AIR Tools - A MATLAB Package of Algebraic Iterative Reconstruction Techniques

Version 1.0 for Matlab 7.8

Hansen, Per Christian; Saxild-Hansen, Maria

Publication date:
2010

Document Version
Publisher's PDF, also known as Version of record

AIR Tools

A MATLAB Package of Algebraic Iterative Reconstruction Techniques

Version 1.0 for MATLAB 7.8

Per Christian Hansen

Maria Saxild-Hansen

Department of Informatics and Mathematical Modelling
Building 321, Technical University of Denmark
DK-2800 Lyngby, Denmark

October 2010

Abstract

This collection of MATLAB software contains implementations of several Algebraic Iterative Reconstruction methods for discretizations of inverse problems. These so-called row action methods rely on semi-convergence for achieving the necessary regularization of the problem. Two classes of methods are implemented: Algebraic Reconstruction Techniques (ART) and Simultaneous Iterative Reconstruction Techniques (SIRT). In addition we provide a few simplified test problems from medical and seismic tomography. For each iterative method, a number of strategies are available for choosing the relaxation parameter and the stopping rule. The relaxation parameter can be fixed, or chosen adaptively in each iteration; in the former case we provide a "training" algorithm that finds the optimal parameter for a given test problem. The stopping rules provided are the discrepancy principle, the monotone error rule, and the NCP criterion; for the first two methods "training" can be used to find the optimal discrepancy parameter. The corresponding manuscript is:

- P. C. Hansen and M. Saxild-Hansen, *AIR Tools – A MATLAB Package of Algebraic Iterative Reconstruction Techniques*, submitted to Journal of Computational and Applied Mathematics.

Notation

All vectors are columns vectors, a_j is the jth column of A, a^i is the transposed of the ith row of A, $\langle x, y \rangle = x^T y$ is the standard inner product, and $\rho(\cdot)$ is the spectral radius (the largest positive eigenvalue).

Acknowledgements

This work is part of the project CSI: Computational Science in Imaging, supported by grant no. 274-07-0065 from the Danish Research Council for Technology and Production Sciences. We are grateful to Jakob Heide Jørgensen for providing efficient MATLAB code to compute the sparse matrix in the test problems, and to Klaus Mosegaard for suggesting the seismic travel-time tomography test problem. We also thank Tommy Elfving for encouragement and advice during the development of the package, and Jim Nagy for pointing out the need for nonnegativity constraints.

Overview of the Package

Iterative ART Methods	
`kaczmarz`	Kaczmarz's method, aka the Algebraic Reconstruction Technique (ART)
`randkaczmarz`	The randomized Kaczmarz method
`symkaczmarz`	The symmetric Kaczmarz method

Iterative SIRT Methods	
`cav`	Component Averaging (CAV) method
`cimmino`	Cimmino's method
`drop`	Diagonally Relaxed Orthogonal Projections (DROP) method
`landweber`	Landweber's method
`sart`	Simultaneous Algebraic Reconstruction Technique (SART)

Training Routines	
`trainDPME`	Training strategy to find the best parameter τ when discrepancy principle or monotone error rule is used as stopping rule
`trainLambdaART`	Training strategy to find the best constant relaxation parameter λ for a given ART method
`trainLambdaSIRT`	Training strategy to find the best constant relaxation parameter λ for a given SIRT method

Test Problems	
`fanbeamtomo`	Creates a 2-D fan-beam tomography problem
`paralleltomo`	Creates a 2-D parallel-beam tomography problem
`seismictomo`	Creates a 2-D seismic tomography problem

Demo Scripts	
`ARTdemo`	Illustrates the simple use of the ART methods
`nonnegdemo`	Illustrates the use of nonnegativity constraints
`SIRTdemo`	Illustrates the simple use of the SIRT methods
`trainingdemo`	Illustrates the use of the training routines as pre-processors for the SIRT and the ART methods

Auxiliary Routines	
`calczeta`	Calculates the roots of the polynomial $g_k(z)$ of degree k
`rzr`	Removes zero rows from A and corresponding elements of b

The Demo Scripts

This MATLAB package includes four demo scripts which illustrate the use of the remaining functions in the package.

The demo `ARTdemo` illustrates the use of the three ART methods `kaczmarz`, `symkaczmarz` and `randkaczmarz`. First the demo creates a parallel-beam tomography test problem using the test problem `paralleltomo`. For this test problem noise is added to the right-hand side and the noisy problem is then solved using the ART methods with 10 iterations. The result is shown as four images, where one contains the exact solution and the remaining images illustrate the solutions computed by means of the three ART methods.

The demo `nonnegdemo` illustrates the use of nonnegativity constraints in the `cimmino` and `kaczmarz` methods. The demo creates a parallel-beam test problem, adds noise and solves the problem without and with the constraints. The exact solution and the results from the methods are shown.

The demo `SIRTdemo` illustrates the use of the five SIRT methods `landweber`, `cimmino`, `cav`, `drop`, and `sart`. First the demo creates a parallel-beam tomography test problem using the test problem `paralleltomo`. For this test problem noise is added to the right-hand side and the noisy problem is then solved using the SIRT methods with 50 iterations. The result is shown as seven images, where one contains the exact solution and the remaining images illustrate the solutions computed by means of the five SIRT methods.

The demo `trainingdemo` illustrates the use of the training functions `trainLambdaART`, `trainLambdaSIRT`, and `trainDPME` followed by the use of an ART or a SIRT method. In this demo the used SIRT method is `cimmino` and the used ART method is `kaczmarz`. First the demo function creates a parallel-beam tomography test problem using the test problem `paralleltomo`, and noise is added to the right-hand side. Then the training strategy `trainLambdaSIRT` is used to find the relaxation parameter for `cimmino` and `trainLambdaART` is used to find the relaxation parameter for `kaczmarz`. Including this information the stopping parameter is found for each of the methods, where `cimmino` uses the ME stopping rule and `kaczmarz` uses the DP stopping rule. After this we solve the problem with the specified relaxation parameter and stopping rule. The result is shown as three images, where one contains the exact image and the remaining images illustrate the found solutions.

calczeta

Purpose:

Calculates the roots of a specific polynomial $g(z)$ of degree k.

Synopsis:

```
z = calczeta(k)
```

Description:

This function uses Newton's method to compute the unique root in the interval $(0, 1)$ of the polynomial of degree `k`:

$$g(z) = (2k-1)z^{k-1} - (z^{k-2} + ... + z + 1) = 0.$$

The input `k` can be given as both a scalar or a vector, and the corresponding root or roots are returned in the output `z`.

The function `calczeta` is used in the functions `cav`, `cimmino`, `drop`, `landweber`, `sart`, and `symkaczmarz`.

Examples:

Calculate the roots for the degrees 2 up to 100 and plot the found roots.

```
k = 2:100;
z = calczeta(k);
figure, plot(k,z,'bo')
```

See also:

`cav`, `cimmino`, `drop`, `landweber`, `sart`, `symkaczmarz`.

cav

Purpose:

Component Averaging (CAV) iterative method.

Synopsis:

```
[X info restart] = cav(A,b,K)
[X info restart] = cav(A,b,K,x0)
[X info restart] = cav(A,b,K,x0,options)
```

Algorithm:

For arbitrary $x^0 \in \mathbb{R}^{\mathtt{n}}$ the algorithm for `cav` takes the following form:

$$x^{k+1} = x^k + \lambda_k \, \mathtt{A}^T D_S(\mathtt{b} - \mathtt{A}x^k), \qquad D_S = \mathrm{diag}\left(\frac{w_i}{\sum_{j=1}^{n} s_j a_{ij}^2}\right),$$

where s_j is the number of nonzero elements in column j of `A`.

Description:

The function implements the Component Averaging (CAV) iterative method for solving the linear system $\mathtt{A}\,x = \mathtt{b}$. The starting vector is `x0`; if no starting vector is given then $\mathtt{x0} = 0$ is used.

The numbers given in the vector `K` are iteration numbers, that specify which iterations are stored in the output matrix `X`. If a stopping rule is selected (see below) and $\mathtt{K} = [\,]$, then `X` contains the last iterate only.

The maximum number of iterations is determined either by the maximum number in the vector `K` or by the stopping rule specified in the field `stoprule` in the struct `options`. If `K` is empty a stopping rule must be specified.

The relaxation parameter is given in the field `lambda` in the struct `options`, either as a constant or as a string that determines the method to compute `lambda`. As default `lambda` is set to $1/\tilde{\sigma}_1^2$, where $\tilde{\sigma}_1$ is an estimate of the largest singular value of $D_S^{1/2}\mathtt{A}$.

The second output `info` is a vector with two elements. The first element is an indicator, that denotes why the iterations were stopped. The number 0 denotes that the iterations were stopped because the maximum number of iterations were reached, 1 denotes that the `NCP`-rule stopped the iterations, 2 denotes that the `DP`-rule stopped the iteration and 3 denotes that the `ME`-rule stopped the iterations. The second element in `info` is the number of used iterations.

The struct `restart`, which can be given as output, contains in the field `s1` the estimated largest singular value. `restart` also returns a vector containing the diagonal of the matrix D_S in the field `M` and an empty vector in the field `T`. The struct `restart` can also be given as input in the struct `options` such that the program does not have to recompute the contained values. We recommend only to use this, if the user has good knowledge of MATLAB and is completely sure of the use of `restart` as input.

Use of `options`:

The following fields in `options` are used in this function:

- `options.lambda`:
  - `options.lambda = c`, where `c` is a constant, satisfying $0 \leq \texttt{c} \leq 2/\tilde{\sigma}_1^2$. A warning is given if this requirement is estimated to be violated.
  - `options.lambda = 'line'`, where the line search method is used to compute the value for λ_k in each iteration.
  - `options.lambda = 'psi1'`, where the method `psi1` computes the values for λ_k using the Ψ_1-based relaxation.
  - `options.lambda = 'psi1mod'`, where the method `psi1mod` computes the values for λ_k using the modified Ψ_1-based relaxation with $\nu = 2$. The parameter ν can be changed in line 365 in the code.
  - `options.lambda = 'psi2'`, where the method `psi2` computes the values for λ_k using the Ψ_2-based relaxation.
  - `options.lambda = 'psi2mod'`, where the method `psi2mod` computes the values for λ_k using the modified Ψ_2-based relaxation with $\nu = 1.5$. The parameter ν can be changed in line 412 in the code.
- `options.nonneg` Logical; if true then nonnegativity in enforced in each iteration.
- `options.restart`
  - `options.restart.M =` a vector with the diagonal of D_S.
  - `options.restart.s1 =` $\tilde{\sigma}_1$, the estimated largest singular value of $D_S^{1/2}A$.
- `options.stoprule`
  - `options.stoprule.type`
    - `options.stoprule.type = 'none'`, where no stopping rule is given and only the maximum number of iterations is used to stop the algorithm. This choice is default.
    - `options.stoprule.type = 'NCP'`, where the optimal number of iterations k_* is chosen according to Normalized Cumulative Periodogram.
    - `options.stoprule.type = 'DP'`, where the stopping index k_* is determined according to the discrepancy principle (DP).
    - `options.stoprule.type = 'ME'`, where the stopping index k_* is determined according to the monotone error rule (ME).
  - `options.stoprule.taudelta` $= \tau\delta$, where δ is the noise level and τ is user-chosen. This parameter is only needed for the stoprule types `DP` and `ME`.
- `options.w` $= w$, where w is an m-dimensional vector of weights.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, computes 50 `cav` iterations and show the last iterate:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
X = cav(A,b,1:50);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`cimmino`, `drop`, `landweber`, `sart`.

References:

1. Y. Censor, D. Gordon, and R. Gordon, *Component averaging: An efficient iterative parallel algorithm for large sparse unstructured problems*, Parallel Computing, 27 (2001), pp. 777–808.

cimmino

Purpose:

Cimmino's iterative projection method.

Synopsis:

```
[X info restart] = cimmino(A,b,K)
[X info restart] = cimmino(A,b,K,x0)
[X info restart] = cimmino(A,b,K,x0,options)
```

Algorithm:

For arbitrary $x^0 \in \mathbb{R}^n$ the algorithm for `cimmino` take the following form:

$$x^{k+1} = x^k + \lambda_k \mathtt{A}^T D(b - \mathtt{A}x^k), \qquad D = \frac{1}{m}\mathrm{diag}\left(\frac{w_i}{\|\mathtt{A}(\mathtt{i},:)\|_2^2}\right).$$

Description:

The function implements Cimmino's iterative projection method for solving linear systems $\mathtt{A}\,x = \mathtt{b}$. The starting vector is `x0`; if no starting vector is given, then $\mathtt{x0} = 0$ is used.

The numbers given in the vector `K` are iteration numbers that specify which iterations are stored in the output matric `K`. If a stopping rule us selected (see below) and $\mathtt{K} = [\,]$, then `X` contains the last iterate only.

The maximum number of iterations is determined either by the maximum number in the vector `K` or by the stopping rule specified in the field `stoprule` in the struct `options`. If `K` is empty a stopping rule must be specified.

The relaxation parameter is given in the field `lambda` in the struct `options`, either as a constant or as a string that determines the method to compute `lambda`. As default `lambda` is set to $1/\tilde{\sigma}_1^2$, where $\tilde{\sigma}_1$ is an estimate of the largest singular value of $D^{1/2}\mathtt{A}$.

The second output `info` is a vector with two elements. The first element is an indicator that denotes why the iterations were stopped. The number 0 denotes that the iterations were stopped because the maximum number of iterations were reached, 1 denotes that the `NCP`-rule stopped the iterations, 2 denotes that the `DP`-rule stopped the iteration and 3 denotes that the `ME`-rule stopped the iterations. The second element in `info` is the number of used iterations.

The struct `restart`, which can be given as output, contains in the field `s1` the estimated largest singular value. `restart` also returns a vector containing the diagonal of the matrix M in the field `M` and an empty vector in the field `T`. The struct `restart` can also be given as input in the struct `options`, such that the program do not have to recompute the contained values. We recommend only to use this, if the user has good knowledge of MATLAB and is completely sure of the use of `restart` as input.

Use of `options`

The following fields in options are used in this function:

- `options.lambda`:

- `options.lambda = c`, where `c` is a constant, satisfying $0 \leq \texttt{c} \leq 2/\tilde{\sigma}_1^2$. A warning is given if this requirement is estimated to be violated.
- `options.lambda = 'line'`, where the line search method is used to compute the value for λ_k in each iteration.
- `options.lambda = 'psi1'`, where the method `psi1` computes the values for λ_k using the Ψ_1-based relaxation.
- `options.lambda = 'psi1mod'`, where the method `psi1mod` computes the values for λ_k using the modified Ψ_1-based relaxation with $\nu = 2$. The parameter ν can be changed in line 371 in the code.
- `options.lambda = 'psi2'`, where the method `psi2` computes the values for λ_k using the Ψ_2-based relaxation.
- `options.lambda = 'psi2mod'`, where the method `psi2mod` computes the values for λ_k using the modified Ψ_2-based relaxation with $\nu = 1.5$. The parameter ν can be changed in line 418 in the code.

- `options.nonneg` Logical; if true then nonnegativity in enforced in each iteration.

- `options.restart`
  - `options.restart.M` = a vector with the diagonal of M.
  - `options.restart.s1` $= \tilde{\sigma}_1$, the estimated largest singular value of $M^{1/2}A$.

- `options.stoprule`
  - `options.stoprule.type`
    - `options.stoprule.type = 'none'`, where no stopping rule is given and only the maximum number of iterations is used to stop the algorithm. This choice is default.
    - `options.stoprule.type = 'NCP'`, where the optimal number of iterations k_* is chosen according to Normalized Cumulative Periodogram.
    - `options.stoprule.type = 'DP'`, where the stopping index k_* is determined according to the discrepancy principle (DP).
    - `options.stoprule.type = 'ME'`, where the stopping index k_* is determined according to the monotone error rule (ME).
  - `options.stoprule.taudelta` $= \tau\delta$, where δ is the noise level and τ is user-chosen. This parameter is only needed for the stoprule types `DP` and `ME`.

- `options.w` $= w$, where w is an m-dimensional vector of weights.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, computes 50 `cimmino` iterations and show the last iterate:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
```

```
X = cimmino(A,b,1:50);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`cav`, `drop`, `landweber`, `sart`.

References:

1. G. Cimmino, *Calcolo approssimato per le soluzioni dei sistemi di equazioni lineari*, La Ricerca Scientifica, XVI, Series II, Anno IX, 1 (1938), pp. 326–333.

2. C. D. Meyer, *Matrix Analysis and Applied Linear Algebra*, SIAM, Philadelphia, 2000.

drop

Purpose:

Diagonally Relaxed Orthogonal Projections (DROP) iterative method.

Synopsis:

```
[X info restart] = drop(A,b,K)
[X info restart] = drop(A,b,K,x0)
[X info restart] = drop(A,b,K,x0,options)
```

Algorithm:

For arbitrary $x^0 \in \mathbb{R}^n$ the algorithm for the `drop` method takes the following form:

$$x^{k+1} = x^k + \lambda_k S^{-1} \mathtt{A}^T M(b - \mathtt{A}x^k), \qquad M = \operatorname{diag}\left(\frac{w_i}{\|\mathtt{A}(\mathtt{i},:)\|_2^2}\right),$$

where $S = \operatorname{diag}(s_j)$ and s_j is the number of nonzero elements in column j of `A` for $\mathtt{i} = 1, \ldots, \mathtt{m}$.

Description:

The function implements the Diagonally Relaxed Orthogonal Projections (DROP) iterative method for solving the linear system $\mathtt{A}\,x = \mathtt{b}$. The starting vector is `x0`; if no starting vector is given, then $\mathtt{x0} = 0$ is used.

The numbers given in the vector `K` are the iteration numbers, that specify which iterations are stored in the output matrix `X`. If a stopping rule is selected (see below) and $\mathtt{K} = [\,]$, then `X` contains the last iterate only.

The maximum number of iterations is determined either by the maximum number in the vector `K` or by the stopping rule specified in the field `stoprule` in the struct `options`. If `K` is empty a stopping rule must be specified.

The relaxation parameter is given in the field `lambda` in the struct `options`, either as a constant or as a string that determines the method to compute `lambda`. As default `lambda` is set to $1/\tilde{\rho}$, where $\tilde{\rho}$ is an estimate of the spectral radius of $S^{-1}\mathtt{A}^T M\,\mathtt{A}$.

The second output `info` is a vector with two elements. The first element is an indicator, that denotes why the iterations were stopped. The number 0 denotes that the iterations were stopped because the maximum number of iterations were reached, denotes that the `NCP`-rule stopped the iterations, 2 denotes that the `DP`-rule stopped the iterations and 3 denotes that the `ME`-rule stopped the iterations. The second element in `info` is the number of used iterations.

The struct `restart`, which can be given as output, contains in the field `s1` the estimated largest singular value. `restart` also returns a vector containing the diagonal of the matrix M in the field `M` and the diagonal of the matrix S in the field `T`. The struct `restart` can also be given as input in the struct `options`, such that the program do not have to recompute the contained values. We recommend only to use this, if the user has good knowledge of MATLAB and is completely sure of the use of `restart` as input.

Use of `options`

The following fields in options are used in this function:

- `options.lambda`:
  - `options.lambda = c`, where `c` is a constant, satisfying $0 \leq$ `c` $\leq 2/\tilde{\rho}$. A warning is given if this requirement is estimated to be violated.
  - `options.lambda = 'line'`, where the line search method is used to compute the value for λ_k in each iteration.
  - `options.lambda = 'psi1'`, where the method `psi1` computes the values for λ_k using the Ψ_1-based relaxation.
  - `options.lambda = 'psi1mod'`, where the method `psi1mod` computes the values for λ_k using the modified Ψ_1-based relaxation with $\nu = 2$. The parameter ν can be changed in line 381 in the code.
  - `options.lambda = 'psi2'`, where the method `psi2` computes the values for λ_k using the Ψ_2-based relaxation.
  - `options.lambda = 'psi2mod'`, where the method `psi2mod` computes the values for λ_k using the modified Ψ_2-based relaxation with $\nu = 1.5$. The parameter ν can be changed in line 428 in the code.
- `options.nonneg` Logical; if true then nonnegativity in enforced in each iteration.
- `options.restart`
  - `options.restart.M` = a vector containing the diagonal of M.
  - `options.restart.T` = a vector containing the diagonal of S^{-1}.
  - `options.restart.s1` $= \tilde{\sigma}_1$, where $\tilde{\sigma}_1 = \sqrt{\tilde{\rho}}$.
- `options.stoprule`
  - `options.stoprule.type`
    - `options.stoprule.type = 'none'`, where no stopping rule is given and only the maximum number of iterations is used to stop the algorithm. This choice is default.
    - `options.stoprule.type = 'NCP'`, where the optimal number of iterations k_* is chosen according to Normalized Cumulative Periodogram.
    - `options.stoprule.type = 'DP'`, where the stopping index is determined according to the discrepancy principle (DP).
    - `options.stoprule.type = 'ME'`, where the stopping index is determined according to the monotone error rule (ME).
  - `options.stoprule.taudelta` $= \tau\delta$, where δ is the noise level and τ is user-chosen. This parameter is only needed for the stoprule types `DP` and `ME`.
- `options.w` $= w$, where w is an m-dimensional vector of weights.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, computes 50 `drop` iterations and show the last iterate:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
X = drop(A,b,1:50);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`cav`, `cimmino`, `landweber`, `sart`.

References:

1. Y. Censor, T. Elfving, G. Herman, and T. Nikazad, *On diagonally relaxed orthogonal projection methods*, SIAM J. Sci. Comput., 30 (2007/08), pp. 473–504.

fanbeamtomo

Purpose:

Creates a two-dimensional fan-beam tomography test problem.

Synopsis:

```
[A b x theta p R w] = fanbeamtomo(N)
[A b x theta p R w] = fanbeamtomo(N,theta)
[A b x theta p R w] = fanbeamtomo(N,theta,p)
[A b x theta p R w] = fanbeamtomo(N,theta,p,R)
[A b x theta p R w] = fanbeamtomo(N,theta,p,R,w)
[A b x theta p R w] = fanbeamtomo(N,theta,p,R,w,isDisp)
```

Description:

This function creates a two-dimensional tomography test problem using fan beams. A 2-dimensional domain is divided into `N` equally spaced intervals in both dimension creating $\texttt{N}^2$ cells. For each specified angle `theta`, given in degrees, a source is located with distance `RN` to the center of the domain. From the sources `p` equiangular rays penetrate the domain with a span of `w` between the first and the last ray. The default values for the angles is `theta` = 0:359. The number of rays `p` have the default value equal to $\texttt{round}(\sqrt{2}\texttt{N})$. The distance from the center of the domain to the sources is given in the unit of side lengths and default value of `R` is 2. The default value of the span `w` is calculated such that from (0,`RN`) the first ray hits the point (−`N`/2,`N`/2) and the last hits (`N`/2,`N`/2). If the input `isDisp` is different from 0 then the function also creates an illustration of the problem with the used angles and rays etc. As default `isDisp` is 0.

The function returns a coefficient matrix `A` with the dimension $\texttt{length(theta)} \cdot \texttt{p} \times \texttt{N}^2$, the right hand side `b`, and the phantom head reshaped as a vector `x`. The figure below illustrates the phantom head for `N` = 100. In case that default values are used the function also returns the used angles `theta`, the number of used rays for each angle `p`, the used distance from the source to the center of the domain given in side lengths `R` and the used span of the rays `w`.

Algorithm:

The element a_{ij} is defined as the length of the ith ray through the jth cell with $a_{ij} = 0$ if ray i does not go through cell j. The exact solution of the head phantom is reshaped as a vector and the ith element in the right hand side b_i is

$$b_i = \sum_{j=1}^{\texttt{N}^2} a_{ij} x_j, \quad i = 1, \ldots, nA \cdot \texttt{p}.$$

Examples:

Create a test problem and visualize the solution:

```
N = 64; theta = 0:5:359; p = 2*N; R = 2;
[A b x] = fanbeamtomo(N,theta,p,R);
```

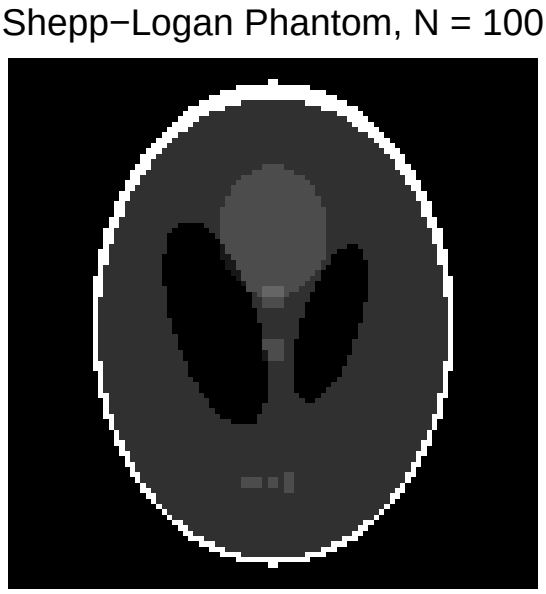
```
imagesc(reshape(x,N,N))
colormap gray, axis image off
```

See also:

`paralleltomo`, `seismictomo`.

References:

1. A. C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging*, SIAM, Philadelphia, 2001.



kaczmarz

Purpose:

Kaczmarz's iterative method also known as algebraic reconstruction technique (ART).

Synopsis:

```
[X info] = kaczmarz(A,b,K)
[X info] = kaczmarz(A,b,K,x0)
[X info] = kaczmarz(A,b,K,x0,options)
```

Algorithm:

For arbitrary starting vector $x^0 \in \mathbb{R}^n$ one iteration of the algorithm `kaczmarz` takes the following form:

$$x \leftarrow x + \lambda_k \frac{b_i - \langle a^i, x \rangle}{\|a^i\|_2^2} a^i$$

Description:

The function implements Kaczmarz's iterative method for solving the linear system $\mathtt{A}\,x = \mathtt{b}$. The starting vector is `x0`; if no starting vector is given then $\mathtt{x0} = 0$ is used.

The numbers given in the vector `K` are iteration numbers, that specify which iterations are stored in the putput matrix `X`. If a stopping rule is selected (see below) and $\mathtt{K} = [\,]$, then `X` contains the last iterate only.

The maximum number of iterations is determined either by the maximum number in the vector `K` or by the stopping rule specified in the field `stoprule` in the struct `options`. If `K` is empty a stopping rule must be specified.

The relaxation parameter is given in the field `lambda` in the struct `options` as a constant. As default `lambda` is set to 0.25.

The second output `info` is a vector with two elements. The first element is an indicator that denotes why the iterations were stopped. The number 0 denotes that the iterations were stopped because the maximum number of iterations were reached, 1 denotes that the `NCP`-rule stopped the iterations and 2 denotes that the `DP`-rule stopped the iterations.

Use of `options`:

The following fields in options are used in this function:

- `options.lambda` $=$ `c`, a constant satisfying $0 \leq$ `c` ≤ 2. A warning is given if this requirement is estimated to be violated.

- `options.nonneg` Logical; if true then nonnegativity in enforced in each step.

- `options.stoprule`

    - `options.stoprule.type`

- `options.stoprule.type = 'none'`, where no stopping rule is given and only the maximum number of iterations is used to stop the algorithm. This choice is default.
- `options.stoprule.type = 'NCP'`, where the optimal number of iterations k_* is chosen according to Normalized Cumulative Periodogram.
- `options.stoprule.type = 'DP'`, where the stopping index is determined according to the discrepancy principle (DP).

- `options.stoprule.taudelta` $= \tau\delta$, where δ is the noise level and τ is user-chosen. This parameter is only needed for the stoprule type `DP`.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, computes 10 `kaczmarz` iterations and show the last iterate:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
X = kaczmarz(A,b,1:10);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`randkaczmarz`, `symkaczmarz`.

References:

1. G. T. Herman, *Fundamentals of Computerized Tomography, Image Reconstruction from Projections*, Springer, New York, 2009.

2. S. Kaczmarz, *Angenäherte Auflösung von Systemen linearer Gleichungen*, Bulletin de l'Académie Polonaise des Sciences et Lettres, A35 (1937), pp. 355–357.

landweber

Purpose:

The Classical Landweber iterative method.

Synopsis:

```
[X info restart] = landweber(A,b,K)
[X info restart] = landweber(A,b,K,x0)
[X info restart] = landweber(A,b,K,x0,options)
```

Algorithm:

For arbitrary $x^0 \in \mathbb{R}^{\mathtt{n}}$ the algorithm for `landweber` takes the following form:

$$x^{k+1} = x^k + \lambda_k \, \mathtt{A}^T(\mathtt{b} - \mathtt{A}x^k).$$

Description:

The function implements the Classical Landweber iterative method for solving the linear system $\mathtt{A}\,x = \mathtt{b}$. The starting vector is `x0`; if no starting vector is given then $\mathtt{x0} = 0$ is used.

The numbers given in the vector `K` are iteration numbers, that specify which iterations are stored in the output matrix `X`. If a stopping rule is selected (see below) and $\mathtt{K} = [\,]$, then `X` contains the last iterate only.

The maximum number of iterations is determined either by the maximum number in the vector `K` or by the stopping rule specified in the field `stoprule` in the struct `options`. If `K` is empty a stopping rule must be specified.

The relaxation parameter is given in the field `lambda` in the struct `options`, either as a constant or as a string that determines the method to compute `lambda`. As default `lambda` is set to $1/\tilde{\sigma}_1^2$, where $\tilde{\sigma}_1$ is an estimate of the largest singular value of `A`.

The second output is a vector with two elements. The first element is an indicator, that denotes why the iterations were stopped. The number 0 denotes that the iterations were stopped because the maximum number of iterations were reached, 1 denotes that the `NCP`-rule stopped the iterations, 2 denotes that the `DP`-rule stopped the iterations and 3 denotes that the `ME`-rule stopped the iterations. The second element is `info` is the number of used iterations.

The struct `restart`, which can be given as output, contains in the field `s1` the estimated largest singular value. `restart` also returns an empty vector in both the fields `M` and `T`. The struct `restart` can also be given as input in the struct `options`, such that the program does not have to recompute the contained values. We recommend only to use this, if the user has good knowledge of MATLAB and is completely sure of the use of `restart` as input.

Use of `options`:

The following fields in options are used in this function:

- `options.lambda`:

- `options.lambda = c`, where `c` is a constant, satisfying $0 \leq \texttt{c} \leq 2/\tilde{\sigma}_1^2$. A warning is given if this requirement is estimated to be violated.
- `options.lambda = 'line'`, where the line search method is used to compute the value for λ_k in each iteration.
- `options.lambda = 'psi1'`, where the method `psi1` computes the values for λ_k using the Ψ_1-based relaxation.
- `options.lambda = 'psi1mod'`, where the method `psi1mod` computes the values for λ_k using the modified Ψ_1-based relaxation with $\nu = 2$. The parameter ν can be changed in line 308 in the code.
- `options.lambda = 'psi2'`, where the method `psi2` computes the values for λ_k using the Ψ_2-based relaxation.
- `options.lambda = 'psi2mod'`, where the method `psi2mod` computes the values for λ_k using the modified Ψ_2-based relaxation with $\nu = 1.5$. The parameter ν can be changed in line 353 in the code.

- `options.nonneg` Logical; if true then nonnegativity in enforced in each iteration.

- `options.restart`

  - `options.restart.s1` $= \tilde{\sigma}_1$, where $\tilde{\sigma}_1$ is the estimated largest singular value of A.

- `options.stoprule`

  - `options.stoprule.type`
    - `options.stoprule.type = 'none'`, where no stopping rule is given and only the maximum number of iterations is used to stop the algorithm. This choice is default.
    - `options.stoprule.type = 'NCP'`, where the optimal number of iterations k_* is chosen according to Normalized Cumulative Periodogram.
    - `options.stoprule.type = 'DP'`, where the stopping index k_* is determined according to the discrepancy principle (DP).
    - `options.stoprule.type = 'ME'`, where the stopping index k_* is determined according to the monotone error rule (ME).

    `options.stoprule.taudelta` $= \tau\delta$, where δ is the noise level and τ is user-chosen. This parameter is only needed for the stoprule types `DP` and `ME`.

Examples:

We generate a "noisy" 50×50 parallel beam tomography problem, computes 50 `landweber` iterations and show the last iterate:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
X = landweber(A,b,1:50);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`cav`, `cimmino`, `drop`, `sart`

References:

1. L. Landweber, *An iteration formula for Fredholm integral equations of the first kind*, American Journal of Mathematics, 73 (1951), pp. 615–624.

paralleltomo

Purpose:

Creates a two-dimensional parallel-beam tomography test problem.

Synopsis:

```
[A b x theta p w] = paralleltomo(N)
[A b x theta p w] = paralleltomo(N,theta)
[A b x theta p w] = paralleltomo(N,theta,p)
[A b x theta p w] = paralleltomo(N,theta,p,w)
[A b x theta p w] = paralleltomo(N,theta,p,w,isDisp)
```

Description:

This function creates a two-dimensional tomography test problem using parallel beams. A 2-dimensional domain is divided into `N` equally spaced intervals in both dimensions creating $\texttt{N}^2$ cells. For each specified angle `theta`, given in degrees, `p` parallel rays, arranged symmetrically around the center of the domain, such that the width from the first to the last ray is `w`, penetrate the domain. The default values for the angles are `theta` = 0:179. The number of rays `p` has the default value equal to $\texttt{round}(\sqrt{2}\texttt{N})$. The default value of the width between the first and the last ray `w` is $\sqrt{2}\texttt{N}$. If the input `isDisp` is different from 0 then the function also creates an illustration of the problem with the used angles and rays etc. As default `isDisp` is 0.

The function returns a coefficient matrix `A` with the dimension $\texttt{length(theta)} \cdot \texttt{p} \times \texttt{N}^2$, the right hand side `b`, and the phantom head reshaped as a vector `x`. The figure below illustrates the phantom head for `N` = 100. In case the default values are used, the function also returns the used angles `theta`, the number of used rays for each angle `p`, and the used width of the rays `w`.

Algorithm:

The element a_{ij} is defined as the length of the ith ray through the jth cell with $a_{ij} = 0$ if ray i does not go through cell j. The exact solution of the head phantom is reshaped as a vector and the i'th element in the right hand side b_i is

$$b_i = \sum_{j=1}^{\texttt{N}^2} a_{ij} x_j, \quad i = 1, \ldots, nA \cdot \texttt{p}.$$

For further information see the paper.

Examples:

Create a test problem and visualize the solution:

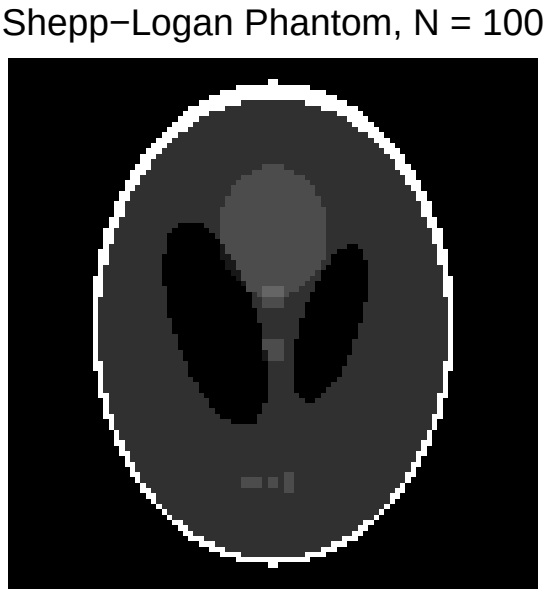
```
N = 64; theta = 0:5:179; p = 2*N;
[A b x] = paralleltomo(N,theta,p);
imagesc(reshape(x,N,N))
colormap gray, axis image off
```

See also:

`fanbeamtomo`, `seismictomo`.

References:

1. A. C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging*, SIAM, Philadelphia, 2001.



randkaczmarz

Purpose:

The randomized Kaczmarz iterative method.

Synopsis:

```
[X info] = randkaczmarz(A,b,K)
[X info] = randkaczmarz(A,b,K,x0)
[X info] = randkaczmarz(A,b,K,x0,options)
```

Algorithm:

For arbitrary starting vector $x^0 \in \mathbb{R}^n$ one step of the algorithm for `randkaczmarz` takes the following form:

$$x \leftarrow x + \lambda \frac{b_{r(i)} - \langle a^{r(i)}, x \rangle}{\|a^{r(i)}\|_2^2} a^{r(i)},$$

where $r(i)$ is chosen from the set $\{1, \ldots, m\}$ randomly with probability proportional with $\|a^i\|_2^2$. One iteration is defined as m steps.

Description:

The function implements the randomized Kaczmarz iterative method for solving the linear system $\mathtt{A}\,x = \mathtt{b}$. The starting vector is `x0`; if no starting vector is given then $\mathtt{x0} = 0$ is used.

The numbers given in the vector `K` are iteration numbers, that specify which iterations are stored in the output matrix `X`. If a stopping rule is selected (see below) and $\mathtt{K} = [\,]$, then `X` contains the last iterate only.

The maximum number of iterations is determined either by the maximum number in the vector `K` or by the stopping rule specified in the field `stoprule` in the struct `options`. If `K` is empty a stopping rule must be specified.

The relaxation parameter is given in the field `lambda` in the struct `options` as a constant. As default `lambda` is set to 1, since this corresponds to the original method.

The second output `info` is a vector with two elements. The first element is an indicator that denotes why the iterations were stopped. The number 0 denotes that the iterations were stopped because the maximum number of iterations were reached, 1 denotes that the `NCP`-rule stopped the iterations and 2 denotes that the `DP`-rule stopped the iterations.

Use of `options`:

The following fields in options are used in this function:

- `options.lambda` $=$ `c`, a constant satisfying $0 \leq \mathtt{c} \leq 2$. A warning is given if this requirement is estimated to be violated.
- `options.nonneg` Logical; if true then nonnegativity in enforced in each step.
- `options.stoprule`
  - `options.stoprule.type`

- `options.stoprule.type = 'none'`, where no stopping rule is given and only the maximum number of iterations is used to stop the algorithm. This choice is default.
- `options.stoprule.type = 'NCP'`, where the optimal number of iterations k_* is chosen according to Normalized Cumulative Periodogram.
- `options.stoprule.type = 'DP'`, where the stopping index is determined according to the discrepancy principle (DP).

- `options.stoprule.taudelta` $= \tau\delta$, where δ is the noise level and τ is user-chosen. This parameter is only needed for the stoprule type `DP`.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, compute 10 `randkaczmarz` iterations and show the last iterate:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
X = randkaczmarz(A,b,1:10);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`kaczmarz`, `symkaczmarz`.

References:

1. T. Strohmer and R. Vershynin, *A randomized Kaczmarz algorithm for linear systems with exponential convergence*, J. Fourier Analysis Appl., 15 (2009), pp. 262–278.

sart

Purpose:

The Simultaneous Algebraic Reconstruction Technique (SART) iterative method.

Synopsis:

```
[X info restart] = sart(A,b,K)
[X info restart] = sart(A,b,K,x0)
[X info restart] = sart(A,b,K,x0,options)
```

Algorithm:

For arbitrary $x^0 \in \mathbb{R}^n$ the algorithm for `sart` takes the following form:

$$x^{k+1} = x^k + \lambda_k V^{-1} \mathtt{A}^T W^{-1} (\mathtt{b} - \mathtt{A} x^k),$$

where $V = \operatorname{diag}\left(\|a^i\|_1\right)$ and $W = \operatorname{diag}\left(\|a_j\|_1\right)$.

Description:

The function implements the SART (Simultaneous Algebraic Reconstruction Technique) iterative method for solving the linear system $\mathtt{A}\,x = \mathtt{b}$. The starting vector is `x0`; if no starting vector is given then $\mathtt{x0} = 0$ is used.

The numbers given in the vector `K` are iteration numbers, that specify which iterations are stored in the output matrix `X`. If a stopping rule is selected (see below) and $\mathtt{K} = [\,]$, then `X` contains the last iterate only.

The maximum number of iterations is determined either by the maximum number in the vector `K` or by the stopping rule specified in the field `stoprule` in the struct `options`. If `K` is empty a stopping rule must be specified.

The relaxation parameter is given in the field `lambda` in the struct `options`, either as a constant or as a string that determines the method to compute `lambda`. The spectral radius is $\rho(V^{-1}\mathtt{A}^T W^{-1}\mathtt{A}) = 1$, and as default `lambda` is set to 1.

The second output `info` is a vector with two elements. The first element is an indicator, that denotes why the iterations were stopped. The number 0 denotes that the iterations were stopped because the maximum number of iterations were reached 1 denotes that the `NCP`-rule stopped the iterations, 2 denotes that the `DP`-rule stopped the iterations and 3 denote that the `ME`-rule stopped the iterations. The second element in info is the number if used iterations. The second element in `info` is the number of used iterations.

The struct `restart`, which can be given as output, contains in the field `s1` the estimated largest singular value. `restart` also returns a vector containing the diagonal of the matrix W^{-1} in the field `M` and the diagonal of the matrix V^{-1} in the field `T`. The struct `restart` can also be given as input in the struct `options`, such that the program do not have to recompute the contained values. We recommend only to use this, if the user has good knowledge of MATLAB and is completely sure of the use of `restart` as input.

Use of `options`:

The following fields in options are used in this function:

- `options.lambda`:
  - `options.lambda = c`, where `c` is a constant, satisfying $0 \leq$ `c` ≤ 2. A warning is given if this requirement is estimated to be violated.
  - `options.lambda = 'line'`, where the line search method is used to compute the value for λ_k in each iteration.
  - `options.lambda = 'psi1'`, where the method `psi1` computes the values for λ_k using the Ψ_1-based relaxation.
  - `options.lambda = 'psi1mod'`, where the method `psi1mod` computes the values for λ_k using the modified Ψ_1-based relaxation with $\nu = 2$. The parameter ν can be changed in line 362 in the code.
  - `options.lambda = 'psi2'`, where the method `psi2` computes the values for λ_k using the Ψ_2-based relaxation.
  - `options.lambda = 'psi2mod'`, where the method `psi2mod` computes the values for λ_k using the modified Ψ_2-based relaxation with $\nu = 1.5$. The parameter ν can be changed in line 411 in the code.
- `options.nonneg` Logical; if true then nonnegativity in enforced in each iteration.
- `options.restart`
  - `options.restart.M` = a vector containing the diagonal of W^{-1}.
  - `options.restart.T` = a vector containing the diagonal of V^{-1}.
  - `options.restart.s1` = 1.
- `options.stoprule`
  - `options.stoprule.type`
    - `options.stoprule.type = 'none'`, where no stopping rule is given and only the maximum number of iterations is used to stop the algorithm. This choice is default.
    - `options.stoprule.type = 'NCP'`, where the optimal number of iterations k_* is chosen according to the Normalized Cumulative Periodogram.
    - `options.stoprule.type = 'DP'`, where the stopping index is determined according to the discrepancy principle (DP).
    - `options.stoprule.type = 'ME'`, where the stopping index is determined according to the monotone error rule (ME).
  - `options.stoprule.taudelta` $= \tau\delta$, where δ is the noise level and τ is user chosen. This parameter is only needed for the stoprule types `DP` and `ME`.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, compute 50 `sart` iterations and show the last iterate:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
X = sart(A,b,1:50);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`cav`, `cimmino`, `drop`, `landweber`.

References:

1. A. H. Andersen and A. C. Kak, *Simultaneous algebraic reconstruction technique (SART): A superior implementation of the ART algorithm*, Ultrasonic Imaging, 6 (1984), pp. 81–94.

seismictomo

Purpose:

Creates a two-dimensional seismic tomography test problem.

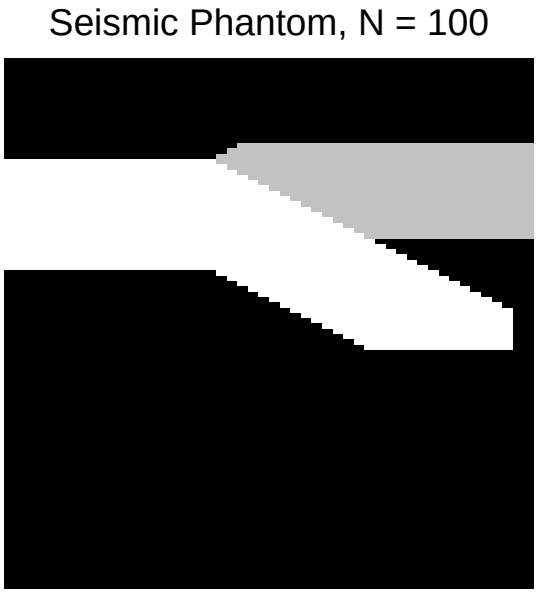
Synopsis:

```
[A b x s p] = seismictomo(N)
[A b x s p] = seismictomo(N,s)
[A b x s p] = seismictomo(N,s,p)
[A b x s p] = seismictomo(N,s,p,isDisp)
```

Description:

This function creates a two-dimensional seismic tomography test problem. A two-dimensional domain illustrating a cross section of the subsurface is divided into `N` equally spaced intervals in both dimensions creating $\mathtt{N}^2$ cells. On the right boundary `s` sources are located and each source transmits waves to the `p` seismographs or receivers, which are scattered on the surface and on the left boundary. As default `N` sources and 2`N` receivers are chosen. If the input `isDisp` is different from 0 then the function also creates an illustration of the problem with the used angles and rays etc. As default `isDisp` is 0.

The function returns a coefficient matrix `A` with the dimensions $\mathtt{p} \cdot \mathtt{s} \times \mathtt{N}^2$, the right hand side `b` and a created phantom of a subsurface as the vector `x` reshaped. The figure below illustrates the subsurface created when $\mathtt{N} = 100$. In case the default values are used, the function also returns the used number of sources `s` and the used number of receivers `p`.



Algorithm:

The element a_{ij} is defined as the length of the ith ray through the jth cell with $a_{ij} = 0$ if ray i does not go through cell j. The exact solution of the subsurface phantom is reshaped as a vector and the i'th element in the right hand side b_i is

$$b_i = \sum_{j=1}^{N^2} a_{ij} x_j, \quad i = 1, \ldots, \mathtt{s} \cdot \mathtt{p}.$$

Examples:

Create a test problem and visualize the solution:

```
N = 100; s = N; p = 2*N;
[A b x] = seismictomo(N,s,p);
imagesc(reshape(x,N,N))
colormap gray, axis image off
```

See also:

`fanbeamtomo`, `paralleltomo`.

symkaczmarz

Purpose:

The symmetric Kaczmarz iterative method.

Synopsis:

```
[X info] = symkaczmarz(A,b,K)
[X info] = symkaczmarz(A,b,K,x0)
[X info] = symkaczmarz(A,b,K,x0,options)
```

Algorithm:

For arbitrary starting vector $x^0 \in \mathbb{R}^n$ one iteration of the algorithm for `symkaczmarz` takes the following form:

$$x \leftarrow x + \lambda_k \frac{b_i - \langle a^i, x \rangle}{\|a^i\|_2^2} a^i, \qquad i = 1, \ldots, m-1, m, m-1, \ldots, 1.$$

Description:

The function implements the symmetric Kaczmarz iterative method for solving the linear system `A` $x =$ `b`. The starting vector is `x0`; if no vector is given then `x0` $= 0$ is used.

The numbers given in the vector `K` are iteration numbers, that specify which iterations are stored in the output matrix `X`. If a stopping rule is selected (see below) and `K` $= [\,]$, then `X` contains the last iterate only.

The maximum number of iterations is determined either by the maximum number in the vector `K` or by the stopping rule specified in the field `stoprule` in the struct `options`. If `K` is empty a stopping rule must be specified.

The relaxation parameter is given in the field `lambda` in the struct `options`, either as a constant or as a string that determines the method to compute `lambda`. As default `lambda` is set to 0.25.

The second output `info` is a vector with two elements. The first element is an indicator, that denotes why the iterations were stopped. The number 0 denotes that the iterations were stopped because the maximum number of iterations were reached, 1 denotes that the `NCP`-rule stopped the iterations, and 2 denotes that the `DP`-rule stopped the iterations.

Use of `options`:

The following fields in options are used in this function:

- `options.lambda`:
  - `options.lambda = c`, where `c` is a constant, satisfying $0 \leq$ `c` ≤ 2. A warning is given if this requirement is estimated to be violated.
  - `options.lambda = 'psi1'`, where the method `psi1` computes the values for λ_k using the Ψ_1-based relaxation.
  - `options.lambda = 'psi2'`, where the method `psi2` computes the values for λ_k using the Ψ_2-based relaxation.

- `options.nonneg` Logical; if true then nonnegativity in enforced in each step.
- `options.stoprule`
  - `options.stoprule.type`
    - `options.stoprule.type = 'none'`, where no stopping rule is given and only the maximum number of iterations is used to stop the algorithm. This choice is default.
    - `options.stoprule.type = 'NCP'`, where the optimal number of iterations k_* is chosen according to Normalized Cumulative Periodogram.
    - `options.stoprule.type = 'DP'`, where the stopping index is determined according to the discrepancy principle (DP).
  - `options.stoprule.taudelta` $= \tau\delta$, where δ is the noise level and τ is user-chosen. This parameter is only needed for the stoprule type `DP`.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, computes 10 `symkaczmarz` iterations and show the last iterate:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
X = symkaczmarz(A,b,1:10);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`kaczmarz`, `randkaczmarz`.

References:

1. Å. Björck and T. Elfving, *Accelerated projection methods for computing pseudoinverse solutions of systems of linear equations*, BIT, 19 (1979), pp. 145–163.

trainDPME

Purpose:

Training strategy to estimate the best parameter when the discrepancy principle or the monotone error rule is used as stopping rule.

Synopsis:

```
tau = trainlambda(A,b,x_exact,method,type,delta,s)
tau = trainlambda(A,b,x_exact,method,type,delta,s,options)
```

Description:

This function implements the training strategy for estimation of the parameter τ, when using the discrepancy principle or the monotone error rule as stopping rule. From test solution `x_exact` and the corresponding noise free right-hand side `b` `s` noisy samples are generated with noise level `delta`. From each sample the solutions for the given method `method` are calculated and according to which type of stopping rule is chosen in `type` an estimate of `tau` is calculated and returned.

A default maximum number of iterations is chosen for the SIRT methods to be 1000 and for the ART methods to 100. If the this is not enough it can be changed in line 74 for the SIRT methods and in line 87 for the ART methods.

Use of `options`:

The following fields in options are used in this function.

- `options.lambda`: See the chosen method `method` for the choices of this parameter.
- `options.restart`: Only available when `method` is a SIRT method. See the specific method for correct use.
- `options.w`: If the chosen method `method` allows weights this parameter can be set.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem. Then the parameter `tau` is found using training for DP and this parameter is used with DP to stop the iterations and the last iterate is shown.

```
[A b x] = paralleltomo(50,0:5:179,150);
delta = 0.05;
tau = trainDPME(A,b,x,@cimmino,'ME',delta,20);
e = randn(size(b)); e = e/norm(e);
b = b + delta*norm(b)*e;
options.stoprule.type = 'ME';
options.stoprule.taudelta = tau*delta;
[X info] = cimmino(A,b,200,[],options);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`cav`, `cimmino`, `drop`, `kaczmarz`, `landweber`, `randkaczmar`, `sart`, `symkaczmarz`.

References:

1. T. Elfving and T. Nikazad, *Stopping rules for Landweber-type iteration*, Inverse Problems, 23 (2007), pp. 1417–1432.

trainLambdaART

Purpose:

Strategy to find the best constant relaxation parameter λ for a given ART method.

Synopsis:

```
lambda = trainLambdaART(A,b,x_exact,method)
lambda = trainLambdaART(A,b,x_exact,method,kmax)
```

Description:

This function implements the training strategy for finding the optimal constant relaxation parameter λ for a given ART method, that solves the linear system $\mathtt{A}\,x = \mathtt{b}$. The training strategy builds on a two part strategy.

In the first part the resolution limit is calculated using `kmax` iterations of the iteration ART method given as a function handle in `method`. If `kmax` is not given or empty, the default value is 100. The first part of the strategy is to determine the resolution limit for the a specific value of λ.

The second part of the strategy is a modified version of a golden section search in which the optimal value of λ is found within the convergence interval of the specified iterative method. The method returns the optimal value in the output `lambda`.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, train to find the optimal value of λ for the ART method `kaczmarz` and use the found value, when 10 iterations of the method are computed. Finally the last iterate is shown:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
lambda = trainLambdaART(A,b,x,@kaczmarz);
options.lambda = lambda;
X = kaczmarz(A,b,1:10,[],options);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`trainLambdaSIRT`

trainLambdaSIRT

Purpose:

Strategy to find best constant relaxation parameter λ for a given SIRT method.

Synopsis:

```
lambda = trainLambdaSIRT(A,b,x_exact,method)
lambda = trainLambdaSIRT(A,b,x_exact,method,kmax)
lambda = trainLambdaSIRT(A,b,x_exact,method,kmax,options)
```

Description:

This function implements the training strategy for finding the optimal constant relaxation parameter λ for a given SIRT method, that solves the linear system $\mathtt{A}x = \mathtt{b}$. The training strategy builds on a two part strategy.

In the first step the resolution limit is calculated using `kmax` iterations of the iteration SIRT method given as a function handle in `method`. If `kmax` is not given or empty, the default value is 1000. To determine the resolution limit the default value of λ is used together with the contents of `options`. See below for correct use of `options`.

The second part of the strategy is a modified version of a golden section search in which the optimal value of λ is found within the convergence interval of the specified iterative method. The method returns the optimal value in the output `lambda`.

Use of `options`:

The following fields in `options` are used in this function.

- `options.restart`: See the specific method for correct use.
- `options.w`: If the chosen mehtod `method` allows weigths this parameter can be set.

Examples:

Generate a "noisy" 50×50 parallel beam tomography problem, train to find the optimal value of λ for the SIRT method `cimmino` and use the found value, when 50 iterations of the method is computed. Finally the last iterate is shown:

```
[A b x] = paralleltomo(50,0:5:179,150);
e = randn(size(b)); e = e/norm(e);
b = b + 0.05*norm(b)*e;
lambda = trainLambdaSIRT(A,b,x,@cimmino);
options.lambda = lambda;
X = cimmino(A,b,1:50,[],options);
imagesc(reshape(X(:,end),50,50))
colormap gray, axis image off
```

See also:

`trainLambdaART`