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A Hybrid PO - Higher-Order Hierarchical MoM Formulation using Curvilinear Geometry Modeling

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1. Introduction

Physical optics (PO) is an efficient method for a class of electromagnetic scattering problems involving electrically large smooth regions. However, the PO current is not very accurate close to wedges, corners, highly curved regions, and regions not directly illuminated. On the other hand, the method of moments (MoM) is very accurate and can be applied to general geometries including those where PO fails. However, the MoM is not feasible for electrically large scatterers due to a prohibitively large memory requirement. Consequently, it seems natural to formulate an efficient hybrid PO-MoM [1]-[3]. In these works, an essential step was to the expand the PO current in terms of MoM basis functions which allowed to include the coupling between the MoM and PO regions. This also provides a convenient way to impose continuity at the PO-MoM boundary. In [1]-[3], the low-order RWG [4] basis functions on flat triangular patches were chosen. However, RWG functions require a relatively large number of unknowns, approximately 120 per square wavelength.

This paper presents a very effective hybrid PO-MoM solution based on higher-order hierarchical basis functions defined on higher-order curved quadrilateral patches. In comparison with [1]-[3], this reduces the number of unknowns by approximately a factor of 4 and the required memory by a factor of 16. At the same time, the use of hierarchical higher-order functions combines the advantages of both low-order and higher-order methods by enabling an adaptively selected expansion order and thus allowing small and large ($\approx 2\lambda$) patches in the same mesh. The hierarchical functions used here were carefully designed to be near-orthogonal while maintaining simplicity and allow the use of very high expansion orders, e.g., 10th order, without causing the MoM matrix to be ill-conditioned [5]. The hybrid approach applied here is similar to that of [1]-[3] which requires the PO currents to be projected onto the MoM basis functions. This is not straightforward when higher-order functions are applied and this issue is treated in Section 4. Numerical results are presented for an offset shaped reflector antenna configuration.

2. Higher-Order Hierarchical MoM

The scatterer is divided into p quadrilateral patches of arbitrary order with an associated parametric curvilinear coordinate system defined by $-1 \leq u, v \leq 1$. The patches can be 4-node bilinear quadrilaterals, 9-node curved quadrilaterals, or even higher orders, and the order determines the expression for the position vector $\mathbf{r}(u, v)$. The surface current on each patch is represented as $\mathbf{J}_s = J_s^u \mathbf{a}_u + J_s^v \mathbf{a}_v$ where $\mathbf{a}_u$ and $\mathbf{a}_v$ are the co-variant unitary vectors, $\mathbf{a}_u = \frac{\partial \mathbf{r}}{\partial u}$ and

$\mathbf{a}_v = \frac{\partial \mathbf{r}}{\partial v}$. Without loss of generality we consider only u-directed currents in the following and note that the corresponding expressions for the v-directed current are obtained by interchanging u and v. The higher-order expansion is then

$$J_s^u(u,v) \;=\; \sum_{m=0}^{M^u}\sum_{n=0}^{N^v} b_{mn} g_{mn}(u,v), \quad g_{mn}(u,v) = \frac{\widetilde{P}_m(u)\, P_n(v)}{\mathcal{J}_s(u,v)}, \qquad (1)$$

where $\mathcal{J}_s(u,v) = |\mathbf{a}_u \times \mathbf{a}_v|$ is the surface Jacobian, b_{mn} are unknown coefficients, and $P_n(v)$ are orthogonal Legendre polynomials. $\widetilde{P}_m(u)$ are the expansion polynomials along the direction of current flow,

$$\widetilde{P}_m(u) = \begin{cases} 1-u, & m=0 \\ 1+u, & m=1 \\ P_m(u) - P_{m-2}(u), & m \geq 2 \end{cases}, \qquad (2)$$

that allow to enforce the normal current continuity while maintaining almost perfect orthogonality. These basis functions have several desirable features which are described in [5]. Note that the coefficients b_{0n} and b_{1n} are associated with the rooftop-like functions $1 \pm u$ which are zero at either $u = -1$ or $u = 1$. These functions must be matched with similar functions on neighboring patches to enforce normal continuity. All functions with $m \geq 2$ are zero at $u = \pm 1$ and do not contribute to the normal current component flowing across the edge. The higher-order basis functions are implemented in the electric field integral equation (EFIE) using Galerkin testing. Details can be found in [5].

3. Hybrid PO-MoM

The hybrid approach applied here is derived in the same way as that of [1]-[3]. The scatterer is divided into a PO region and a MoM region. The PO region consists of the smooth part of the scatterer and the MoM region is the rest.Both the MoM and the PO currents are discretized using the higher-order basis functions defined in (1). The EFIE with Galerkin testing is then applied in the MoM region, while taking into account the PO region. In matrix form this leads to the equation

$$\left(\overline{\mathbf{Z}}^{MoM} + \overline{\mathbf{Z}}^{MoM,PO}\overline{\mathbf{P}}^{PO,MoM}\right)\mathbf{I}^{MoM} = \mathbf{V} - \overline{\mathbf{Z}}^{MoM,PO}\mathbf{I}^{PO,inc} \qquad (3)$$

in which $\overline{\mathbf{Z}}^{MoM}$ and $\mathbf{V}$ are the standard MoM matrix and excitation vector, respectively, and $\mathbf{I}^{MoM}$ is a vector with the unknown coefficients for the MoM current. $\mathbf{I}^{PO,inc}$ is a vector obtained by projecting the part of the PO current related to the incident field onto the PO basis functions. The matrix $\overline{\mathbf{Z}}^{MoM,PO}$ results from testing the electric field radiated by the PO basis function with the MoM basis functions. The matrix $\overline{\mathbf{P}}^{PO,MoM}$ results from projecting the magnetic field radiated by the MoM basis functions onto the PO basis functions. The product of these matrices accounts for infinitely many interactions between the MoM and PO regions. Eq. (3) is then solved for $\mathbf{I}^{MoM}$. The current in the PO region is obtained by adding $\mathbf{I}^{PO,inc}$ and the PO current resulting from the magnetic field radiated by $\mathbf{I}^{MoM}$, i.e., $\mathbf{I}^{PO} = \mathbf{I}^{PO,inc} + \overline{\mathbf{P}}^{PO,MoM}\mathbf{I}^{MoM}$. The matrix-matrix product in (3) is computationally expensive and should never be evaluated explicitly. Instead, (3) is solved by applying an iterative solver directly or through the iterative procedure described in [3] involving multiple interactions. If interactions between the PO and MoM regions are negligible $\overline{\mathbf{P}}^{PO,MoM}$ is set to zero and (3) reduces to the EFIE with an excitation vector including the field radiated by the PO currents. $\overline{\mathbf{Z}}^{MoM,PO}$ is then needed only once and is not stored.

4. Projection of PO Currents onto Higher-Order Basis Functions

The projection of PO currents onto the PO basis functions used when computing $\overline{\mathbf{P}}^{PO,MoM}$ and $\mathbf{I}^{PO,inc}$ in (3) is not straightforward with higher-order basis functions. The PO current is written in terms of the contravariant projection

$$\mathbf{J}_{PO} = J^u_{PO}\mathbf{a}_u + J^v_{PO}\mathbf{a}_v, \quad J^u_{PO} = \mathbf{J}_{PO}\cdot\mathbf{a}^u \text{ and } J^v_{PO} = \mathbf{J}_{PO}\cdot\mathbf{a}^v\,, \qquad (4)$$

where $\mathbf{a}^u$ and $\mathbf{a}^v$ are the contravariant unitary vectors. In order to project J^u_{PO} and J^v_{PO} onto the basis in (1), we initially determine the coefficients in the expansion

$$J^u_{PO} \approx \sum_{m=0}^{M^u}\sum_{n=0}^{N^v} a_{mn} f_{mn}(u,v), \text{ where } f_{mn}(u,v) = \frac{P_m(u)P_n(v)}{\mathcal{J}(u,v)}. \qquad (5)$$

The functions f_{mn} are mutually orthogonal if weighted with $\mathcal{J}(u,v)$, i.e.,

$$\int_S \mathcal{J}(u,v) f_{mn}(u,v) f_{ij}(u,v)\, dS = 0\,, \quad m \neq i \text{ or } n \neq j\,. \qquad (6)$$

Thus, we can find the coefficients a_{mn} as

$$a_{mn} = \frac{\int_S \mathcal{J}(u,v) f_{mn}(u,v) J_u^{PO}\, dS}{\int_S \mathcal{J}(u,v) f_{mn}^2(u,v)\, dS} \qquad (7)$$

It is straightforward to write up a matrix that shifts between the bases in (5) and (1), and vice versa. Consequently, having obtained the a_{mn}'s, we can obtain the b_{mn}'s. Note that this projection does not require a linear system to be solved although the basis functions are not orthogonal. Also note that the evaluation of (7) is similar to the testing procedure in the MoM/Galerkin method.

5. Numerical Results

We illustrate the accuracy of the hybrid method by calculating the radiation pattern of the offset shaped reflector antenna specified in Fig. 1. The MoM region is only one patch wide which in this case is approximately 1.8λ. The highest polynomial degree is 7 and the average basis function density is 28 per square wavelength. This resulted in 2265 MoM basis functions and 7791 PO basis functions. For this problem the coupling term in (3) can be neglected and the required memory was only 40 MByte. The incident field is computed from a spherical-wave expansion of the field from a corrugated horn. The directivity obtained with MoM and the hybrid PO-MoM is shown in Fig. 1. The PO and PO-PTD results obtained with the software package GRASP8 are

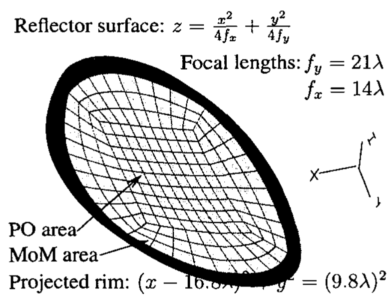

Figure 1: *Parameters and mesh of offset shaped reflector antenna.*

also shown. The hybrid PO-MoM and the PO-PTD agree very well with the MoM which illustrates the excellent accuracy of both these methods. However, the hybrid PO-MoM can be applied to much more general problems where a PTD solution is inaccurate or not possible. Furthermore, the accuracy of the hybrid PO-MoM can be improved by increasing the size of the MoM region.

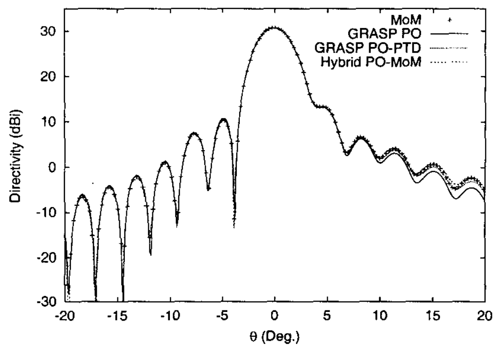

Figure 2: *E-plane directivity ($\phi = 0$) of offset shaped reflector antenna obtained with MoM, PO and PO-PTD from the software package GRASP8, and the hybrid PO-MoM presented here.*

6. Conclusions

A very efficient hybrid PO-MoM method has been presented. In contrast to existing methods, the present solution employs higher-order hierarchical basis functions to discretize the MoM and PO currents. This allows to reduce the number of basis functions in both the PO and MoM regions considerably which implies a very modest memory requirement. Nevertheless, the hierarchical feature of the basis functions maintains the ability to treat small geometrical details efficiently. In addition, the scatterer is modelled with higher-order curved patches which allows accurate modelling of curved surfaces with a low number of patches. A numerical result for an offset shaped reflector antenna illustrated the accuracy of the method.

References

[1] U. Jakobus and F. M. Landstorfer, "Improved PO-MM hybrid formulation for scattering from three-dimensional perfectly conducting bodies of arbitrary shape," *IEEE Trans. Ant. Propagat.*, vol. 43, no. 2, pp. 162–169, Feb. 1995.

[2] F. Obelleiro, J. M. Taboada, J. L. Rodríguez, J. O. Rubiños, and A. M. Arias, "Hybrid moment-method physical optics formulation for modeling the electromagnetic behavior of on-board antennas," *Microwave and Opt. Tech. Letters*, vol. 27, no. 2, pp. 88–93, Oct. 2000.

[3] R. E. Hodges and Y. Rahmat-Samii, "An iterative current-based hybrid method for complex structures," *IEEE Trans. Ant. Propagat.*, vol. 45, no. 2, pp. 265–276, Feb. 1997.

[4] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Ant. Propagat.*, vol. AP-30, no. 3, pp. 409–418, May 1982.

[5] E. Jørgensen, J. L. Volakis, P. Meincke, and O. Breinbjerg, "Higher-order hierarchical Legendre basis functions for iterative integral equation solvers with curvilinear surface modeling," *IEEE Ant. Propagat. Symp.*, San Antonio, TX, USA, June 2002, vol. IV, pp. 618–621.