td>6.147</td>
</tr>
<tr>
<td>GGDH(^2)</td>
<td>&quot;</td>
<td>180.4</td>
<td>192.6</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>WET(^2)</td>
<td>&quot;</td>
<td>181.2</td>
<td>193.6</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>k-(\varepsilon)</td>
<td>59687.6</td>
<td>132.0</td>
<td>146.5</td>
<td>138.5</td>
<td>7.047</td>
</tr>
<tr>
<td>SED(^2)</td>
<td>59372.5</td>
<td>125.2</td>
<td>132.5</td>
<td>137.9</td>
<td>6.574</td>
</tr>
<tr>
<td>GGDH(^2)</td>
<td>&quot;</td>
<td>134.7</td>
<td>144.1</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>WET(^2)</td>
<td>&quot;</td>
<td>135.0</td>
<td>144.7</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>k-(\varepsilon)</td>
<td>40046.1</td>
<td>93.7</td>
<td>104.5</td>
<td>100.6</td>
<td>7.827</td>
</tr>
<tr>
<td>SED(^2)</td>
<td>39658.0</td>
<td>90.0</td>
<td>94.3</td>
<td>99.9</td>
<td>7.398</td>
</tr>
<tr>
<td>GGDH(^2)</td>
<td>&quot;</td>
<td>90.5</td>
<td>97.1</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>WET(^2)</td>
<td>&quot;</td>
<td>90.5</td>
<td>97.3</td>
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<tr>
<td>k-(\varepsilon)</td>
<td>22773.3</td>
<td>57.7</td>
<td>63.4</td>
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<td>9.123</td>
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<tr>
<td>SED(^2)</td>
<td>23082.4</td>
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<td>60.8</td>
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<tr>
<td>GGDH(^2)</td>
<td>&quot;</td>
<td>52.4</td>
<td>55.3</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>WET(^2)</td>
<td>&quot;</td>
<td>52.0</td>
<td>55.3</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

### Table 2. Local (Nu\(_x\)), overall Nu-number and Fanning friction factors for a trapezoidal duct with h/b=2 and 45\(^\circ\) base-angle.

<table>
<thead>
<tr>
<th>Type</th>
<th>Re</th>
<th>Nu(_x)</th>
<th>Nu(_{ov})</th>
<th>Nu(_{circ})</th>
<th>f x 10(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-(\varepsilon)</td>
<td>59977.3</td>
<td>118.1</td>
<td>141.1</td>
<td>139.0</td>
<td>6.728</td>
</tr>
<tr>
<td>SED(^2)</td>
<td>59086.9</td>
<td>117.7</td>
<td>130.0</td>
<td>137.4</td>
<td>6.499</td>
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<tr>
<td>GGDH(^2)</td>
<td>&quot;</td>
<td>120.8</td>
<td>137.9</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>WET(^2)</td>
<td>&quot;</td>
<td>121.0</td>
<td>138.4</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>k-(\varepsilon)</td>
<td>30004.0</td>
<td>61.8</td>
<td>73.0</td>
<td>79.9</td>
<td>7.842</td>
</tr>
</tbody>
</table>
Table 3. Local (Nu_x), overall Nu-number and Fannings friction factor for a trapezoidal duct with different h/b ratio and with 60° base-angle.

<table>
<thead>
<tr>
<th>Type</th>
<th>a=h/b</th>
<th>Re</th>
<th>Nu_x</th>
<th>Nu_ov</th>
<th>Nu_circ</th>
<th>f x 10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-ε^1</td>
<td>1</td>
<td>60892.2</td>
<td>129.6</td>
<td>140.5</td>
<td>140.7</td>
<td>6.685</td>
</tr>
<tr>
<td>SED^2</td>
<td>&quot;</td>
<td>61403.2</td>
<td>128.6</td>
<td>136.4</td>
<td>141.7</td>
<td>6.574</td>
</tr>
<tr>
<td>GGDH^2</td>
<td>&quot;</td>
<td>138.8</td>
<td>149.5</td>
<td>&quot;</td>
<td>&quot;</td>
<td></td>
</tr>
<tr>
<td>WET^2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>139.2</td>
<td>149.9</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>k-ε^1</td>
<td>0.5</td>
<td>60627.1</td>
<td>129.1</td>
<td>145.5</td>
<td>140.2</td>
<td>6.788</td>
</tr>
<tr>
<td>SED^2</td>
<td>&quot;</td>
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<td>128.0</td>
<td>142.5</td>
<td>140.1</td>
<td>6.805</td>
</tr>
<tr>
<td>GGDH^2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>154.2</td>
<td>174.3</td>
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<td>&quot;</td>
</tr>
<tr>
<td>WET^2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>154.5</td>
<td>174.6</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

1Nu-number calculated by Simple Eddy Diffusivity (SED) model.
2The turbulent shear stresses calculated from Speziale’s non-linear k-ε

\( y^+ \) (\( y^+ \) at the point adjacent to the wall) for all calculations is between 40 and 50.

To obtain a appropriate \( y^+ \), the distance between the wall and the point adjacent to the wall should increase as Re-number decreases. This presents a strong non-orthogonality (triangle formed control-volume) at the base corner which in turn makes convergence of the temperature field difficult due to many source terms. This is the reason why calculations at very low Re-numbers could not be carried out. It should also be noted that the results calculated by the hybrid, QUICK, van Leer, MUSCL, and upwind schemes are almost identical. The results in this paper have been calculated by the hybrid scheme.

The most important conclusion from the tables is that GGDH and WET predicts higher Nu-number than SED at high Re-numbers but they predict lower Nu-number than SED at low Re-numbers. Further investigations are needed to reveal which model is most accurate.

Another conclusion from the tables is that the Fanning friction factor (based on the hydraulic diameter) does not change significantly as the shape of the cross-section or base-corner angle changes. It has been found that the Fanning friction factor follows:

\[
f = 0.10349 \text{Re}^{-0.25} = 1.31 \times 0.079 \text{Re}^{-0.25}
\]

very well for all cases considered.

Appendix 1
The values of the coefficients in equations (4), (5), (9) and (11) are: $\sigma_k = 1.0$, $\sigma_{\varepsilon} = 1.314$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $C_D = C_E = 1.68$ and $C_\mu = 0.09$. The turbulent Prandtl-number ($\sigma_\tau$) is set to 0.89.
Fig. 2. Secondary velocity flow with corresponding main flow contours for a base-
corner angle 60° and various h/b ratio.

8. REFERENCES