The Barkhausen Criterion (Observation ?)

Lindberg, Erik

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The Barkhausen Criterion (Observation)

Erik Lindberg, IEEE Life member
Faculty of Electrical Eng. & Electronics,
348 Technical University of Denmark,
DK-2800 Kongens Lyngby, Denmark
Email: el@elektro.dtu.dk

Abstract—A discussion of the Barkhausen Criterion which is a necessary but NOT sufficient criterion for steady state oscillations of an electronic circuit. An attempt to classify oscillators based on the topology of the circuit. Investigation of the steady state behavior by means of the time-varying linear approach ("frozen eigenvalues").

I. INTRODUCTION

Oscillators occur all over in nature and in man-made systems. Their behavior is characterized by size (amplitude) and period (frequency). They are controlled by the basic principle of nature which says that a system always try to go to a minimum energy state. We observe oscillators varying in size from \(1 \cdot 10^{+31}\) for the galaxies in space to \(1 \cdot 10^{-31}\) for the super-strings proposed in physics. Steady state oscillations may be limit cycle oscillations or chaotic oscillations.

Autonomous oscillators are non-linear oscillating systems which are only influenced by a constant energy source. When two oscillating systems are coupled they try to synchronize in order to obtain the minimum energy state.

Electronic oscillators are man-made non-linear circuits which show steady state oscillating behavior when powered only by dc power supplies. The behavior may be limit cycle behavior or chaotic behavior. The order of the circuit is the number of independent memory elements (capacitive, inductive or hysteretic).

For many years we have seen that some basic circuit theory textbooks introduce the Barkhausen Criterion as the necessary and sufficient criterion for an electronic circuit to be an oscillator. Also the concept of linear steady state oscillators is introduced. The aim of this discussion is to point out that steady state oscillators must be non-linear circuits and linear oscillators are mathematical fictions.

In some textbooks you may also find statements like: “an oscillator is an unstable amplifier for which the nonlinearities are bringing back the initial poles in the right half plane of the complex frequency plane, RHP, to the imaginary axis”. This statement is not true [1]. When you solve the implicit non-linear differential equations modeling an electronic circuit the kernel of the numerical method is the solution of a linear circuit. By means of Taylor evaluation the nonlinear components are replaced with linear approximations and iteration is performed until a solution is obtained. The iteration is based on Picard (static) or Newton-Raphson (dynamic) methods. In each integration step a small-signal model is found for the circuit corresponding to a linearization of the Jacobian of the differential equations.

II. BARKHAUSEN’S OBSERVATION

In 1934 H. Barkhausen (1881-1956) [2] pointed out that an oscillator may be described as an inverting ampli-
fier (a vacuum tube) with a linear frequency determining feedback circuit (fig. 1). The non-linear amplifier is a two-port with a static gain-factor equal to the ratio between the signals at the ports. The linear feedback circuit is a two-port with a feed-back-factor equal to the ratio between the port signals. It is obvious that the product of the two factors becomes equal to one. The product is called the Barkhausen Criterion or the Allgemeine Selbstregungsformel in German language.

![Fig. 2. Barkhausen’s Criterion. Characteristic polynomial](image)

Barkhausens figure may be redrawn as shown in fig. 2 where the non-linear amplifier is assumed to be a perfect amplifier with infinite input impedance, zero output impedance and linear time-varying gain A. The feedback circuit is assumed to be a linear, lumped element, time-invariant passive two-port with a rational transfer function H(s). It is obvious that the closed-loop gain is always equal to one (1) and the phase-shift is equal to a multiple of $360^\circ$ ($2\pi$). Furthermore it is seen that the Barkhausen Criterion is just an expression for the characteristic polynomial of the circuit as function of the amplifier gain. For zero gain the characteristic polynomial becomes equal to the denominator of H(s). For infinite gain the characteristic polynomial becomes equal to the numerator of H(s).

You may open the loop and study another circuit closely related to the oscillator circuit. This circuit has a time independent bias-point. You may perform the normal linear small-signal analysis (ac analysis) and study the natural frequencies (poles, eigenvalues). You may design the open-loop gain to be one (1) and you may also make the closed-loop circuit unstable with poles in the right half of the frequency plane, RHP, in the hope that the circuit will start to oscillate. However when you close the loop the bias-point of the amplifier will change and you have no guarantee that oscillations start up. The conclusion is that you must base your design on the characteristic polynomial of the closed-loop circuit.

![Fig. 3. Proper Barkhausen topology with H(s) as a modified full graph admittance circuit](image)

Figure 3 shows a realization of the closed-loop circuit where the feed-back circuit is represented with a modified full graph admittance circuit. The admittance YE between node 6 and node 7 is deleted and the admittance YF between node 4 and node 5 is deleted.

The characteristic polynomial with a full graph feedback admittance circuit may be found from

$$
YE \times (YA + YD + YC + YB) +
(YA + YB) \times (YD + YC) +
A \times (YA \times YC - YB \times YD) = 0 \quad (1)
$$

where the admittances are functions of the complex frequency s. The admittance YF does not occur because it is in parallel with the output voltage source of the amplifier.

The amplifier is a voltage controlled voltage source (VCVS) and the output signal is returned to the input by positive (YA, YB) and negative (YC, YD) voltage division. This structure has been used to investigate various oscillator families [3], [4], [5].

When you study the poles (eigenvalues) of the linearized Jacobian of the non-linear differential equations you may observe that they move around in the complex frequency plane as function of time. The signals are increasing when the poles are in RHP (the right half plane). The signals are decreasing when the poles are in LHP (the left half plane). You may observe how a complex pole pair in RHP goes to the real axis and splits-up into two real poles of which one goes towards zero and the other towards infinity. The two real poles meet again in LHP and leave the real axis as a complex pole pair [6].

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The basic mechanism behind the behavior of an oscillator is a balance of the energy received from the power source when the poles are in RHP with the energy lost when the poles are in the LHP. The real part of the poles may go between $+\infty$ and $-\infty$. At a certain instant the frequency is determined by the imaginary part of the complex pole pair. The phase noise observed corresponds to the part of the period where the instantaneous frequency deviates from the dominant frequency, the oscillator frequency \[7].

III. CLASSIFICATION OF OSCILLATORS

So far classification of oscillators is not found in the basic electronics textbooks in a proper way. You may classify with respect to waveform as relaxation, sinusoidal, multi-frequency or chaotic. You may classify with respect to application as e.g. used to synchronize systems (clock of computers), used to communication (carrier of waveforms, audio) or used to test of systems (instrumentation). You may classify with respect to implementation as e.g. voltage controlled (VCO), integrated or lumped element. However a given oscillator may fall into several of these classes. Classification based on structure (topology) seems to be the only proper way, see e.g. [8] where oscillators are classified according to number of memory elements.

Based on the topology of the circuit oscillators may be classified as belonging to one of the following classes.

Class I: Proper Barkhausen Topology.

Proper Barkhausen topology is a loop of an amplitude determining inverting non-linear amplifier and a passive frequency determining linear feed-back circuit.

The two circuits in the loop are 4-terminal or 3-terminal two-ports (fig. 1 and fig. 2). The bias point of the amplifier vary with time.

It is obvious that the power source limits the amplitude of the oscillator. The following question should be discussed: Can you separate the design of the non-linearity from the design of the gain and the linear frequency determining sub-circuit when designing an oscillator?

Class II: Modified Barkhausen Topology.

Modified Barkhausen topology is a loop of an inverting linear amplifier and a passive amplitude and frequency determining two-port non-linear feed-back circuit.

From mathematical point of view a linear amplifier with constant gain is easy to implement for analysis and design purposes but a number of questions should be discussed. Is it possible to create a linear real world amplifier which does not influence frequency and amplitude responding to a complex pole pair on the imaginary axis are: $C_A = C_B = C = 10\,\text{nF}$, $R_A = R_B = R = 20\,\text{k}\Omega$, $R_D = 3\,\text{k}\Omega$ and $R_C = 6\,\text{k}\Omega$. The frequency becomes $795.8\,\text{Hz}$ and $\omega_0 = 5\,\text{krad/sec}$. The poles of the linear Wien Bridge oscillator are found as function of resistor $R_C$. If $R_C$ is amended with a large resistor $R_{CL}$ in series with a non-linear element made from two diodes in antiparallel as shown in fig. 4 you have a mechanism for controlling the movement of the poles between RHP and LHP so you can avoid making use of the non-linear gain. The circuit becomes a Class II oscillator with modified Barkhausen topology. For $R_C = 7\,\text{k}\Omega (> 6\,\text{k}\Omega)$, $D_1 = D_2 = D1N4148$ and three values of

![Wien Bridge Oscillator](image)

\[\text{Fig. 4. Wien Bridge Oscillator}\]
$R_{CL}: R_{CL} = \infty$, $R_{CL} = 38k\Omega$ and $R_{CL} = 17.5k\Omega$ it is demonstrated that you may control both frequency and amplitude of the oscillator. When you change the perfect linear $A = 100k$ amplifier to an $AD712$ operational amplifier with a dominant pole at $12Hz$ and a high-frequency pole at $15MHz$ the non-linear control in the feed-back circuit is overruled by the non-linearities of the amplifier and the circuit becomes a Class III oscillator.

The circuit is now scaled to low frequencies by means of new capacitor values $C_A = C_B = C = 10\mu F$ and a new value $R_C = 6.010k\Omega$ ($> 6k\Omega$). Figure 5 shows that it is possible to adjust the circuit into a Class II oscillator with an almost linear amplifier. In order to start-up oscillations the initial conditions for the capacitors were chosen as $V(C_A) = -0.17406342924 V$ and $V(C_B) = +0.044747527689 V$ i.e. values close to an instant time of the steady state. Figure 6 shows how the harmonics are reduced. Figure 7 shows the dynamic and the static gain as functions of time. It is seen how the dynamic gain is almost constant in a large part of the period.

V. CONCLUSION

It is demonstrated that the Barkhausen Criterion is a necessary but not sufficient criterion for steady state oscillations of an electronic circuit. Barkhausen did not "open the loop"! Oscillators may be classified into three groups based on the Barkhausen Observation. A Wien bridge oscillator with an almost linear inverting amplifier and a nonlinear passive amplitude and frequency determining feed-back circuit is investigated by means of the time-varying linear approach ("frozen eigenvalues").

REFERENCES


Fig. 7. Dynamic and static gain
The Barkhausen Criterion (Observation ?)

Erik Lindberg, IEEE Life member,  
348 Technical University of Denmark,  
Email: el@elektro.dtu.dk

Faculty of Electrical Eng. & Electronics,  
DK-2800 Kongens Lyngby, Denmark

Abstract:

A discussion of the Barkhausen criterion which is a necessary but NOT sufficient criterion for steady state oscillations of an electronic circuit. An attempt to classify oscillators based on the topology of the circuit. Investigation of the steady state behavior by means of the time-varying linear approach (”frozen eigenvalues”).
Barkhausen Observations (Oscillators)
Barkhausen's Criterion
Classification of Oscillators
An example (Wien Bridge Oscillator)
Conclusion
H. Barkhausen (1881-1956)
Heinrich Georg Barkhausen (December 2, 1881 – February 20, 1956), born at Bremen, was a German physicist.

He studied at the Technical University of Munich (1901), TU Berlin (1902) and University of Munich (1903) and Berlin before obtaining a doctorate at the University of Göttingen in 1907.

He became Professor for Electrical Engineering at the Technische Hochschule Dresden in 1911 at the age of 29, thus obtaining the world's first chair in this discipline.
He discovered in 1919 an effect named after him, the **Barkhausen effect**, which suggested that ferromagnetic materials contain regions of like-oriented atoms. Induced changes in the magnetic orientation of these domains affect the whole domain and not individual atoms. With suitable equipment, these changes of orientation (jumps) can be heard.

The **Barkhausen stability criterion** states that an oscillator will oscillate when the total phase shift from input to output back to input is an integral multiple of 360 degrees and the system gain is equal to 1.
The Barkhausen Stability Observation states that when an oscillator oscillates with steady state signals, then the total phase shift around the loop from input to output and back to input is an integral multiple of 360 degrees and the loop gain is equal to 1.
- Barkhausen
- Observations (Oscillators)
- Barkhausen's Criterion
- Classification of Oscillators
- An example
  - (Wien Bridge Oscillator)
- Conclusion
OSCILLATORS

- An oscillator is a system which show oscillations

- Oscillators occur all over in nature and in man-made systems

- You may observe steady state, damped or unstable oscillators
a famous damped oscillator (almost linear ;-)

Pendule de Foucault
OSCILLATORS

• Linear steady state oscillators are mathematical fiction

• Poles must be on the imaginary axis all the time

• You can not balance on the razors edge

Steady state oscillators must be non-linear circuits
OSCILLATORS

- Steady state oscillations may be of chaotic nature or limit cycle nature

- Size (amplitude) and Period (frequency)

- Minimum energy state
OSCILLATORS

- Autonomous oscillators are non-linear oscillating systems which are only influenced by a constant energy source.
- When two oscillating systems are coupled, they try to synchronize in order to obtain the minimum energy state.
- Entrainment is defined as the tendency for two oscillators to lock into phase so that they vibrate in harmony.
When you solve the implicit non-linear differential equations modeling an electronic circuit, the kernel of the numerical method is the solution of a linear circuit.

By means of Taylor evaluation, the nonlinear components are replaced with linear approximations and iteration is performed until a solution is obtained.
When you study the poles (eigenvalues) of the linearized Jacobian of the non-linear differential equations you may observe that they move around in the complex frequency plane as function of time.

The signals are increasing when the poles are in RHP (the right half plane).

The signals are decreasing when the poles are in LHP (the left half plane).
The kernel of analyzing nonlinear circuits is the solution of a linear circuit.

All elements may be modelled by means of controlled sources.

During the iterations the elements may be approximated with either a dynamic value or a static value.
NUMERICAL INTEGRATION with Variable Integration Step and Variable Order Polynomial Approximation.

Newton-Raphson Iteration

Replace with dynamic value

Picard Iteration

Replace with static value

\[ g = \frac{di}{dv} \]

\[ G = \frac{I}{V} \]
Nonlinear controlled source $i = G(v)$

Static:
$G = I/V$

Dynamic:
$g = \frac{di}{dv}$
Non-linear circuits may be treated as time-varying linear circuits so it make sense to study the eigenvalues as function of time in order to better understand the mechanisms behind the behavior of an oscillator.
- Barkhausen
- Observations (Oscillators)
- Barkhausen's Criterion
- Classification of Oscillators
- An example
  (Wien Bridge Oscillator)
- Conclusion
Barkhausen Criterion –
the necessary and sufficient criterion for an electronic circuit
to be an oscillator? 

NOT sufficient!

Steady state oscillators must be non-linear circuits.

Linear oscillators are mathematical fictions.
In a real oscillator, there is no input signal $x_s$ at all. Here it is included to explain the principle of operation. The oscillator operates under positive feedback, and hence the feedback signal $x_f$ is summed to the input.

The closed loop gain is given as: $A_f = \frac{A(s)}{1 - A(s)\beta(s)}$

The open loop gain is given as: $L(s) = A(s)\beta(s)$
Physically, it means that we have a zero input for a finite output.

In this case, we can remove the input signal, as the circuit regenerates itself, and oscillates.

Thus the requirements for an oscillator are:

1. The magnitude of loop gain is unity, i.e.
   \[ L(s) = A(s) \beta(s) = 1 \]

2. The phase shift of the loop gain should be zero or a multiple of \( 2\pi \), i.e. \( 2n\pi \) where \( n = 0, 1, 2, \ldots \)

This is known as the Barkhausen Criterion.
The Barkhausen Criterion (Observation?)

Bild 1. Allgemeines Schema einer Rückkopplung

(1) $U_a = -B U_g$; $B = -\frac{U_a}{U_g}$ = „Verstärkungsfaktor“

(2) $\beta = -\frac{U_k}{U_a}$ = „Rückkopplungsfaktor“

(3) $\alpha = \frac{1}{\beta}$ oder $\alpha \beta = 1$ (Allgemeine Selbstregungsformel.)

Fig. 1 Barkhausen’s original statement
Barkhausen’s criterion (Observation ?).

\[ V(6,7) = H(s) \cdot V(4,5) = H(s) \cdot V(3,5) = H(s) \cdot A \cdot V(1,2) \]
\[ V(6,7) = -A \cdot V(2,1) \cdot H(s) = -A \cdot H(s) \cdot V(6,7) \]

Closed loop gain = \(-A \cdot H(s) = 1 = -A \cdot N(s) / D(s)\)

Characteristic polynomial

\[ D(s) + A \cdot N(s) = 0 \]
Proper Barkhausen topology with $H(s)$ as a modified full graph admittance circuit

Fig. 3
The characteristic polynomial may be found from:

\[(YA + YB)(YD + YC) + A[(YA + YB)YC - (YD + YC)YB] = 0\]

\[(YA + YB)(YD + YC) + A(YA*YC - YD*YB) = 0\]

Fig. 3
You may open the loop and study another circuit closely related to the oscillator circuit.

However when you close the loop the bias-point of the amplifier will change and you have no guarantee that oscillations start up.

The conclusion is that you must base your design on the characteristic polynomial of the closed-loop circuit.
Making open-loop measurements, if done properly, is a very effective method of determining such things as stability margins, the investigation of conditional stability, etc. and a great deal of linear systems theory derived via Nyquist Criterion enables many practical multi-loop systems to be investigated, including systems which are open-loop unstable but

it is the closed-loop system which defines the actual performance
The bias-point of the amplifier vary with time
i.e. the gain $A$ vary with time
i.e. the characteristic polynomial vary with time

The **Barkhausen Observation** is a **starting-point** for **oscillator design**
Barkhausen
Observations (Oscillators)
Barkhausen's Criterion
Classification of Oscillators
An example
(Wien Bridge Oscillator)
Conclusion
- You may classify with respect to **waveform** as relaxation, sinusoidal, multi-frequency or chaotic.

- You may classify with respect to **application** as e.g. used to synchronize systems (clock of computers), used to communication (carrier of waveforms, audio) or used to test of systems (instrumentation).

- You may classify with respect to **implementation** as e.g. voltage controlled (VCO), integrated or lumped element.
However a given oscillator may fall into several of these classes.

Classification based on structure (topology) seems to be the only proper way, see e.g.


where oscillators are classified according to number of memory elements
Class I: Proper Barkhausen Topology is a loop of an amplitude determining inverting non-linear amplifier and a passive frequency determining linear feed-back circuit.

Class II: Modified Barkhausen Topology is a loop of an inverting linear amplifier and a passive amplitude and frequency determining two-port non-linear feed-back circuit.

Class III: A topology different from I and II non-linear amplifier and non-linear feed-back circuit.
<table>
<thead>
<tr>
<th>Class</th>
<th>Amplifier</th>
<th>Feed-back circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>non-linear</td>
<td>linear</td>
</tr>
<tr>
<td>II</td>
<td>linear</td>
<td>non-linear</td>
</tr>
<tr>
<td>III</td>
<td>non-linear</td>
<td>non-linear</td>
</tr>
</tbody>
</table>
A number of questions should be discussed in connection with the second class

- Is it possible to create a linear real world amplifier which does not influence frequency and amplitude?
- Is the dc bias point of the amplifier time-invariant?
- What kind of passive non-linearity should be introduced in the feedback circuit?
Questions in connection with oscillator design

- Design of amplitude and frequency independently?
- What are the mechanisms behind oscillation?
- Do we have an escape mechanism like the one we find in connection with the pendulum clock?
- What is the difference in behavior between tube amplifiers and semiconductor amplifiers?
Questions in connection with oscillator design

- What is phase noise?
- How to minimise phase noise?
- How to minimise harmonics?
- How does the bias point of the amplifier vary with time?
- Can we design an amplifier which is linear even with time-varying bias point?
Conclusions in connection with classification of oscillators

Class I oscillators with proper Barkhausen topology are still of interest (Hartly, Colpitts, Phase-Shift, Wien-Bridge, etc.)

Class II oscillators with a linear amplifier might be of interest

Class III oscillators with more than one amplifier and multiple loops are of interest (Active filter oscillators, Double integrator oscillators, etc.)
• Barkhausen
• Observations (Oscillators)
• Barkhausen's Criterion
• Classification of Oscillators
• An example
  (Wien Bridge Oscillator)
• Conclusion
the characteristic polynomial

\[ s^2 + 2\alpha s + \omega_0^2 = 0 \]

\[ 2\alpha = \frac{1}{RA \cdot CA} + \frac{1}{RB \cdot CB} + \frac{1}{RB \cdot CA} \cdot \frac{RC \cdot (1 - A) + RD}{RD \cdot (1 + A) + RC} \]

\[ \omega_0^2 = \frac{1}{RA \cdot CA \cdot RB \cdot CB} \]

with \( RA = RB = R \) \( RC = 2 \cdot RD \) \( \alpha = 0 \)

and \( CA = CB = C \) \( RC > 2 \cdot RD \) \( \alpha < 0 \)
The poles or the natural frequencies of the circuit - the eigenvalues of the Jacobian of the linearized differential equations - are the roots of the characteristic polynomial

\[ p_{1,2} = -\alpha \pm j \sqrt{\omega_0^2 - \alpha^2} \]

\[ = -\alpha \pm j \omega \]
Citation from ref. [9] E. Lindberg, “Oscillators - an approach for a better understanding”, ECCTD 2003:

"The resistor RC is crucial for the sign of the loss coefficient 2*alpha. If RC is amended with a large resistor in series with a nonlinear element made from two diodes in antiparallel you have a mechanism for controlling the movement of the poles between RHP and LHP so you can avoid making use of the nonlinear gain. In this way you may control both frequency and amplitude of the oscillator."
but

When you change the perfect linear $A = 100k$ amplifier to an AD712 operational amplifier with a dominant pole at 12Hz and a highfrequency pole at 15MHz the circuit becomes a Class III oscillator at 795.8 Hz.

In order to obtain a Class II oscillator the circuit is scaled to a frequency of 0.7958 Hz well below the dominant pole at 12 Hz.
Wien Bridge Oscillator

Fig. 4
.tran 0 1000 100 1m, RELTOL = 1e-5, RC = 6.010e+3, RCL = 17.5e+3,
AD712 SPICE Macro-model 4/92, Rev.B: POLE at 12Hz, dc power supply:
COMMON-MODE GAIN ZERO at 300 Hz, POLE at 15MHz, +10V / +10V

\[ V(3) = -A \times V(1,2) \]

Fig. 5, Dynamic transfer characteristic of the amplifier with almost constant bias point
Fig. 6, Frequency spectrum of amplifier output

RC = 6.010e+3,  RCL = ∞

RC = 6.010e+3,  RCL = 17.5e+3,
Dynamic gain = $\frac{dV(3)}{dV(1,2)}$

Static gain = $\frac{V(3)}{V(1,2)}$

$RC = 6.010e+3, \quad CA = 10e-6 \quad ic=-1.7406342924E-01$

$RCL = 17.500e+3, \quad CB = 10e-6 \quad ic=+4.4747527689E-02$

Fig. 7
WBO

Fig. 8

\begin{align*}
\text{GIRC1} & : 0 & 50 & \quad \text{value} = \{V(2,3)/I(VRC)\} \\
\text{R50} & : 50 & 0 & 1 & ; & V(50) = RC(t)
\end{align*}

V(2, 3) = 0
I(VRC) = 0
\[ GC = \frac{1}{RC}, \quad I(VRC) = GC \times V(2,3) \] static value

\[ V(2,3) = 0, \quad I(VRC) = 0 \] error
It is an open question whether it is possible to create Class II oscillators or not!

\[ V(3) = -A \times V(1,2) \]

Fig. 5, Dynamic transfer characteristic of the amplifier with almost constant bias point.
- Barkhausen
- Observations (Oscillators)
- Barkhausen's Criterion
- Classification of Oscillators
- An example
  (Wien Bridge Oscillator)
- Conclusion
The Barkhausen Criterion should be called the Barkhausen Observation. It is a necessary but NOT sufficient criterion for steady state oscillations of an electronic circuit.

Oscillators may be divided into two groups: "Proper Barkhausen Topology" and "Not Proper Barkhausen Topology."

Insight in the mechanisms behind the behavior of an oscillator may be obtained by means of the time-varying linear approach ("frozen eigenvalues").
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I am very grateful to

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for many nice discussions on
oscillators
WITH A "KISS" ;-) 

THANK YOU FOR YOUR ATTENTION 

the slides are available from 

erik.lindberg@ieee.org