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CHAOTIC DUFFING TYPE OSCILLATOR WITH INERTIAL DAMPING

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Abstract—*A novel Duffing–Holmes type autonomous chaotic oscillator is described. In comparison with the well–known non–autonomous Duffing–Holmes circuit it lacks the external periodic drive, but includes two extra linear feedback subcircuits, namely a direct positive feedback loop, and an inertial negative feedback loop. SPICE simulation and hardware experimental results are presented.*

I. INTRODUCTION

The Duffing–Holmes non–autonomous oscillator is a classical example of a nonlinear dynamical system exhibiting complex also chaotic behaviour [1–3]. It is given by the second–order differential equation with an external periodic drive term:

$$\ddot{x} + b\dot{x} - x + x^3 = a\sin\omega_1 t\,. \tag{1}$$

Three different techniques are used to solve the Duffing–Holmes equation and/or to process its solutions electronically. *The first* approach is a hybrid one making use of integration the equation in a digital processor and of the digital–to–analogue conversion of the digital output for its further analogue processing, analysis and display [4, 5]. *The second* method employs purely analogue hardware based on analogue computer design [6–9]. For example, analogue computer has been used to simulate Eq. (1) and to demonstrate the effect of scrambling chaotic signals in linear feedback shift registers [6–8]. Later analogue computer has been suggested for demonstration of chaos from Eq. (1) for the undergraduate students [9]. *The third* technique is based on building some specific analogue electrical circuit imitating dynamical behaviour of Eq. (1). The Young–Silva oscillator [10] described in more details in [11] and used to demonstrate the effect of resonant perturbations for inducing chaos [12, 13] is an example. Recently the Young–Silva circuit has been essentially modified and used to test the control methods for unstable periodic orbits [14, 15] and unstable steady states [16] of dynamical systems. The modified version has been characterized in details both numerically and experimentally in [17].

Evidently the first and the second techniques are rather general and can be applied to other differential equations as well. In contrast, the third approach is limited to a specific equation. Despite this restrict-ion the “intrinsic” electrical circuits have an attractive advantage due to their extreme simplicity and cheapness.

One may think of such analogue electrical circuits as of analogue computers. This is true from a mathematical and physical point of view in the sense that the underlying equations are either exactly the same or very similar also that the dynamical variables in the both cases are represented by real electrical voltages and/or currents. However, the circuit architecture of an analogue computer, compared to an “intrinsic” nonlinear circuit, is rather different. Any analogue computer is a standard collection of the following main processing blocks: inverting RC integrators, inverting adders, inverting and non-inverting amplifiers, multipliers, and piecewise linear nonlinear units. Meanwhile the specific analogue circuits comprise only small number of electrical components: resistors, capacitors, inductors, and semiconductor diodes. In addition, they may include a single operational amplifier (in some cases several amplifiers). The differences between the “intrinsic” analogue electrical circuits, simulating behaviour of the specific dynamical systems, and the common analogue computers are discussed in [18].

In this paper, we introduce, as an alternative for the non–autonomous Eq. (1), an autonomous version of the Duffing type oscillator given by

$$\ddot{x} - b\dot{x} - x + x^3 + kz = 0, \qquad \dot{z} = \omega_f(\dot{x} - z) \tag{2}$$

or equivalently by a set of three first order differential equations

$$\dot{x} = y, \qquad \dot{y} = x - x^3 + by - kz, \qquad \dot{z} = \omega_f(y - z). \tag{3}$$

Here z is the third independent dynamical variable, ω_f is its characteristic rate, and k is the feedback coefficient. We emphasize in Eq. (2) an opposite sign of the damping term, compared to Eq. (1). The negative damping, $-b\dot{x}$ in Eq. (2), or $+by$ in Eq. (3) yields an additional spiral instability. Also we propose a specific electrical circuit imitating solutions of Eq. (3).

II. Circuitry

First of all we recall to the *non–autonomous* circuit shown in Fig. 1 for comparison. The novel *autonomous* circuit is presented in Fig. 2.

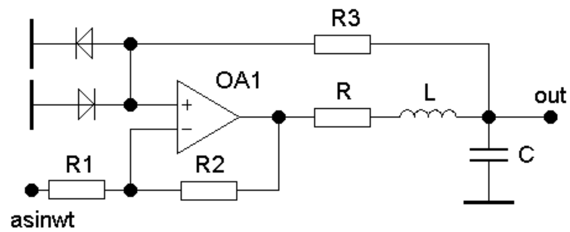

Fig. 1. Circuit implementation of the *non–autonomous* Duffing–Holmes oscillator [17].

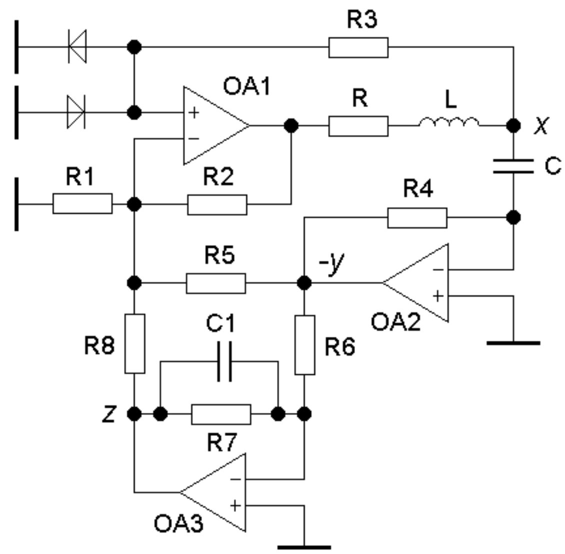

Fig. 2. Circuit implementation of the *autonomous* Duffing–Holmes type oscillator. Any of the nodes lettered as ‘x’, ‘$-y$’ or ‘z’ can be taken for the output.

The non–autonomous oscillator contains a single *nonlinear* positive feedback loop introduced by the resistor R3, two diodes, and the operational amplifier OA1. The external periodic drive is given by $a\sin\omega t$.

The autonomous oscillator lacks the external periodic drive, but includes two additional linear feedback loops. The circuit composed of the OA2 based stage and the resistor R5 introduce the positive feedback loop, specifically negative damping in Eq. (2). While the circuit including the OA2–OA3 stages (note a capacitor C1 in the latter stage) and the resistor R8 compose the inertial negative feedback, specifically the inertial damping term kz in Eq. (2).

III. Simulation Results

The oscillator in Fig. 2 has been simulated using the *Electronics Workbench* package (*SPICE* based software) and the results are shown in Figs. 3–5.

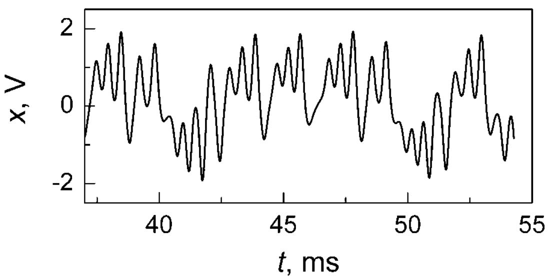

Fig. 3. Snapshot of typical chaotic waveform of $x(t)$ from the autonomous Duffing type oscillator. L = 19 mH, C = 470 nF, C1 = 20 nF, R = 20 Ω, R1 = 30 kΩ, R2 = 10 kΩ, R3 = 30 kΩ, R4 = 820 Ω, R5 = 75 kΩ, R6 = R7 = 10 kΩ, R8 = 20 kΩ. The OA1 to OA3 are the LM741 type or similar operational amplifiers, the diodes are the 1N4148 type or similar general–purpose devices.

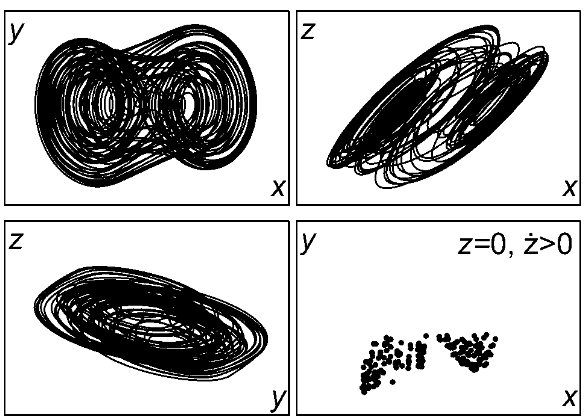

Fig. 4. Simulated phase portraits and Poincaré section (bottom right). Element values are the same as in Fig. 3.

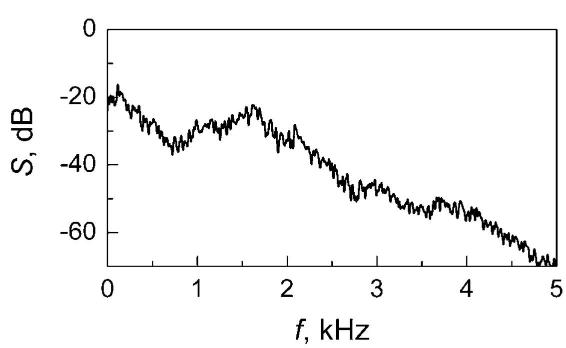

Fig. 5. Simulated power spectrum S from the variable $x(t)$. Circuit element values are the same as in Fig. 3.

IV. Hardware Experiments

The autonomous oscillator has been built using the elements described in the caption to Fig. 3. Typical experimental results are presented in Figs. 6–8.

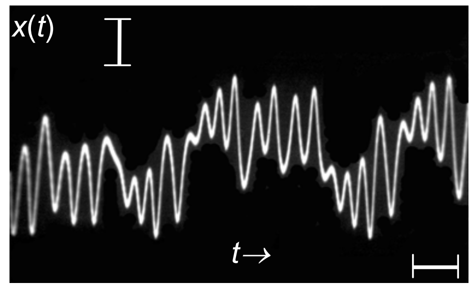

Fig. 6. Experimental snapshot of chaotic waveform $x(t)$. Horizontal scale 2 ms/div. Vertical scale 1 V/div. Element values are the same as in Fig. 3, except R5 = 68 kΩ.

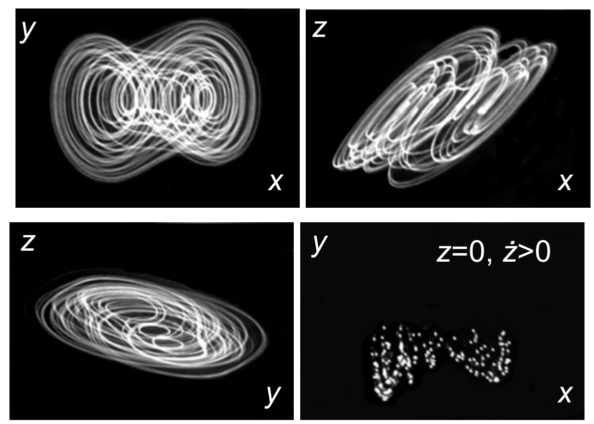

Fig. 7. Experimental phase portraits and Poincaré section (bottom right). Circuit element values are the same as in Fig. 3, except R5 = 68 kΩ.

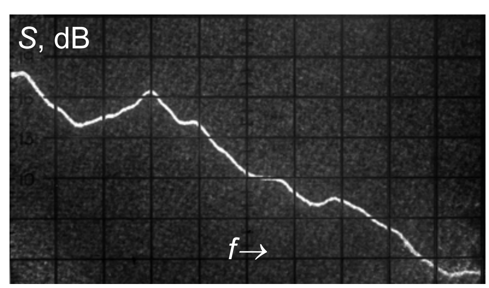

Fig. 8. Experimental power spectrum S from the output signal $x(t)$. Frequency range 0 to 5 kHz. Horizontal scale 500 Hz/div., resolution 100 Hz. Vertical scale 10 dB/div. Circuit element values are the same as in Fig. 3, except R5 = 68 kΩ.

V. Concluding Remarks

We have designed and built a novel Duffing–Holmes type autonomous third–order chaotic oscillator. In comparison with the common non–autonomous Duffing–Holmes type oscillator the autonomous circuit has an internal positive feedback loop instead of an external periodic drive source. In addition, it is supplemented with an RC inertial damping loop providing negative feedback. The circuit has been investigated both numerically and experimentally. The main characteristics, including the time series, phase portraits, Poincaré sections, and power spectra have been calculated using the SPICE based software, also taken experimentally. Fairly good agreement between the simulation and the hardware experimental results is observed (Figs. 3–8). Some discrepancy (about 10%) between the model and the hardware prototype, namely R5 = 75 kΩ in the model (Figs. 3–5) and R5 = 68 kΩ in the experimental circuit (Figs. 6–8) can be explained in the following way. The inductive element in the model is an ideal device in the sense that L = const. Meanwhile the inductance of a real inductor, e.g. a coil wound on a ferrite core has a slight dependence on the current $L = L(I)$.

Finally, we emphasize that the described autonomous oscillator is not simply a formal alternative to the classical non–autonomous Duffing–Holmes oscillator. An externally driven chaotic oscillator has a sharp and ≈ 20 dB high peak in the power spectrum at the drive frequency $f_1=\omega_1/2\pi$. While autonomous oscillator exhibits no peaks but essentially smoother spectra (Figs. 5, 8). This feature may have an advantage in practical applications.

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