



## The Smallest Transistor-Based Nonautonomous Chaotic Circuit

Lindberg, Erik; Murali, K.; Tamasevicius, Arunas

*Published in:*

I E E E Transactions on Circuits and Systems. Part 2: Express Briefs

*Link to article, DOI:*

[10.1109/TCSII.2005.850766](https://doi.org/10.1109/TCSII.2005.850766)

*Publication date:*

2005

*Document Version*

Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*

Lindberg, E., Murali, K., & Tamasevicius, A. (2005). The Smallest Transistor-Based Nonautonomous Chaotic Circuit. *I E E E Transactions on Circuits and Systems. Part 2: Express Briefs*, 52(10), 661-664. <https://doi.org/10.1109/TCSII.2005.850766>

---

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

# The Smallest Transistor-Based Nonautonomous Chaotic Circuit

Erik Lindberg, *Life Member, IEEE*, K. Murali, and Arunas Tamasevicius

**Abstract**—A nonautonomous chaotic circuit based on one transistor, two capacitors, and two resistors is described. The mechanism behind the chaotic performance is based on “disturbance of integration.” The forward part and the reverse part of the bipolar transistor are “fighting” about the charging of a capacitor. Measurements and PSpice experiments are presented.

**Index Terms**—Chaos, chaos mechanism.

## I. INTRODUCTION

ALL electrical and electronic engineers know not to try to interrupt the current in a coil or short-circuit the voltage of a capacitor in no time. If one attempts this, one may expect nasty behavior of the circuit. These mechanisms are used for design of chaotic circuits [1]. Apparently, the kernel of all chaotic oscillators is an oscillator of some kind [2]. If one disturbs the performance of the oscillator by adding some nasty circuit composite with local activity and memory [3], [4], the result may be chaos. Chaos sets in when the circuit drifts out of synchronization, i.e., if two circuits are coupled which are not in harmony, chaos may result. Very little is reported about the mechanisms behind chaos. Often it is just assumed that a transistor or a diode-coupled transistor in the nasty composite behave as an ideal switch. The aim of this contribution is to report on a kind of chaos that is not directly based on the two mechanisms mentioned above but merely on a kind of general disturbance of the charging of a capacitor. Fig. 1 shows an ideal sinusoidal oscillator VS loaded with an RC series circuit in connection with the collector of a transistor with grounded emitter [5]. The base of the transistor is loaded with a grounded capacitor C2, which is charged by a constant dc voltage source VDC in series with a large resistor, i.e., integration of an almost constant dc current source. The time constant of the charging of C2 (0.68 ms) is large compared with the period of VS (0.10 ms). When the transistor “goes ON” the integration of the current is disturbed by the “short circuit” of C2. The two independent sources VDC and VS are “fighting” concerning the charging of the capacitor. In the following, a modified version of this circuit is investigated. The dc source is removed

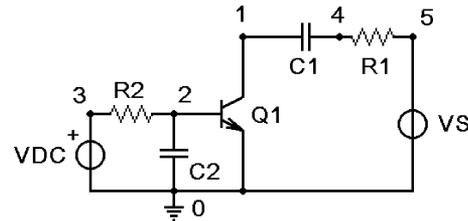


Fig. 1. Oscillator VS with an active RC load composite.  $V_{DC} = 12$  V,  $R_2 = 1$  M $\Omega$ ,  $C_2 = 680$  pF,  $C_1 = 4.7$  nF,  $R_1 = 1$  k $\Omega$ ,  $V_S = 12$  V/10 kHz, and  $Q_1 = BC107$ .

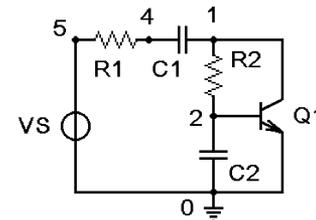


Fig. 2. Oscillator VS with an active RC load composite.  $V_S = 10$  V/10 kHz,  $R_1 = 1$  k $\Omega$ ,  $C_1 = 4.7$  nF,  $R_2 = 994$  k $\Omega$ ,  $C_2 = 1.1$  nF, and  $Q_1 = 2N2222A$ .

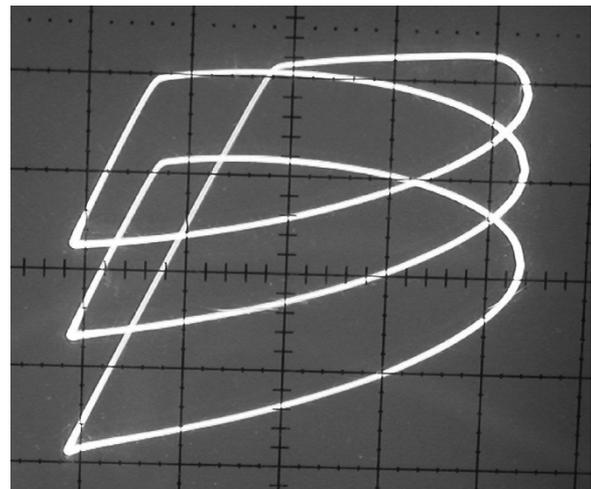


Fig. 3. Measured limit cycle (Fig. 2).  $V_S : 10$  V/3 kHz.  $y : V(2) = V(C_2)$ , 0.5 V/div,  $x : V(5) = V(V_S)$ , 2.0 V/div.

and the large resistor  $R_2$  is connected from the base to the collector of the transistor.

## II. CHAOS BASED ON DISTURBANCE OF INTEGRATION

Fig. 2 shows an ideal sinusoidal oscillator VS loaded with an active RC composite. Figs. 3 and 4 show measurements on the circuit. By varying the frequency (or the amplitude)

Manuscript received September 21, 2004; revised January 28, 2005. This paper was recommended by Associate Editor T. Ueta.

E. Lindberg is with Ørsted-DTU Department, Technical University of Denmark, Lyngby DK2800, Denmark (e-mail: el@oersted.dtu.dk).

K. Murali is with Department of Physics, Anna University, Chennai-600 025, India.

A. Tamasevicius is with the Semiconductor Physics Institute, LT 2600 Vilnius, Lithuania.

Digital Object Identifier 10.1109/TCSII.2005.850766

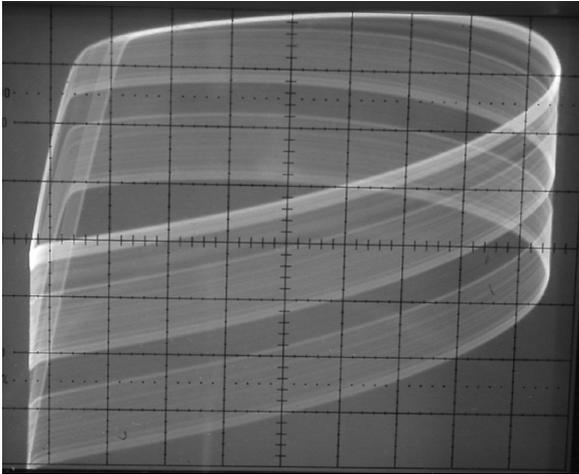


Fig. 4. Measured chaotic attractor (Fig. 2).  $V_S$ : 10 V/10 kHz.  $y$ :  $V(2) = V(C2)$ , 0.5 V/div,  $x$ :  $V(5) = V(VS)$ , 2.0 V/div.

of the oscillator, limit cycle behavior or chaotic behavior is easily found. In order to study the mechanism, PSpice simulations have been made. Chaotic behavior is easily found but apparently it is very difficult to find limit cycle behavior by means of simulation because it is very time-consuming to vary the frequency by fractions of a hertz (or the amplitude by fractions of a volt). If you neglect the transistor Q1, then the time constant for the passive load of VS becomes  $\tau = RC = (R1 + R2)((C1C2)/(C1 + C2)) = 0.886922$  ms. By choosing a frequency with a period almost equal to the time constant, limit cycle behavior may be found. Fig. 5 shows the collector current and the base voltage as functions of time, and Fig. 6 shows the same signals as functions of the collector voltage. It is seen that, in time intervals of apparently random size, the collector current is negative and the base voltage is increasing regularly, corresponding to integration of the reverse current source  $IAR = AR * I(DBC)$  of the Ebers–Moll injection model built from two diodes ( $DBC, DBE$ ) and two current-controlled current sources ( $IAF, IAR$ ). When the base voltage reach approximately 0.65 V, the transistor “goes ON” and the collector current becomes a large positive “spike” followed by a large negative “spike.” Fig. 6 shows that sometimes the forward and sometimes the reverse active region contains the instant bias point of the transistor. In short, the forward and the reverse part of the transistor “fights” concerning the charging of the capacitor  $C2$ .

Experiments with a transistor model without memory indicate that the junction and diffusion capacitances are not of importance for the qualitative behavior of the circuit.

The Lyapunov exponents have been calculated from the PSpice times series using the TISEAN software [6]. The values are +0.203 353, -0.046 210 23, and -0.553 241 7, thus confirming the chaotic nature of the output.

### III. EIGENVALUE INVESTIGATION

Experiments with piecewise linear modeling and investigation of the linearized Jacobian of the differential equations [7]

have been made. The instant piecewise linear model may be described by the following input file for ANP3:

```
*circuit
*title lmt-01.anp
:
source 5 0 v
detector 2 0 v
R1 5 4 1e+3
C1 4 1 4.7e-9
R10 1 0 1.0000e+20 : PSpice resistor
R2 1 2 994e+3
C2 2 0 1.1e-9
: transistor, Ebers-Moll injection
model
GBC 2 1 1 : ON
:GBC 2 1 0.5736e-12 : OFF, G = IS/VT
GBE 2 0 1 : ON
:GBE 2 0 0.5736e-12 : OFF, G = IS/VT
IF 1 2 0.9961074348 I GBE
:AF = BF/(1 + BF), alpha forward
IR 0 2 0.8589960519 I GBC
:AR = BR/(1 + BR), alpha reverse
*output
```

The following ANP3 output file indicate two real poles which move on the negative real axis between  $-4.7e-5$  close to origo when the base-emitter diode is OFF and far out to  $-1.3e+8$  when both diodes are ON as expected [8]:

```
*modify
GBC 1 : ON
GBE 1 : ON
*output
POLES Sigma
P -1.312D + 08
P -2.125D + 05
*modify
GBC 1 : ON
GBE 0.5736e-12 : OFF, G = IS/VT
*output
POLES Sigma
P -3.406D + 05
P -4.695D - 05
*modify
GBC 0.5736e-12 : OFF, G = IS/VT
GBE 1 : ON
*output
POLES Sigma
P -4.405D + 06
P -4.412D + 04
*modify
GBC 0.5736e-12 : OFF, G = IS/VT
GBE 0.5736e-12 : OFF, G = IS/VT
*output POLES Sigma
P -1.127D + 03
P -9.889D - 05
```

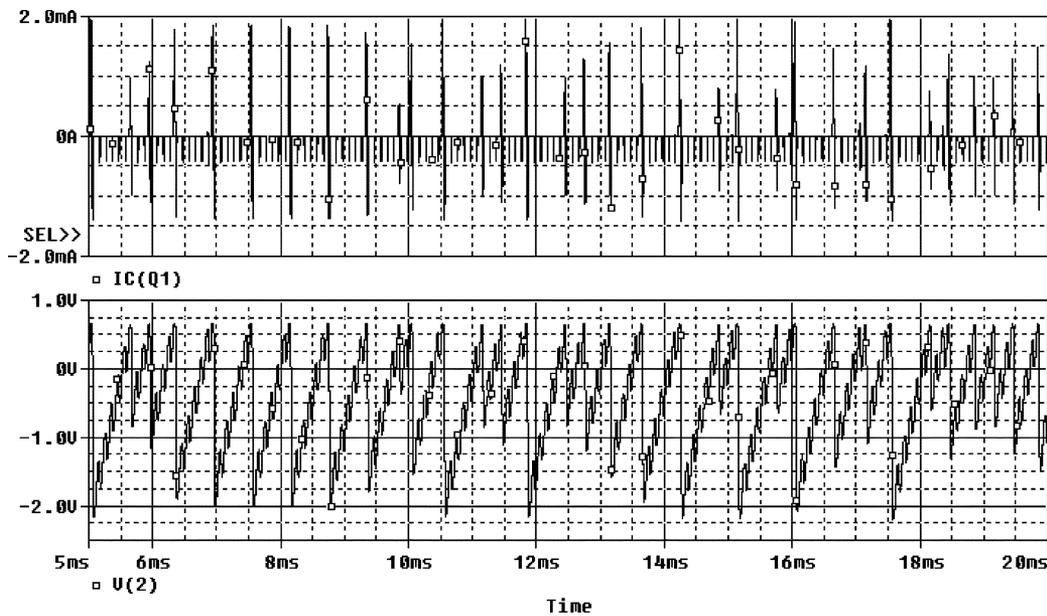


Fig. 5. Calculated collector current  $IC(Q1)$  and base voltage  $V(2)$  as functions of time.  $V_S$ : 10 V/10 kHz. Transistor  $Q1$ : 2N2222A. Time: 5–20 ms.

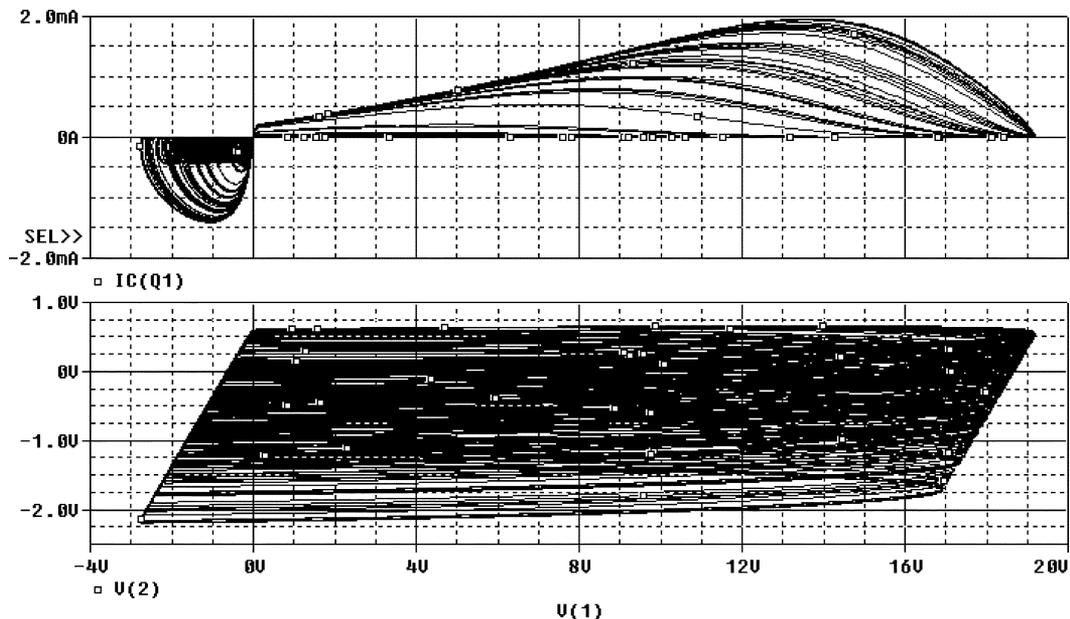


Fig. 6. Calculated collector current  $IC(Q1)$  and base voltage  $V(2)$  as functions of collector voltage  $V(1)$ .  $V_S$ : 10 V/10 kHz. Transistor  $Q1$ : 2N2222A. Time: 5–20 ms.

#### IV. CONCLUSION

So far, the simplest dissipative nonautonomous chaotic circuit is the MLC circuit [9], [10] which is based on one inductor, one capacitor and a Chua diode. The mechanism behind the chaotic behavior is “flip-flop” of a current source in parallel to a capacitor. The Chua diode may be implemented by means of one opamp, two diodes, and seven resistors. In the circuit presented here, chaos may be observed in a nonautonomous circuit based on one transistor, two resistors, and two capacitors. The mechanism behind the chaotic behavior is based on capacitor charging

by means of “forward-reverse fighting” of the transistor. The resistor in series with the ideal sinusoidal voltage source driving the circuit is not of importance for the behavior so it is claimed that the circuit is the smallest nonautonomous chaotic circuit found to date. We call it the Lindberg–Murali–Tamasevicius (LMT) circuit.

We acknowledge the reviewers comments [11], [12] concerning the “diode resonator” nonautonomous chaotic circuit being a “smaller” circuit. The diode resonator is a series connection of one resistor, one inductor, one nonlinear capacitor diode (e.g., varactor, IMPATT, and Schottky), and a sinusoidal

voltage source. The basic mechanism behind the chaotic behavior seems to be the interrupt of the current in the inductor. The parasitic capacitances and parallel losses in connection with the inductor must be taken into account and the concept of local activity is also a crucial matter. To our opinion the diode resonator is more complicated than the LMT circuit.

#### REFERENCES

- [1] G. Chen and T. Ueta, Eds., *Chaos in Circuits and Systems*. Singapore: World Scientific, 2002, vol. 11, p. 630.
- [2] A. E. Elwakil and M. P. Kennedy, "Design Methodology for autonomous chaotic oscillators," in *Chaos in Circuits and Systems*, Singapore: World Scientific, 2002, pp. 23–49.
- [3] J. L. Huertas, W.-K. Chen, and R. N. Madan, Eds., *Visions of Nonlinear Science in the 21st Century*, Singapore: World Scientific, 1999.
- [4] L. O. Chua, "CNN: A paradigm for complexity," in *Visions of Nonlinear Science in the 21st Century*, Singapore: World Scientific, 1999, ch. 13, pp. 529–837.
- [5] K. Hoh, T. Irita, T. Tsujita, and M. Fujishima, "Generation of chaos with simple sets of semiconductor devices," in *Proc. 2nd Int. Conf. Knowledge-Based Intelligent Electronic Systems*, Adelaide, Australia, Apr. 1998, pp. 250–259.
- [6] TISEAN—Nonlinear Time Series Analysis [Online]. Available: [http://www.mpiikps-dresden.mpg.de/~tisean/TISEAN\\_2.1/index.html](http://www.mpiikps-dresden.mpg.de/~tisean/TISEAN_2.1/index.html)
- [7] E. Lindberg, "Is the quadrature oscillator a multi-vibrator?," *IEEE Circuits Devices*, vol. 20, no. 6, pp. 23–28, Nov./Dec. 2004.
- [8] ———, "Colpitts, eigenvalues and chaos," in *Proc. 5th Int. Specialist Workshop Nonlinear Dynamics of Electronic Systems*, Moscow, Russia, Jun. 1997, pp. 262–267.
- [9] K. Murali, M. Lakshmanan, and L. O. Chua, "The simplest dissipative nonautonomous chaotic circuit," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 41, no. 6, pp. 462–463, Jun. 1994.
- [10] E. Lindberg, "An eigenvalue study of the MLC circuit," in *Proc. IEEE Int. Conf. Electronics, Circuits and Systems*, Lisboa, Portugal, 1998, pp. 521–524.
- [11] P. S. Linsay, "Period doubling and chaotic behavior in a driven anharmonic oscillator," *Phys. Rev. Lett.*, vol. 47, no. 19, pp. 1349–1352, 1981.
- [12] E. R. Hunt, "Stabilizing high-period orbits in a chaotic system: the diode resonator," *Phys. Rev. Lett.*, vol. 67, no. 15, pp. 1953–1955, 1991.