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Comment on "Direct Measurement of the Oscillation Frequency in an Optical-Tweezers Trap by Parametric Excitation"

In [1], Hooke's constant for an over-damped harmonic oscillator at finite temperature is modulated harmonically with 5% relative amplitude. It is claimed that a narrow and distinct resonance peak is observed in the oscillator's mean squared position as function of the modulation frequency, located at twice the cyclic frequency ω_0 of the undamped and unperturbed oscillator. Damping dominates dynamics, however, so inertial motion is not resolved. Instead, the authors invoke the fluctuation-dissipation theorem to explain their Fig. 2 [2]. This Comment explains on four different levels why the observed resonance cannot originate in the system's dynamics.

First, ω_0 cannot define the scale of anything observed, because it is defined in terms of the inertial mass. Effects of this mass are unobservable, because momentum is lost to friction with characteristic time $\lambda^{-1} = 0.5\ \mu$s, while positions are sampled at 16.4 kHz, i.e., only every 60 μs.

Second, the theory for the system studied is very well known and easily solved. It is Einstein's theory for Brownian motion from 1905. So the claimed resonance peak should occur in this theory as well and should supposedly be connected with its fluctuation-dissipation theorem. It does not, however. The system consists of a polystyrene microsphere in water at room temperature, trapped optically with modulated Hooke's constant $k_{\rm trap}(t) = k_0[1 + h\cos(\gamma t)]$, $k_0 \simeq 8\ \mu$N/m. The microsphere's diameter is 3 μm, its drag coefficient $\beta = 3 \times 10^{-8}$ N s/m according to Stokes law for rectilinear motion, and its mass $m = 1.5 \times 10^{-14}$ kg. Consider the Ornstein-Uhlenbeck theory of Brownian motion: the x coordinate of the microsphere's center, as measured from the trap's center, satisfies the Langevin equation

$$m\ddot{x} = -\beta\dot{x} - k_{\rm trap}(t)x + F_T, \quad (1)$$

where $F_T(t) = (2k_BT\beta)^{1/2}\xi(t)$ and $\xi(t)$ is a normalized white noise [3]. This theory is not fully correct hydrodynamically [3], but it is a convenient starting point, and it gives the value above for $\lambda^{-1} = m/\beta$.

The left-hand side of Eq. (1) is well approximated by zero, resulting in Einstein's simple theory for Brownian motion in the modulated trap. For $\langle x^2(t)\rangle$ it gives

$$t_{\rm trap}\frac{d}{dt}\langle x^2(t)\rangle + 2[1 + h\cos(\gamma t)]\langle x^2(t)\rangle = \frac{2k_BT}{k_0}, \quad (2)$$

where $t_{\rm trap} = \beta/k_0 = 4$ ms is the characteristic time of the average drift of the microsphere towards the center of the trap *and* the average time $\langle x^2\rangle/D$ that it takes for the microsphere to diffuse one trap radius, since $\langle x^2\rangle = k_BT/k_0$ for $h = 0$ and $D = k_BT/\beta$. For comparison, the resonance peak occurs at $\gamma = 2\omega_0$, i.e., for modulation of the trap with period $\pi/\omega_0 = 0.14$ ms $= 0.035t_{\rm trap}$. Modulation at such a high frequency cancels its own effect. After transients have died out, $\langle x^2(t)\rangle$ in Eq. (2) is periodic with period $2\pi/\gamma$, so

$$\langle x^2(t)\rangle = \sum_{n=\infty}^{\infty} e^{in\gamma t}c_n, \quad (3)$$

and Eq. (2) reads

$$(2 + in\gamma t_{\rm trap})c_n + h(c_{n+1} + c_{n-1}) = 2k_BT/k_0\delta_{n,0}, \quad (4)$$

where c_0 is the time average of $\langle x^2(t)\rangle$, which is supposed to resonate as function of γ. For $h = 0$, $c_0 = k_BT/k_0$ as it should. For $h > 0$, $c_n = \mathcal{O}(h^{|n|})$, and Eq. (4) is solved perturbatively for c_0,

$$c_0 = \frac{k_BT}{k_0}\left(1 + \frac{2h^2}{4 + (t_{\rm trap}\gamma)^2}\right) + \mathcal{O}(h^4). \quad (5)$$

This expression does contain a Lorentzian function of γ, but the peak occurs at $\gamma = 0$, rises only 1/800 over the background, and its full width at half maximum is $4/t_{\rm trap} = 1$ kHz.

Third, the full theory in Eq. (1) gives

$$c_0 = \frac{k_BT}{k_0}\left(1 + \frac{2\omega_0^2[(2\omega_0)^2 - \gamma^2]h^2}{(2\omega_0)^4 + 4(\lambda^2 - 2\omega_0^2)\gamma^2 + \gamma^4}\right) + \mathcal{O}(h^4) \quad (6)$$

which also has just a single maximum at $\gamma = 0$, unless $\lambda^{-1} > t_{\rm trap}$, which is a factor 10^4 away from being the case.

Fourth, the hydrodynamically correct theory for the system studied here shows [Ref. [3], Eqs. (29), (30)] that the effective inertial mass due to entrained fluid of a sphere doing linear harmonic motion with frequency f is $[3/2 + 9/4(f_\nu/f)^{1/2}]m$, when fluid and sphere have approximately the same density as here. The same equations show that the friction is $\beta[1 + (f/f_\nu)^{1/2}]$. Here $f_\nu = 140$ kHz for a 3 μm-diameter sphere. At $f = 8$ kHz this entrainment increases the inertial mass by a factor 11 and the friction by a factor 1.2. This effect does not change any time scales enough to matter. Even if it did, the effect is gradual and monotonic in f, hence unable to cause a sharp resonance.

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