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Using disjunctive cuts in a branch-and-cut-and-price algorithm for the capacitated vehicle routing problem

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Abstract

This talk presents computational results that show the usefulness of the general-purpose valid inequalities *disjunctive cuts* when applied to the CVRP. Results indicate that the disjunctive cuts are able to reduce the gap between lower bound and upper bound more than state-of-the-art problem specific inequalities. Results also indicate that introducing the cuts leads to a smaller branch and bound tree and faster solution times overall.

1   Disjunctive cuts for the capacitated vehicle routing problem

This talk presents another entry in the quest for proven optimal solutions to the *capacitated vehicle routing problem* (CVRP). The CVRP can be described as follows. We are given a set of customers $\{1, \ldots, n\}$ that each has a certain demand d_i. The customers must be served by a fleet of K homogeneous vehicles with capacity C. All vehicles are based in a common depot denoted 0. For each pair $(i, j) \in \{0, 1, \ldots, n\}$ we are given a travel cost c_{ij}. The objective of the problem is to construct K vehicle routes of minimum total cost. All vehicle routes must start and end at the depot, all customers must be visited exactly once and each route must satisfy the capacity limit of the vehicle. For details see [8]. Exact solution of the CVRP have intrigued researchers for decades and in recent years we have witnessed a substantial improvement of solution methods for the CVRP, see for example [7], [4] and [3].

The aim of our work is to investigate the effect of using *disjunctive cuts* in a branch-and-cut-and-price algorithm for the CVRP. The branch-and-cut-and-price algorithm itself is is based

on a set-partitioning formulation of the CVRP arising from an Dantzig-Wolfe reformulation of a standard CVRP model as described in [4].

Disjunctive cuts dates back to a 1974 technical report [1] by E. Balas. The inequalities were later shown to be useful for solving *mixed integer programming* (MIP) problems by cutting plane algorithms [2] and has since been integrated into commercial (MIP) solvers like CPLEX and XPRESS MP.

Disjunctive cuts are general-purpose valid inequalities that can be applied to any MIP problem. In [5] it is shown that disjunctive cuts can be useful for solving combinatorial optimization problems by showing that several well-known facet-defining inequalities for problems like the asymmetric traveling salesman problem and the max-cut problem can be obtained as disjunctive cuts. In this talk we presents results that empirically evaluate the effect of using disjunctive cuts in a branch-and-cut-and-price algorithm for the CVRP. The results indicate that disjunctive cuts are useful for improving lower bounds for the CVRP as the following table shows. The table reports gap in the root node in the second column (averaged over a large set of standard CVRP instances). The gap for an instance is calculated as $(\bar{z} - \underline{z})/\bar{z}$ where $\bar{z}$ is the optimal solution to the instance and $\underline{z}$ is the lower bound. The four rows show the gap obtained with four different configurations of the branch-and-cut-and-price algorithm. In the row CC only *capacity constraints* are enabled. Capacity constraints (see e.g. [7]) are the basis of most (all?) successful branch-and-cut algorithms for the CVRP and the table show that this inequality along with the set-partitioning relaxation already provide tight lower bounds. In the row CC + LLE04 capacity constraints and a number of problem specific valid inequalities described in [6] and [7] are enabled. The third column show how large a fraction of the gap that is closed with this set of inequalities compared to the configuration with just the capacity inequalities. In the row CC + DC the capacity constraints and disjunctive cuts are enabled and we see that the disjunctive cuts are able to close a larger fraction of the gap than the problem specific inequalities were able to. This demonstrate the strength of disjunctive cuts when applied to the CVRP. The last row shows the performance when all inequalities are enabled.

Cut type	Gap(%)	Gap closed (%)
CC	0.86	
CC + LLE04	0.76	10.9
CC + DC	0.60	29.9
CC + LLE04 + DC	0.56	34.9

In the talk we will provide results from solving standard CVRP instances to optimality when using the disjunctive cuts and we will compare these results to state-of-art results from [3] and [4]. We conclude that disjunctive cuts show potential for improving branch-and-cut-and-price algorithms, not only for the CVRP, but also for other combinatorial optimization problems because of the

general-purpose nature of the cuts. However, further experiments are necessary to evaluate the effect on other problem types.

References

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