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Kymmel, Mogens; Andersen, Henrik Weisberg

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ROBUST GEOMETRIC CONTROL OF A DISTILLATION COLUMN

Mogens Kümmel and Henrik Weisberg Andersen

Chemical Engineering Department
Technical University of Denmark
DK-2800 Lyngby, DENMARK

ABSTRACT

A frequency domain method, which makes it possible to adjust multivariable controllers with respect to both nominal performance and robustness, is presented. The basic idea in the approach is that the designer assigns objectives such as steady-state tracking, maximum resonance peaks, bandwidth, minimum stability margin, steady-state sensitivity and maximum sensitivity to modelling errors. For a given control structure the parameters are found which minimize an objective function consisting of the weighted sum of deviations between desired and obtained values of these objectives. This method is used to examine and improve geometric control of a binary distillation column.

INTRODUCTION

Distillation column control is an important operation in many chemical plants, partly because it is so energy intensive. Consequently, the topic has received much attention over the last many years.

The control of both top and bottom product compositions is clearly a multivariable control problem due to the coupling between the control loops. Thus, advanced modern control strategies, such as optimal control, modal control and geometric control have the potential of improving the performance compared to classical control schemes.

Takamatsu et al. (1979) designed a geometric controller for a binary distillation column. This controller yields total nominal disturbance rejection in the top and bottom compositions towards disturbances in the feed composition and total nominal disturbance rejection in the top composition towards disturbances in the feed flow rate. However, the design of geometric controllers relies heavily on the linear time invariant state space model, which is only an approximate description of the real plant. Thus, it is uncertain how well this controller will perform when it is implemented on the real plant. This causes the need of a method for analysis and design of multivariable controllers with respect to robustness as well as nominal performance.

Doyle and Stein (1981) presented a multivariable frequency analysis of the output sensitivity to modelling errors and the stability margin. Arkun et al. (1984) used this method to analyse different decoupling schemes in distillation control. Palazoglu and Arkun (1985) presented a multivariable frequency analysis of nominal setpoint tracking. In Kümmel and Andersen (1986,a), these ideas were used to develop a multivariable frequency analysis of nominal disturbance attenuation. Thus, the analysis contains the following items:

- Nominal setpoint tracking
- Nominal disturbance attenuation
- Output sensitivity to modelling errors
- Stability margin

Moreover the nominal stability is analysed by evaluation of closed-loop eigenvalues.

In Kümmel and Andersen (1986), this analysis was used to compare the nominal performance and robustness of geometric control, optimal control and PI control applied to distillation column control.

In this paper we will treat both the analysis and the design problem. To be more precise, this paper addresses the problem: "Given a control structure. It is desired to assign values of the control parameters which will give a satisfactory trade off between nominal performance and robustness". Note that this is a local optimization since the control structure is predetermined. In order to perform this control adjustment, the items in the analysis are quantified. This is done mainly by the use of classical frequency domain terms such as steady-state error, bandwidth, resonance peaks and minimum stability margin. Based on the designer's knowledge of the control objectives, he can specify desired values of these terms. An objective function is then evaluated as the weighted sum of the squared deviation between desired and obtained objectives. The designer defines the structure of the controller. Moreover, the control parameters must be initialized so that the nominal closed-loop system is stable. He might assign some of the control parameters in order to predetermine desired properties of the controlled system. For example some of the control parameters could be fixed in order to obtain complete nominal disturbance rejection through geometric control (Takamatsu et al. 1979). The objective function is then minimized by adjustment of the unassigned control parameters. The following analysis will show to which extent the desired objectives have been achieved. If some performance items need to be improved, the designer might choose to weight these higher in the objective function or to develop a new control structure which can be adjusted to achieve the desired objectives.

This method has been used to adjust selected control parameters in order to improve the robustness of basic geometric control. This can be done without changing the nominal disturbance rejection provided by the basic geometric controller. Thus, the scope of this method is that we can obtain a geomtric controller with improved robustness properties and at the same time preserve the attractive nominal performance.

ANALYSIS

This section is a review of the frequency analysis described in Kümmel & Andersen (1986). We have used the following standard state space notation:

$$\dot{x}(t) = Ax(t) + Bu(t) + Dq(t)$$
$$y(t) = Cx(t)$$
$$u(s) = G(s)(HC^T(CC^T)^{-1} r_y(s)-f(s))$$
$$f(t) = Hx(t) + v(t)$$

Nominal performance

The first objective is to obtain satisfactory nominal performance. Here we will focus on how to evaluate setpoint tracking and disturbance rejection in the frequency domain.

Setpoint tracking. The continuous time closed-loop frequency response between the outputs $y(iw)$ and their corresponding setpoints $r_y(iw)$ is given as:

$$y(iw) = C(iwI-A+BG(iw)H)^{-1}BG(iw)HC^T(CC^T)^{-1}r_y(iw) = T1(iw)r_y(iw) \quad (1)$$

Since T1 is a square matrix we have:

$$\underline{\sigma}(T1(iw)) \le \frac{\|y(iw\|}{\|r_y(iw)\|} \le \bar{\sigma}\ (T1(iw)) \quad (2)$$

where $\bar{\sigma}$, $\underline{\sigma}$ respectively denote the maximum and the minimum singular value.

Disturbance rejection. The closed-loop frequency response between the outputs $y(iw)$ and the external disturbances $q(iw)$ is given as:

$$y(iw) = C(iwI-A+BG(iw)H)^{-1}\ Dq(iw) = T2(iw)q(iw) \quad (3)$$

Depending on the dimension of y (m) and q (d) we have the following vector gain bounds:

$$m \ge d \quad \underline{\sigma}\ (T2(iw)) \le \frac{\|y(iw)\|}{\|q(iw)\|} \le \bar{\sigma}\ (T2(iw)) \quad (4)$$

$$m \le d \quad 0 \le \frac{\|y(iw)\|}{\|q(iw)\|} \le \bar{\sigma}\ (T2(iw)) \quad (5)$$

Robustness

Output sensitivity to modelling errors. This analysis is based on results from Cruz and Perkins (1964). The sensitivity is a relative measurement between a closed-loop error and an open-loop error. This is a reasonable approach since the purpose of using closed-loop control is to obtain a lower sensitivity than that of open-loop control. The following relation exists between the closed-loop and the open-loop error:

$$e_{cx}(s) = (I+P'(s)G(s)H)^{-1}\ e_{ox}(s) \quad (6)$$

where $x_c(s) = x_o(s)$ and $P'(s) = P(s) + \Delta P(s)$.

Thus the closed-loop error in the outputs can be expressed as:

$$e_{cy}(s) = C(I+P'(s)G(s)H)^{-1}\ e_{ox}(s)=S'(s)e_{ox}(s) \quad (7)$$

Here we will only evaluate the nominal sensitivity:

$$e_{cy}(s) = S(s)e_{ox}(s) \quad (8)$$

The open-loop error in the outputs can be expressed as:

$$e_{oy}(s) = Ce_{ox}(s) \quad (9)$$

Using singular values we can express the vector gain between $e_{cy}(iw)$ and $e_{ox}(iw)$ as:

$$\frac{\|e_{cy}(iw)\|}{\|e_{ox}(iw)\|} \le \bar{\sigma}\ (S(iw)) \quad (9)$$

and the gain between $e_{oy}(iw)$ and $e_{ox}(iw)$ as:

$$\frac{\|e_{oy}(iw)\|}{\|e_{ox}(iw)\|} \le \bar{\sigma}\ (C(iw)) \quad (10)$$

Selecting the worst case open-loop output error yields:

$$\frac{\|e_{cy}(iw)\|}{\|e_{oy}(iw)\|} \le \bar{\sigma}\ (S(iw))/\bar{\sigma}\ (C) \quad (11)$$

Stability. The stability margin is calculated for input multiplicative errors (Doyle & Stein, 1981).

$$P'(s) = P(s)\ (I+\Delta P(s)) \quad (12)$$

Thus, the stability margin is calculated as:

$$l_{mr}(w) \le \underline{\sigma}\ (I + (G(iw)HP(iw))^{-1}) \quad (13)$$

This stability margin does only examine the influence of input multiplicative errors. Thus, it might be too optimistic since output and input multiplicative and additive errors might occur simultaneously. Moreover, it might be too conservative since the error is only described as norm-bounded regardless of its structure. Improved calculation of the stability margin might be obtained by structured singular values as suggested by Doyle (1982).

DESIGN OBJECTIVES

Normal performance

From the classical Bode gain analysis it is well known that terms such as steady-state error, bandwidth and resonance peaks contain important information about the transient response of the controlled system. A similar procedure can be used in multivariable frequency analysis. In the following is described how these terms can be quantified.

<u>Setpoint tracking</u>. We can measure a bound for the steady-state setpoint tracking error by evaluation of the gain at a suitable low frequency w_1:

$$c_{sp1} = \bar{\sigma}\ (T1(iw_1)) \tag{14}$$

$$c_{sp2} = \underline{\sigma}\ (T1(iw_1)) \tag{15}$$

A conservative indication of the overshoot and oscillation can be found by evaluation of the maximum resonance peak (if present):

$$c_{sp1max} = \max_{w} \bar{\sigma}\ (T1(iw)) \tag{16}$$

To measure the response time of the closed-loop system we will use the classical concept of bandwidth:

$$\text{Max.bandw. } w_{c1}\text{: } \bar{\sigma}(T1(iw_1)) - \bar{\sigma}(T1(iw_{c1})) = 3\text{dB} \tag{17}$$

$$\text{Min.bandw. } w_{c2}\text{: } \underline{\sigma}(T1(iw_1)) - \underline{\sigma}(T1(iw_{c2})) = 3\text{dB} \tag{18}$$

<u>Disturbance rejection</u>. A conservative bound for the steady-state error can be evaluated at a suitable low frequency:

$$c_{d1} = \bar{\sigma}\ (T2(iw_1)) \tag{19}$$

The maximum peak (if present) indicate the maximum possible disturbance in the output signals:

$$c_{d1max} = \max_{w} \bar{\sigma}\ (T2(iw)) \tag{20}$$

<u>Robustness</u>

<u>Output sensitivity to modelling errors</u>.

The steady-state sensitivity can be evaluated at a suitable low frequency:

$$c_{s1} = \bar{\sigma}\ (S(iw_1))/\bar{\sigma}\ (C) \tag{21}$$

The maximum peak (if present) determine the maximum possible sensitivity:

$$c_{s1max} = \max_{w} (\bar{\sigma}\ (S(iw))/\bar{\sigma}\ (C)) \tag{22}$$

<u>Stability</u>. The critical margin (if present) is calculated as:

$$c_{smmin} = \min_{w} \underline{\sigma}\ (I+(G(iw)HP(iw))^{-1}) \tag{23}$$

Above a certain frequency the stability curve will draw off towards infinity. This is an important point, since above this region in frequency we can tolerate large modelling errors. We define a separation between the two regions from:

$$\underline{\sigma}\ (I+(G(iw_{off})HP(iw_{off}))^{-1}) - \underline{\sigma}\ (I+(G(iw_1)HP(iw_1))^{-1}) = 3\text{dB} \tag{24}$$

ADJUSTMENT OF CONTROL PARAMETERS

In order to shape the closed-loop frequency response, desired values and weights can be assigned to the different design objectives. Thus, we can evaluate an objective function consisting of the weighted difference between the actual and desired values of the individual objectives. The individual objective functions fall into three categories.

In order to evaluate the low frequency setpoint tracking we will use an objective function of the following form:

$$F_i = v_i\ (\log(d_i) - \log(c_i))^2 \tag{25}$$

This selection is reasonable in the case where we have selected a desired low frequency setpoint tracking to be 1. In this case both a positive and a negative deviation will result in an error. The same objective function is used to evaluate the obtained bandwidth.

To evaluate the high and low frequency disturbance attenuation we will use the following objective function:

$$\text{If } c_i < d_i \text{ then } F_i = 0 \quad \text{else } F_i = v_i\ (\log(d_i) - \log(c_i))^2 \tag{26}$$

This is relevant since a better disturbance attenuation than specified will always be desirable. The same objective function is used to evaluate the high frequency setpoint tracking, the high and low frequency sensitivity to modelling errors and the draw off.

To evaluate the high and low frequency stability margin we will use the following objective function:

$$\text{If } c_i > d_i \text{ then } F_i = 0 \quad \text{else } F_i = v_i\ (\log(d_i) - \log(c_i))^2 \tag{27}$$

This is reasonable since a higher stability margin than specified is always desirable.

The total objective function is now calculated as the sum of the individual functions. By assigning the structure of the controller we can minimize this objective function through adjustment of the control parameters.

This optimization is clearly a nonlinear minimization. In this work an IMSL subroutine named ZXMIN (The IMSL Library, 1982) has been applied. The complete program package used in the solution is presented in (Andersen, 1986).

One problem in this optimization is whether the nominal closed-loop stability of the initial controller guarantees convergence to a stabilizing controller. When shifting from a stable to an unstable closed-loop system there exists an intermidiate controller which yields poles on the imaginary axis.

For this controller we will have unbounded objective function. An exeption from this occur when we have pole-zero cancellation on the imaginary axis.

Performing the optimization in finite steps will make it possible to cross this unbounded barrier. However, in practise we have not experienced problems of this type. A safe approach would be to check the eigenvalues in each step and then decrease the step length if the resulting closed-loop system has unstable poles.

GEOMETRIC CONTROL OF A DISTILLATION COLUMN

Distillation column model. We use the binary distillation column model of Takamatsu (1979). The column consists of 9 plates, reboiler, and total condenser. The feed, liquid at its boiling point, is entered at plate 5 (state 6). Output variables are the composition in the condenser (state 1) and in the reboiler (state 11). Manipulated variables are the reflux flow and the vapour flow. Disturbances occur in the feed flow and the feed composition. The state space model is shown in table 1. For further details, see Takamatsu et al. (1979).

Basic geometric control. The geometric controller was designed by Takamatsu et al. (1979). Nominally, this controller yields total disturbance rejection at the top and bottom composition towards disturbances in the feed composition. Disturbances in the feed flow rate are completely rejected at the top composition, but not at the bottom composition. According to the control law $u = -GHx$, the feedbacks and gains are given by:

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 330.06 & -470.17 \\ 251.47 & -632.04 \end{bmatrix} \qquad (28)$$

This controller is denoted G1.

The results of the analysis for G1 are shown in table 2 and in Figs. 1, A-D.

The sensitivity can be shown to be above 1 for all frequencies. This is clearly unacceptable since this sensitivity is equal to that of an open-loop controller.

From Fig. 1B it is seen that there is no critical frequency where the stability margin reaches a minimum. In fact, the stability margin is greater than 100% for all frequencies. Thus, basic geometric control of this distillation column should have excellent stability robustness properties.

Since there is no feedback from the outputs (states 1 and 11), both setpoint tracking curves are zero (Fig. 1C).

The nominal disturbance attenuation is shown in Fig. 1D. The basic geometric controller is only represented with one curve. This is due to the fact that nominally the top and bottom concentrations are completely decoupled from disturbances in the feed composition and consequently, the corresponding transfer functions are identical to zero. The top concentration is decoupled from disturbances in the feed flow rate, but the bottom composition is not. Thus, the curve in Fig. 1D corresponds to the transfer function between bottom composition and disturbances in the feed flow rate. The steady-state error is around 0.2% and there is no resonance peak.

Design objectives. In order to adjust the controller we will select the desired frequency domain properties of the controlled system. At $w = 0.01$ we have chosen the desired upper bound on the low frequency sensitivity to be 1%. The desired maximum sensitivity is chosen to 100%. Since the sensitivity curve approach 100% for high frequencies, this selection corresponds to a desired elimination of the maximum on the sensitivity curve.

The desired low frequency stability margin (at $w=0.01$) and the desired minimum stability margin are both selected to 100%. The desired draw off point is chosen to be at $w = 1$.

The desired low frequency setpoint tracking (at $w=0.01$) is chosen to 100% for both the upper and the lower curve. The desired maximum value is also selected to 100%, (this corresponds to a desired elimination of the resonance peak). The desired minimum and maximum bandwidths are both selected to be at $w=1$.

The desired low frequency disturbance rejection (at $w=0.01$) and the maximum disturbance rejection are both chosen to be less than 0.1%.

Improvements by constant gain feedback.

The following structure is obtained for the geometric controller by Kümmel and Foldager (1986):

$$H = I_{11} \quad G = \begin{bmatrix} a & a & f & 0 & 0 & 0 & 0 & 0 & 0 & f & r \\ a & a & f & 0 & 0 & 0 & 0 & 0 & 0 & f & r \end{bmatrix} \qquad (29)$$

a: The control parameter can be assigned an arbitrary value without affecting the nominal disturbance rejection provided by the basic geometric controller.

f: The control parameter is fixed by the basic geometric controller.

0: The control parameter is fixed to zero by the basic geometric controller (i.e. no feedback is allowed from these states).

r: The control parameters must satisfy the following relation:

$$g_{1,j} = -(b_{2,2}/b_{2,1})g_{2,j} \qquad (30)$$

where g is an element in G, and b is an element in B.

In order to reduce the worst case sensitivity of G1 (Fig. 1A) below 1, it is necessary to include feedback from at least states 1 and 11 (the outputs). G2 makes use of all the feedback gains that can be used without affecting the disturbance rejection provided by the basic geometric controller. The gains have been adjusted in order to minimize the described objective function.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (31)$$

$$G = \begin{bmatrix} 12560 & 875 & 330.06 & -470.17 & -4558 \\ 10190 & 731 & 251.47 & -632.04 & -6077 \end{bmatrix}$$

$g_{1,5}$ and $g_{2,5}$ have been adjusted under the restriction (30).

The results of the analysis of G2 is shown in Figs. 1,A-D and in table 2. The output sensitivity (Fig. 1A) has been improved significantly for G2 compared to G1. The steady-state sensitivity has been reduced from 120% (G1) to 16% (G2).

The minimum stability margin (Fig. 1B) has been reduced from 100% (G1) to 70% (G2) indicating a trade-off between nominal performance, output sensitivity and stability robustness. The draw off point has been slightly increased from 1.85 (G1) to 3.63 (G2).

Opposite to G1 the setpoint tracking problem of states 1 and 11 is addressed by G2 (Fig. 1C). For G2 the steady-state gain is 87% for the upper curve and 86% for the lower curve. The upper bandwidth 1.46 (G2) and the lower bandwidth 1.19 (G2) is close to the desired value 1.

The steady-state disturbance attenuation (Fig. 1D) has been improved from 0.2% (G1) to 0.02% (G2), thus the design objective of 0.1% is satisfied for G2.

Transient simulations for G1 and G2. To illustrate the disturbance attenuation the transient response of a step change in the feed flow rate ($\Delta L_F = 1.5$ mol/min) was simulated. Figures 2 and 3 show the nominal as well as a perturbed response of states 1 and 11 for G1 and G2.

The perturbed responses were simulated for a perturbation in the obtained vapour flow rate. This perturbation is described by:

$$\Delta V(\text{obtained}) = E*\Delta V(\text{set}) \tag{32}$$

where ΔV(set) is the vapour flow ordered by the controller. In Fig. 2 the perturbed response was obtained for $E=0.9$.

Nominally, both G1 and G2 yield complete disturbance rejection in state 1 (Fig. 2). However, the sensitivity of G1 is clearly larger than that of G2.

State 11 is not decoupled from disturbances in the feed flow rate (Fig. 3). The nominal steady-state error for G1 is 0.19% and 0.027% for G2 (compare with Fig. 1D). Again, it is seen that the sensitivity of G1 is larger than that of G2.

Improvements by integral action.

The selected design objectives impose high demands on the low frequency behaviour of the controlled system. These will be better addressed by combining the geometric controller with integral action in the feedback loops from states 1 and 11. Since the gains in the feedback from state 1 can be assigned arbitrarily without affecting the nominal disturbance rejection, we can immediately introduce integration in the loop from state 1 to control signal 1. The gains in the feedback from state 11 should obey equation (30). Thus, we have to select the same integral time in the feedback from state 11 to control signal 1 and in the feedback to control signal 2. The following controller was selected as an initial geometric controller with integration:

$$H = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{33}$$

$$\begin{bmatrix} 3000(1+1/3s) & 330.06 & -470.17 & -750(1+1/8s) \\ 0.0 & 251.47 & -632.04 & -1000(1+1/8s) \end{bmatrix}$$

This controller is denoted GPI1.
Note that $g_{1,4}$ and $g_{2,4}$ satisfy restriction (30).

The results of the analysis for GPI1 is shown in Figs. 1, A-D and in table 2.

Figure 1A shows the output sensitivity. The integration in the feedback from both outputs eliminates the steady-state sensitivity. At the low frequency 0.01 the sensitivity is 10%. The curve has an undesired maximum of 268%.

From Fig. 1B it is seen that the stability margin has a minimum of 41%. The draw off point is at $w = 3.64$.

Figure 1C shows the setpoint tracking. The upper curve has an undesired maximum of 250%. The minimum and maximum bandwidths are 1.105 and 1.952 respectively. As for basic geometric control we have only one disturbance attenuation curve (Fig. 1D). As for basic geometric control this curve represents the transfer function between disturbances in the feed flow rate and state 11. Due to the integration the steady-state error has been eliminated.

Adjustment of gains and integration times. Using the control parameters of GPI1 as initial values and the design objectives as described for basic geometric control, we arrive at the controller GPI2 after adjustment of the control parameters. The adjusted controller GPI2 contains the following control parameters:

$$H = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{34}$$

$$G = \begin{bmatrix} 5916(1+1/8.0s) & 671 & 330.06 & -470.17 & -2654(1+1/1.5s) \\ 4282 & 513 & 251.47 & -632.04 & -3539(1+1/1.5s) \end{bmatrix}$$

Note that $g_{1,5}$ and $g_{2,5}$ have been adjusted under the restriction (30).

The results of the analysis of GPI2 are shown in Fig. 1, A-D and in table 2.

The low frequency sensitivity ($w = 0.01$) has been reduced from 10% (GPI1) to 1.3% (GPI2). The maximum has been reduced from 268% (GPI1) to 132% (GPI2).

From Fig. 1B it is seen that the minimum stability margin is increased from 41% (GPI1) to 59% (GPI2). Moreover, the draw off point has been reduced from 3.64 (GPI1) to 1.9 (GPI2).

The maximum on the upper setpoint tracking curve (Fig. 1C) has been reduced from 250% (GPI1) to 140% (GPI2). The maximum and minimum bandwidths are 1.19 and 0.67 respectively. Even though the performance towards external disturbances are in the desired range for GPI1, it has been improved by the adjustment (Fig. 1D).

Simulation of geometric control with integration. Figures 4 and 5 show the nominal and perturbed responses of states 1 and 11 towards a step disturbance in the feed flow rate for GPI1 and GPI2. From Fig. 4 it is seen that state 1 is nominally decoupled and that the sensitivity is equal for the two controllers. Figure 5 shows that the nominal disturbance attenuation and the sensitivity of state 11 has been improved by the adjustment.

Figure 6 shows the transient responses of states 1 and 11 for a setpoint change (0.05) in state 1 for the controllers G2, GPI1 and GPI2. As expected G2 exhibits an offset in state 1. GPI1 eliminates this offset, however, it has an undesired overshoot and oscillation. The adjustment (GPI2) has significantly improved the setpoint tracking. As expected the overshoot and oscillation have been reduced, so that the transient responses are satisfactorily damped.

Figures 7 and 8 show the nominal and perturbed responses of states 1 and 11 towards a step disturbance in the feed flow rate for G1, G2, and GPI2. The major improvements obtained by introducing integration is that the steady-state sensitivity of state 1 has been eliminated (Fig. 7) and that the steady-state error of state 11 has been eliminated (Fig. 8).

CONCLUSION

The attractive property of geometric control is the complete nominal disturbance rejection. However, the basic geometric controller has an output sensitivity which equals that of open-loop control, and this controller is likely to fail when it is implemented on a real distillation column. In order to improve the robustness we have presented a multivariable frequency domain analysis and design approach. The problem is solved by local optimization; for a given control structure values of the control parameters are chosen which minimize the objective function.

The first step in the design is the selection of closed-loop properties in the frequency domain. The formulation of these objectives is well known from classical frequency analysis. The variety of objectives should not be considered as an unnecessary complication of the design problem. On the contrary, they enable us to include detailed information about the desired performance and robustness, which again is believed to yield a control system design with reliable desired properties.

The weighting of the different objectives in the evaluation of the objective function enables the designer to impose mandatory demands on the control design. These should be thought of as design parameters which can be adjusted in parallel with the adjustment of a given control structure in order to get close enough to the mandatory properties. Selecting other control structures is another approach to the problem of getting close enough to our defined objectives. The step by step approach used in this paper illustrates the feasibility of this method. Starting out with a simple control structure, which typically provides some desired nominal performance, and then include possible extensions in a discrete fashion, we can evaluate which extensions are necessary in order to satisfy our objectives.

By the use of this method we have obtained geometric controllers with significantly improved setpoint tracking and robustness behaviour. Note that these controllers preserve the attractive nominal disturbance rejection properties of basic geometric control. These properties of the extended geometric controllers allow us to expect that these controllers will yield improved performance on industrial distillation columns compared to conventional control.

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$$
A = (\text{min})^{-1}\begin{vmatrix}
-0.174 & 0.105 & & & & & & & & & \\
0.522 & -0.943 & 0.469 & & & & & & & & \\
 & 0.522 & -0.991 & 0.529 & & & & & & & \\
 & & 0.522 & -1.051 & 0.569 & & & & & & \\
 & & & 0.522 & -1.118 & 0.662 & & & & & \\
 & & & & 0.522 & -1.584 & 0.718 & & & & \\
 & & & & & 0.922 & -1.640 & 0.799 & & & \\
 & & & & & & 0.922 & -1.721 & 0.901 & & \\
 & & & & & & & 0.922 & -1.823 & 1.021 & \\
 & & & & & & & & 0.922 & -1.943 & 1.142 \\
 & & & & & & & & & 0.115 & -0.171
\end{vmatrix}
$$

$$
B = (\text{mol})^{-1}\begin{vmatrix}
0 & 0 \\
0.00328 & -0.00244 \\
0.00384 & -0.00288 \\
0.00400 & -0.00304 \\
0.00376 & -0.00280 \\
0.00308 & -0.00232 \\
0.00236 & -0.00312 \\
0.00288 & -0.00382 \\
0.00308 & -0.00412 \\
0.00300 & -0.00306 \\
0.00032 & -0.00042
\end{vmatrix}
\quad
(\text{min})^{-1}\, D = (\text{mol})^{-1}\begin{vmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0.4 & 0 \\
0 & 0.00236 \\
0 & 0.00288 \\
0 & 0.00308 \\
0 & 0.00300 \\
0 & 0.00032
\end{vmatrix}
\quad
C = \begin{vmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{vmatrix}
$$

Table 1. Linearized mathematical model of binary distillation column.

	Design object.		G1		G2		GPI1		GPI2	
Low freq. analysis.	Desired	Weight	Value	Object	Value	Object	Value	Object	Value	Object
Upper setp. trac.	1.0000	1.0000	0.0000	∞	0.8670	0.0038	1.0200	0.0000	1.0030	0.0000
Lower setp. trac.	1.0000	1.0000	0.0000	∞	0.8516	0.0049	0.9760	0.0001	0.9973	0.0000
Dist. attenuation.	0.0010	1.0000	0.0019	0.0736	0.0002	0.0000	0.0001	0.0000	0.0000	0.0000
Upper. sensitivity.	0.0100	1.0000	1.2040	4.3290	0.1614	1.4590	0.1015	1.0130	0.0135	0.0167
Stability magin.	1.0000	1.0000	0.9736	0.0001	0.9944	0.0000	0.9975	0.0000	0.9990	0.0000
High freq. analysis.										
Max. upper setp.	1.0000	1.0000					2.4950	0.1577	1.4010	0.0214
Max. dist. att.	0.0010	1.0000					0.0009	0.0000	0.0004	0.0000
Max. upper sens.	1.0000	1.0000			1.1990	0.0062	2.6760	0.1827	1.3240	0.0149
Min. stab. margin.	1.0000	1.0000			0.6985	0.0024	0.4135	0.1471	0.5939	0.0512
Max. bandwidth.	1.0000	1.0000			1.4560	0.0266	1.9520	0.0844	1.1930	0.0059
Min. bandwidth.	1.0000	1.0000			1.1870	0.0055	1.1046	0.9613	0.6710	0.0300
Draw off.	1.0000	1.0000	1.8580	0.0724	3.6310	0.3136	3.6400	0.3149	1.9010	0.0778
Total object func.				∞		1.8440		2.8612		0.2179

Table 2. Desired and obtained objectives together with the accumulated object function for G1, G2, GPI1 and GPI2.

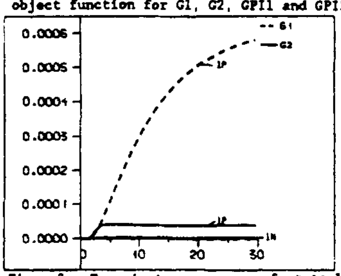

Fig. 2. Transient responses of state 1 for G1 and G2 towards a step disturbance in the feed flow rate. 1N / 1P: Nominal / perturbed response of state 1.

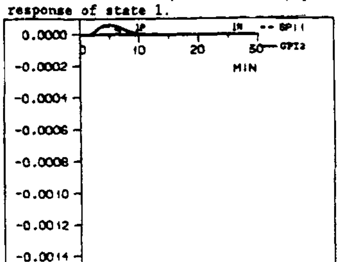

Fig. 4. Transient responses of state 1 for GPI1 and GPI2 towards a step disturbance in the feed flow rate. 1N / 1P: Nominal / perturbed response of state 1.

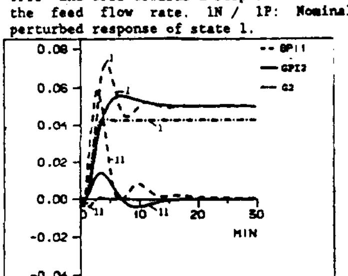

Fig. 6. Nominal transient responses of state 1 and 11 for GPI1, GPI2 and G2 towards a step change in the setpoint for state 1.

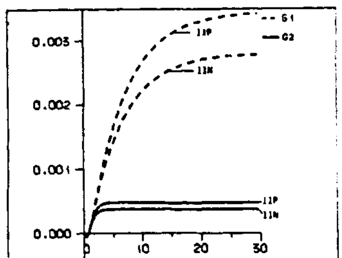

Fig. 3. Transient responses of state 11 for G1 and G2 towards a step disturbance in the feed flow rate. 11N / 11P: Nominal / perturbed response of state 11.

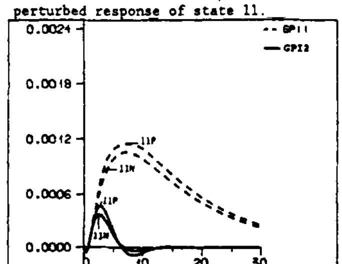

Fig. 5. Transient responses of state 11 for GPI1 and GPI2 towards a step disturbance in the feed flow rate. 11N / 11P: Nominal / perturbed response of state 11.

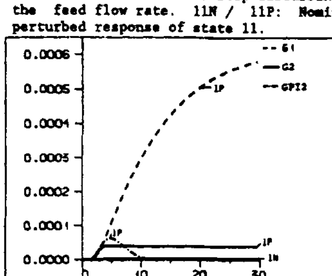

Fig. 7. Transient responses of state 1 for G1, G2 and GPI2 towards a step disturbance in the feed flow rate. 1N / 1P: Nominal / perturbed response of state 1.

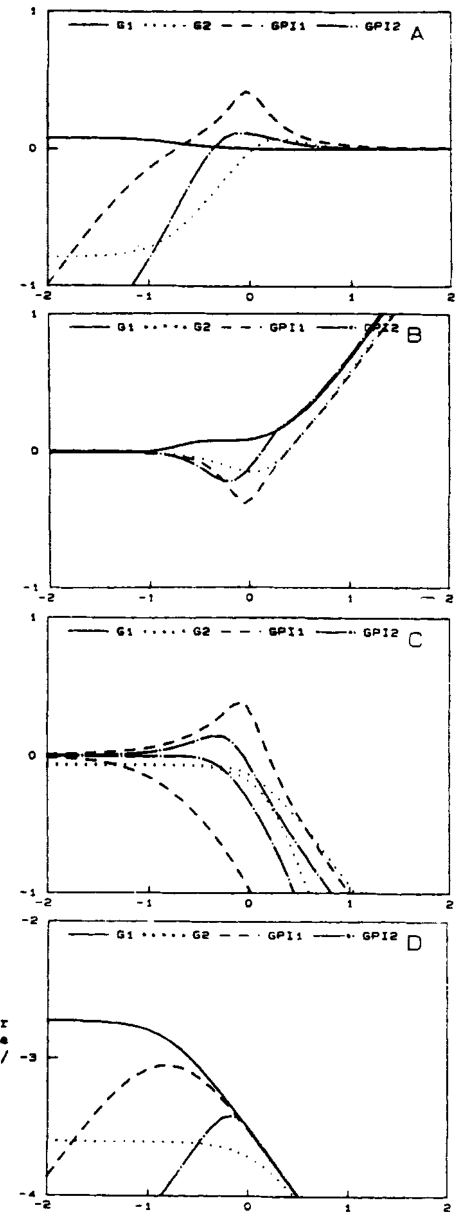

Fig. 1. Analysis of G1, G2, GPI1 and GPI2.
A : Output sensitivity.
B : Stability.
C : Setpoint tracking.
D : Disturbance attenuation.

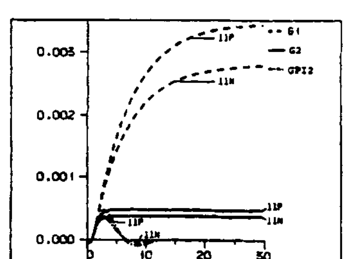

Fig. 8. Transient responses of state 11 for G1, G2 and GPI2 towards a step disturbance in the feed flow rate. 11N / 11P: Nominal / perturbed response of state 11.