Search methods for rate 1/N convolutional codes used in concatenated systems

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Abstract - Different criteria for selection of good convolutional codes have been investigated and the interesting conclusion is that the Viterbi bound on BER evaluated at a higher $E_b/N_0$ value is the best guide when selecting codes for use on low $E_b/N_0$ channels.

I. INTRODUCTION
Conventionally, selection of codes have been based on optimal (i.e. minimal BER) asymptotic performance. However, as mentioned by Lee [1] the optimal asymptotic performance may not be the goal for practical purposes. Among applications concatenated coding systems with a convolutional code of rate $R=1/N$ and a Reed-Solomon (RS) outer code carries great weight. For such systems the best inner code with a given rate and constraint length becomes the code which minimizes the required signal-to-noise-ratio $E_b/N_0$ for a given symbol-error-rate (SER) in the interval $10^{-4} < \text{SER} < 10^{-5}$. In most systems the symbol size $J$ of the RS code is selected such that $8 < J < 12$.

II. CODE SEARCH TECHNIQUE
There are two ways to follow in a search for codes minimizing the event-error-rate (EER), which can be shown to be closely related to the SER. Another way is to use parameters aimed at minimizing the bit-error-rate (BER) in the interval $5 \cdot 10^{-4} < \text{BER} < 10^{-5}$, since numerous simulations indicate that codes that are good according to the BER criterion are also good according to a SER criterion. To establish efficient search methods we have tested the following parameters:
1. optimum $d_i$, with a minimum number $s(d_i)$ of adversary paths of weight $d_i$.
2. optimum $d_i$, with a minimum sum $b(d_i)$ of weights of input sequences generating the adversary paths of weight $d_i$.
3. optimum $d_i$ and an optimum distance profile.
4. optimum increase in extended-row-distance function.
5. minimum upper bound on BER for a specific $E_b/N_0$.
6. minimum upper bound on EER for a specific $E_b/N_0$.

Many researchers have reported on codes which are optimum according to one of the criteria 1-6 above, but to our knowledge no one has investigated the feasibility of these criteria for selection of codes according to our criterion. We therefore performed a test to judge the different criteria and find the one best suited for a search. For $N=4$ and $M=5$ we selected at random 200 codes, calculated the parameters for each criterion and sorted the codes. Each criterion was then judged by comparing the 10 best codes according to that criterion, with the 10 best codes according to computer simulations of SER. If for a certain criterion, a large intersection of "best" codes is found, then that criterion is possibly a good choice when selecting codes which are optimum according to our chosen criterion.

The interesting result is that although Viterbi's union bounds on BER and EER are completely unfit for prediction of the actual error rates in the intervals we selected, then the bounds used for a somewhat higher $E_b/N_0$ ($3.0 \text{ dB}$ for $N=4$, $M=5$) are by far the best guide among the criteria to select codes according to our chosen criterion. The $E_b/N_0$ value should in principle be selected as the smallest possible value before the bounds diverge to infinity, and tests with higher $E_b/N_0$ values (e.g. $E_b/N_0=5 \text{ dB}$) result in a smaller intersection of "best" codes. Thus the $E_b/N_0$ value used to evaluate the bounds should be decreased when we search for longer constraint length codes, but further tests with such codes indicate that it becomes less critical to use the smallest possible $E_b/N_0$ when the constraint length is increased.

Tests for different values of $M$ indicate that the BER and EER criteria are equally well suited for selecting codes according to our minimum SER criteria (i.e. 8), but since calculating the bound on BER requires less computations than calculating the bound on EER it is our suggestion to use BER as selection criterion.

The straightforward method of calculating Viterbi's union bounds involves the timeconsuming step of finding a large (since we want significant results) number of components of input-weight- or weight spectrum (depending on whether to calculate BER or EER) by an algorithm described by Cedervall and Johannesson [2]. Lee has in [3] given a more direct method of calculating BER (this method can also easily be adapted to calculate EER). Both methods were tested and Lee's method has by far the best performance of the two.

III. CONCLUSION
Our conclusion is that the best criterion for selecting convolutional codes for use in concatenated systems, is BER evaluated by Lee's method at minimum possible $E_b/N_0$ for the specific $N$ and $M$.

Since the ensemble of codes is of size $2^{N(M+1)}$ it is impossible to perform exhaustive searches, even for moderate $N$ and $M$, and we therefore have to reduce the subsets, but in such a way that the probability of missing (near) optimal codes according to our criterion is very small. Furthermore the subsets must be found in advance such that only codes in the subsets need to be generated. To do this we used methods similar to what have been done in earlier code searches. Exhaustive computer searches were then carried out in the subsets, and we present a number of new codes, although most of them only marginally better than codes previously known.

REFERENCES.