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Rank, Mike Lind

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Robust \mathcal{H}_2 Performance for Sampled-data Systems

M.L. Rank*

F. Paganini†

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Abstract

Robust \mathcal{H}_2 performance conditions under structured uncertainty, analogous to well known methods for \mathcal{H}_∞ performance, have recently emerged in both discrete and continuous-time. This paper considers the extension into uncertain sampled-data (SD) systems, taking into account inter-sample behavior. Convex conditions for robust \mathcal{H}_2 performance are derived for different uncertainty sets.

1 Introduction & Background

In Dullerud [Dul95], a thorough study of robustness analysis for sampled-data systems was undertaken. In the configuration of Figure 1, G is a continuous time linear time-invariant (LTI) system, controlled by a discrete-time LTI controller K_d by means of ideal sample and hold devices with synchronized period h . This makes the nominal closed loop map M (from (p, w) to (q, z)) periodically time varying (PTV), instead of LTI as is usual in robust control. The system is affected by dynamic uncertainty Δ , which has spatial structure and can be LTI, PTV, or arbitrarily time-varying (LTV). Methods for robust stability and \mathcal{H}_∞ performance evaluation were studied in [Dul95], extending the standard theory for continuous or discrete time systems.

In this paper we consider the question of robust \mathcal{H}_2 performance for sampled data systems, following recent results in [Pag96a, Pag96b] in the standard case, which closely resemble the robust \mathcal{H}_∞ theory. For sampled data systems, we extend these conditions for both PTV and LTV perturbations.

1.1 Lifting

For a general introduction see [CF95] and references therein. The Laplace, Lift and the Λ (or Z) trans-

*Department of Automation, Technical University of Denmark, Building 326, DK-2800, Lyngby, Denmark. Tlph: (+45) 45 25 35 79, Fax: (+45) 45 88 12 95. E-mail: mlr@iau.dtu.dk

†Electrical Engineering Department, UCLA, Los Angeles, CA 90095, USA. E-mail: paganini@ee.ucla.edu

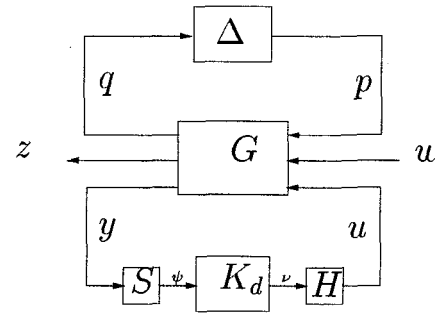


Figure 1: Uncertain Sampled-data System

forms between the signal spaces are related as seen below, we use the same accents (from [Dul95]) for operators mapping within the domains.

$$\begin{array}{ccc} \check{f}(\lambda) \xleftarrow{\Lambda} \check{f}(k) \in \ell_2 \triangleq \ell_{\mathcal{L}_2[0;h]} & & \\ \Lambda L \mathcal{L}^{-1} \uparrow & & \uparrow L \\ \hat{f}(s) \xleftarrow{\mathcal{L}} f(t) \in \mathcal{L}_2 & & \end{array} \quad (1)$$

The lifting technique converts the PTV operator M to LTI in the lifted domain. In the Λ -domain, it amounts to the operator $\check{f}(\lambda) \mapsto \check{M}(\lambda)\check{f}(\lambda)$, where at each λ in the unit disk, $\check{M}(\lambda)$ is an operator on $\mathcal{L}_2[0; h]$.

1.2 \mathcal{H}_2 Performance for TV systems

For LTI systems the \mathcal{H}_2 norm is given by

$$\|T\|_2^2 \triangleq \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{trace}(\hat{T}(j\omega)^* \hat{T}(j\omega)) d\omega = \sum_{i=1}^n \|T\delta e_i\|_{\mathcal{L}_2}^2$$

where $T\delta e_i$ is the impulse response for the i -th input channel. For TV systems, this impulse response varies in time; one possible generalization of the \mathcal{H}_2 norm used in [BJ92] (a different one is given in [Pag96b]) is to average over time. For PTV systems, we average over the period:

$$\|T\|_{\mathcal{H}_2}^2 = \frac{1}{h} \int_0^h \sum_{i=1}^n \|T\delta_\tau e_i\|_{\mathcal{L}_2}^2 d\tau \quad (2)$$

Going to the Λ domain, this is equivalent to the form given in (3), where $\|\cdot\|_{\mathbb{H}_S}$ is the Hilbert-Schmidt norm of an operator on $\mathcal{L}_2[0; h]$, see [BJ92, CF95].

$$\|\check{T}\|_{\mathcal{H}_2}^2 = \frac{1}{2\pi} \int_0^{2\pi} \|\check{T}(e^{j\theta})\|_{\mathbb{H}_S}^2 d\theta \quad (3)$$

For arbitrary LTV systems (2) may be generalized by taking the limit as $h \rightarrow \infty$.

2 Robust \mathcal{H}_2 Performance SD for TV uncertainty

The sets of full block structured LTV and PTV perturbations (of period h) are given by

$$\begin{aligned} \Delta_{LTV} &\triangleq \{\Delta = \text{diag}(\Delta_1, \dots, \Delta_F) : \Delta_k \in \mathfrak{L}(\mathcal{L}_2^{m_k})\} \\ \Delta_{PTV} &\triangleq \{\Delta \in \Delta_{LTV} : D_h \Delta = \Delta D_h\} \end{aligned}$$

$T_{zw}(\Delta)$ denotes the map from w to z (see Fig. 1).

2.1 PTV perturbation case

Let $\Delta \in \Delta_{PTV}$. At each $\lambda = e^{j\theta}$, we introduce a scaling which commutes with $\check{\Delta}(e^{j\theta})$, $X(\theta) \in \mathbb{X} \triangleq \{X = \text{diag}[x_1 I_{m_1}, \dots, x_F I_{m_F}], x_k > 0\}$ (constant matrix multiplication operator on $\mathfrak{L}(\mathcal{L}_2[0; h])$).

Condition 1 *There exists functions $X(\theta) \in \mathbb{X}$ and $Y(\theta) \in \mathfrak{L}(\mathcal{L}_2[0; h])$, such that*

$$\check{M}(e^{j\theta})^* \begin{bmatrix} X(\theta) & 0 \\ 0 & I \end{bmatrix} \check{M}(e^{j\theta}) - \begin{bmatrix} X(\theta) & 0 \\ 0 & Y(\theta) \end{bmatrix} < 0 \quad (4)$$

for all $\theta \in [0; 2\pi[$ and

$$\int_0^{2\pi} \text{tr} Y(\theta) \frac{d\theta}{2\pi} = \int_0^{2\pi} \int_0^h \text{trace} Y_\theta(t, t) dt \frac{d\theta}{2\pi} < 1. \quad (5)$$

Remark 1 *In (5) we use the trace of an operator $Y \in \mathfrak{L}(\mathcal{L}_2[0; h])$; this is defined as*

$$\text{tr} Y \triangleq \sum_{i=1}^{\infty} \langle Y b_i, b_i \rangle = \int_0^h \text{trace} Y(t, t) dt, \quad (6)$$

where b_i is any orthonormal basis of $\mathcal{L}_2[0; h]$, and $Y(t, \tau)$ is the kernel representation of Y .

Proposition 1 *If Condition 1 holds and $\Delta \in \mathcal{B}_{\Delta_{PTV}}$, then the system is robustly stable and*

$$\sup_{\Delta \in \mathcal{B}_{\Delta_{PTV}}} \|T_{zw}(\Delta)\|_{\mathcal{H}_2} < 1. \quad (7)$$

Proof: See [RP97].

Remark 2 *This sufficient condition is convex in the unknowns $X(\theta)$, $Y(\theta)$. The “frequency” and “time” dependence of M is reflected in $Y_\theta(t, t) \in \mathbb{C}^{m \times m}$. A finite dimensional approximation can be obtained by gridding. Clearly, this condition also holds for LTI perturbations, however, the LTI behaviour can be further exploited (see [RP97]).*

2.2 LTV perturbation case

Proposition 2 *If Condition 1 holds for a constant function $X(\theta) \equiv X \in \mathbb{X}$, and $\Delta \in \mathcal{B}_{\Delta_{LTV}}$, then the uncertain system is robustly stable and*

$$\sup_{\Delta \in \mathcal{B}_{\Delta_{LTV}}} \|T_{zw}(\Delta)\|_{\mathcal{H}_2} < 1. \quad (8)$$

3 Conclusion and further directions

Conditions for robust \mathcal{H}_2 performance for sampled-data systems have been derived under time-varying uncertainty (PTV or arbitrary LTV). Only sufficiency was shown; it is expected that necessity results will follow if one adopts the notion of \mathcal{H}_2 performance in [Pag96a, Pag96b], and replaces PTV uncertainty by a “quasi-PTV” notion (see [Dul95]). Further work includes state-space computations for these conditions, and more refined conditions for the case of purely LTI uncertainty.

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