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Uncertainty Estimation in SiGe HBT Small-Signal Modeling

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(Student Paper)

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***Abstract*— An uncertainty estimation and sensitivity analysis is performed on multi-step de-embedding for SiGe HBT small-signal modeling. The uncertainty estimation in combination with uncertainty model for deviation in measured S-parameters, quantifies the possible error value in de-embedded two-port parameters (Y and Z - parameters). The analysis is applied to a 0.35μm 60GHz f_T SiGe HBT in frequency range 45MHz to 26GHz.**

I. Introduction

In device modeling and small-signal model verification for circuit simulation, model parameters are extracted from experimental data taken at high frequencies using scattering parameters (S-parameters). Ideally, the model parameter should present a constant behavior versus frequency, thus making it irrelevant at which frequencies their true values are extracted. In practice, stochastic deviations are superimposed on true values. The calculated model parameters thus present stochastic behavior versus frequency. On-wafer S-parameter measurements at high frequencies are not accurate due to limited dynamic range and accuracy of measurement equipment. The stochastic model parameter deviations originate from uncertainties in the S-parameter measurements.

While estimating model parameter values of transistor like SiGe HBT, parasitics introduced by pad structures and interconnect lines should be de-embedded from experimental data before extracting any model parameter. Multi-step de-embedding methods are used to de-embed these pads and interconnect parasitics. The inaccuracies in the measurement system propagate through the process of de-embedding and results in stochastic deviations in extracted small-signal parameters.

Despite critical applications of SiGe HBT, very little work has been reported on how to find uncertainties with which model parameters can be extracted. Most of the work is reported for FET [1], [2], [3]. In [4], Taher *et al.* gives quantitative figures for the sensitivities of intrinsic SiGe model parameters. Analytical expression are used, but since sensitivity analysis is performed only for single step de-embedding and only numerical figures are given at single frequency without taking into account the inaccuracy of measurement system, it is difficult to draw any general conclusion from these results. Estimation of uncertainties in the extraction of model parameter for SiGe HBT is not reported.

In this paper, uncertainties in de-embedded two-port parameters for SiGe HBT are calculated using analytical expressions for sensitivities and uncertainty model of the measured S-parameters. Uncertainty model of the S-parameters quantifies the inaccuracies in measurement system. The parameter uncertainties are used to perform optimal, minimum uncertainty parameter extraction without any prior knowledge of device frequency characteristics.

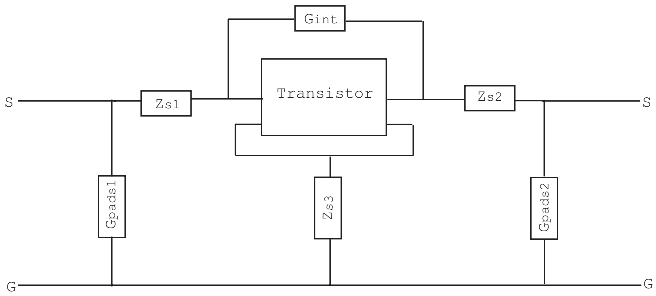

Fig. 1. Equivalent circuit representation of pad and interconnect line parasitic associated with transistor.

Sensitivity analysis identifies the stability of different multi-step de-embedding methods by their response to relative error in the measured S-parameters. Sensitivity analysis along with uncertainty estimation can also be used to select the approach (Y and Z-parameter) in de-embedding method for optimal extraction of model parameters with less uncertainty.

II. Uncertainty Estimation

A. Three-Step De-embedding Method

When devices are fabricated on Si-substrate, they are surrounded by contact pads and interconnect lines. These structures (pads and interconnect lines), introduce parasitic effects in form of series and shunt components to the device. For accurate extraction of small-signal model parameters for SiGe HBT transistor, the parasitics introduced by pad structures and interconnect lines should be properly de-embedded by test fixtures. For this purpose methods reported in [5], [6] have been traditionally used. A model of transistor with pad and interconnect lines parasitic is presented, in Fig. 1. To de-embed these parasitic, a three-step de-embedding method [6] is used. Two test-structures are required to perform de-embedding, one open standard and one short standard. The open test structure is used to de-embed pad parasitic and shunt parasitic of interconnect lines and the short structure is used to de-embed series parasitic of interconnect lines. The de-embedding method used here is modified as compared to the one reported in [6]. The reported method uses four standards; open, thru, short1 and short2 standard but in present analysis only two standard is used; open and short standard. De-embedding equations for SiGe HBT are

$$Y_{d1} = Y_{meas} - Y_{pads} \tag{1}$$

$$Y_{d2} = \left(Y_{d1}^{-1} - (Y_{short} - Y_{pads})^{-1}\right)^{-1} \quad (2)$$

$$Y_{dut} = Y_{d2} - Y_{intc} \quad (3)$$

where Y_{meas} is the measured Y-parameters for the SiGe HBT transistor, and

$$Y_{pads} = \begin{bmatrix} Y_{11open} + Y_{12open} & 0 \\ 0 & Y_{12open} + Y_{22open} \end{bmatrix} \quad (4)$$

$$Y_{intc} = \begin{bmatrix} -Y_{12open} & Y_{12open} \\ Y_{12open} & -Y_{12open} \end{bmatrix} \quad (5)$$

In Z-parameter approach of three-step de-embedding method, the Y-parameters are transformed to the Z-parameters [7].

B. Sensitivity Analysis

The relative sensitivity, K, in a parameter Y (port parameter of de-embedded matrix) for relative changes in parameter S is defined as [1]

$$K_S^Y \triangleq \frac{\partial Y}{\partial S}\frac{S}{Y} \quad (6)$$

The relative sensitivity K tells the percentage error in Y for 1% error in the parameter S. Relative change in Y is related to relative change in S using sensitivity.

$$\frac{\partial Y}{Y} \cong K_S^Y \frac{\partial S}{S} \quad (7)$$

As port parameter depends on more than one S-parameters, multi-parameter sensitivity [8] is defined, which represents the total relative error in port variables as total derivative due to error in all four S-parameters,

$$\frac{\Delta Y}{Y} = \sum_{\forall m,n\epsilon\{1,2\}} K_{S_{mn}}^Y \frac{\Delta S_{mn}}{S_{mn}} \quad (8)$$

where small errors in S-parameter are assumed. S-parameters are complex quantities with real and imaginary values, so sensitivity analysis is performed with respect to real and imaginary parts of the S-parameters. As the S-parameters are measured for three configuration, transistor with parasitics, open and short standard, the sensitivity of port parameter in three-step de-embedding method is defined with respect to all three configurations' S-parameters (real and imaginary) as

$$\begin{aligned}\frac{\Delta Y}{Y} = \sum_{\forall m,n\epsilon\{1,2\}} \Bigg(& K_{S_{mnr,m}}^Y \frac{\Delta S_{mnr,m}}{S_{mnr,m}} + K_{S_{mni,m}}^Y \frac{\Delta S_{mni,m}}{S_{mni,m}} \\ & + K_{S_{mnr,op}}^Y \frac{\Delta S_{mnr,op}}{S_{mnr,op}} + K_{S_{mni,op}}^Y \frac{\Delta S_{mni,op}}{S_{mni,op}} \\ & + K_{S_{mnr,sh}}^Y \frac{\Delta S_{mnr,sh}}{S_{mnr,sh}} + K_{S_{mni,sh}}^Y \frac{\Delta S_{mni,sh}}{S_{mni,sh}} \Bigg) \quad (9)\end{aligned}$$

where the subscripts *m*, *op*, and *sh* refer to the transistors with parasitics, open and short standards, respectively.

C. Uncertainty Calculation

The S-parameter deviations arise from measurement uncertainties that are random and thus described by probability distributions. The exact distributions are usually unknown, however, they are often assumed to be normal distributed. If S-parameter deviations are assumed to be normal-distributed, having zero mean, and being uncorrelated, makes it possible to use sensitivities from equation (9) to express the variance in Y in terms of S-parameter variances [1],

$$\begin{aligned}\sigma_Y^2 = \sum_{\forall m,n\epsilon\{1,2\}} \Bigg(& \left(K_{S_{mnr,m}}^Y\right)^2 \sigma_{S_{mnr,m}}^2 + \left(K_{S_{mni,m}}^Y\right)^2 \sigma_{S_{mni,m}}^2 \\ & + \left(K_{S_{mnr,op}}^Y\right)^2 \sigma_{S_{mnr,op}}^2 + \left(K_{S_{mni,op}}^Y\right)^2 \sigma_{S_{mni,op}}^2 \\ & + \left(K_{S_{mnr,sh}}^Y\right)^2 \sigma_{S_{mnr,sh}}^2 + \left(K_{S_{mni,sh}}^Y\right)^2 \sigma_{S_{mni,sh}}^2 \Bigg) \quad (10)\end{aligned}$$

D. Uncertainty Model for Measured S-Parameters

To estimate parameter uncertainties, deviations in the S-parameters should be known. An empirical model is therefore being developed for S-Parameters' magnitude and phase uncertainties (equation (11,12)) from the VNA specifications[9]. As the model is being developed for the magnitude and phase uncertainties, the uncertainties are then converted to real and imaginary values to estimate parameter uncertainty.

$$\sigma_{|S_{11,22}|} = \sigma_{\angle S_{11,22}} = k_1 + k_2 S_{11,22} + k_3 {S_{11,22}}^2 + k_4 e^{-k_5 S_{11,22}} \quad (11)$$

$$\begin{aligned}\sigma_{|S_{12,21}|} = \sigma_{\angle S_{12,21}} = & k_6 + k_7 {S_{12,21}}^{k_8} + k_9 {S_{12,21}}^{k_{10}} \\ & + k_{11} {S_{12,21}}^{k_{12}} + k_{13} log {S_{12,21}}^{k_{14}} \\ & + k_{15} e^{-(k_{16} S_{12,21})} \quad (12)\end{aligned}$$

$S_{11,22}$ and $S_{12,21}$ represent either magnitude or phase. Parameter k_n can take different values ranging from 0 to any positive and negative real number. Different values of k_n describes uncertainty profiles at different frequency intervals

E. Small-Signal Model Uncertainty Calculation

Small-signal equivalent circuit models for SiGe HBT devices can naturally be divided into an intrinsic part describing transistor action in a vertical structure underneath the emitter and an extrinsic part due to unavoidable parasitics associated with the device [10]. To determine intrinsic model parameter uncertainties using (10), sensitivities of intrinsic two-port parameters should be known. For intrinsic two-port parameter sensitivities, extrinsic series resistances are extracted in saturation mode [11] and de-embedded from the model. De-embedding of extrinsic elements effects two-port parameter sensitivities. As sensitivities of extrinsic resistances are negligible, their effect on intrinsic model is neglected. Thus sensitivities of intrinsic two-port parameters can be represented by (13).

$$K_S^{Y_{int}} = K_S^{Y_{dut}} \frac{Y_{dut}}{Y_{int}} \quad (13)$$

Intrinsic two-port sensitivities can be used to determine uncertainties in small-signal incremental base resistance R_{bi}. In [12], Johansen *et al.* gives analytical expression to calculate incremental base resistance for SiGe HBT using intrinsic two-port parameters Y_{int},

$$a = \frac{1}{|Y_{21} - Y_{12}|_{\omega\to 0}} \quad (14)$$

$$ang = \arctan\left[\frac{\sqrt{\frac{1}{|Y_{int,21} - Y_{int,12}|^2} - a^2}}{a}\right] \quad (15)$$

$$X = \frac{Y_{int,11} + Y_{int,12}}{Y_{int,21} - Y_{int,12}} e^{\jmath\left(\angle(Y_{int,21} - Y_{int,12}) + ang\right)} \tag{16}$$

$$R_{bi} = \frac{\sqrt{\frac{1}{|Y_{int,21} - Y_{int,12}|^2} - a^2} - aIm\left[\frac{Y_{int,22}+Y_{int,12}}{Y_{int,11}+Y_{int,21}}\right]}{Im\left[X\right] - Re\left[X\right] Im\left[\frac{Y_{int,22}+Y_{int,12}}{Y_{int,11}+Y_{int,21}}\right]} \tag{17}$$

Sensitivity of R_{bi} can be derived in terms of intrinsic two-port parameter sensitivities as

$$K_S^{R_{bi}} = \frac{\partial R_{bi}}{\partial S} \frac{S}{R_{bi}} \tag{18}$$

$$\frac{\partial (Y_{int,11} + Y_{int,12})}{\partial S} S = K_S^{Y_{int,11}} Y_{int,11} + K_S^{Y_{int,12}} Y_{int,12} \tag{19}$$

$$\frac{\partial (Y_{int,11} + Y_{int,21})}{\partial S} S = K_S^{Y_{int,11}} Y_{int,11} + K_S^{Y_{int,21}} Y_{int,21} \tag{20}$$

$$\frac{\partial (Y_{int,21} - Y_{int,12})}{\partial S} S = K_S^{Y_{int,21}} Y_{int,21} - K_S^{Y_{int,12}} Y_{int,12} \tag{21}$$

$$\frac{\partial (Y_{int,22} + Y_{int,12})}{\partial S} S = K_S^{Y_{int,22}} Y_{int,22} + K_S^{Y_{int,12}} Y_{int,12} \tag{22}$$

Uncertainty in R_{bi} is determined from (10) using (18).

III. Results

A. Sensitivity Analysis

Fig. 2 and Fig. 3 shows the sensitivities of Y_{12} and Z_{12} during the process of the three-step de-embedding method with respect to the real S-parameters for the open standard.

The sensitivity of any two-port parameter strongly depends on the form of de-embedded equations. Y_{12} parameter of the two-port matrix in the first-step of de-embedding method is in-sensitive to the S-parameter variations. As the first-step de-embedding equation (1) is linear in Y-parameter approach, therefore sensitivity calculations mask out the effect of open standard S-parameters variations. However, the second and third-step parameters are sensitive to these variations and their sensitivities are slightly increased from the second-step to third-step de-embedding (Fig. 2). Y_{12} parameter sensitivities show increasing profiles for higher frequencies. It means any small error in the S-parameter can result in considerable amount of error in Y_{12} parameter at higher frequencies.

For the Z-parameter approach, the Z_{12} parameter sensitivities also increase with the steps of de-embedding method. However, in the Z-parameter approach, Z_{12} parameter in the first-step of de-embedding method is sensitive to the variations of S-parameter for the open standard. This is due to the nature of de-embedding equation in the Z-parameter approach, which introduces this sensitivity.

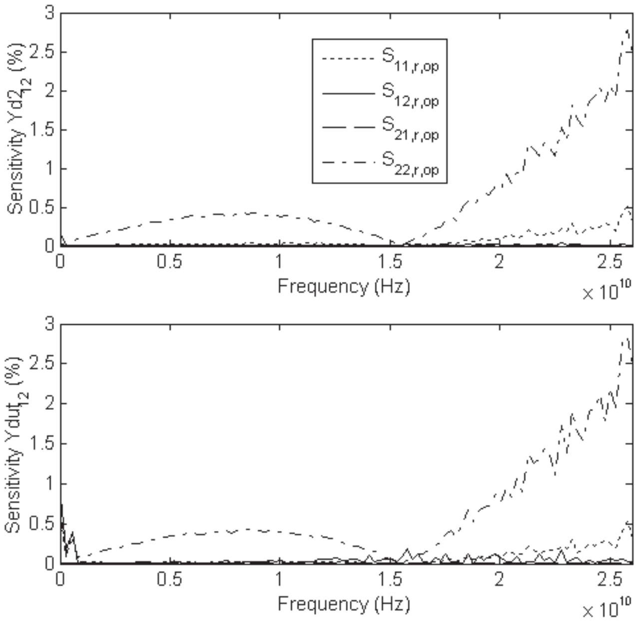

Fig. 2. Sensitivity distribution of $Yd2_{12}$ and $Ydut_{12}$ w.r.t variations in all four open standard real S-parameters versus frequency (45MHz to 26GHz).

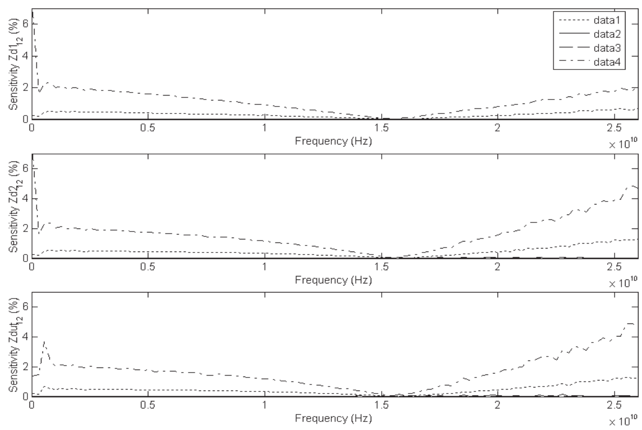

Fig. 3. Sensitivity distribution of $Zd1_{12}$,$Zd2_{12}$, and $Zdut_{12}$ w.r.t variations in all four open standard real S-parameters versus frequency (45MHz to 26GHz).

B. Uncertainty Estimation

Fig. 4 presents real values third-step de-embedding (Y and Z-parameter) and their uncertainty level distribution over frequency range (45MHz to 26GHz). Both approaches, the Y-parameter and Z-parameter result in nearly same uncertainty level but the Z-parameter approach shows more inconsistent behavior at high frequencies. Uncertainty level increases with frequency. These results reflect that the Y-parameter approach is more suitable to extract small-signal model parameters of SiGe HBTs.

C. Base Resistance Uncertainty

Fig. 5 presents incremental base resistance R_{bi} values and its uncertainty level distribution over frequency range (45MHz to 26GHz). This distribution defines frequency range for optimal extraction of R_{bi} by indicating uncertainty level in R_{bi} over complete frequency range. R_{bi} can be extracted optimally

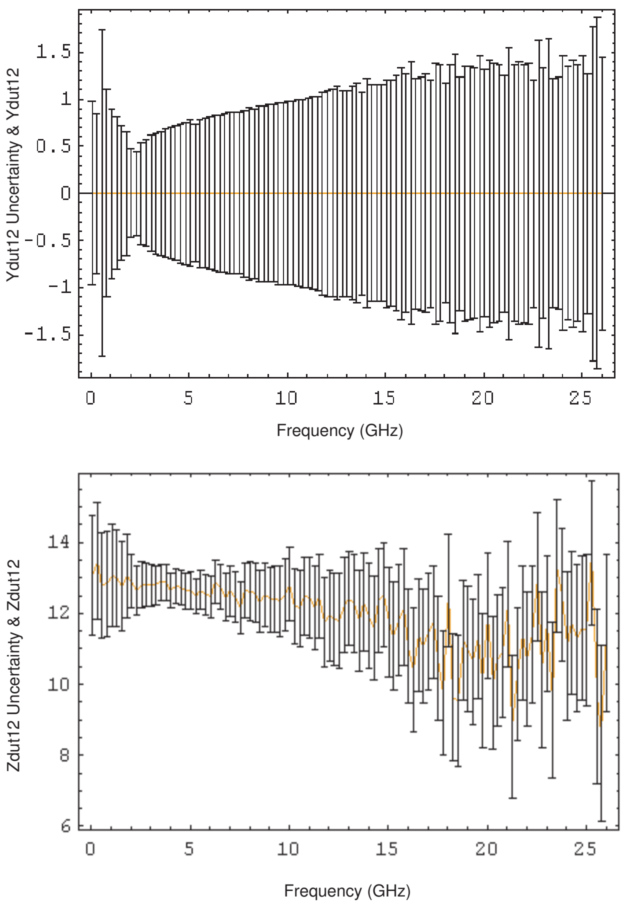

Fig. 4. Real $Ydut_{12}$ and $Zdut_{12}$ (Y and Z-parameter based three-step de-embedding approach) with their uncertainty distribution versus frequency (45MHz to 26GHz).

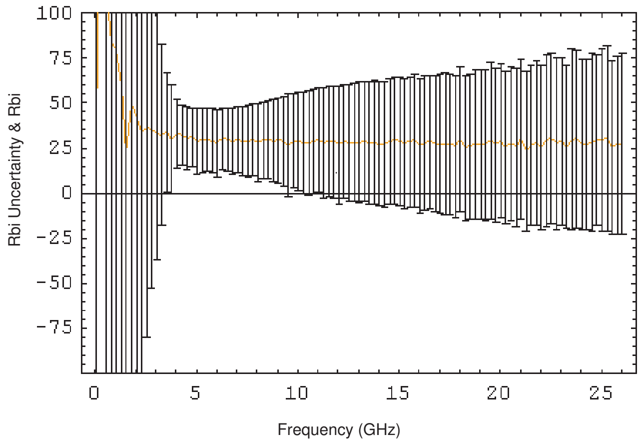

Fig. 5. R_{bi} and its uncertainty distribution versus frequency (45MHz to 26GHz).

from 4GHz to 7GHz with uncertainty level $\pm 15\Omega$. Uncertainty level for R_{bi} is high for low and high frequency range.

IV. Conclusions

An analytical derivation of two-port parameters' sensitivities and uncertainties in three-step de-embedded method was presented. The sensitivity analysis reflects that parameter sensitivities depend strongly on measured data and de-embedded equations and it increases with steps of de-embedding. Uncertainty estimation identifies the frequency range where parameters could be extracted with less uncertainty. These estimation expressions can be used for any small-signal model parameter extractions method (direct extraction and numerical optimization). Uncertainty estimation and sensitivity analysis using both Y and Z-approach produces nearly same results but Y-parameter approach is consistent.

References

[1] C. Fager, L. J. Peter, and J. C. Pedro, "Optimal parameter extraction and uncertainty estimation in intrinsic FET small-signal models," *IEEE Trans Microwave Theory Tech*, vol. 50, pp. 2797–2803, Dec. 2002.

[2] F. D. King, P. Winson, A. D. Sinder, L. Dunleavy, and D. P. Levinson, "Math methods in transistor modeling: Condition numbers for parameter extraction," *IEEE Trans Microwave Theory Tech*, vol. 46, pp. 1313–1314, Sept. 1998.

[3] P. Walters, R. Pollard, J. Richardson, P. Gamand, and P. Suchet, "On-wafer measurement uncertainty for 3-terminal active millimeter-wave devices," in *GaAs IC Symp. Dig*, 1992, pp. 55–58.

[4] H. Taher, D. Schreurs, S. Vandenbergs, and B. Nauwelaers, "Influence of de-embedding on the extraction sensitivity of SiGe HBT," in *INMMIC Roma*, 2004.

[5] M. Koolen, J. Geelen, and M. Versleijen, "An improved de-embedding technique for on-wafer high-frequency characterization," in *IEEE BCTM*, 1991, pp. 188–191.

[6] H. Cho and D. Burk, "A Three-Step Method for the De-Embedding of High-Frequency S-Parameter Measurements," *IEEE Transaction on Electron Devices*, vol. 38, no. 6, pp. 1371–1375, Jun. 1991.

[7] D. M. Pozar, *Microwave Engineering*, 3rd ed. John Willey Sons Inc, 2003.

[8] J. Vlach and K. Singhal, *Computer Methods for Circuit Analysis and Design*. Van Nostrand Rienhold Company, 1983.

[9] *"PNA Microwave Network Analyzer Data Sheet"*, Agilent Technologies, Oct, 2004.

[10] T. K. Johansen, J. Vidkjær, and V. Krozer, "Substrate Effects in SiGe HBT Modeling," in *Proc. GAAS2003*, Oct 2003, pp. 445–448.

[11] Y. Gobert, P. J. Tasker, and K. H. Becham, "A Physical, Yet simple, Small-Signal Equivalent Circuit for Hetrojunction Bipolar Transistor," *IEEE Trans Microwave Theory Tech*, vol. 45, pp. 149–153, Jan. 1997.

[12] T. Johansen, "Monolithic Microwave Integrated Circuits for Wideband SAR Syatem," Ph.D. dissertation, Technical University of Denmark, Dept. of Electromagnetics, April. 2003.