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**TEMPORAL PHOTON DIFFERENTIALS**

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Abstract: The finite frame rate also used in computer animated films is cause of adverse temporal aliasing effects. Most noticeable of these is a stroboscopic effect that is seen as intermittent movement of fast moving illumination. This effect can be mitigated using non-zero shutter times, effectively, constituting a temporal smoothing of rapidly changing illumination. In global illumination temporal smoothing can be achieved with distribution ray tracing (Cook et al., 1984). Unfortunately, this, and resembling methods, requires a high temporal resolution as samples has to be drawn from in-between frames. We present a novel method which is able to produce high quality temporal smoothing for indirect illumination without using in-between frames. Our method is based on ray differentials (Igehy, 1999) as it has been extended in (Sporring et al., 2009). Light rays are traced as bundles creating footprints, which are used to reconstruct indirect illumination. These footprints expand into the temporal domain such that light rays interacting with non-static scene elements draw a path reacting to the elements movement.

1 INTRODUCTION

Rendering animated scenes with global illumination methods produces some interesting problems, where the perhaps most prominent problem is aliasing caused by the fact that the temporal domain is discretized at a often very low resolution.

Feature films usually shot a frame rate around 25 frames per second (fps). Despite the fact that the human eye is much more perceptible than 25 fps, this frame rate still gives a seemingly fluid motion when a film is produced with a traditional movie camera. However, were we to produce a feature film of an animated virtual scene using an unmodified global illumination method at a frame rate of 25 fps, fluid motion is not guaranteed. A typical unmodified global illumination method produces images at instant time in the temporal domain. This procedure can induce temporal aliasing, which is seen as an adverse stroboscopic effect, where the illumination changes rapidly over time. A traditional-camera produced feature film will avoid this problem because the camera has non-zero exposure time. This means that camera-perceived illumination is averaged over the exposure time. In effect, high frequency motion is blurred and therefore seems fluid. This effect is often called motion blur. A temporal aliasing defect not solved by this blurring is the wagon-wheel effect, which is seen as a backwards turning of the spokes of a wheel due to temporal under sampling. However, as our method does not address this particular problem, it will not be discussed further here.

A survey paper for global illumination and rendering solutions exploiting temporal coherence is given in (Tawara et al., 2004).

Brute force methods, such as the accumulation buffer (Haaberli and Akeley, 1990), average together in-between frames in order to achieve motion blur. These methods can achieve arbitrary high accuracy but are often prohibitively expensive as full renderings typically have to be made of a large number of in-between frames.

Different global illumination methods also address temporal aliasing by simulating motion blur. Distribution ray tracing (Cook et al., 1984) achieves motion blur by stochastically sampling the temporal
domain as well as the spatial. In (Myszkowski et al., 2001) the authors adaptively controlled the temporal and spatial sampling resolution by examining local variations of indirect illumination over time and space in a pilot estimate. In (Egan et al., 2009) motion blur is modeled as a wedge in the frequency domain. Ray tracing methods using ray differentials have been studied in (Igehy, 1999; Christensen et al., 2003; Durand et al., 2005; Gjøl et al., 2008).

With time dependent photon mapping (Cammarano and Jensen, 2002), photon mapping was extended such that indirect illumination was estimated using a four dimensional photon map that expanded into the temporal domain. In this manner photons were filtered not only based on their spatial position but also their temporal.

A problem common to the discussed methods is that they all rely on information from in-between frames in order to achieve motion blur. Having this information available places certain restrictions on the animated scene; because a scene description is needed at arbitrary time steps, movement of scene elements either needs to be described as a analytic function, or movement has to be interpolated between frames. The analytical approach is by far the best but demands a certain complexity of the animation application, as well as a tight link to the rendering software. The interpolative approach is error prone, since the animation curve might not be linear. Furthermore, some accelerator for ray-object intersection (such as a bsp-tree) typically needs to be either rebuilt, or at least updated an extra number of times equal to the number in-between frames needed.

In this paper, we propose a method that produces motion blur, and which neither needs in-between frames, nor to over-smooth indirect illumination with high temporal frequency. Our proposed method is an extension of photon differentials (Schjøth et al., 2007). It takes advantage of ray differentials (Igehy, 1999) and their extension into the temporal domain (Sporring et al., 2009), and since velocity is a relative concept, our methods handles camera as well as object motion. We call this method temporal photon differentials.

2 TEMPORAL PHOTON DIFFERENTIALS

In our method each photon represents a beam of light that expands, contracts and reshapes in space and time as it propagates through the scene. We keep track of a photon’s coherence by deriving the first order structure of its direction and position with respect to both time and space as it traverse the scene.

Representing a photon as a parameterized ray with origin \( \mathbf{x} \) and the direction \( \mathbf{\omega} \), we describe the derivatives of a photon with two Jacobian matrices; one for the positional derivatives and one for the directional derivatives. The positional derivatives are then given by

\[
D\mathbf{x} = \begin{bmatrix}
\frac{\partial \mathbf{x}}{\partial u}
\frac{\partial \mathbf{x}}{\partial v}
\frac{\partial \mathbf{x}}{\partial t}
\end{bmatrix} = \begin{bmatrix}
D_u \mathbf{x}
D_v \mathbf{x}
D_t \mathbf{x}
\end{bmatrix},
\]


where \( D\mathbf{x} \) is the Jacobian of the positional derivatives, and \( D_u \mathbf{x}, D_v \mathbf{x} \) and \( D_t \mathbf{x} \) are column vectors that describe the positional derivatives with respect to the scalar variables \( u, v \), and \( t \). The spatial variables \( u \) and \( v \) relate to light sources, from which the photon differential was emitted.

We write the directional derivatives of the photon as

\[
D\mathbf{\omega} = \begin{bmatrix}
D_u \mathbf{\omega}
D_v \mathbf{\omega}
D_t \mathbf{\omega}
\end{bmatrix}.
\]

Exactly, as with photon differentials, \( D_u \mathbf{\omega}, D_v \mathbf{\omega}, D_t \mathbf{\omega} \) are column vectors with respect to time are new to photon differentials; they are noted as \( D_t \mathbf{\omega} \) and \( D_t \mathbf{x} \) in the above equations. For a scene, in which the light sources are static, the time dependent directional vectors will, initially, be zero. If the scene, additionally, is completely static, they will remain zero-vectors throughout the photon’s traversal of the scene. In this specific case, our method will behave exactly as ordinary photon differentials: the photons will expand and contract depending on the reflections and refraction encountered during tracing, and their spatial dependent positional vectors will form a footprint, which is used in the reconstruction of the indirect illumination.

On the other hand, if we have a dynamic scene, then photon differentials interacting with a non-static scene element will attain non-zero time dependent differential vectors. In this case, the derivatives of a dynamic scene-element’s surface positions or normals with respect to time will be non-zero:

\[
D_t \mathbf{n} \neq \mathbf{0},
\]

or

\[
D_t \mathbf{q} \neq \mathbf{0},
\]

where \( \mathbf{n} \) is a surface normal to the element and \( \mathbf{q} \) is a position on the element’s surface. This again will affect the time dependent derivatives of a photon interacting with the scene element.
Figure 1: Spatial filter kernel shaped by the positional differential vectors, $D_u x$ and $D_v x$.

Sporring et al. evaluates the full differentials for a parameterized ray. This allows for an extension of parameters such that the derivatives of a ray can be considered with respect to time. From Sporring et al.’s equations for transfer, reflection and refraction, we observe that non-zero time-dependent element differentials (e.g. $D_t q$) propagate through these interactions to the differentials of the interacting photon. We exploit this behavior such that a footprint from a photon differential traveling in a dynamic scene not only describes the spatial coherence of the ray but also the temporal coherence of the ray.

When a photon differential hits a surface, its positional differential vectors are projected onto the surface’s tangent plane at the intersection point. The spatial footprint of the photon differential is the area on the tangent plane of a parallelogram spanned by the positional differential vectors. The spatial footprint can be used to shape an anisotropic filter kernel as illustrated in Figure 1.

The time dependent positional differential vector, $D_t x$, tells us either how the photon’s footprint is going to behave over consecutive frames, or how the footprint has behaved in former frames. In the former case, the direction of $D_t x$ predicts the direction on the surface that the footprint will move, and the magnitude of the vector predicts how far the footprint is likely to move. Basically, the magnitude and the direction of $D_t x$ depends on the estimation method used to calculate the time derivatives of an element, which again depends on the geometry representation. In the present method, we simply use finite differences and triangle meshes. Except for the last frame, in which we use backward differences, we estimate the time dependent differentials using forward differences. When we want to predict how a footprint is going to behave, having intersected a moving element, we estimate the element’s positional time derivatives by

$$D_t q_f = e(q_{f+1} - q_f),$$

where $D_t q_f$ is the derivative of the vertex $q_f$ with respect to time at frame step $f$, and $e$ is the exposure. The exposure is a parameter for how much we trust our prediction. Generally, it works as a smoothing parameter for the time dependent footprint that decides how much motion blur we induce. Its unit is given in frames as it depends on the movement of the scene elements between frames. The exposure is related to the exposure time by the frame rate such that the exposure time is equal to the exposure divided by the frame rate.

The time dependent footprint constitutes an integration of the spatial footprint over the time dependent differential vector such that the spatial footprint is elongated along the vector. We achieve this by translating the spatial footprint along the time dependent differential vector. As in the spatial case, the time dependent footprint describes a filter kernel. In Figure 2(a), $D_t x_{pd}$ is the time dependent differential vector, $x_{pd}$ is the center of the spatial kernel, and $x$ is the estimation point, for which the kernel weight is estimated.

The kernel is translated along $D_t x_{pd}$ to the point, $x'_{pd}$, on the line segment, $(x_{pd} \rightarrow x_{pd} + D_t x_{pd})$, where $x'_{pd}$ is the point on the segment having the shortest distance to the estimation point, $x$. Using $x'_{pd}$ as center for the spatial kernel, the resulting time dependent kernel will achieve an elongated shape as illustrated in Figure 2(b).

The irradiance of the time dependent photon differential is estimated as

$$E_{pd} = \Phi_{pd}/A_{pd},$$

where $\Phi_{pd}$ is the radiant flux carried by the photon, and $A_{pd}$ is the surface area, to which the radiant flux
is incident. For the time dependent photon differential, this area is the area of the time dependent kernel. Referring to Figure 3 this area is calculated as

$$A_{pd} = \frac{1}{4}\pi|D_u x \times D_v x| + l|D_v x|, \quad (7)$$

where the first term is the area of the spatial kernel and the second term is the area of a rectangle. One side of the rectangle is the length of the time dependent differential vector and the other is the length of the spatial kernel in a direction perpendicular to the time dependent differential vector.

Having defined the time dependent kernel as well as the irradiance of the photon differential, we can now formulate a radiance estimate for temporal photon differentials.

2.1 THE TEMPORAL RADIANCE ESTIMATE

Reflected radiance from temporal photon differentials can be estimated by

$$\hat{L}_r(x, \omega) = \sum_{pd=1}^{n} f_r(x, \omega_{pd}, \omega)E_{pd}(x, \omega_{pd})$$

$$K_s((x - x'_{pd})^T M_{pd}^T M_{pd}(x - x'_{pd})),$$  \quad (8)

where $x$ is a position on an illuminated surface, $\omega$ is reflection direction considered, $f_r$ is the bi-directional reflectance function, $\omega_{pd}$ is the incident ray direction, $x'_{pd}$ is the translated center of spatial kernel, $E_{pd}$ is the irradiance of the temporal photon differential, $K_s$ is a bivariate kernel function from Table 1, and $M_{pd}$ is a matrix that transforms from world coordinates to

<table>
<thead>
<tr>
<th>Kernel</th>
<th>$K_s(y)$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1 if $y &lt; 1,$ 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>Epanechnikov</td>
<td>$2(1 - y)$ if $y &lt; 1,$ 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>Biweight</td>
<td>$3(1 - y)^2$ if $y &lt; 1,$ 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>Gaussian</td>
<td>$\frac{1}{2} \exp\left(-\frac{y}{2}\right)$</td>
<td></td>
</tr>
</tbody>
</table>

The temporal radiance estimate can be extended as to include filtering in time. One intuitive approach is to weight the part of the differential, which is closest in time the highest, where the time is estimated form the photon hit point. This can be achieved using a simple univariate kernel as those presented in Table 2. To the kernel, we input a distance along the time dependent differential vector, $D_t x_{pd}$, relative to furthest point of the kernel along negative $D_v x_{pd}$. This is illustrated in Figure 3.

With time filtering the temporal radiance estimate is formulated as

$$\hat{L}_r(x, \omega) = \sum_{pd=1}^{n} f_r(x, \omega_{pd}, \omega)E_{pd}(x, \omega_{pd})$$

$$K_t((x - x'_{pd})^T M_{pd}^T M_{pd}(x - x'_{pd}))$$

$$K_t((x_t - x'_{pd})^T (x_t - x'_{pd})), \quad (9)$$

where $K_t$ is a bivariate kernel function (See Table 1), $K_t$ is a univariate kernel function, $h_t$ is the length of the temporal kernel along $D_t x_{pd}$, and $x_t$ is the furthest point of the kernel in the direction $-D_t x_{pd}$. With the formulation of the temporal radiance estimate, we now have a method, which reconstructs indirect illumination based on a virtual scenes dynamics. This allows for motion blur. In the following we will make a simple analysis of the method.
3 RESULTS

We first test our proposed method using a case study. The case study is a simple animated scene, in which a sinusoidal wave moves horizontally in a direction perpendicular to the wave crests. The wave is illuminated from above by collimated light, which it refracts such that the light forms caustics on a plane beneath the wave. A virtual camera is placed such that the caustics are clearly visible.

We have rendered the scene using temporal photon differentials, and Cammarano and Jensen’s time dependent photon mapping. The images in Figure 5 are renderings of the same frame but at different exposures, \( \epsilon \). They were rendered using temporal photon differentials and a photon map containing only 1000 photons. From the images we notice that the temporal photon differentials assume the expected behavior. As the exposure increases the caustics are blurred acquiring a comets tail away from the direction of movement. This is the behavior chosen at implementation time. We could just as well have placed the time dependent kernel centered over the photon intersection point and likewise have centered the time filtering or we could just have centered the filtering. As it is, the time differential is ‘trailing’ after the photon both in respect to placement and filtering. As we shall see, though it is hardly visible, the same strategy has been implemented for time dependent photon mapping.

The renderings in Figure 6 have all except (a) been created with time dependent photon mapping. Additionally, all images were rendered using the same exposure, \( \epsilon = 1.0 \) frame. Figure 6(a) has been included for comparison, it was rendered using temporal photon differentials and is a copy of the image in Figure 5 with an exposure 1.0 frame. First of all, what we see from image 6(b) is that the bias versus variance tradeoff provided by time dependent photon mapping is too poor to produce palpable caustics. For this reason a much higher number photons have been used to render the images in the three rightmost columns. Of these, the top row is based on a photon map containing as much as 480 000 photons while the bottom row is based on a photon map contain 40 000 photons. From left to right the temporal resolution increases from 0 to 2 to 10 in-between frames. The spatial bandwidth for the renderings was chosen as to decrease noise to an acceptable level. This leads to the perhaps most important observation, namely that a low temporal resolution produces visible bands that can only be removed by filtering beyond what removes normal noise. This complicates matter, as an increase of photons no longer is a guarantee for high quality illumination.

Photon differentials are free of this concern as the blurring is based on the first order derivatives of object

Table 2: Univariate kernel functions

<table>
<thead>
<tr>
<th>Kernel</th>
<th>( K_c(y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>( \begin{cases} \frac{1}{4} &amp; \text{if } y &lt; 1, \ 0 &amp; \text{otherwise} \end{cases} )</td>
</tr>
<tr>
<td>Epanechnikov</td>
<td>( \begin{cases} \frac{1}{4}(1-y) &amp; \text{if } y &lt; 1, \ 0 &amp; \text{otherwise} \end{cases} )</td>
</tr>
<tr>
<td>Biweight</td>
<td>( \begin{cases} \frac{1}{16}(1-y)^2 &amp; \text{if } y &lt; 1, \ 0 &amp; \text{otherwise} \end{cases} )</td>
</tr>
<tr>
<td>Gaussian</td>
<td>( \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) )</td>
</tr>
</tbody>
</table>

Figure 5: Renderings of the case study scene using temporal photon differentials. The number under the renderings indicate exposure, \( \epsilon \). All images are rendered at the same frame step using a photon map containing 1 000 photons.
resolution of photon map
temporal resolution

Figure 6: Comparing temporal photon differentials with
time dependent photon differentials. Rendering in (a)
shows temporal photon differentials using 1000 photons,
(b) shows time dependent photon differentials using 1000
photons, (c)-(e) shows time dependent photon differentials
using 480000 photons with 0, 2, and 10 in-between frames
respectively, and (f)-(g) shows time dependent photon dif-
fferentials using 40000 photons with 0, 2, and 10 in-between
frames respectively. The exposure for all images is 1.0
frame.

Figure 8: Projected time differentials from a rotating glass
cylinder.

time dependent photon mapping. All images were
rendered with the same number of photons contained
in the photon map. However, the two latter images
were rendered with different bandwidths. From these
two images we see that at this obviously low temporal
resolution an increase in bandwidth can help remove
the temporal bands that time dependent photon map-
ing is prone to. The price, however, is an unwanted
blurring of the front of the caustic.

Finally, Figure 8 solely depicts the photons’ time
differentials as they are projected down on the plane
beneath the cylinder. A high exposure time has been
used as to facilitate the illustration. The image con-
firms that the time differential vectors become longer
when refracted from the high-speed end of the cylin-
der, thus elongating the time dependent kernel used in
the temporal radiance estimate.

4 CONCLUSION

In this paper we have presented temporal photon dif-
ferentials – a global illumination method for render-
ing animated scenes. Temporal photon differentials
elegantly handles time filtering such that frames can
be rendered on a one to one basis. Since velocity
is a relative concept, our method handles camera as
well as object motion, further in contrast to simi-
lar dynamic scene renderers, temporal photon dif-
ferentials does not need in-between frames in order to
avoid temporal aliasing. Finally, temporal photon dif-
Figure 7: Rendering of a rotating cylinder. All images were rendered with an exposure of 1.0 frames, using a photon map containing 5000 photons. Figure 7(a) was rendered using temporal photon differentials. Figure 7(b) and 7(c) were rendered using time-dependent photon mapping the former using 100 photons per radiance estimate and the latter using 250 photons in the radiance estimate.
differentials includes photon differential’s ability to efficiently and sharply model caustics in still frames, and it further improves the computationally efficiency for animations by reusing photons from earlier frames through the first order spatial-temporal model of photon incidence.

REFERENCES


