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## Three-Dimensional Microwave Imaging for Breast-Cancer Detection using the Log-Phase Formulation

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The log-phase formulation is applied for the reconstruction of images from a simulation of a three-dimensional imaging system. By using this formulation, a clear improvement in the quality of the reconstructed images is achieved compared to the case in which the usual complex phasor notation is employed.

### Introduction

Microwave imaging is emerging as a tool for biomedical imaging [1–5]. In particular, microwave imaging is a promising tool for screening for breast cancer due to the high contrast between malignant tumors and the healthy breast tissue [1–3, 5].

One of the most challenging aspects of microwave imaging is to reconstruct images of large, high-contrast objects since such objects cause the associated inverse scattering problem to become non-linear. Such image-reconstruction problems are often encountered when imaging the breast where, in addition to any tumors, large inclusions of fibroglandular tissue, with constitutive parameters very different from those found in the rest of the breast, may be present. An algorithm for imaging of the breast must therefore be capable of coping with such multiple large, high-contrast inclusions and still be able to detect the smaller tumors. At Dartmouth College, an iterative Gauss-Newton based imaging algorithm, in which the reconstruction problem is solved in 2-D, is currently being tested on data collected from breast-cancer patients [6]. For this algorithm it has been found that to cope with the complexity of the imaging problem it is useful to reformulate the inverse scattering problem from using the usual complex phasor notation of the measured signals to involve the logarithm of the magnitude of the signals and their unwrapped phase. This is referred to as the log-phase formulation [1, Sec. II].

At the Technical University of Denmark, the Physical Anomaly Tomography (PAT) Scanner is currently being developed. This imaging system operates in a three-dimensional setup and to reconstruct the images, an algorithm based on the iterative Gauss-Newton scheme is applied. In this paper, the improvement in the image quality obtained using the log-phase formulation of the measured scattering data as opposed to the complex phasor formulation in a three-dimensional microwave-imaging setup is shown for the first time. The two different formulations are tested on simulated data and it is shown that even for objects with relatively low contrasts, the log-phase formulation clearly improves the capability of the algorithm to reconstruct the images.

## Imaging System and Algorithm

The PAT Scanner consists of 32 antennas arranged in a hemispherical setup. During patient exams, the patient is to lie prone on an exam bed above the imaging system with her breast suspended through an aperture in the lid of the imaging system as seen in Figure 1. The breast and antennas are immersed in a coupling liquid to maximize the energy coupled from the antennas to the inside of the breast. The scattering (S) parameters of every possible combination of transmitting and receiving antenna are then measured, yielding a total of 992 measurements.

In the imaging algorithm currently applied at the Technical University of Denmark, the constitutive parameters of the imaging domain is represented by the squared complex wave numbers

$$k^2(\mathbf{r}) = \mu_0\omega^2\epsilon(\mathbf{r}) + i\mu_0\omega\sigma(\mathbf{r}). \quad (1)$$

The reconstruction of the images is then performed by discretizing the imaging domain into  $N_v$  voxels and solving the non-linear optimization problem

$$\underline{k}^2 = \operatorname{argmin} \left\{ \|\underline{S}^{\text{meas}} - \underline{S}^{\text{calc}}(\underline{k}^2)\|_2 \right\} \quad (2)$$

using an iterative Gauss-Newton algorithm. In (2), the 1984-element vectors  $\underline{S}^{\text{meas}}$  and  $\underline{S}^{\text{calc}}$  contains the measured and calculated data, respectively, in either the complex phasor formulation or in the log-phase formulation. The vector  $\underline{k}^2$  holds the squared complex wave numbers. When the complex phasor formulation is used, the elements in the vectors holding the measured and calculated data are given by  $\Re\{S_{rt,\text{object}}\} - \Re\{S_{rt,\text{empty}}\}$  and  $\Im\{S_{rt,\text{object}}\} - \Im\{S_{rt,\text{empty}}\}$ , that is, the real and imaginary parts of the difference in the measured or calculated S parameters between a measurement/calculation with an object in the system and a measurement/calculation with no object in the system. When using the log-phase formulation, the elements of the vectors are given by  $\log|S_{rt,\text{object}}| - \log|S_{rt,\text{empty}}|$  and  $\angle S_{rt,\text{object}} - \angle S_{rt,\text{empty}}$ . For high-contrast targets, the change in phase may lie outside  $\pm\pi$ , and the phase is unwrapped using a method similar to that outlined in [1].

In each iteration of the Gauss-Newton algorithm, the forward data  $\underline{S}^{\text{calc}}$  is calculated using a volume integral equation [7] and from the results, the corresponding Jacobian is determined using perturbation theory. The Jacobian  $\underline{J}_n$  is then used for solving for the update of the constitutive parameters  $\underline{\Delta k}_n$  using

$$\underline{J}_n \underline{\Delta k}_n^2 = \underline{S}^{\text{meas}} - \underline{S}^{\text{calc}}(\underline{\Delta k}_n^2). \quad (3)$$

This equation is ill-posed and regularization must therefore be applied to determine the update vector. The updated values of the constitutive parameters are then found as

$$\underline{k}_{n+1}^2 = \underline{k}_n^2 + \underline{\Delta k}_n^2. \quad (4)$$

Depending on the formulation of the problem, the Jacobian must be given in either its complex form or its log-phase formulation. The latter is easily found from the complex form by application of the chain rule.

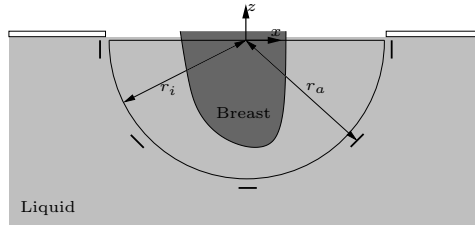


Figure 1: Schematic of the PAT Scanner. The 32 antennas are positioned in a hemispherical setup with radius  $r_a = 8$  cm and the imaging domain has a radius of  $r_i = 7.5$  cm. The breast is suspended through an aperture in the lid of the measurement tank.

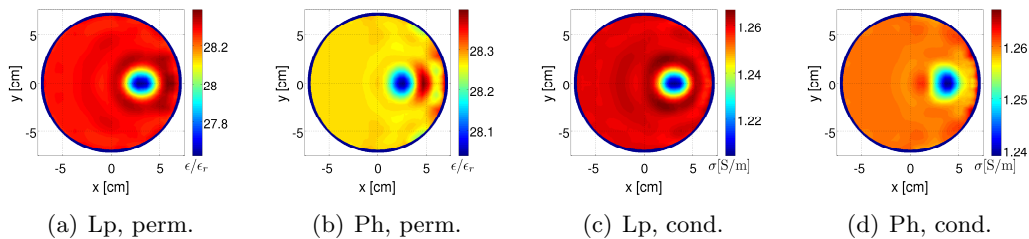


Figure 2: Reconstructed permittivity and conductivity images of the scatterer ( $x$ - $y$  plane for constant  $z = -2$  cm). The images obtained using the log-phase formulation are shown in (a) and (c) and are seen to reconstruct the actual constitutive parameters and position of the scatterer better than the images obtained using the complex phasor notation shown in (b) and (d). Note the different color scales.

## Results

The effect of using the log-phase formulation is illustrated by the reconstruction of a simulated data set using the frequency 1100 MHz. The coupling liquid has a relative permittivity of  $\epsilon_{r,cl} = 28.27$  and conductivity  $\sigma_{cl} = 1.26$  S/m. A local coordinate system with origin at the center of the aperture in the lid of the imaging system is introduced and a single, spherical scatterer with a radius of 1 cm is positioned with its center at  $(x_{c_1}, y_{c_1}, z_{c_1}) = (3, 0, -2)$  cm. The scatterer has a relative permittivity of  $\epsilon_{r,s_1} = 27$  and a conductivity of  $\sigma_{s_1} = 1.15$  S/m and is thus a low-contrast scatterer. The results of the reconstructions are seen in Figure 2 where the color scales have been chosen individually for the two reconstructions to better show the effect of the log-phase algorithm. It is seen that both algorithms detect the position of the scatterer and reconstruct the approximate size of the scatterer. The log-phase formulation, however, is closer to reconstructing the actual constitutive parameters and position of the scatterer. Furthermore, when using the complex phasor notation, the relative level of the artifacts in the images is greater than when using the log-phase formulation.

The results shown here are typical for small scatterers in the lossy coupling liquid where the log-phase formulation benefits from emphasizing the small absolute but large relative changes in the scattering parameters measured between antennas on opposite sides of the imaging system, containing the most information on the scatterer. For larger objects, the log-phase formulation benefits not only from emphasizing the large relative changes but also from the additional information available

in the unwrapped phase. The algorithm using the complex phasor notation, on the other hand, fails completely when large, high-contrast objects are present in the imaging system. For imaging problems in a lossless environment, the two algorithms have similar performances as long as the change in phase is within  $\pm\pi$ . For changes in phase outside this interval, the log-phase formulation still performs better due to the additional information available in the unwrapped phase.

## Conclusion

The log-phase formulation has been applied to a three-dimensional microwave imaging problem. When compared to the images obtained using the usual complex phasor notation, the log-phase formulation shows a significant improvement in the quality of the reconstructed images.

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