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On Lagrange Multipliers in Work with Quality and Reliability Assurance

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Abstract—**In optimizing some property of a system, reliability say, a designer usually has to accept certain constraints regarding cost, completion time, volume, weight, etc. The solution of optimization problems with boundary constraints can be helped substantially by the use of Lagrange multipliers techniques (LMT). With representative examples of increasing complexity, the wide applicability of LMT is illustrated. Two particular features are put in focus. First, an easy to follow yet powerful new graphical approach is presented. Second, the concept of Fuller–Polya maps is shown to be helpful in the areas of sales promotion and teaching. These maps illuminate the logic structure of solution sequences. One such map is shown, illustrating the application of LMT in one of the examples.**

I. Introduction

LAGRANGE multipliers techniques, LMT, have been used successfully to solve constrained optimization problems in both engineering and economics. In the field of reliability engineering, use of LMT is well documented by Tillman, Hwang, and Kou [1]. LMT's are fundamental to the applied mathematics field best known as optimization theory or mathematical programming (see Vidal [2]). However, few engineers appear to be familiar with the wide applicability of LMT; hence this paper.

During the past twenty years some effort has been made to obtain a better interaction between the theory and practice of optimization. This paper is offered as a modest contribution in this direction.

We present the LMT by introducing some fundamental concepts of optimization theory. Some elementary problems are solved emphasizing the geometrical understanding of the solution procedure. The limitations of the classical LMT are discussed.

This paper presents a new approach, a graphical (nonalgorithmic) method, to solve a family of simple resource-allocation problems. The method is illustrated within the context of a simple reliability problem.

The LMT can be displayed in the method problem structure mapping, developed by Fuller from a suggestion by Polyá [3]. The Fuller–Polya map is a mechanism for systemizing and organizing the problem solving procedure. It shows the structure of the solution procedure independent of the mathematical manipulations. It focuses the attention on the solution method rather than on the specific answer. Once constructed the Fuller–Polya map also shows the structure of the solution of all similar problems. The last section of this paper shows a Fuller–Polya diagram for an optimization problem solved by LMT.

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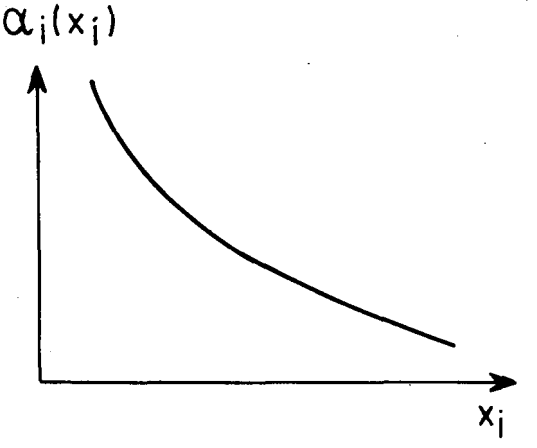

Fig. 1. Failure rate, $\alpha_i(x_i)$, versus coolant flow, x_i, for the ith subsystem.

Some important aspects of LMT are not discussed in this paper. One of these is the case of the geometrical interpretation of the optimal conditions of a constrained optimization problem resulting in the well-known Kuhn–Tucker conditions (see Vidal [2]).

II. Optimal Distribution of a Coolant to a Cooling System

The LMT will be applied in the analysis of a cooling system to determine the optimum distribution of coolant to the subsystems for maximum system reliability [4]. The subsystems are assumed to be cooled in series. Only the cooling of electronic equipment will be considered. Fig. 1 shows a typical curve of failure rate, $\alpha_i(x_i)$, of the ith subsystem, $i = 1, \cdots, n$, as a function of the coolant flow rate, x_i.

The reliability of the ith subsystem is assumed to decrease exponentially with time: $R_i(t) = \exp[-\alpha_i(x_i)t]$. Assuming that all subsystems are statistically independent and connected in series (no redundancy is allowed) the system reliability is

$$R(t) = \prod_{i=1}^{n} R_i(t) = \exp\left[-t\left(\sum_{i=1}^{n} \alpha_i(x_i)\right)\right].$$

Maximizing $R(t)$ is equivalent to maximizing $\ln[R(t)] = F(W)$:

$$F(W) = \max \left(- \sum_{i=1}^{n} \alpha_i(x_i) \right) \tag{1}$$

subject to

$$\sum_{i=1}^{n} x_i = W; \tag{2}$$

n is the number of subsystems and W is the total flow rate allowed.

This constrained optimization problem can be transformed into an unconstrained one by introducing a *Lagrange multiplier*, λ; that is we introduce a penalty for not satisfying (2).

The unconstrained optimization problem now takes the form:

$$\max_{x_i} L(x, \lambda) = - \sum_{i=1}^{n} \alpha_i(x_i) + \lambda \left(W - \sum_{i=1}^{n} x_i \right).$$

The problem can be decomposed into n similar problems each in one variable, x_i,

$$\max L_i(x_i) = [-\alpha_i(x_i) - \lambda x_i].$$

If $\alpha_i(x_i)$ is a *differentiable convex function*, as for example, k_i/x_i, then a necessary and sufficient condition for optimality is

$$-\frac{d\alpha_i(x_i)}{dx_i} = \lambda. \tag{3}$$

Equation (3) illustrates a well-known principle in microeconomics, which states that the marginal utility of each subsystem is equal to a particular constant, the Lagrange multiplier. *For the case* $\alpha_i(x_i) = k_i/x_i$, (3) specializes to (4),

$$x_i = \sqrt{\frac{k_i}{\lambda}} = f_i(\lambda). \tag{4}$$

The n values can be substituted into (2) which is satisfied for $\lambda = \lambda^*$.

$$\lambda^* = \frac{\left(\sum_{i=1}^{n} \sqrt{k_i} \right)^2}{W^2}. \tag{5}$$

By specializing (4) for $\lambda = \lambda^*$, we obtain the optimal solution

$$x_i^* = \frac{\sqrt{k_i}}{\sum_{i=1}^{n} \sqrt{k_i}} W, \quad \text{i.e.,} \quad \alpha_i(x_i^*) = \frac{\sqrt{k_i}}{W} \cdot \sum_{i=1}^{n} \sqrt{k_i}$$

The *perturbation function*, (1), becomes

$$F(W) = - \frac{\left(\sum_{i=1}^{n} \sqrt{k_i} \right)^2}{W} \tag{6}$$

This function expresses the negative of the optimal total failure rates as a function of the total flow rate, W; $F(W)$ is a concave differentiable function. Recalling (5), it is verified that

$$\frac{dF(W)}{dW} = \lambda. \tag{7}$$

A popular interpretation of λ is that of a *shadow price*; a shadow price is defined as the marginal increase (or decrease) in utility due to a marginal increase (or decrease) in resource.

In this example, the optimal solution could be found analytically using the LMT. In [5] a similar approach is utilized to optimize the reliability of some simple circuit problems.

The n constraints $x_i \geq 0$ are not explicitly included in (1) and (2), but due to the structure of the problem they will automatically be satisfied.

III. The Geometric Argument

Let our starting point be the n functions $\alpha_i' = -\alpha_i(x_i)$ all of which are differentiable, strictly concave and increasing functions; see fourth quadrant of Fig. 2. Minus one times the slope of the failure rate function is called $s_i = -d\alpha_i(x_i)/dx_i$; see first quadrant of Fig. 2. Next, we draw the straight line $y_i = \lambda x_i$, $\lambda > 0$; see first quadrant of Fig. 2.

The (negative) distance, $L_i(x_i)$, from α_i' to λ_i is

$$L_i(x_i) = -\alpha_i(x_i) - \lambda x_i$$

For a given value of λ, $L(x_i)$ has its maximum (i.e., numerically speaking the distance is as short as possible) at the point $x_i^*(\lambda)$ as shown in Fig. 2, at this point the slope of α_i', s_i, happens to be λ. For the $(n-1)$ other α_i' functions we may likewise find a $x_i^*(\lambda)$-value. Obviously, these $x_i^*(\lambda)$-values yield the solution to our problem, (1) and (2), for $W = W^*(\lambda)$, where

$$W^*(\lambda) = \sum_{i=1}^{n} x_i^*(\lambda) \tag{8}$$

and the perturbation function becomes

$$F(W^*(\lambda)) = \sum_{i=1}^{n} [-\alpha_i(x_i^*(\lambda))]. \tag{9}$$

That is, *the arbitrarily chosen λ-value thus turns out to be the Lagrange multiplier value which maximizes the sum of α_i-values for $W = W^*(\lambda)$*. By choosing smaller and smaller λ-values we can consequently find larger and larger maximas, the $F(W^*(\lambda))$'s, for larger and larger $W^*(\lambda)$-values.

As illustrated in Fig. 2, first quadrant, the $x_i^*(\lambda)$-value may also be found as the intersection between the horizontal line $s_i = \lambda$ and the curve $s_i = -d\alpha_i(x_i)/dx_i$, this is the condition stipulated in (3). If we add the ns_i-curves horizontally, the sumcurve, S, will also be intersected by the *horizontal* line, this time at $W^*(\lambda)$. The sumcurve, S,

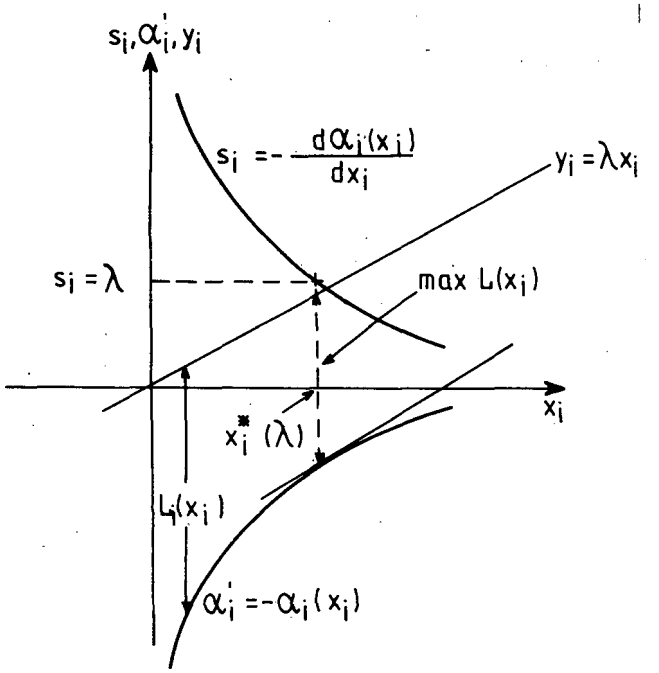

Fig. 2. The graph of s_i, α_i', and y_i.

TABLE I
SEVEN $\alpha_i(x_i)$-FUNCTIONS, AND THEIR TRANSFORM:

$$x_i = A_i + B_i \cdot \theta(\lambda)$$

A_i AND B_i ARE FUNCTIONS OF THE $\alpha_i(x_i)$-PARAMETER VALUES, AND INDEPENDENT OF THE λ-VALUE; $\theta(\lambda)$ IS A FUNCTION OF THE ACTUAL λ-VALUE, AND INDEPENDENT OF THE VALUE OF i.

$\alpha_i(x_i)$	$\theta(\lambda)$	A_i	B_i
$p_i(1-k_i/x_i)$ $p_i, k_i > 0$	$\frac{1}{\sqrt{\lambda}}$	0	$\sqrt{p_i k_i}$
$-p_i x_i^k$ $p_i > 0, k > 2$	$\lambda^{1/(k-1)}$	0	$\frac{1}{(-p_i k)^{1/(k-1)}}$
$p_i/(a_i - x_i)$ $p_i, a_i > 0$	$\frac{1}{\sqrt{\lambda}}$	a_i	$-\sqrt{p_i}$
$p_i \log_e(1+m_i x_i)$ $p_i, m_i > 0$	$\frac{1}{\lambda}$	$-\frac{1}{m_i}$	p_i
$p_i x_i/(a_i + x_i)$ $p_i, a_i > 0$	$\frac{1}{\sqrt{\lambda}}$	$-p_i$	$\sqrt{p_i a_i}$
$2p_i\sqrt{x_i}$ $p_i > 0$	$\frac{1}{\lambda^2}$	0	p_i^2
$p_i(1-e^{-k_i x_i})$ $p_i, k_i > 0$	$\ln(\lambda)$	$\frac{\ln(p_i k_i)}{k_i}$	$-\frac{1}{k_i}$

once it has been generated, can be used with any number of λ-values.

Generating a sumcurve, clearly, is particularly simple *if all the added curves are straight lines* or piecewise linear functions. As pointed out by Vidal [6] such linear relationships may be achieved by a suitable transformation whenever (3) may be solved for x_i and given the form: $x_i = A_i + B_i \cdot \theta(\lambda)$.

Seven cases are listed in Table I [6]. More often than not λ^* can only be determined by numerical methods, and not analytically; e.g., with inventory problems [7] and allocation of cooling resources [8].

IV. Economic Fuel Dispatch

In this section, we examine a more difficult problem. It is desired to minimize the total fuel consumption for a given configuration of a power generation system composed of n units. The total power production of the system should be equal to a load L. Moreover, the ith unit's output x_i (MW) is subjected to a set of special constraints that impose an upper bound and a lower bound on each x_i. Assuming that the fuel consumption at each unit is a quadratic function of x_i, the optimization model is

$$F(L) = \max_{x_i} \left[-\sum_{i=1}^{n} (a_i + b_i x_i + c_i x_i^2) \right] \tag{10}$$

subject to:

$$\sum_{i=1}^{n} x_i = L \tag{11}$$

$$LB_i \le x_i \le UB_i, \quad i = 1, 2, \cdots, n. \tag{12}$$

This problem is also decomposable, and for each i we have to

$$\max_{x_i} [-(a_i + b_i x_i + c_i x_i^2) - \lambda x_i]$$

subject to:

$$LB_i \le x_i \le UB_i.$$

If the optimal x_i, x_i^*, is an interior point then

$$d\left\{\max_{x_i}\right\}/dx_i = 0$$

for

$$x_i^* = -\frac{b_i + \lambda}{2c_i}; \tag{13}$$

if $x_i^* < LB_i$, reset $x_i^* = LB_i$, and if $x_i^* > UB_i$ reset $x_i^* = UB_i$.

Now, an iterative procedure can be developed. For a given λ, (13) uniquely determines a set of x_i^*'s, if (11) is satisfied, then we have an optimal solution. Otherwise, λ should be suitably modified. This is actually the essence of the LMT, which also can be applied to more complex problems; e.g., in [9] a fuel dispatch problem is handled by this approach. Note that the approach suggested in Section III can be modified to solve the problem in this section, since (13) already defines a linear relationship like those from Table I.

Does the LMT always work? Let us look at a third example.

V. Least Cost Allocations of Reliability Investments

The problem is one of optimizing the redundancy of an n-stage system; each stage consists of a number x_i of par-

allel (redundant) components of cost c_i and reliability a_i. The separate stages are taken to be in series, so that the system is operable if, and only if, every stage contains at least one operable component. The allocation problem is then to choose the stage redundancies (x_i's) in such a manner as to maximize the system reliability subject to constrained total cost. The states of all components are assumed to be statistically independent. The system's reliability is given by

$$\prod_{i=1}^{n} [1 - (1 - a_i)^{x_i}]$$

Since maximizing the logarithm of a function maximizes the function, we take our payoff to be the logarithm of the reliability. Our problem is to determine

$$F(C) = \max_{x_i} \left[\sum_{i=1}^{n} \ln [1 - (1 - a_i)^{x_i}] \right] \quad (14)$$

subject to:

$$\sum_{i=1}^{n} c_i x_i = C \quad (15)$$

$$x_i \geq 1 \quad \text{(integer)}, \qquad i = 1, 2, \cdots, n. \quad (16)$$

This optimization problem differs from the two discussed earlier in that the x_i's ought to be integers. The (generalized) LMT can be applied in a similar way as shown above. The problem is decomposable, so we have to

$$\max [\ln [1 - (1 - a_i)^{x_i}] - \lambda c_i x_i]$$

subject to:

$$x_i \geq 1 \quad \text{(integer)}.$$

For given λ, this problem is readily solved, for instance by simple enumeration.

Now, the iterative procedure developed in the last section can also be applied to solve (14), (15), and (16). That is, λ is modified iteratively until the values of the x_i's satisfy (15).

Does such a λ-value that generate the x_i's always exist? The answer is *no*. The examples we have shown in the earlier sections are so-called "nice" problems because: 1) the variables to be found were *continuous*, 2) the payoff function to be maximized was *concave*, and 3) the region of feasible values was *convex*. In such situations the perturbation function, $F(\ )$, is *concave* and the existence of λ^* corresponds to the existence of support hyperplanes; this is illustrated in Fig. 3. For the example in Section V the perturbation function is illustrated in Fig. 4.

Fig. 4 illustrates that for the problem of (14) a support hyperplane cannot always be found. This corresponds to the fact that, in general, there exists no λ which can generate the right-hand side of (15). These regions of C-values are denominated *gaps*. Fortunately, for large-scale problems the gaps are small, and near-optimal solutions can be found, as shown in (10). The LMT as applied to

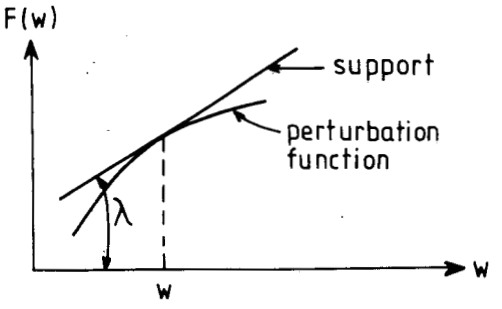

Fig. 3. Perturbation function for a "nice" problem, see (7).

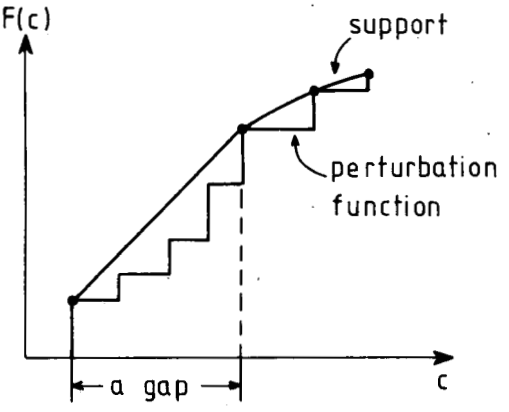

Fig. 4. Perturbation function for the problem in Section V.

the field of optimization are usually referred to as Lagrange relaxation.

VI. Nonlinear Lagrangian Functions

Many optimization problems in engineering are not "nice," and often the problem to be solved might have many gaps. The LMT can be further generalized to work with support functions instead of support hyperplanes. This is equivalent to generalizing the classical Lagrangian function (linear in λ)

$$L(x, \lambda) = f(x) - \sum_{i=1}^{m} \lambda_i g_i(x)$$

for the problem $\{\max_x f(x) | g_i(x) \leq 0, i = 1, 2, \cdots, m\}$, to a generalized Lagrangian function

$$L(x, \gamma) = f(x) - \sum_{i=1}^{m} \gamma_i(g_i(x))$$

to be maximized. The idea is still the same to transform a constrained problem into a sequence of unconstrained problems. Many forms of the functions $\gamma_i(\ )$ have been suggested, see for instance [2]. The authors have good experiences with a quadratic function

$$\gamma_i(g_i(x)) = a_i(g_i(x))^2 + \lambda_i g_i(x)$$

to solve engineering problems.

The iterative procedure will be similar to the LMT, but we have now two multipliers (a_i, λ_i) for each constraint. The main disadvantage resides in the fact that the generalized Lagrangian function is not decomposable.

VII. Optimal Control Problems

LMT can also be used to solve dynamic optimization problems [14]. Let us see a simple example, an optimal control problem. A student beginning a course which lasts n weeks wants to plan his study time to pass the course without working more than necessary. His knowledge level k increases as the work invested, x, increases; it is assumed that $\Delta k = ax^{\beta}$, $a > 0$, $0 < \beta < 1$. However, his memory is imperfect; from one week to the next he retains only a fraction b $(0 < b < 1)$ of his knowledge. If k_i is his knowledge level at the end of the ith week and x_i is the work done during the ith week, then

$$k_i = a \cdot x_i^{\beta} + b \cdot k_{i-1}, \qquad i = 1, 2, \cdots, n$$

The student wishes to minimize the total work invested

$$F(K) = \max_{x_i} \left[-\sum_{i=1}^{n} x_i \right]$$

subject to:

$$k_i - ax_i^{\beta} - bk_{i-1} = 0, \qquad i = 1, 2, \cdots, n$$

and

$$k_o = 0, \qquad k_n = K;$$

k_o is the initial knowledge level and K is the level requirement to pass the examination. This problem is a generalization of the one discussed in [11] for $\beta = \frac{1}{2}$.

We use n multipliers, λ_i, one per constraint. The (linear) Lagrangian function L is

$$L(x, k, \lambda) = \sum_{i=1}^{n} [-x_i - \lambda_i(k_i - ax_i^{\beta} - bk_{i-1})];$$

Note that the problem is decomposable. L has to be maximized with respect to x_i and to k_i.

$$\partial L/\partial x_i = 0 \quad \text{for:} \quad x_i = (a\beta\lambda_i)^{1/(1-\beta)}, \qquad i = 1, 2, \cdots, n.$$

$$\partial L/\partial k_i = 0 \quad \text{for:} \quad \lambda_i = b\lambda_{i+1} = 0, \qquad i = 1, 2, \cdots, n.$$

Consequently,

$$x_{i+1}^{*} = x_i^{*} \cdot b^{1/(\beta-1)}, \qquad i = 1, 2, \cdots, n-1.$$

That is, the student's weekly work should be increased by the constant factor $Q = b^{1/(\beta-1)}$ each week. The optimal work schedule is completely determined from the boundary conditions $k_o = 0$ and $k_n = K$.

$$F(K) = -x_1 - Q \cdot x_1 - \cdots - Q^{n-1} \cdot x_1$$

$$= -x_1 \cdot [Q^n - 1]/[Q - 1].$$

A formal application of the LMT to optimal control problems will result in the well-known (linear) maximum principle as shown in [12]. Moreover, in [13] a generalized (nonlinear) maximum principle is shortly discussed.

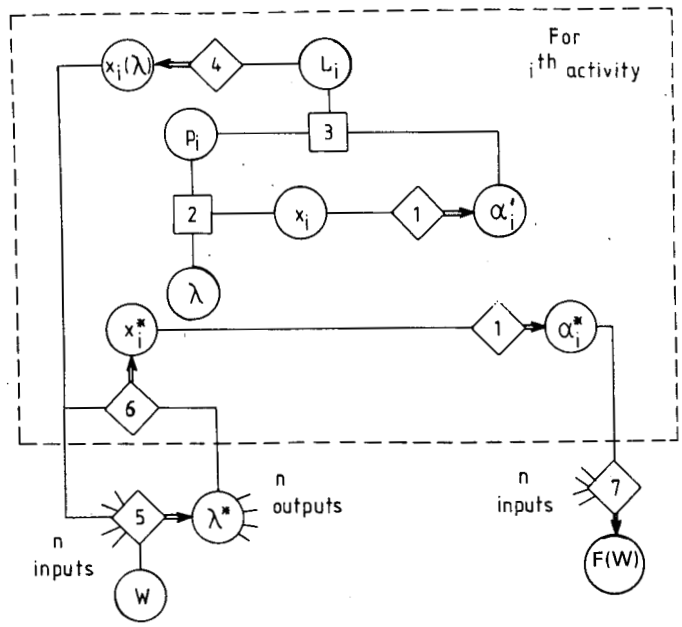

Fig. 5. Fuller–Polya map for the decomposable problem from Section II. The variables are drawn as circles Ⓧ; the algorithms are identified by number; completely reversible algorithms are shown as 3 ; irreversible (or nearly so) algorithms are shown as ◇6.

1) $\alpha_i' = \alpha_i(x_i)$

2) $p_i = \lambda x_i$

3) $L_i = \alpha_i' - p_i$

4) $\dfrac{dL_i}{dx_i} = 0 \Rightarrow x_i(\lambda)$

5) $\sum_{i=1}^{n} x_i(\lambda) = W \Rightarrow \lambda^*$

6) $x_i^* = x_i(\lambda^*)$

7) $F(W) = \sum_{i=1}^{n} -\alpha_i'^*$

These new results may be applied to a number of practical engineering problems.

VIII. Constructing a Fuller–Polya Map

Following Kardos [3], the preliminaries to construction of a Fuller–Polya map are the same as for any problem. The problem must first be understood. The data must be identified, the unknown expressed explicitly; and the physical and economical principles, with their mathematical algorithms must be identified. This is done to the best of one's abilities; but it does not need to be completed right away. The construction of the Fuller–Polya map itself will demonstrate if there are sufficient connections among the known data and the unknown quantities, and also indicate what may be missing.

Let us assume that all closed form problem solution structures have two sets of components: 1) *variables* (data, intermediate, solution); 2) *algorithms*, which related the variables to one another (they may be reversible) and/or table look-ups (which usually are irreversible).

The Fuller–Polya map for the complete problem is con-

structed by joining the algorithms together by the way of their variables, *each variable appearing only once*.

Fig. 5 shows the Fuller–Polya map for the decomposable problem solved in Section II by the LMT. The elaboration of these maps within the context of optimization can be quite powerful tools in teaching and sales-promoting activities.

Acknowledgment

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