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Four point bending setup for characterization of semiconductor piezoresistance

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We present a four point bending setup suitable for high precision characterization of piezoresistance in semiconductors. The compact setup has a total size of 635 cm³. Thermal stability is ensured by an aluminum housing wherein the actual four point bending fixture is located. The four point bending fixture is manufactured in polyetheretherketon and a dedicated silicon chip with embedded piezoresistors fits in the fixture. The fixture is actuated by a microstepper actuator and a high sensitivity force sensor measures the applied force on the fixture and chip. The setup includes heaters embedded in the housing and controlled by a thermocouple feedback loop to ensure characterization at different temperature settings. We present three-dimensional finite element modeling simulations of the fixture and discuss the possible contributions to the uncertainty of the piezoresistance characterization. As a proof of concept, we show measurements of the piezocoefficient $\tau_{44}$ in $p$-type silicon at three different doping concentrations in the temperature range from $T = 30 \, ^\circ\text{C}$ to $T = 80 \, ^\circ\text{C}$. The extracted piezocoefficients are determined with an uncertainty of 1.8%. © 2008 American Institute of Physics. [DOI: 10.1063/1.2908428]

I. INTRODUCTION

Since the pioneering work of Smith in 1954,¹ piezoresistivity of silicon has attracted attention from both academia²–⁴ and industry.⁵,⁶ Smith experimentally determined the three piezoresistance coefficients of lightly doped silicon. The piezoresistance coefficients for more heavily doped silicon were later experimentally determined by other research groups,²,³,¹³ and still, today, the piezoresistance coefficients of silicon and other materials are topics of interest in both academia²,⁴,⁷,⁸ and industry. The continued academic interest is partly due to the scarcity of reliable measurements and partly due to a discrepancy between theoretical models and available measurements especially for $p$-type silicon.⁹–¹²

Essentially, the piezoresistance effect is a change in the resistivity tensor (second order) caused by an applied stress.¹³ The effect is characterized by a fourth order piezoresistivity tensor, which, in the case of silicon due to symmetry, has three independent coefficients.¹⁴ The piezoresistance coefficients are dependent on sample temperature, doping level, and doping type.² In order to measure these coefficients and characterize the effect, it is necessary to apply a well controlled stress to the silicon sample with well defined resistors and measure the relative change in resistance of these.

In the original experiments by Smith,¹ silicon rods were pulled to apply a uniform uniaxial stress. Machined pull samples with through holes were used in Ref. 2 and a pull force was applied by pins inserted in the through holes.

For microfabricated thin film devices, it is more convenient to use a four point bending (4PB) fixture.⁸,¹⁵–¹⁹ In Refs. 16 and 18 an optical method is used to measure the deflection and curvature of the chip. The stress is applied to the chip using a piezoelectric actuator and a translation stage, respectively. In Ref. 17, the displacement of the chip is known at the contact points between chip and 4PB fixture and this enables a calculation of the applied stress in the chip. Reference 8 and 15 use simple loads to apply the force and have no external measurement of the applied force. This is a cumbersome and time consuming method, especially for characterization at different temperatures.

We present a four point bending method where a motorized stepper actuator is used to apply a displacement while the force on the chip sample is measured with a dedicated force sensor. With this method, the measured force can be directly applied to calculate the stress. Thus, Young’s modulus is not included in the stress calculation as is the case when a deflection is measured. The compact setup has a total volume of 635 cm³.

The four point bending setup is designed and fabricated to analyze the piezoresistance coefficients of embedded piezoresistors located on a dedicated silicon chip. The main focus is to characterize the piezoresistivity of $p$-type silicon and other related semiconductor materials, e.g., Si under tensile strain and compressively strained SiGe.⁸ In this paper, we present measurements of the piezocoefficient $\tau_{44}$ in $p$-type silicon with several different doping concentrations in the temperature range $70–80 \, ^\circ\text{C}$ as an example of use of the setup. Boron doped silicon is the preferred piezoresistive material in commercial micro electromechanical systems (MEMS) due to the large piezocoefficient $\tau_{44}$ and the very low values of the two other piezocoefficients $\pi_{11}$ and $\pi_{12}$. When the piezoresistors are directed along certain crystal directions, (110), and placed in a Wheatstone bridge configu-
area between the rails and the 4PB fixture is very small by small rails in the bottom plate of the setup. The contact form heat distribution in the aluminum casing.

The setup comprises cartridge heaters (1) that are embedded in the bottom plate of the Al housing surrounding the 4PB fixture (2) consisting of base and slider, the chip (3), and the force sensor (4). The actuator motor (5) is placed outside the Al housing to prevent heating of the motor.

We characterize the 4PB setup using analytical expressions, finite element modeling (FEM), calibration measurements, and an application specific stress sensor chip. The stress distribution in a chip in the 4PB setup is thoroughly investigated and this analysis is used to estimate the uncertainties of the measured piezoeffectives.

II. APPARATUS

The piezoresistance characterization setup consists of a 4PB fixture with integrated thermocouples and temperature control. An actuator applies a displacement to the fixture and the force is measured by a force sensor. A schematic of the setup is shown in Fig. 1.

The fixture is placed in an aluminum housing including a metal lid (not shown in the figure) to stabilize temperature and shield off light. The thickness of the aluminum bottom plate and sidewalls are 10 and 20 mm, respectively. Aluminum has a very high thermal conductivity of 239 W/m K\(^{-1}\) (Ref. 20) compared to air (\(\approx 0.02\) W/m K\(^{-1}\)), ensuring a uniform heat distribution in the aluminum casing.

The 4PB fixture is made from the thermoplastic material polyetheretherketon (PEEK) which is a semicrystalline material. Young’s modulus of PEEK is \(Y=3.5\) GPa and the mechanical properties are quite stable in the temperature range from \(-64\) to \(250\) °C.\(^8\)

The vertical alignment of the chip to the 4PB fixture is ensured by resting the chip on the two steel pins in the setup (see Fig. 2). Horizontal alignment of the chip is done by visual inspection. The stress is considered constant in the area between the two inner blades on the slider. The dimensions of the resistor are much smaller than the 12 mm distance between the two inner blades. Thus, the resistors which are located in the middle region between the two inner blades experience a uniform stress.

B. Force sensor

The force sensor is a Strain Measurement Devices s415 button cell.\(^22\) It consists of a plate with four sputter defined resistors in a Wheatstone bridge configuration. The force sensor is fastened to the setup casing. An input voltage, \(V_{in}=10\) V, is applied to the bridge. The output voltage of the Wheatstone bridge, \(V_o\), depends linearly on the force \(F\), as described by

\[
\sigma_{xx} = \frac{6Fax}{wh^3},
\]
The force sensor offset is a result of imbalance in the force sensor and the actuator. This actuator hysteresis is caused by the actuator. This actuator hysteresis linearly depends on the applied force, we conclude that the temperature dependence of the actuator motion due to a hysteresis in the actual actuator position.

The constant \( \alpha \) is measured in a calibration setup where the force sensor is horizontally placed and well known forces are applied using weights of different masses. The calibration curve at \( T=30 \, ^\circ C \) is shown in Fig. 3, where the calibration constant, \( \alpha \), is determined as the slope of the linear fit. The force sensor showed a small hysteresis in the output when increasing and decreasing the applied force. The hysteresis is described by a 0.3% change in the calibration constant and this will not significantly contribute to the uncertainty. The force sensor was calibrated at different temperatures (see Table I). The temperature dependence of \( \alpha \) is significant and the values in Table I are included in the analysis.

The force sensor offset is a result of imbalance in the thin film Wheatstone bridge and the actual force sensor temperature. This offset has no influence on the measurements since the force sensor is offset compensated before the actual measurement.

C. Actuator

The motorized Newport NSA12 microstep actuator\(^2\) used in the setup has a resolution of 0.3 \( \mu \text{m} \) and a maximum loading capacity of 25 N. The actual force on the actuator is of no interest since the force on the chip is measured by an independent force sensor. Characterization of the actuator has shown severe hysteresis during increasing and subsequent decreasing loads. The displacement of the actuator is affected by the force thus the actual position of the actuator is not reliable (see Fig. 4). The actuator is, in turn, increasing the load on the chip and decreasing the load while the output voltage on the force sensor is measured. Since previous characterization in Fig. 3 showed that the force sensor signal linearly depends on the applied force, we conclude that the hysteresis is caused by the actuator. This actuator hysteresis has no influence on the piezoresistance measurements since the force sensor signal (and not the actuator displacement) is used in the characterization.

D. Temperature monitor and control

The setup contains two integrated thermocouples. One thermocouple is placed in air close to the chip and reads the temperature near the chip. The other thermocouple is attached to the aluminum casing. This thermocouple supplies a feedback signal to the temperature controller. The thermocouples have small thermal masses resulting in a fast response time. The temperature is read using a Pico Technologies data logger.\(^2\)

A Watlow series 96 temperature controller is used to control the temperature and a Watlow solid state DIN-AMITE power relay supplies bias current to three Watlow cartridge heaters embedded in the aluminum plate placed below the setup, as seen in Fig. 1. The actuator has a temperature operating range between 5 and 40 \( ^\circ C \), thus cooling of the actuator is necessary. This is done using an ARX Cera Dyne fan and a heat sink on the actuator. A metal shield between the fan and the metal casing prevents significant cooling of the setup casing.

The time to reach a given temperature is measured to be less than \( t_{\text{eq}}=20 \) min. This is done by measuring the temperature inside the Al housing as a function of time at a

\[
F = \frac{1}{\alpha}(V_o - V_{\text{off}}),
\]

where \( 1/\alpha \) is the constant of proportionality and \( V_{\text{off}} \) is the offset voltage.

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The resulting final doping concentrations in the implanted doses were \( D = 1.5 \times 10^{13}, 1.5 \times 10^{14}, \) and \( 1.5 \times 10^{15} \text{ cm}^{-2} \), respectively, all at an energy of 50 keV. The resulting final doping concentrations in the samples are then \( N_A = 1.5 \times 10^{17}, 2.0 \times 10^{18}, \) and \( 2.2 \times 10^{19} \text{ cm}^{-3} \), respectively. The fabrication of the piezoresistors was performed by using a long postoxidation annealing which activates the acceptors and results in an extremely uniform doping profile in the piezoresistors, as verified in a simulation using the SILVACO ATHENA process simulator.\(^{25}\)

The piezoresistors are patterned using UV lithography and reactive ion etching (RIE). Contact windows in the oxide are formed using buffered HF on a photoresist mask. This mask is also used to pattern an additional high dose \( 5 \times 10^{15} \text{ cm}^{-2} \) boron ion implant to improve the contact resistance. A Ti/Al metal layer is deposited in a lift-off process to form interconnects and electrical contact to the piezoresistors. Finally, the chips are diced in a deep RIE using the Bosch process\(^{26}\) with an etch angle of 90° ± 1° to accurately define the chip direction with respect to the crystal orientation. A cross-sectional schematic of the chip is shown in Fig. 6.

The chip layout is sketched in Fig. 7. This test chip is designed to measure the piezoefficient \( \pi_{44} \) and the sum of the two other coefficients \( \pi_{11} + \pi_{12} \) in p-type silicon. The relative resistance change, \( \Delta R / R \), in a resistor with an applied uniaxial stress, \( \sigma_{xx} \), is given by\(^{8}\)

\[
\frac{\Delta R}{R} = \frac{\sigma_{xx}}{2} \left[ \pi_{11} + \pi_{12} + \pi_{44} \cos(2\theta) \right],
\]

where \( R = \frac{V_i}{I_i} \) is the resistance according to Figs. 6 and 7(b) and \( \theta \) is the angle of the resistor direction with respect to the \( \sigma_{xx} \) stress direction, i.e., [110] according to Fig. 7(a). By plotting the relative resistance change as a function of the applied stress, we obtain a value of the bracketed piezoefficient linear combination in Eq. (3) for each resistor. This value is plotted as a function of \( \cos(2\theta) \) to determine the piezoefficient \( \pi_{44} \) as the slope of a linear fit and the sum \( \pi_{11} + \pi_{12} \) as the offset of a linear fit.

### E. Chip design

The silicon chip to be inserted in the setup is a 4 cm long and 5.3 mm wide beam. The resistors are fabricated on 350 \( \mu \text{m} \) (001) silicon on insulator wafers with a device layer of 2 \( \mu \text{m} \). The device layer is thinned down to 500 nm by oxidation thinning. The piezoresistors are formed in the 500 nm thick device layer by boron doping using ion implantation. The implanted doses were \( D = 1.5 \times 10^{13}, 1.5 \times 10^{14} \), and \( 1.5 \times 10^{15} \text{ cm}^{-2} \), respectively, all at an energy of 50 keV. The resulting final doping concentrations in the samples are then \( N_A = 1.5 \times 10^{17}, 2.0 \times 10^{18}, \) and \( 2.2 \times 10^{19} \text{ cm}^{-3} \), respectively. The fabrication of the piezoresistors was performed by using a long postoxidation annealing which activates the acceptors and results in an extremely uniform doping profile in the piezoresistors, as verified in a simulation using the SILVACO ATHENA process simulator.\(^{25}\)

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### F. Electrical measurements

The full electrical setup is sketched in Fig. 8. A thermocouple placed in the Al housing supplies the signal to a feedback loop for the heaters in the bottom of the Al housing through the temperature controller. A Keithley 2400 sourcemeter and a Keithley 2700 multimeter are used for the electrical measurements on the chip with a simple four ter-
minal measurement, as illustrated in Fig. 6. The force sensor is connected to a power supply and a multimeter. All instruments are controlled via a NATIONAL INSTRUMENTS LABVIEW software interface.

The chip is contacted using zero insertion force flat flexible cable (FFC) connectors [Molex Electronics, part No. 52746-1090 (Ref. 27)], as shown in Fig. 9. The connectors do not influence the stress distribution in the chip and allow for mechanical movement of the chip.

III. ERROR ESTIMATES

The accuracy of the piezoresistance measurements depends on several factors associated with the 4PB fixture. First, the inaccuracy of the intended uniaxial stress caused by force and geometry errors is discussed. Second, deviations from the assumed uniaxial stress distribution caused by model insufficiencies and alignment errors are analyzed using COMSOL MULTIPHYSICS 3.3. In the simulation, the inner blades of PEEK are constrained in the z direction on the bottom surface of the PEEK mass. The outer blades each have a distributed force, \( F/2 \) on the surface plane. The chip is assigned the elastic parameters Young’s modulus \( Y = 170 \) GPa and Poisson’s ratio \( \nu = 0.07 \) from Ref. 28, since the chip is stressed along the [110] direction. Figure 10(a) shows the stress distribution \( \sigma_{xx} \) in the chip at a force of \( F = 2.5 \) N. The resistor area in the center region of the chip is sketched by the dashed square (3 x 3 mm²). The in-plane stress distribu-
analyze the effect (see illustration in Fig. 11). The investigation is summarized in Table II, where the ratio of the stress components are listed for different blade configurations. The ratios listed are the maximum values obtained in the resistor area. To obtain the maximum value of $\sigma_{xy}/\sigma_{xx}$, the $(x,y)$ coordinate is $(1.5 \text{ mm}, 0)$. The maximum $\sigma_{xy}/\sigma_{xx}$ value is found in the coordinate $(1.5, 1.5 \text{ mm})$.

Notice, that the $\sigma_{xx}$ stress in all cases varies less than 0.5% with respect to the analytical expression in Eq. (1). It is also seen that the transverse stress, $\sigma_{xy}$, does not depend on the blade misalignment. Thus, we assume that this stress is constant and less than 0.8% of the $\sigma_{xx}$ stress. A misalignment of the outer blades, A and D in Fig. 11 does not have a significant impact on the stress distribution. However, a misalignment of the inner blades does change the shear stress distribution. As listed in Table II, the largest shear stress, $\sigma_{xy}/\sigma_{xx}=3.5\%$, is obtained with a rotation in the same direction of the two inner blades.

The above values are extracted for a 2° horizontal rotation of the blades. This rotation is very large compared to the realistic value, but it is used in order to illustrate the influence. The milling machine used to fabricate the 4PB fixture has a very high precision (precision of 1 $\mu$m), thus, it is not expected that a horizontal misalignment influences the stress distribution in the chip.

### D. Vertical blade misalignment

A vertical rotation of the two inner blades as sketched in Fig. 12 results in a pure torsion of the beam. The resulting shear stress in the surface can be described by

![FIG. 12. Schematic of chip exposed to torsion due to vertical misalignment](image)

TABLE II. Results of a FEM analysis of blade rotation. The blades are rotated with a worst case misalignment of $\phi=\pm 2^\circ$ according to Fig. 11. Column 2: the relative difference in the extracted FEM $\sigma_{xx}$ and the analytically calculated stress $\sigma_{xx,max}$ at the chip surface in $(x,y)=(1.5 \text{ mm}, 0)$. Column 4: $\sigma_{xy}/\sigma_{xx}$ at the chip surface in $(x,y)=(1.5,1.5 \text{ mm})$.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\Delta \sigma_{xx}$</th>
<th>$\sigma_{xx,max}$</th>
<th>$\sigma_{xy}$</th>
<th>$\sigma_{xy,max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A,C,D: $\phi=0^\circ$</td>
<td>0.1%</td>
<td>0.8%</td>
<td>0.3%</td>
<td></td>
</tr>
<tr>
<td>B,C,D: $\phi=0^\circ$; A: $\phi=2^\circ$</td>
<td>0.2%</td>
<td>0.8%</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td>B,C: $\phi=0^\circ$; A,D: $\phi=2^\circ$</td>
<td>0.1%</td>
<td>0.8%</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td>B,C: $\phi=0^\circ$; A: $\phi=2^\circ$; D: $\phi=-2^\circ$</td>
<td>0.2%</td>
<td>0.8%</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td>A,C,D: $\phi=0^\circ$; B: $\phi=2^\circ$</td>
<td>0.2%</td>
<td>0.8%</td>
<td>1.9%</td>
<td></td>
</tr>
<tr>
<td>A,D: $\phi=0^\circ$; B,C: $\phi=2^\circ$</td>
<td>0.4%</td>
<td>0.7%</td>
<td>3.5%</td>
<td></td>
</tr>
<tr>
<td>A,D: $\phi=0^\circ$; B: $\phi=2^\circ$; C: $\phi=-2^\circ$</td>
<td>0.2%</td>
<td>0.7%</td>
<td>0.3%</td>
<td></td>
</tr>
</tbody>
</table>

### FEM analysis of blade rotation

FEM is used to describe the influence of a possible misalignment of the blades. The simulations are done with a total misalignment of $\phi=\pm 2^\circ$ on each blade in order to...
misalignment results in a rather large shear stress component.

Thus, we assume that all resistors experience a shear stress.

The actual misalignment angle can be measured by increasing the stress even further than what is seen in Fig. 14. Thus, if the shear stress is caused by a vertical misalignment of the inner blades, we conclude that the misalignment is at least $\Delta \psi = 0.07^\circ$. Another contribution to the shear stress may rise from an in-plane misalignment of the inner blades, as described in Sec. III B. A misalignment of $\phi = 2^\circ$ of both inner blades need to be present in order to accommodate the measured shear stress and considering the precision of the mechanical equipment used to machine the 4PB fixture (precision $\pm 1 \mu m$), this is not possible. A third contribution is a rotation of the whole PEEK 4PB fixture. The distance between the guiding rails in the bottom of the aluminum housing is 0.15 mm larger than the width of the 4PB base to accommodate thermal expansion of the two materials. Thus, a rotation of the 4PB fixture is possible. However, due to the very small air gap, the rotation angle is at most 0.3$^\circ$ and this does not significantly contribute to the shear stress. A fourth contribution to the shear stress is a misalignment of the slider and the base. However, since the two steel pins and guiding holes are specifically fabricated to fit each other with a very small air gap, this is not expected to induce a significant shear stress. Thus, we conclude that the present shear stress in the chip is an effect from a vertical rotation of the inner blades in the 4PB fixture.
function of the velocity \( v \) in a stick-slip fashion. At rest, the magnitude of the applied force must exceed the static stiction-friction force, \( F_{f00} \), to initiate motion, while in motion, the frictional force may be assumed to have a static and a dynamic component, \( F_f=F_f(\dot{v})=F_{f0}v/|v|−\beta v \), where \( F_{f0} \) and \( \beta \) is a viscous friction coefficient. In a steady state sequence of measurement steps in a given direction, the stiction-friction force is unimportant, as is the viscous friction, since the important frictional force is found when motion stops. Thus, we expect the relation, \( F_{\text{sensor}}=F_\delta \pm F_{f0} \), between the force, \( F_{\text{sensor}} \), measured on the force sensor and the actual force, \( F_\delta \), on the sample in the 4PB fixture, where the sign depends on the direction of motion.

The force \( F_{f0} \) can be estimated from the width of the hysteresis loop in the stress direction, \( \Delta \sigma \approx 2\sigma_{f0} \approx 8.2 \) MPa, as seen in Fig. 15. This corresponds to a frictional force of \( F_{f0}=0.11 \) N. If this frictional force is load independent, it does not affect the piezoresistance characterization since only the slopes of the linear fits to the measured resistance change during increasing or decreasing loads are used. We do, however, slightly find different slopes for increasing and decreasing loads, such that they equal the mean slope \( \pm 0.6\% \). This uncertainty must be included in the total uncertainty derived in Sec. III A.

### F. Electronic setup

The electronic instruments used to measure the voltage drop and to inject the current are all high precision instruments. The Keithley 2700 multimeter with multiplexer has a resolution of 1 \( \mu \text{V} \) at a voltage of 1 V and the Keithley 2400 sourcemeter has a current source accuracy of 0.03% at 100 \( \mu \text{A} \). These uncertainties are significantly smaller than the uncertainties described in Sec. III A.

### G. Friction

Frictional forces are expected to be present in the four point bending fixture since it consists of two parts where one is moving (slider) with respect to the other (base). Figure 15 shows a characterization of two resistors during sequential increasing and decreasing loads where the resistance change is measured at each applied load. For both resistors, a hysteresis loop is seen where the measured data are shifted to the right during increasing loads and to the left during decreasing loads. This behavior is explained by the frictional force, \( F_f \), between the slider and the base, while we conclude from the force diagram sketched in Fig. 16 that the frictional force, \( F_{\text{dyn}} \), between the 4PB base and the aluminum housing does not affect the measurements. The only significant frictional force contribution is from the movement of the slider on the steel pins. The frictional force may be a multivalued

\[
\Delta \sigma = \frac{\sigma_{xx}}{2R} = \sigma_{xx} \left( \frac{\pi_1 + \pi_{12} + \pi_{44} \cos(2\theta)}{2} \right)
+ \sigma_{yy} \left( \frac{\pi_1 + \pi_{12} - \pi_{44} \cos(2\theta)}{2} \right)
+ \sigma_{xy}(\pi_{11} - \pi_{12}) \sin(2\theta).
\]

In \( p \)-type silicon, the piezocoefficient \( \pi_{44} \) is much larger than \( \pi_{11} \) and \( \pi_{12} \), i.e., \( |\pi_{11}/\pi_{44}| \approx 0.8\% \) and \( |\pi_{12}/\pi_{44}| \approx 4.3\% \). Moreover, the maximum value of the shear stress in the chip was measured to be 3.6% of \( \sigma_{xx} \). Thus, it follows from Eq. (8) that the shear stress causes an insignificant error.

Moreover, the contribution from the stress \( \sigma_{xy} \) cannot be neglected since its effect is proportional to the large piezocoefficient \( \pi_{44} \) in Eq. (8). From the FEM analysis, we conclude that the contribution from \( \sigma_{xy} \) is approximately 0.8% of the contribution from \( \sigma_{xx} \). Including this contribution (and the contribution from the uncertainty of the linear fits from Sec. III G) in the uncertainty calculation of the magnitude of \( \sigma_{xx} \) in Sec. III A, we conclude that the piezocoefficient \( \pi_{44} \) is determined with an uncertainty of 1.8%.

### IV. SAMPLE CHARACTERIZATION

The chips presented in Sec. II E have been characterized in the 4PB setup. An example of a measurement series at \( T=30 \text{ °C} \) on a chip with a doping concentration of \( N_d=1.5 \times 10^{17} \text{ cm}^{-3} \) is shown in Fig. 17 for increasing loads. The
The analysis in Sec. III concluded that errors due to shear stress are negligible, the error contribution from \( \sigma_{yy} \) is at most 0.8%, and the uncertainty in the \( \sigma_{xx} \) value is 1.5%. The slope of the linear fit to the data is found with an uncertainty of 0.6%, thus, the presented 4PB setup allows measurements of the piezoelectric coefficient \( \pi_{44} \) in p-type silicon at different doping concentrations and temperatures with an uncertainty of 1.8%.

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