Mathematical models and methods for analysis of distributed power generation on market conditions

Schaumburg-Müller, Camilla

Publication date: 2008

Document Version
Publisher’s PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
Mathematical models and methods for analysis of distributed power generation on market conditions

Camilla Schaumburg-Müller

Kongens Lyngby 2008
IMM-PHD-2008-177
The liberalisation of electricity markets around the world which has taken place in recent years – and is still ongoing – has had several consequences for the various players in the markets affected. Typically, the tasks of production, transmission, and distribution of electricity which were often handled by so-called vertically integrated monopolies have been separated to varying degrees and are in liberalised systems handled by different players.

In the Nordic system, electricity is traded as a commodity on a day-ahead spot market where suppliers and consumers submit their bids for the following day and a common hourly electricity spot price is found. Intra-day markets for balancing power also exist. The raison d’etre for this type of market is that although supply and demand are balanced on a day-ahead basis, actual demand is impossible to forecast with complete accuracy. Thus on the day of operation actual demand and planned supply never match precisely. The system operator must then procure so-called balancing power in the intra-day market to maintain the physical balance of the system at all times.

The present thesis considers the effects of large amounts of distributed electricity generation in a power system subject to a liberalised market. In particular, the Danish electricity system is analysed in terms of four different focus topics which are considered in the six research papers presented and commented on in the thesis. The analyses range from planning the operation and/or bidding of single-technology units such as wind power turbines and local combined heat and power plants to analyses from a system point of view such as the interaction between the natural gas, district heating, and electricity systems, and the system operator dilemma of procuring reserve power well in advance as opposed to
purchasing the needed volumes in the intra-day balancing market.

The thesis itself provides an introduction to the Nordic power system and market with emphasis on the Danish situation. After presenting a few classic topics in power system operation, the situation post-liberalisation of the electricity markets is analysed and a literature review is given of the major topics of the thesis, setting the contributions of the thesis into perspective of previous work on related topics. Subsequently, the papers included in the thesis are summarised and commented upon and the main contributions are listed, before the thesis is concluded upon.
Den liberelisering af elmarkeder, der har fundet sted de seneste år – og som stadig pågår – har haft adskillige konsekvenser for de forskellige aktører på de ramte markeder. Typisk har production, transmission og distribution af el været håndteret af såkaldte vertikalt integrerede monopoler, men disse opgaver er i liberaliserede systemer delt op og håndteres af forskellige markedsaktører.

I det nordiske system handles elektricitet som en vare på et såkaldt day-ahead marked, hvor producenter og forbrugere indsender deres bud for det efterfølgende døgn og en fælles elektricitetsspotpris findes for hver time. Der findes også intra-dag markeder for balancekraft. Sådanne markeders eksistensgrundlag er, at selvom udbud og efterspørgsel for en given dag er udbalanceret dagen før, så er det umuligt at forudsige det faktiske forbrug med fuldkommen nøjagtighed. Derfor stemmer det faktiske forbrug og den planlagte produktion aldrig helt overens i driftdøgnet. For at rette op på de ubalancer, der opstår af denne grund, må elsystemoperatøren skaffe såkaldt balancekraft på intra-dag markedet for at opretholde den fysiske balance i elsystemet til enhver tid.

Nærværende afhandling omhandler konsekvenserne af store mængder decentral produktion i et liberaliseret elsystem. Helt specifikt analyseres det danske elsystem i perspektivet af fire forskellige fokusemner, der er indeholdt i de seks artikler, som præsenteres og kommenteres i afhandlingen. Analyserne spænder fra planlægning af drift og/eller budgivning af enkelttæknologier som vindkraftanlæg og decentral kraftvarmeværker til analyser fra et systemperspektiv som for eksempel samspilliet mellem naturgas-, fjernvarme- og elsystemerne samt systemoperatørens dilemma om at sikre sig reservekraft på forhånd eller satse på, at de nødvendige mængder kan skaffes i intra-dag balancemarkedet.
Selve afhandlingen indeholder en introduktion til det nordiske elsystem og elmarked med fokus på den danske situation. Efter at præsentere nogle få af de klassiske emner indenfor drift af elsystemer, analyseres situationen post-liberalisering og en litteraturgennemgang gives for afhandlingens hovedemner, som sætter afhandlingen forsørgningsbidrag i perspektiv i forhold til tidligere forskning indenfor relaterede emner. Slutteligt resumeres og kommenteres de vedfægtede artikler og hovedbidragene opremes inden der rundes af med en konklusion.
This thesis was prepared at Informatics Mathematical Modelling, the Technical University of Denmark in partial fulfillment of the requirements for acquiring the Ph.D. degree in engineering. It was financed by the Western Danish power system operator, Eltra\(^1\), for which I am grateful.

The thesis considers different aspects of mathematical modelling of the Nordic electricity system (with emphasis on the Danish system), primarily issues regarding distributed generation. The main focus is on modelling local combined heat and power (CHP) plants acting on the electricity spot market, however aspects of wind power prognosis making and power reserve management as well as the interaction between the electricity, district heating, and natural gas systems via local CHP plants are also considered.

The thesis consists of a summary report and a collection of six research papers written during the period 2003–2007, and published elsewhere.

Kgs. Lyngby, February 2008

Camilla Schaumburg-Müller

\(^1\)Now Energinet.dk
Papers included in the thesis


Acknowledgements

I gratefully acknowledge the help and inspiration provided by my de facto supervisor Hans Ravn without whom the present work would never have come into being. Thanks are also due to Peter Børre Eriksen, Henning Parbo, and particularly Jens Pedersen at Energinet.dk for their patience in answering questions regarding the intricacies of the Danish electricity system.

Thank you to Professor Lennart Söder and Senior Researcher Hannele Holttinen for accepting the task of external examiners at my thesis defense – or “grill party” as Lennart would put it. And, naturally, thank you to Lennart’s lovely mother Barbro who housed me during my stay in Stockholm and to Lennart’s group of ph.d. students at KTH who took me in as one of their own.

My gratitude also goes to my various co-authors – Hans Ravn, Claus S. Nielsen, Trine Krogh Kristoffersen (who also showed me a lovely time in Aarhus), Lars Bregnbæk, Jannik Blok-Riisom and Sarah Nørholm Straarup. To the latter three I am particularly grateful for letting me impose my taste in music on them during their separate times as my ‘roommates’ at DTU.

Thanks are also due to the OR group at IMM for innumerable lively lunches and general good company. Both will be sorely missed. Further thanks go again to Hans Ravn and to my parents for patiently proofreading the thesis, providing valuable input and corrections.

Finally, Morten, thank you for bearing with me during this whole process, for swinging the whip when needed (which towards the end was often), and for being such a wonderful shoulder to lean on. This thesis might still be languishing in
limbo if not for you. Joakim, thank you for your infallible ability to cheer me up, for your cuddles and smiles and crazy antics. And last, but certainly not least, my gratitude to Joakim’s little sister who has been kind and understanding enough to stay unborn while I put the last revisional touches on the thesis and sent it off to print.

Til Morten, Joakim & Joakim’s lillesøster.
Contents

Summary
Resumé
Preface
Papers included in the thesis
Acknowledgements

1 Introduction
  1.1 Background and motivation
  1.2 The Nordic power system
  1.3 Focus topics
  1.4 Overview

2 Topics of optimisation in power systems
## Contents

2.1 Classic problems in power system operation ........................................... 15
2.2 Problems arising in liberalised power systems ................................. 19

### 3 Contributions

3.1 Wind power prognosis making .............................................................. 27
3.2 Operation of local CHP on market terms ............................................. 30
3.3 Power reserve management ................................................................. 39

### 4 Conclusions

4.1 Summary ................................................................................................... 43
4.2 Main contributions .................................................................................. 45
4.3 Closure ..................................................................................................... 47

### A Two wind power prognosis criteria and regulating power costs 49

A.1 Abstract .................................................................................................. 49
A.2 Introduction ............................................................................................ 50
A.3 Wind power prognosis and criteria .......................................................... 50
A.4 Analysis of regulating power costs ............................................................ 54
A.5 Numerical results .................................................................................... 56
A.6 Concluding remarks ................................................................................ 60

### B Modelling Danish local CHP on market conditions 61

B.1 Abstract .................................................................................................. 61
B.2 Introduction ............................................................................................. 62
B.3 Modelling a local CHP plant ........................................ 64
B.4 Analysis ................................................................. 68
B.5 Numerical examples ................................................... 75
B.6 Simulation of an entire year .......................................... 81
B.7 Concluding remarks .................................................. 86
B.8 Acknowledgement ...................................................... 88
B.9 List of symbols .......................................................... 88

C A Stochastic Unit Commitment Model for a Local CHP Plant 89
C.1 Abstract ................................................................. 89
C.2 Introduction ............................................................ 90
C.3 Uncertainties when bidding on the spot market ................. 92
C.4 Unit commitment model .............................................. 94
C.5 Immersion heater ....................................................... 97
C.6 Simulations .............................................................. 99
C.7 Conclusions ............................................................ 103

D A Partial Load Model for a Local Combined Heat and Power
Plant ................................................................. 105
D.1 Abstract ................................................................. 105
D.2 Nomenclature .......................................................... 106
D.3 Introduction ............................................................ 107
D.4 Problem formulation .................................................. 109
D.5 Case study .............................................................. 117
### CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.6</td>
<td>Conclusions</td>
<td>124</td>
</tr>
<tr>
<td>E</td>
<td>Local CHP Plants between the Natural Gas and Electricity</td>
<td>127</td>
</tr>
<tr>
<td>E.1</td>
<td>Abstract</td>
<td>127</td>
</tr>
<tr>
<td>E.2</td>
<td>Introduction</td>
<td>128</td>
</tr>
<tr>
<td>E.3</td>
<td>The Natural Gas System</td>
<td>129</td>
</tr>
<tr>
<td>E.4</td>
<td>The Electricity and District Heating Systems</td>
<td>130</td>
</tr>
<tr>
<td>E.5</td>
<td>Combined Heat and Power Technologies</td>
<td>132</td>
</tr>
<tr>
<td>E.6</td>
<td>Modelling the Interconnected Systems</td>
<td>134</td>
</tr>
<tr>
<td>E.7</td>
<td>Simulation Results</td>
<td>136</td>
</tr>
<tr>
<td>E.8</td>
<td>Discussion and Conclusion</td>
<td>140</td>
</tr>
<tr>
<td>F</td>
<td>Power Reserve Management by Two-Stage Stochastic Program-</td>
<td>141</td>
</tr>
<tr>
<td>F.1</td>
<td>Abstract</td>
<td>141</td>
</tr>
<tr>
<td>F.2</td>
<td>Introduction</td>
<td>142</td>
</tr>
<tr>
<td>F.3</td>
<td>Power reserves</td>
<td>143</td>
</tr>
<tr>
<td>F.4</td>
<td>The power reserve management problem</td>
<td>145</td>
</tr>
<tr>
<td>F.5</td>
<td>The stochastic programming problem</td>
<td>151</td>
</tr>
<tr>
<td>F.6</td>
<td>Solution procedure</td>
<td>153</td>
</tr>
<tr>
<td>F.7</td>
<td>Computation results</td>
<td>155</td>
</tr>
<tr>
<td>F.8</td>
<td>Further research</td>
<td>158</td>
</tr>
</tbody>
</table>

**Bibliography** 159
This chapter provides a general introduction to the thesis. Initially, the background and motivation for the thesis are given and the Nordic electricity system is described in short both in physical and market terms. Also, the challenges facing the system are mentioned. The contributions of the thesis are briefly listed and finally an overview of the thesis is given.

1.1 Background and motivation

Originally electricity generation took place close to or at the site-of-use. Later it became technology feasible to connect production units with ever more distant load and this lead to the gradual interconnection of the transmission systems. Over time electricity generation, transmission, and distribution in most countries were operated as vertically integrated\(^1\) monopolies obligated to secure the supply of electricity at a reasonable price to all consumers. The obligation included frequency and voltage control, generation unit dispatch, and maintaining adequate reserve capacity. This type of system was later termed the regulated power system and the power generation units were often quite large and situated centrally in relation to consumers.

---

\(^1\)When generation, transmission, and distribution is handled by a single provider.
The oil crises in the 1970’s emphasised the high dependence of most western countries on fossil fuels from abroad which had become prevalent and this realisation proved the nursing ground for new ideas as to how power systems should be structured. More precisely, the idea that electricity could be viewed as a commodity like any other and be bought and sold at a market. The theory was that inefficiencies or bad investment decisions in the system were paid by the consumers and the monopolies had become complacent in their handling of the situation. The solution, it seemed, was a liberalisation of the power systems by dividing the vertically integrated monopolies into separate entities for transmission, distribution, and generation, and by creating electricity markets to administer transactions between producers and consumers. This was assumed to e.g. help lower electricity prices by increasing price competition and in general improve efficiency (see also [3] and [37] for further details). It should be kept in mind, however, that at present many liberalised electricity markets are oligopolistic in nature and by necessity carefully watched by competition authorities for abuse of market power.

Another issue which has only had increasing focus since its emergence is the impact of electricity generation (and other heavy industries) on the environment. The oil crises forced utilities to consider different fuel types. This paved the way for research into renewable energy sources (RES), which in Denmark mainly meant wind power, and ways of producing energy more efficiently both financially and environmentally, such as combined heat and power (CHP) fuelled by e.g. waste and biomass. Both wind power and CHP have become markedly prevalent in the Danish power system and while wind power has received some notice, particularly in the form of how to predict the output of a given wind turbine or farm, CHP has led a fairly shadowed existence in literature. To the knowledge of the author no research has yet been done regarding the consequences of market liberalisation for CHP units.

The primary aim of the thesis is to model some of the most pressing issues that have arisen in conjunction with the liberalisation of the Danish electricity system. In particular these issues are the planning the operation of local combined heat and power on markets terms, the influence of wind power prognoses on regulating power demand, and the procurement of reserves by advance agreements versus purchase on the balancing, or regulating, power market.

1.2 The Nordic power system

The characteristics presented above apply to most liberalised power systems. The focus in the papers A-F, however, is on applications in the Nordic (partic-
1.2 The Nordic power system

1.2.1 The physical system

The Nordic countries, with the exception of Iceland, (i.e. Denmark, Sweden, Norway, and Finland) are directly connected via a common transmission grid, which may be seen in detail in Figure 1.1. Note, that the eastern and western parts of the Danish AC/DC electricity system are not directly connected, only via the neighbouring countries\(^2\). The western part (Jutland and Funen) which is part of the UCTE\(^3\) is connected to Norway, Sweden, and Germany and the eastern part (Zealand and the smaller isles) is a part of the Nordel\(^4\) system and connected to Sweden and Germany.

Power production in the Nordic countries is primarily a combination of hydro, nuclear, other thermal (e.g. coal, gas, oil, biomass), and wind power. The allocation of the various production types is depicted in Figure 1.2. Demand is generally higher in winter than in summer with typical diurnal and weekly variations as depicted in Figure 1.3.

Combined heat and power

In Denmark a great majority of the thermal power production is provided by combined heat and power units either in the form of one of sixteen central plants placed in or near the major cities or one of the approximately 415 local CHP plants which range in size from less than 100 kW to nearly 100 MW electricity output. Figure 1.4(a) illustrates the evolution from 1995 to 2005 in the distribution of production facility types and Figure 1.4(b) shows the same for fuel types used for power production.

The study of combined heat and power is not a topic that has had much prevalence in energy optimisation. This is probably partly due to the fact that not many countries possess the level of penetration of CHP that is present in Denmark. In the 1990’s there was a boom in the number of local CHP plants (cf. Figure 1.5), mainly due to government subsidies. The reasoning behind this

\(^2\)Although a connection across the Great Belt is scheduled by 2010 [40].
\(^3\)Union for the Co-ordination of Transmission of Electricity. UCTE is the association of transmission system operators in continental Europe.
\(^4\)The organisation of the transmission system operators in Iceland, Norway, Sweden, Denmark, and Finland
Figure 1.1: The Nordic transmission grid (image source: Nordel [76]).

Figure 1.2: Electricity production in the Nordic countries by type (2005 data, source: Nordel [77])
1.2 The Nordic power system

Figure 1.3: Total consumption in Denmark during a winter and summer week in 2006 (source: Energinet.dk [40])

Figure 1.4: Evolution of power production in Denmark by production unit type (left) and fuel type (right) during the years 1995–2005 (source: The Danish Energy Authority [29]).
promotion of local CHP was largely due to environmental concerns although there were also economic benefits from combining heat and power production\(^5\). Quite a large number of local (municipally owned) heat production facilities and district heating networks already existed throughout the country which could advantageously be used for combined heat and power production, utilising the heat, which is a waste product of power production, for heating purposes. The reality that a large percentage of the local CHP plants in existence in Denmark today are former local district heating facilities also has consequences for how the plants are operated. In most cases, the primary raison d’être for the small CHP plants is to supply heat to the local area. The local CHP plants are either owned municipally or by a local group of heat consumers.

Prior to the reform instituted in 2005 [80], producing and selling electricity at the feed-in tariff\(^6\) was often seen as little more than a means of compensation for the expenses incurred by heat production. As many local CHP plants consist of one or more CHP units, heat only boilers, and a heat storage facility, planning production before 2005 was a straightforward task: taking into account production and storage limitations, produce maximal output on the CHP units during peak and/or high price hours, store any heat not directly utilised to use for later demand and, if needed, produce any extra heat needed on the boilers. After January 1st, 2005, however, planning production for local CHPs is no simple task, the reason being that electricity prices are no longer known in advance to the local CHP producers and they must now take into account the fluctuating prices on the power exchange that may or may not be sufficient to cover their marginal production costs of CHP production of heat. The alternative is to produce the heat, which the local CHP plants are often contractually bound to supply, on purely heat producing units such as a boiler or by utilising their heat storage facility\(^7\) (if such a unit exists).

The transition of the local CHP plants from producing according to the fixed feed-in tariff to producing on market terms is the main motivation for the thesis. Indeed, four of the six papers included examine various instances of that very problem.

---

\(^5\)As the excess heat emitted when producing electricity is captured and utilised, CHP plants potentially reach total efficiencies of 85% or more compared to around 40–50% for conventional power production plants. Thus less fuel needs to be consumed to obtain the same amount of useful energy and, further, less pollution is produced for a given economic benefit.

\(^6\)The tariff was denoted the three-stage tariff due to the three load levels it encompassed (low, high, and peak load) each corresponding to a price. The pattern was the same all year round but for an evening peak during the winter half of the year. See Figure B.2 for visualisation.

\(^7\)Typically a large hot water tank.
1.2 The Nordic power system

Wind power

Denmark has traditionally\(^8\) been a pioneer with regards to wind power and this form of renewable energy along with combined heat and power plants which may be fuelled by waste or biomass have comprised the majority of RES in Denmark. A general policy of conserving energy when possible by e.g. isolating houses with increasing efficiency has meant that Denmark has had very little increase in total energy consumption over the last 30 years, despite large economic growth during the last 10 years. The current government plans to maintain this course by, among other initiatives, increasing the share of RES in the Danish system to 30% (cf. [28]).

As the majority of this increase is expected to be wind power there are certain existing issues that have arisen with a large share of wind power in the system which may be exacerbated. One of the most pressing issues is the impact of fluctuating and occasionally hard to predict wind power production on power system imbalances and consequently on the imbalance prices in the regulating market.

The term imbalance is in the thesis defined as follows. In the Nordic power system expected production and consumption are scheduled and reported to the

\(^8\)For example, in 1891 the Dane, Poul LaCour, was the first to build a wind turbine that generated electricity, cf. [4]
relevant transmission system operator so that the physical system is balanced (i.e. consumption is matched to supply) prior to the day of operation. However, as consumption is impossible to predict exactly and production units need not supply as scheduled, imbalances, i.e. mismatches between actual and planned consumption and supply, occur and it is the task of the TSO to constantly monitor the system and increase or decrease actual supply using the regulating market and/or power system reserves to match actual consumption.

This impact of predicted wind power production on system imbalances (as defined in the previous paragraph) and the consequences for the way the system operator manages reserves are the topics of the remaining two papers of the thesis.

### 1.2.2 The Nordic electricity market

In the Nordic\(^9\) electricity market there are two main types of trading: bilateral (i.e. trading directly between players) and via the Nordic power exchange, Nord Pool.

Nord Pool was initiated by Norway in 1993 with the other Nordic countries joining one by one: Sweden in 1996 (resulting in the first multi-national power exchange), Finland in 1998, western Denmark in 1999, and finally eastern Denmark in 2000. The markets of primary interest in this thesis are the so-called physical markets, namely the spot market and the regulating power market, both provided by Nord Pool.

The spot market is the day-ahead market for trading electricity. Hourly demand and supply bids for the coming 12-36 hours must be submitted no later than noon on the day prior to operation. The hourly intersection of demand and supply then determines the spot price. Occasionally there is a power deficit in an area ex ante and bottlenecks in the grid make transmission of sufficient power to the area impossible. Then the area is designated a high-price area, with a price higher than the system price. Correspondingly, an area with a power surplus that cannot be exported due to transmission grid bottlenecks is designated a low-price area, with a price lower than the system price. The typical diurnal and weekly variations in the demand are also often evident in the spot price. As hydro power comprises such a large part of the supply in the Nordic power system, however, the general level of the spot price relies heavily on the amount of water in the reservoirs, and thus on precipitation. Variations are typically designated dry, normal, and wet years.

---

\(^9\)In the following, 'Nordic' refers to Denmark, Sweden, Norway, and Finland, but not Iceland.
1.2 The Nordic power system

The regulating power market is an intra-day market that acts as a tool for the Nordic transmission system operators\(^\text{10}\) (TSOs) to balance the total consumption and generation of electricity real time\(^\text{11}\), as imbalances invariably occur during the 24-hour day of operation. If there is a deficit of power in an area there is a need for up-regulation and, conversely, if there is a power surplus there is a need for down-regulation. The TSOs buy up- and down-regulating power as needed during operation and an imbalance settlement follows post operation where the players responsible for the imbalances compensate the TSOs for their regulating power purchase expenses.

A more detailed description of the Nordic power exchange may be found in \([42]\).

1.2.3 Challenges

The transition from regulated to deregulated markets in many countries incurred new planning challenges\(^\text{12}\) for both the newly formed transmission system operators (TSOs) and production companies and some of these challenges are scrutinised in the thesis. Where previously dispatch planning of generation units was done by the system operator in order to meet expected demand, the focus of the production companies is to minimise expenses and maximise profits when bidding to the open electricity market. This applies to all production companies, both the oligopolistic companies with whole portfolios of generation units at their disposal and the price-takers in charge of a few or single units.

The liberalisation of the Danish power system began in 1997 with the subdivision of the western Danish utility, Elsam, into separate companies for transmission and generation, soon followed by a similar process for the eastern Danish utility, Elkraft. In contrast to many other oligopolistic power systems, Denmark actually has a rather large penetration of distributed generation in the form of local combined heat and power (CHP) plants and wind turbines. However, until recently these production types were termed ‘prioritised’ which meant that whenever they produced, the system operator was obligated to buy that production for later resale on the spot market. The local CHP plants sold their power at a fixed feed-in tariff and therefore produced accordingly, regardless of the market conditions, as previously described. The combination of power generated by the local CHP plants and the wind power could be quite substantial (particularly in the western Danish area – at times even adequate to supply the entire load. In 2006 this happened during 177 hours (cf. Figure 1.6)),

\(^{10}\)Which are the following: Statnett (Norway), Svenska Kraftnät (Sweden), Fingrid (Finland), and Energinet.dk (Denmark).

\(^{11}\)In order to maintain a stable frequency of 50 Hz.

\(^{12}\)See e.g. \([37]\) for an early view of the situation in the United States.
which created problems for the TSOs with surplus generation resulting in low (occasionally even zero) prices in the affected area.

As an attempt to rectify the situation, and as a part of the liberalisation still in progress, new legislation decreed that the feed-in tariff system would be abolished and all local CHP plants with a capacity larger than 10 MW should act on market terms from January 1st, 2005 (cf. [80], [2]). The transition to market terms was to be gradual, so plants with a capacity of less than 10 MW were given a two-year grace period from January 1st, 2005 where they might decide individually whether to operate on market terms or continue using the three-stage tariff. After the two years plants with a capacity larger than 5 MW were then obliged to operate on market terms while smaller plants could still use the three-stage tariff. Despite these changes the local CHP plants would still receive subsidies equivalent to what they received when using the three-stage tariff, regulated according to the electricity spot price.

A further change in the power system was that the two separate power system operators (Elkraft-System and Elitra) were to unite with each other as well as the natural gas transmission system operator (Gastra) to one single TSO for both electricity and natural gas called Energinet.dk (cf. [1]). The latter event was also a part of the similar liberalisation occurring for the Danish natural gas utility. The goal of having more distributed generation units acting on market terms was to increase competition, ideally resulting in lower prices for the consumers. The local CHP plants, rather than simply selling their power at the favourable feed-in tariff, were now faced the daunting task of bidding to the electricity spot market at a time when prices were unknown, while meeting the local heat demand which they were contracted to supply. It is precisely this issue which takes up the majority of the thesis (papers B–E).

The aforementioned large wind power capacity in the system coupled with the fact that although weather forecasting techniques have become quite advanced it is still very hard to accurately predict wind power production, often leads to imbalances between scheduled and realised production and consumption during the day of operation. In addition, the Danish government has recently declared a goal of at least 30% of consumption should be provided by renewable energy sources (RES) by 2025 [28]. It is very likely that the majority of those RES will be in the form of wind power, thus further exacerbating the above-mentioned imbalances. As previously explained, the TSO usually covers these imbalances with the use of regulating power. Paper A investigates the difference between focussing on predicting the energy content of the wind (i.e. the wind power produced) and focussing on the cost of buying regulating power to compensate for the prognosis errors.

Another consequence of the liberalisation process is the change in the way the
1.3 Focus topics

The main contributions of the thesis are presented in the papers A-F. These are hinted at in the previous section and described in detail in Chapter 3 but may briefly be listed as follows.

- Illuminating the multi-criteria nature of the problem of optimising wind power bidding when taking the influence on regulating prices into account (paper A).
• Modelling of a price-taking CHP unit on market terms with successive degrees of complexity, taking price uncertainties into account where so far CHP units have only been handled as general dispatch or without uncertainty, cf. Chapter 3 (papers B, C, D)\textsuperscript{13}.

• Study (bottom-up approach) of the interaction between the Danish electricity, district heating, and natural gas systems via the local CHP plants (paper E).

• Modelling the reserve vs. regulation question from the point of view of the TSO (paper F).

As may be seen from the list, the focus of the thesis lies on analyses relating to issues arising from distributed power generation in a liberalised electricity market. The papers presented in the thesis originate with planning the operation and bidding of single distributed technology types such as wind power and local combined heat and power and evolves to consider the consequences at a system level of such technologies acting on market terms. Figure 1.7 illustrates this evolution.

As regards the above-mentioned focus topics, the papers presented in the thesis concentrate on the modelling of the mentioned issues. With the exception of paper F the papers primarily rely on algebraic modelling tools, letting well-designed commercial software handle the solution of the models. The advantage of this approach is its flexibility, as even significant changes in a model do not require recoding a solution algorithm.

1.4 Overview

The thesis comprises four chapters and six research papers. Apart from the present introductory chapter there are two main chapters, and a concluding chapter. Of the six papers, four were published in conference proceedings (papers A-C), one has been accepted and one has been submitted for publication in journals.

The remainder of the thesis is organised as follows:

\textsuperscript{13}A slightly revised version of the unit commitment model presented in the papers C and D has in fact been implemented in practise as a trial with two local CHP plants of approx. 10 MW power production capacity each situated in western Denmark. This was done as a demonstration project overseen by Energinet.dk during October of 2003.
1.4 Overview

Figure 1.7: From single technology types to the system level – the evolutionary structure of the papers included in the thesis.

Chapter 2 gives a description of the topics of optimisation in power systems that are most relevant to the thesis, comparing the situations prior to and post liberalisation of the market.

In Chapter 3 the contributions of the thesis are presented in detail. The papers A-F are expounded upon and set into perspective of previous work in their respective areas of research.

Finally, Chapter 4 summarises and concludes the thesis.
Introduction
Chapter 2

Topics of optimisation in power systems

This chapter initially gives a short description of a few of the classic problems of operating a power system and how they have been handled in a regulated system. Subsequently, the focus is shifted to the present situation in many countries, i.e., the deregulated power system with a liberalised market. The changes in traditional problem types are discussed and new problem types are introduced. The typical challenges of such deregulated systems are presented with emphasis on the Nordic system in general and the Danish system in particular.

2.1 Classic problems in power system operation

The regulated power system is typically a public monopoly in which one (or more) operator(s) is responsible for both power production and transmission. This type of monopoly operates according to non-profit principles. The main task of the system operator is to ensure that demand is met by appropriate supply both on the short and long term (i.e., that the system is balanced) while incurring as few expenses as possible. A power system as the one described above is often termed a vertically integrated power system, as both transmission, generation, and distribution of power is handled by the same utility or by closely
related utilities.

The optimisation problems inherent in the operation of a regulated power system are naturally not limited to the topics listed below. Other problem types include fuel budgeting and planning, optimising power system security, maintenance scheduling, network operation and planning, etc. A more extensive list may be found in [55]. For a thorough description of many of the classic problems in the operation of a regulated power system, see e.g. [105]. In the following a short introduction is given to a few of the classic problems within power system planning and power production planning, namely the economic dispatch problem, the unit commitment problem, and production planning of hydro and hydro-thermal systems.

2.1.1 Economic or optimal dispatch

In a regulated power system the system operator owns multiple power production units\(^1\) must plan the order of dispatch\(^2\) for the units in order to meet a given demand (or load) while minimising operating costs. This problem is termed the economic or optimal dispatch problem and most power planning problems are in some way derived from this classic problem.

The mathematical formulation of the economic dispatch problem may in its simplest form be written as follows. Consider a system consisting of \(N\) power generation units. Let \(x_{it} \in X_{it}\) denote the power produced by unit \(i, i \in I = \{1, \ldots, N\}\) at time \(t, t \in T = \{1, \ldots, T\}\) where \(X_{it}\) is the feasible production space for unit \(i\) at time \(t\), and let \(C(x_{it})\) denote the cost of production on unit \(i, i \in I\) at time \(t, t \in T\) (typically a non-linear function of the production). Further, let \(d_t\) denote the total demand of the system at time \(t\). The question of supplying the required load while minimising production costs may then be formulated as

\[
\min \sum_{i \in I} \sum_{t \in T} C(x_{it}) \tag{2.1}
\]

subject to

\[
\sum_{i \in I} x_{it} \geq d_t, \quad t \in T \tag{2.2}
\]

\[
x_{it} \in X_{it}, \quad i \in I, t \in T. \tag{2.3}
\]

\(^1\)Classically these units are thermal, typically coal- or gas-fired, though later nuclear and hydro units have been included.

\(^2\)I.e. when a power production unit is set to begin operation.
Common extensions of the original economic dispatch problem are the addition of network transmission losses to the demand constraint and/or the inclusion of hydro power units in the power systems considered.

An overview of work performed on the problem from 1977 to 1988 may be found in [23]. Among the earlier work done on the economic dispatch problem, [94] consider real-time economic dispatch using the merit order method and [21] use a network flow approach to solve the problem when taking into account transmission flow limits. Recent years have seen a boom in the use of alternative optimisation methods such as particle swarm optimisation\(^3\) to solve various instances of the economic dispatch problem, see e.g. [50] and [62], and naturally the environmental consequences of the problem have seen increasing popularity, see e.g. [72] and [106]. However this may be, it is clear from the definition of the economic dispatch problem that it is not directly applicable in a liberalised power market, as it is no longer the system operator which handles the power production units.

### 2.1.2 Unit commitment

The unit commitment problem in energy production planning considers the task of scheduling generators with the objective of minimising operating costs subject to meeting system demand and adhering to given physical limitations. A thorough description of what the unit commitment problem may encompass is given in [10].

Where the economic dispatch problem mainly considers the system perspective, the unit commitment problem offers the opportunity of examining the behaviour of individual units in detail. A simple mathematical formulation of the problem may be given as follows. Once again let \(x_{it} \in X_{it}\) denote the power produced by unit \(i, i \in \mathcal{I} = \{1, \ldots, N\}\) at time \(t, t \in \mathcal{T} = \{1, \ldots, T\}\) where \(X_{it}\) is the feasible production space for unit \(i\) at time \(t\), and let \(C(x_{it})\) denote the cost of production on unit \(i, i \in \mathcal{I}\) at time \(t, t \in \mathcal{T}\). Set the binary variable \(z_{it}\) to 1 if unit \(i\) is online at time \(t\) and zero otherwise. Having defined the binary variable, let \(S(z_{it})\) denote the start-up costs for unit \(i\) at time \(t\). Once again letting \(d_{it}\) denote demand, the unit commitment problem may then be stated as

\(^3\)Particle swarm optimisation simulates the kind of social optimisation that takes place when a group of people tries to solve a problem and by interacting with one another in the process of working towards a solution their beliefs, attitudes, and behaviours change by moving toward one another in a sociocognitive space (i.e. towards a common, ideally optimal, solution).
The first constraint concerns the production of the required load, whereas the second constraint connects the production with the binary start-up variables where $x_{it}$ and $\bar{x}_{it}$ are the minimum and maximum production levels. The latter two constraints merely ensure that the variables $x_{it}$ and $z_{it}$ remain in their feasible regions. Further extensions often implemented in the model are ramping constraints, i.e. limiting the rate at which production may increase or decrease, as well as constraints regarding minimum up and down time, and fuel constraints. Another possibility is the inclusion of shut-down costs in the objective function.

A great amount of work has been performed on the unit commitment problem using many different formulations and solution methods. Stochastic programming versions of the problem may be found in e.g. [101] and [19]. In later years [96] solve the short-term unit commitment problem using simulated annealing, [20] use CPLEX and MINOS under GAMS to solve their formulation of the problem in which they approximate the quadratic cost function with a piecewise linear function rendering the unit commitment problem a mixed-integer linear problem, and [56] analyses the value for unit commitment of improved load forecasts. For comprehensively detailed overviews of work performed on the unit commitment problem, see e.g. [93], [57], and [83].

2.1.3 Hydro and hydro-thermal production planning

A type of production planning particularly relevant to the Nordic system is the planning of hydro and hydro-thermal power production units. Planning of hydro and hydro-thermal power production for units with reservoirs differs from pure thermal power production planning in that it to some extent provides the possibility of storing power in the form of controlling the level of the reservoir.
As mentioned previously, one of the extensions of the economic dispatch problem is the inclusion of hydro power in the system which e.g. is considered in the form of short-term hydro-thermal dispatch in [70].

More prominent perhaps are the unit commitment versions of the hydro and hydro-thermal planning problems, many of which employ stochastic programming to account for the uncertainties inherent in planning when to produce and when to store power (using water reservoirs) for later use. For instance, [84], [79], and [17] all consider the aspect of uncertain demand, the former for a pure hydro unit and the latter two for a hydro-thermal unit, whereas [52] extend their unit commitment model for a hydro-thermal with the possibility of considering – in addition to uncertain load – aspects such as reserve policies, staggered fuel prices\(^4\), and non-linear start-up costs.

As the following section will illuminate, however, the liberalisation that has taken and is taking place in many power systems skews the focus of the above-mentioned topics and introduces new ones.

### 2.2 Problems arising in liberalised power systems

The transition from a traditional vertically integrated system to a liberalised system changes the perspective of the issues mentioned above and gives rise to new ones. [81] and [82] explain the restructuring of the Australian power market and [42] details experience with the Nord Pool design and implementation. For further information on the liberalisation process of e.g. the markets of England & Wales, New Zealand, and California, see [3].

The deregulated or liberalised power system typically consists of one or more independent transmission system operators (TSOs) which, among other tasks, are in charge of balancing the physical power and maintaining the correct frequency at all times in a given area, one or more distribution system operators, and (ideally) several independent production companies that act on the electricity market and are connected to either the transmission or distribution networks. The distinction between transmission and distribution networks is typically made by voltage levels with transmission networks being high voltage and distribution networks being low voltage. An early analysis of the impact of the liberalisation process on power systems operation may be found in [37].

\(^4\)The bigger the fuel purchase, the lower the price per unit of fuel.
In line with the increasing amount of wind power in the Nordic (and particularly the Danish) system several issues have arisen which the system operator needs to take into account. These issues primarily concern the integration of large amounts of wind power (typically from off-shore wind farms) into the transmission system, prognosis making in order to predict the amount of wind power on a day-ahead basis and thereby implicitly considering a third issue, namely the influence of wind power on the spot and regulating markets. These issues are discussed in Section 2.2.1.

As it is no longer a task of the system operator to produce power, both the economic dispatch problem and the unit commitment problem in their various incarnations have become the problem of the individual production companies, with the additional motive of maximising profits as opposed to merely meeting the expected demand of the consumers. As opposed to exclusively planning production after a least cost principle, both problem types have now become part of one greater problem, namely how to bid into the electricity spot market. This problem is examined from various angles in Section 2.2.2.

### 2.2.1 Consequences of large-scale wind power production

Wind power has become increasingly popular in recent years as a means to avoid emissions of greenhouse gases from thermal production, especially for those countries for which other renewable energy sources (RES) such as hydro power generation\(^5\) is not practicable. The penetration of wind power is by far the largest in Europe (primarily Germany, Denmark, and Spain)\(^6\) although North America is a respectable second (cf. [4]).

However, such a degree of penetration comes at a price. Large wind power farms (often placed off-shore) must be integrated physically into the power grids and in the Nordic system the large fluctuations of power output from the wind farms also influence the power market. These issues are the topic of the present section.

---

\(^5\) Typically found in countries with mountainous regions in which snow melt provides the main source of power.

\(^6\) As of 2003, around 74% of the worldwide wind power capacity was installed in Europe (cf. [4]). Indeed, the present Danish government has formed a policy of at least 30% of consumption being provided by RES – primarily wind power – by the year 2025 (cf. [28]).
2.2 Problems arising in liberalised power systems

Integration into and influence of wind power on the Nordic power market

The emergence of large-scale off-shore wind farms gives rise to some practical integration issues. It may necessitate reinforcements in the local power grid depending on the capacity of the wind farm such as described in e.g. [88] and [24]. The at times volatile behaviour of wind power may also cause voltage instability or other faults in the system, see e.g. [5], [4], and [61].

As the variable costs for wind power production are almost negligible in comparison to thermal production, large amounts of wind power in the system must influence the day-ahead electricity spot prices in a negative fashion. Thorough discussions of the impact of large-scale wind power on the Nordic system may be found in [58] and [4].

Another consequence of the volatile nature of wind power production is the large imbalances that may occur in the system. These imbalances are handled during the day of operation by the TSO using the regulating market, which is an intra-day market, but ultimately paid for by the production company. In the case of very large imbalances, the TSO also has the opportunity of activating manual reserves although these are costly, as the regulating price must be paid for the volume activated in addition to a reservation payment. In paper F the trade-off between the safe but expensive reserves and the typically cheaper regulating market which cannot guarantee sufficient capacity in the case of major imbalances is analysed.

Limited work has been performed in the area of power reserve management, certainly from the point of view of the TSO. In [104] the authors consider the aspect of power reserve management from the point of view of a producer and a stochastic optimisation problem is formulated for the coordination of bidding strategies in day-ahead and reserves markets. Also, [102] use a stochastic programming approach to optimise bidding as a price-taker to both in the day-ahead market as well as the reserve and balancing markets. There are, however, similarities to the model formulated in paper F, as both market prices and volumes are determined within the model. In [107] the focus is on the pricing as well as the procurement of reserves. A so-called capacity-reliability analysis, which relates the available capacity to the probability of reserves shortage, is performed using a stochastic model based on welfare maximisation. Finally, in [31] a in-depth analysis of the consequences of a large share of wind power production in an interconnected system on the needs of power reserve management is given.
Wind power prognosis making

An alternative approach from the point of view of the producer to meet the consequences of wind power fluctuations is to create a good prognosis in order to anticipate wind power production properly, and thus be spared many of the economic consequences of forecasting wrongly.

Wind power prognosis making is an extensive discipline which is of great use to both regulators/TSOs and production companies. The volatility of the wind power has a consequence for the generators in the form how to bid in the liberalised market and in the form of imbalance costs. The latter arise when e.g. the weather forecast indicates that a wind front will pass the generation units at a given time and wind power is bid to the day-ahead market in specific hours according to the forecast. An error in the forecast as to the timing of the wind front would mean that the generators must compensate the TSO for purchasing positive or negative balancing power during the day of operation according to whether the wind power prognosis had respectively underestimated or overestimated actual production.

Different methods have been implemented to create such prognoses, e.g. time series analysis and artificial intelligence (such as in [95] which uses wind speed and direction forecasts as input parameters). Using the former approach involves considering which criteria to use when minimising the error of the power curve\(^7\). This choice of criteria is analysed in [74] which leads directly to paper A where the consequences of the various choices are analysed. In a similar consideration of bidding wind power to an electricity spot market, [11] uses Markov probabilities to reduce imbalance costs whereas [97] creates a model for trading wind power in competitive markets.

Detailed overviews of short-term prediction for wind power production are given in [65] and [51], and descriptions of various wind power forecasting tools used in the Danish and German systems may be found in Chapter 17 in [4].

2.2.2 Bidding under uncertainty

In line with the fact that the task of power generation has moved from the vertically integrated monopoly to the independent generating companies, so has the problem of production planning. In the liberalised market the electricity spot price is formed by matching demand and supply bids for each hour of a given

\(^7\)I.e. the wind power production of a given turbine as a function of the wind speed. See Section 3.1.1 for details.
2.2 Problems arising in liberalised power systems

day. Where previously power was sold at a price that merely ensured financial break-even, now the production companies focus on maximising their profits. The main concern here is that the bidding to the power market must be performed at a time when prices are unknown. Thus, a new type of uncertainty has been added to the unit commitment problem, which now faces the production companies rather than the system operator.

Optimal bidding for large production companies

Much research has been devoted to considering how production companies with a portfolio of production units should bid to the market. The problem of how to construct optimal bidding strategies for profit maximising production companies trading in a liberalised electricity market was initially addressed in [32]. An exhaustive literature survey of the early work on strategy bidding in competitive electricity markets is given in [33]. The approach to handling the task of bidding varies. Some attempt to approximate the behaviour of competing production companies and take that into account when creating a bid, e.g. by letting the unknown be represented by a probability distribution ([6]) or a supply function ([7], [103]).

The uncertainty inherent in bidding may also be viewed from a different angle as in [78], where stochastic programming is utilised to handle e.g. uncertain load and fuel prices for a hydro-thermal production company. Stochastic programming is also the tool used in [71] where the authors take risks into account for production companies participating in a pool-based single-buyer electricity market and in [64] a two-stage stochastic program is set up where the first stage concerns the unit commitment problem and the second stage is a fuel allocation linear program. The paper [25] explicitly considers an oligopolistic production company, where one of the mentioned tools for maximising profit is "the [production company]'s ability to modify the market clearing price". It should be noted, however, that as many liberalised power markets are oligopolistic in nature they are often carefully monitored by market regulators and/or competition authorities for abuse of market power. In [34] the authors simulate an oligopolistic pool-based market and by identifying different equilibrium patterns according to price-makers versus price-takers. Such tools may be used by the market regulator to identify exercise of market power.
Optimal bidding for price-taking production companies

The papers mentioned in the above paragraph thus consider mainly oligopolistic production companies although it is not always mentioned directly. Extensive research has also been performed on the opposite case, i.e. where the production company is small enough to be considered price-taker in the market. It may be argued that bidding to a spot market as a price taker ought not be difficult, as bids should simply be made at the marginal price of production. However, there are also strategic considerations when bidding to a spot market depending on whether the unit which is bid to the market is expected to be market clearing.

One approach to bidding as a price-taker in the market is to assume the spot price is well-known at the time of bidding and then focus on describing all the complexities of the production unit in the model, such as in [9]. However, the true uncertainty in bidding lies with the unknown spot prices at the time of bidding and this subject is the focus of most research in bidding strategy for price-takers.

In [73] the authors consider the 'offer stack' of the producer and attempt to find an offer stack approximating marginal production cost in a way that maximises profit. The expected spot price is in this case assumed given by a probability distribution. In [91] the authors analyse the difference between risk averse and risk seeking producers by constructing a number of price scenarios, finding the optimal response to each scenario and then letting the producer choose an appropriate offer stack for each hour, thereby constructing a bidding portfolio according to risk averseness. The optimisation implemented in this method thus only covers part of the problem, in that it is used separately for each scenario. In contrast, the method implemented in papers B, C and D takes all the various price scenarios into consideration in a simultaneous optimisation. The approach in [26] is similar to the one implemented in B, C and D although the price alone is considered uncertain.

In contrast to the stochastic programming approach it is also an option to use time series models (cf. [14]) to forecast next-day electricity spot prices as it is done in e.g. [15], [75], and [27]. Then one might utilise the model as deterministic input data in a regular unit commitment model.

---

8 Defined as "consisting of a finite set of price-quantity pairs, indicating that the generator is willing to produce those quantities at the corresponding prices" [73].

9 Shown in previous work [75] to follow an approximate Lognormal distribution.
2.2 Problems arising in liberalised power systems

Demand-side bidding

The above-mentioned work all consider the point of view of a power producer, i.e. the supply side of bidding to an electricity market – only few have actually considered the demand side. The difference between supply and demand bidding lies mainly in the flexibility of the market participant. The producers readily adapt to price signals in the market while still endeavouring to maximise their profits, whereas the majority of the consumers, presently at least, display little to no price elasticity. In [67], the authors use stochastic programming for allocating bids to the relatively stable long-term market versus the more volatile short-term market. [43] considers a retailer acting on both the Norwegian spot and regulating markets. It should be noted that the retailer is assumed to have end users with price-sensitive demand which, as mentioned above, is not often the case. The Norwegian market is also the subject of [85] where the authors conclude that retailers ought to bid for slightly less than their expected demand in order to profit in the regulating market. The reason this behaviour is not observed is that the system operator Statnett requires the purchasers to bid for their expected demand. Similar to [43], [68] is concerned with optimal purchase allocation for dual electricity power markets and conducts a case study based on data from the U.S. Californian market.

Combined heat and power

Comparatively little work has been done regarding production planning of combined heat and power plants – especially in a liberalised market. The issue is however of particular interest in the Danish system as the penetration of both centralised and especially local CHP, which was mentioned in Chapter 1, is quite large.

The work regarding the optimal operation of CHP units has not yet progressed as far as the work on regular thermal units or indeed hydro and hydro-thermal units, which have been examined using e.g. stochastic programming. Indeed, a number of the papers on CHP concern the economic dispatch problem (cf. Section 2.1.1) of a system of CHP units. For example, [54] decompose the problem into subproblems: heat dispatch and power dispatch, and use Lagrangian relaxation techniques to solve the dispatch problem iteratively. [100] implement an ant colony search algorithm to solve the problem, and [16] use a genetic algorithm approach. [22] use stochastic programming where both heat and power demand are treated as random variables and consider the problem multi-objective: minimise the total generation cost as well as the expected deviation for both heat and power generation.
Others focus on probabilistic production simulation of CHP systems (i.e. systems consisting of power and/or heat producing units) such as [99], [47], [66], and [41]. Such simulations are primarily of interest for large utilities and for policy-making and regulatory bodies. Only a few such as [36] and [8] consider the optimal operation of a singular CHP unit, the latter paper taking the planning of the district heating network into account. The point of view presented in the majority of the papers included in the thesis (papers B–D) is that of a single local CHP unit acting on market terms, i.e. with only an expectation of the electricity prices. Whereas this issue has been considered from various angles for regular thermal units as well as for hydro and hydro-thermal units (as described in the above sections), the price-taker CHP unit perspective is a new one.
This chapter lists the contributions made by this thesis via the articles presented in the appendices. The contributions cover three main areas, viz. wind power prognosis making, the operation of local combined heat and power plants under market conditions, and power reserve management from the perspective of the system operator.

### 3.1 Wind power prognosis making

Wind power prognoses usually aim at predicting the physical amount of wind power produced by a given (set of) wind mill(s) and are typically structured as depicted in Figure 3.1. However, the large percentage of wind power in the Danish system (particularly the western part) and the uncertainty inherent in predicting the volumes at the correct times up to 36 hours before production takes place, pose a source of significant system imbalances which the TSO must address during the day of operation.

Indeed, the Danish TSO Energinet.dk (then (in 2004), Eltra) has stated “A change in wind power is by far the most important source of unforeseen regulation...
in the Eltra system.”\textsuperscript{1} The western part of Denmark has the largest share of wind power\textsuperscript{2} and is thus the most affected, but a fair amount of wind power production also takes place in eastern Denmark\textsuperscript{3}. Living up to the proposal of the current government of at least 30% of consumption being provided by renewables – primarily wind power – by 2025 (cf. [28]) only exacerbates the uncertainty of the prognoses and thus the problem of balancing power in the system.

It should be noted that the Danish TSO is obligated to purchase power from a number of wind generation units. The power is subsequently bid to the spot market and the TSO therefore has a marked interest in accurate wind power prognoses for the spot market, as they must compensate for prognosis errors by purchasing regulating power during the day of operation. In paper A, presented below, a prognosis approach is constructed and evaluated which includes consideration of the cost of the consequences of prognosis errors in the form of regulating power purchases.

\textsuperscript{1}[4], p. 226.\textsuperscript{14}
\textsuperscript{2}Wind power constituted between 18 and 25\% of the power production in western Denmark in the years 2005–7 [40].
\textsuperscript{3}Between 9 and 13\% of the power produced in eastern Denmark during the years 2005–7 was wind power [40].
3.1 Wind power prognosis making

3.1.1 Two wind power prognosis criteria and regulating power costs (Paper A)

Typically, wind power prognoses focus on minimising the error compared to the realised production. One of the ways of achieving this is to make the power curve\(^4\) (cf. Figure 3.1) as accurate as possible. Optimising the prognosis thus usually entails minimising the sum of squared deviations (hence termed the quadratic criterion) and/or the sum of absolute deviations. However, the TSO (or indeed any party which bids to the physical spot market) is also interested in minimising the cost of the consequences of prognosis errors, i.e. the cost of purchasing regulating power to compensate for the errors.

The purpose of paper A is via simulations to support the proposition that when optimising wind power bids to the day-ahead spot market the TSO needs to take into account the regulating costs that may be incurred by erroneous prognoses.

The alternative criterion suggested in paper A considers the cost of erroneous predictions specified by a cost when the realised value of the wind power production lies above the prognosis (necessitating down regulation) and another cost when the realised value lies below the prognosis (necessitating up regulation). This is referred to as the cost \((c^+ : c^-)\) criterion and it is compared to the more traditional sum of squared deviations (referred to as the quadratic criterion) using wind measurements from Taastrup in eastern Denmark from April 9 to December 31, 2002. The cost \((c^+ : c^-)\) criterion is formulated for both the eastern and western Danish areas where \(c^+ : c^-\) corresponds to the ratio between the average yearly cost of down resp. up regulating in each area.

A bid curve for a location in eastern Denmark is then estimated. The bid curve is similar to a power curve in that it estimates wind power production of a given facility depending on the wind speed. However, the objective when estimating the bid curve is to discover the optimal power bid based on expected wind speed taking into account consequences for the regulating prices when over- or underestimating the expected wind power production.

The bid curve is estimated using both cost criteria as well as the quadratic criterion and results indicate that at low wind speeds, the cost criteria have a tendency to underestimate the production in comparison to the quadratic criterion and at high wind speeds the case is reversed. Comparing the performances of the two types of criterion it is clear that they are mutually conflicting, which essentially suggests that the objective to minimise when estimating the power

\(^4\)The power curve is the wind power production of a given turbine as a function of the wind speed. It is estimated based on numerous simultaneous measurements of both wind speed and the associated power generation.
curve should be multi-objective and include both cost and quadratic criteria.

In the paper it is assumed that variations in wind power production is the major cause of system imbalances when other factors such as generation unit failures, transmission line failures, or similar may also play a role. Indeed, in western Denmark the share of wind power capacity in the system is significant and – as the quote in Section 3.1 indicates – the assumption made in the paper is not unreasonable. The situation in eastern Denmark is less clear as the share of wind power is not quite as large. Nevertheless it is still significant enough that the assumption may also be justified in that area.

Thus the main contribution of the paper is a prognosis approach that takes into account the cost of regulating power which is incurred by erroneous wind power prognoses. It is shown by an example using Danish data from 2002 that the cost criterion is mutually conflicting with the more traditional quadratic criterion. As both criteria however are important to the TSO, constructing the final prognosis should be considered as a multi-criteria optimisation problem.

### 3.2 Operation of local CHP on market terms

As has previously been mentioned, one of the recent changes which the liberalisation process has wrought in Denmark is the gradual transition decreed to commence as of January 1st, 2005 [80] of local combined heat and power production from being sold to the transmission system operator according to the so-called three-stage tariff to being bid into the electricity market. There are several uncertainties inherent in such an act from the point of view of the local combined heat and power producers, not the least of which is whether the power they plan to produce is sold at all.

With a little over 400 local CHP facilities scattered throughout the country there are many producers for whom the issue of planning production on market terms is immediate. Papers B, C, and D relate directly to production planning of local CHP plants under uncertainty from the point of view of individual producers. The individual local CHPs are all considered price-takers in the market.

Paper E differs somewhat in that it considers a larger perspective. The focus of the paper is the interconnection between the Danish natural gas, district heating, and electricity systems in the form of gas-fired local CHP plants (of which there is quite a large share). The paper analyses the consequences for the natural gas consumption of the transition of local CHP plants to market conditions using simulation, comparing the results to the simulated consumption using the
three-stage tariff and to the historic consumption.

In the following, the papers B–E which in some form consider local CHP on market conditions are described both in terms of content and contributions.

### 3.2.1 Modelling Danish local CHP on market terms (Paper B)

The purpose of paper B is the construction of a model for production planning of a local CHP plant which bids into the electricity spot market on day-ahead basis where actual spot prices are unknown. In the paper, four variations of a basic linear programming model are presented and analysed in depth. In order to evaluate the performance of the model concept, yearly simulations are run on two of the variations and compared with the case of full or perfect information, i.e. when spot prices are known in advance.

The type of local CHP plant analysed in paper B is considered a price-taker in the electricity market and is a typical example of a small-scale Danish plant in that it possesses a CHP unit which produces electricity and heat, a heat boiler which only produces heat, and a heat storage facility in the form of a large hot water tank\(^5\). The primary goal of the local CHP plant is to supply the contracted heat to the area it is connected with. Therefore the objective is to minimise heat production costs and selling electricity produced in conjunction with the heat is a means of reducing that cost. However, as previously mentioned, electricity prices are unknown at the time of planning, wherefore a stochastic approach is adopted.

The cost of power production on the CHP unit differs according to whether the heat produced simultaneously can be utilised or not. It is only relevant for the local CHP to utilise the heat from power production when the profit of expected sale of electricity is greater than the production cost on the CHP unit. This is also compared to the cost of producing the heat demanded on the heat boiler. Thus there are two production costs that are relevant to consider: \( p_1 \) which is the power production cost when the simultaneously produced heat can be utilised and \( p_2 \) is the power production cost when the heat produced cannot be utilised. Note, that \( p_1 < p_2 \).

The hourly bids to be submitted to the power exchange consist of the power volume corresponding to the heat production plan for the CHP unit\(^6\) and the

\(^5\)The plant examples analysed in papers C through E are of a similar type.

\(^6\)Calculated using the fixed electricity-to-heat ratio of the CHP unit.
corresponding cost \( p_1 \) or \( p_2 \) according to expectations of the electricity spot price in the given hour. Spot price expectations are given by a set of scenarios, each of which has an associated probability and consists of 24 hourly spot prices. The heat demand is deterministic, simulated to emulate the typical diurnal and seasonal variations.

Four variations of the same basic model structure are examined. All versions minimise production costs subject to capacity constraints for the production units and meeting heat demand. The variations lie in whether the heat storage is explicitly specified and whether the cooling off of heat produced on the CHP unit is permitted. The latter was not permitted prior to the transition of local CHP to market terms but there has been some discussion of the relaxation of this rule to give the local CHP plants more room to manoeuvre in terms of planning, although such behaviour goes against the principle of combined heat and power production\(^7\). The models yield an optimal heat production plan for both CHP unit and boiler and a plan for the heat storage content. It should perhaps be noted that the models in the paper lack a specified capacity constraint for the heat boiler, however this was in fact implemented in the programming of the simulations and is included in all subsequent modelling of local CHP on market terms (papers C-E).

An analysis of the Lagrangian relaxation of the model with simple storage confirms the greater flexibility of planning that presents itself when cooling is allowed. A simple counter example also shows that the general level of the spot price as well as the pattern of diurnal variation has an influence on the optimisation.

An extensive simulation is then conducted to test the performance of the model with specified heat storage using spot price data from all of 2001 and 2002 as well as a slightly more complex prognosis structure which takes into account the differences between behaviour on weekdays and weekends. Given a chosen number \( N \) of scenarios the prognosis consists of the spot prices of the last \( N - 1 \) days of the same type as the one to be examined (each assigned equal probability) as well as an additional high price scenario (assigned a small probability) to ensure motive for bidding even though the prices in the remaining scenarios are low. Both the number of scenarios used in the prognosis and the general level of the spot price are varied and both the cooling and non cooling cases are examined. The simulations were compared to the perfect information case where the spot prices are known in advance and planning is done accordingly and the comparison yielded the following conclusions.

\(^7\)Note, that cooling off heat is not always physically possible for a given local CHP plant – at least not unlimited cooling
It was shown that the number of scenarios in the prognosis mainly has influence when the general price trend changes, e.g. in dry years when prices typically increase towards the end of the year. In such cases it was clearly a disadvantage to have a large number of scenarios in the prognosis.

The effect of the level of the spot price clearly varies depending on whether cooling is permitted or not. If cooling is permitted there is little deviation when prices are either very low or very high. In the first case nothing is bid and in the latter case full capacity is bid and any excess heat that may not be used directly or stored is simply cooled off. If cooling is not permitted the deviation is still small for low price levels. For high price levels costs for less accurate bids increase, as the heat produced must be either stored or utilised directly, and thus deviation escalates.

Apart from these conclusions, the major contribution of the paper is in fact the modelling of a local combined heat and power plant on market terms which the author has not been able to find any previous examples of.

### 3.2.2 A stochastic unit commitment model for a local CHP plant (Paper C)

The purpose of paper C is to create a unit commitment model for a local CHP plant and to enable the inclusion of additional technologies which work as a reverse of the CHP unit, i.e. technologies which convert electricity to heat. This is done by expanding upon the basic model of paper B.

Apart from the inclusion of unit commitment and associated starting costs for the CHP unit, a minimum production limit for when production does take place on the CHP unit is also implemented. Further, the model incorporates a more sophisticated bid price handling which allows for more graded bid volumes thus abandoning the $p_1/p_2$ price system of the model used in paper B. The objective of the model is to minimise production costs (now including starting costs for the CHP unit) subject to capacity constraints on the production units and the heat storage facility, unit commitment constraints on the CHP unit, and meeting heat demand.

It is assumed that production is constant within the hour, i.e. ramping\(^8\) within the hour is not considered. However, as a large portion of the western Danish local CHP plants are gas-fired and therefore able to ramp up and down very

---

\(^8\)The gradual increase and decrease of production.
The concept of an immersion heater which uses electricity to produce heat is introduced as an addition to the model which may become feasible with the new legislation. The idea is that when electricity prices are low, production of heat using the immersion heater may be preferable to production using the conventionally fuelled heat boiler or the CHP unit. This is illustrated in Figure 3.2. The immersion heater is implemented in the model as a unit basically the opposite of the CHP unit, but the nature of the implementation is such that e.g. a heat pump or similar technology could easily be considered instead – such a change is mainly dependent on altering the electricity-to-heat ratio appropriately.

Simulations are run using spot price data from the whole years 2001–2003 which cover the spectrum of Nordic normal and dry years (in terms of average or a large amount of hydro power available in the system). The simulations are conducted both with and without the immersion heater included in the model in order to analyse the effect of said technology. The simulations clearly indicate that the inclusion of the immersion heater in the model reduces the deviation from the perfect information case, i.e. the case where prices are assumed known in advance. This confirms the intuitive hypothesis that an extra heat production technology, which is cheap to use when the others (i.e. CHP unit and heat boiler)

---

9Rapidly, in this context, meaning less than 15 minutes from zero to full capacity.
3.2 Operation of local CHP on market terms

are not, will at least not increase and at best decrease overall production costs. It should perhaps be noted that cooling off heat is not permitted in the unit commitment model. The reason is that at the time of modelling legislation still did not allow this and the consequences of permitting cooling had already been examined in the linear model (cf. paper B).

A slight variation of the model presented in paper C was used for a demonstration project for Energinet.dk in 2003 where two local CHP plants volunteered to act as if they were operating on market terms. Energinet.dk acted as both system operator and 'Nord Pool' and throughout the month of October the two plants produced according to the schedules devised by the model. Initial difficulties and misunderstandings were brief and the experiment worked quite well. One of the areas where the demonstration project model differs from the one in paper C is that the handling of the bid prices encompasses a fixed set of prices close to the marginal production cost of the units, which the plant operators used when formulating the actual bids. Thus they were ensured that production costs at least were covered if the bids were accepted.

The main contribution of paper C is the unit commitment aspect incorporated in the modelling of a local combined heat and power plant on market terms, with the added possibility of including an immersion heater (or similar conversion technologies) in order to improve flexibility, and thus providing a possibility of reducing operating costs.

3.2.3 A partial load model for a local combined heat and power plant (Paper D)

The purpose of paper D is to advance the unit commitment model for a local CHP plant first presented in paper C by subdividing the variables and relevant parameters pertaining to the CHP unit into segments thus enabling the model to take into account such factors as varying emissions and electricity-to-heat ratio depending on the share of production capacity (the 'load') of the CHP unit. In the previous models (cf. papers B–C) the electricity-to-heat ratio was fixed corresponding to a so-called back pressure production unit. However, other types of CHP units have varying ratios depending on the load level and the model presented in paper D takes this possibility into account.

A further opportunity which arises with the segmentation of the load is the option of assigning specific parameters for emissions depending on the load as well as an associated cost which is included in the objective function. It is typically the case that production at full capacity is more efficient – also pollution-wise – than producing at, say, half load. Including emissions costs in
the objective function affords the option of taking the environment into account when planning production. Even if there is no immediate cost for a given level of emissions the opportunity of imposing a penalty for inefficient production is nevertheless implemented and may be weighed according to the wishes of the plant manager.

The heat demand is still deterministic – however as the heat storage and boilers of the local CHP plants are dimensioned to be able to supply the historical heat demand of the associated area there is not much of a point in creating e.g. stochastic heat demand to be met. Indeed, if the plant operator were insecure about the model utilising the heat storage to its limits, it is easily possible to reduce the ‘window’ of capacity by setting a positive lower limit and decreasing the upper limit from the maximum.

Simulations are conducted for the years 2003–2005 as well as part of 2006 using the same prognosis approach for the scenarios as described and implemented for papers B and C and the value of perfect information (VPI) is examined. The simulations are carried out both for the segmented model and for a version of the model with only one segment. Further, both types of simulations are conducted with both 5, 10, and 20 scenarios included in the model.

The results indicate that the inclusion of segmentation yields a greater flexibility in the model which reduces the VPI in all cases of comparison. Increasing the number of scenarios in the prognosis also improves performance but causes a great increase in computation time, which in a real-life situation might limit the number of scenarios out of necessity. Sudden changes in the price trend cause an immediate increase in VPI but the effect diminishes as soon as the new trend stabilises.

The main contribution of paper D is the segmentation of the parameters and variables associated with the power producing unit which enables the model to take varying efficiency and emissions into account for a combined heat and power plant.

3.2.4 Local CHP plants between the natural gas and electricity systems (Paper E)

The purpose of paper E is to analyse the role of gas-fuelled local CHP plants as the connection point of three systems: natural gas, district heating, and electricity. With the large amount of gas-fired local CHP in the Danish system, the question arises whether the transition to operation on market terms in the power market may cause a significant change in the consumption pattern of natural gas
for power production under the assumption that the heat consumption which the local CHPs are bound to supply is unchanged. This question may be viewed as the first half of a feedback loop between the power and natural gas markets which potentially could originate from the mentioned transition. The second half of the feedback loop relates to whether a change in the gas consumption pattern of the local CHPs may affect natural gas prices. Should variations in the natural gas prices occur this could yet again influence the consumption pattern of the local CHP plants and thus the feedback loop may be perpetuated.

As mentioned, the analysis presented in paper E only considers the first half of the feedback loop between the natural gas and electricity systems, i.e. how changes in market structure of the electricity system may influence changes in natural gas consumption, but not how changes in gas consumption may affect gas prices and thus perhaps the operation of the local CHP plants.

The model developed in paper B is used as the basis of a bottom-up model of the Danish gas-fired local CHP units\textsuperscript{10}. Two versions of the model are used: the stochastic version presented in paper B and a deterministic version which uses the fixed three-stage tariff (see Figure 3.3) as its price input. Both versions are simulated and, using data describing the location, type, and capacity of gas-fired CHP facilities in Denmark, the consumption results are aggregated to a number of network nodes matching the M/R\textsuperscript{11} stations of the natural gas transmission system. This yields an hourly consumption curve for approximately 50 locations throughout Denmark which is compared to historical consumption from four weeks spread out through the year 2003 (January, April, August, and October).

The difference in the consumption pattern for the two model types simulated (i.e. the deterministic tariff model and the stochastic spot price model) is clear in all four cases. During the January week, where heat demand is at its peak, the natural gas consumption in the tariff model is clearly influenced by the profitability (electricity-wise) of the high and peak load hours (see Figure 3.3). A similar consumption pattern is visible for the spot price model results but the deterministic nature of the tariff model allows for far greater volatility in the gas consumption pattern whereas the stochastic model 'hedges its bets' thus creating a more stable gas consumption pattern.

A similar observation may be made for the April week, although the evening consumption peak disappears along with the evening price peak for electricity in the tariff case. During the summer heat demand is practically non-existent. The amount of natural gas consumed in the two cases is practically identical but the patterns are quite dissimilar. Once again tariff consumption is limited to specific

\textsuperscript{10}Three traditionally gas-fired technologies are chosen for the analysis: gas engines, single-cycle steam turbines, and combined cycle facilities.

\textsuperscript{11}Metering and regulation.
(high load) hours, whereas spot price consumption is smooth – indeed, nearly constant. The results for the October week simulations once again confirm the above-mentioned trend.

Thus the simulation results clearly indicate a change in the natural gas consumption pattern of the local CHP plants as a consequence of the new terms of operation. The use of the spot price model leads to a less volatile consumption profile than when using the three-stage tariff model (this is especially clear in the January week) which may be explained by the fact that the spot price model is more ‘cautious’ in planning production whereas the deterministic three-stage tariff model is able to plan production precisely when it is most profitable with regards to selling the electricity and then making good use of the storage and heat boiler facilities to cover heat demand.

A concern expressed by the Danish TSO for natural gas\textsuperscript{12} was that the transition to market terms in the electricity market would make the local CHP plants act more erratically and thereby cause difficulties in operating the interconnected gas storages and transmission system due to the more sluggish physical nature of gas. However, the analysis performed in paper E implies that the transition in fact has the opposite effect. Rather than increasing the volatility of the natural gas transmission system, operation of the gas-fired CHP plants on market terms seems to have a smoothing effect on the natural gas consumption compared to operating according to the earlier three-stage tariff.

\textsuperscript{12}Then Gastra, now Energinet.dk [40].
The major contribution of paper E is the demonstration by analysis that the transition to operating on market terms for local CHP plants has a stabilising influence on their natural gas consumption. Although the paper only covers the first half of the feedback loop described above, it is the first instance of such a joint analysis of the natural gas, district heating, and electricity systems which the author has seen.

3.3 Power reserve management

In a power system, it is the responsibility of the TSO to ensure balance between supply and demand at all times. If there is a surplus of power the TSO must reduce supply or increase demand and if there is a deficit of power the TSO must increase supply or reduce demand. To this purpose the TSO has the choice between reserving power in advance which may then be activated (or deactivated) if needed, or purchasing the needed down- or up-regulating power in the intra-day regulating market\textsuperscript{13}. The former possibility is perhaps safer, as there is no guarantee that the desired volumes are available in the regulating market, but also more expensive.

3.3.1 Power reserve management using two-stage stochastic programming (Paper F)

Paper F presents a two-stage stochastic programming model for handling power reserve management from the point of view of the TSO. There exist several types of reserves in a typical power system which range from automatic reserves activated within a few minutes to emergency start-up plants which help rebuild the system in case of blackout. The reserves which are the focus of the model are the manual regulation reserves which, once contracted by the TSO, are the reserves that suppliers are obligated to bid in the regulating market and which may be activated within 10 minutes.

An example of a case when such reserves may be activated is when there is a large portion of wind power in the system. If the wind speed increases beyond a certain level, the wind turbines are shut down for safety reasons and suddenly a large amount of supply may vanish from the system. As it is virtually impossible to reduce demand sufficiently to meet the reduced supply in such cases, it is critical that manual reserves are available in the regulating market in order to

\textsuperscript{13}In practice it is nearly always supply which is reduced or increased on an intra-day basis, as very few power consumers make bids to the regulating market.
increase supply to the level prior to the safety shut-downs of the wind turbines. However, the TSO could also decide to take the chance that all necessary power in such a case is available in the regulating market. It is the decision where the added expense of reserving power in advance is weighted against the risk of insufficient supply/demand in the regulating market which is the focus of the paper.

The model presented in paper F is constructed in such a way that the first-stage decisions relate to how much power should be reserved in advance and the second-stage decisions pertain to volumes purchased in the regulating market. In both cases both up and down regulation may be procured. At the time paper F was written, the regulating market was a pay-as-bid market which is therefore one version of the model considered. However, plans were underway to change the regulating market in such a way that marginal pricing determines the regulating market prices, wherefore a second version of the model which encompasses such a pricing regime is also constructed.

The scenarios considered in the model are the differences between supply and demand in the system and an autoregressive process is used to create the scenarios. The model is solved for a varying number of scenarios using data from the western Danish system from June 2006, just prior to the transition from pay-as-bid to marginal pricing in the regulating market. Further, for the sake of comparison to a deterministic approach, the expected result of using the expected value solution\textsuperscript{14} (EEV) is calculated.

It was the case for both the regular simulation runs and the EEV runs that the marginal pricing version of the model resulted in higher optimal costs than the pay-as-bid version. Further, although CPU times where insignificant for the EEV problem, the optimal costs were far higher than the stochastic programming problem both for marginal and pay-as-bid pricing. This was despite the fact that no reserve bids were activated in the EEV problem whereas the opposite was the case for the stochastic programming formulation. Thus the computational results indicate that for both pricing versions of the model reserves are highly necessary to cover imbalances in an optimal way.

It may be argued that power reserve management could be said to affect spot market trading, as reserve purchases withhold resources from the spot market. In paper F, however, it is implicitly assumed that the spot market is not seriously affected.

The major contribution of this paper is the modelling of the power reserve management problem from the point of view of the system operator. A further

\textsuperscript{14}Where the stochastic supply and demand differences are replaced by their expected values.
point is that both the traditional pay-as-bid pricing and marginal pricing of the regulating power is considered, rendering the model flexible for different pricing approaches in the market.
This thesis has presented various problems within the area of power and CHP system optimisation, with emphasis on the workings of the Danish system, and proposed mathematical models for solving these problems. This chapter summarises the work, presents the main contributions of the thesis, and outlines future avenues worth investigation.

4.1 Summary

The focus of the thesis has been on analyses in relation to distributed generation in a liberalised electricity market. The papers presented in the thesis examine the subject from various angles. The papers evolve from planning the operation and bidding of single technology types (wind and local CHP) to considering distributed generation and the consequences thereof at a system level. This process is illustrated in Figure 4.1.

Paper A describes the possible consequences of erroneous wind power prediction in Denmark in terms of regulating power costs and considers how to bid wind power to the market when taking these consequences into account. The introduction of local Danish combined heat and power to the market (papers B–D)
Figure 4.1: The evolvement of the papers in the thesis: from single technology types to the system level.

should help compensate for system imbalances\(^1\) incurred by large amounts of wind power in the system. This is a consequence of the fact that when bidding to the electricity spot market, the local CHPs now also take the electricity demand into account, as opposed to merely considering heat demand while being ensured a good price for any power produced during certain hours of the day\(^2\). The consequences at the system level of distributed generation on market terms are examined in papers E and F. In paper E the focus lies on the combined natural gas, district heating, and electricity systems and the changes incurred when the large share of gas-fired local CHP in Denmark begin operating on market terms. In paper F system reserves are considered and the dilemma for the TSO of purchasing reserves in advance and paying extra to activate them versus purchasing the reserves on an intra-day basis in the regulating market (with the risk that supply is insufficient) is examined.

The thesis provides an introduction to the Nordic power system and market with emphasis on the Danish situation. Also, after presenting a few classic topics in power system operation the situation post-liberalisation of the electricity markets is analysed and a literature review is given of the major topics of the thesis, setting the contributions of thesis into perspective of previous work on

---

\(^1\)See Section 2.2.1.

\(^2\)Viz. the afore-mentioned three-stage tariff (see e.g. the section on combined heat and power on page 4).
4.2 Main contributions

Wind power and combined heat and power dominate electricity production in Denmark and while wind power has received a fair amount of attention, particularly in the area of forecasting production, not much research has been conducted on combined heat and power.

The accuracy of wind power prognoses has great importance both to large-scale wind power generators bidding into the spot market and to the Danish TSO which handles the sale of most small-scale wind power production in Denmark. Both wish to avoid imbalances created by erroneous prognoses, in the first case because imbalances may be very costly financially and in the latter case because it increases the risk of system failure. One paper has been presented which concerns the nature of wind power prognoses and the main contribution of the paper be stated as:

**Paper A** An analysis which shows that accurately predicting the energy content of the wind and focusing on the cost of buying regulating power to compensate for prognosis errors are mutually conflicted objectives when estimating the power curve of a given wind power facility for prognosis purposes. As both criteria are desirable, the optimal estimation of the power curve should be considered a multi-objective problem.

The area of optimal operation of local combined heat and power units on market terms is quite new and groundwork in modelling of this topic is the major contribution of the present thesis. More specifically, the contributions regarding local CHP on market terms may for papers B–D in brief be summarised as follows:

**Paper B** The construction of a linear stochastic programming model for production planning under uncertainty of a local CHP plant on market terms. The local CHP plant is considered price-taker in the market and cases both with and without cooling are examined in several simulations using spot price data from the Nord Pool market. Variations in the simulations include the number of scenarios in the spot price prognosis and the general
level of the spot price. Results show that the impact of both types of simulations depended upon which type of year\(^3\) is simulated, as the impact of varying the mentioned factors is less for the normal year (2001) than the dry year (2002) which are simulated.

**Paper C** The construction of a stochastic unit commitment model based on the model from paper B, still with the local CHP acting as price-taker. An additional technology which converts electricity to heat, in this case an immersion heater, is incorporated in the model. Simulations of three years with and without the conversion technology are conducted and the results indicate that in each year costs are lowered with the conversion technology in place.

**Paper D** The extension of the stochastic unit commitment model presented in paper C to one where the production on the CHP unit is segmented in order to facilitate varying levels of efficiency and emissions depending on the level of production. This also enables the possibility of taking environmental issues of CHP production into account. Three and a half years are simulated and the value of perfect information is analysed for varying number of scenarios in the prognosis for the situation with and without segmenting implemented in the model. Although results do improve in some cases with more scenarios in the prognosis, computation times increase dramatically.

With such a large share of local CHP fuelled by natural gas it is relevant to consider the influence on the consumption pattern of natural gas when those local CHP facilities begin operating on market terms rather than selling their power at a fixed tariff. The main contribution of the fifth paper of the thesis is thus

**Paper E** Analysis of the interaction between the Danish district heating, natural gas, and electricity systems, as represented by gas-fired local CHP plants. The analysis is in the form of a comparison between the historic natural gas consumption and results obtained from simulation of a bottom-up model considering the pre and post liberalisation situation. The bottom-up model is based on the linear programming model from paper B using the known three-stage tariff for the pre liberalisation situation and the original stochastic model for the post liberalisation situation. Results indicate a more stable gas consumption pattern post liberalisation on part of the local combined heat and power plants.

\(^3\)I.e. dry, wet, or normal (see Section 1.2.2).
4.3 Closure

In a power system, it is the responsibility of the TSO to ensure balance between supply and demand at all times. If there is a surplus of power during the day of operation in comparison to the day-ahead schedules the TSO must reduce supply. Conversely, if there is a deficit of power during the day of operation in comparison to the day-ahead schedules the TSO must increase supply. To this purpose the TSO has the choice between reserving power in advance which may then be activated (or deactivated) if needs be and purchasing the needed down or up regulating power in the intra-day regulating market. The former possibility is perhaps safer, as there is no guarantee that the desired volumes are available in the regulating market. Reserve costs may be substantial, however there are also cases in which regulating power prices have peaked wildly (both positively and negatively), an event which typically occurs in cases of extreme shortage or surplus of power. The contribution of the thesis in this area is:

**Paper F** The construction of a two-stage stochastic programming model which works as decision support for the TSO in order to determine when to reserve power in advance as opposed to relying on purchasing the needed volumes in the regulating market. The model is formulated in two versions: one which handles pay-as-bid pricing in the regulating market and one which handles marginal pricing in order to accommodate the two different market types in play around the time of writing.

4.3 Closure

Apart from the contributions of the thesis in the form of the work presented in the attached papers A–F, there is another type of contribution amounting from the thesis (indeed, hopefully from any such thesis) which ought to be mentioned, namely in the form of a source of information and inspiration for researchers with similar interests. In fact, the main contributions presented in the thesis lend themselves to several possible extensions, some of which are presently being implemented.

During the research and writing of the papers related directly to optimal planning and bidding to the electricity spot market for a local combined heat and power plant (papers B–D), preliminary research was also conducted regarding the inclusion of bidding to the intra-day regulating market. The question of whether taking the regulating market into account would affect production planning for the spot market is an intriguing one. Also real-time planning for the regulating market during the day of operation while taking into account commitments already made to the spot market the previous day has definite appeal. The Danish TSO, Energinet.dk, continues this research today.
Another topic, on which preliminary research was conducted towards the very end of the study for the present thesis, is the construction of a time series model for the western Danish electricity spot price which, among possible other factors, should take the wind power forecast into account. The reason for considering the forecasted production and not the actual production is that the Nordic spot market is a day-ahead market and thus it is the expected production which is a factor in determining the price – the actual wind power production has influence on the intra-day regulating market. Such a time series model for the spot price could be used as input in the operation planning model for the local CHPs and the results compared to using the stochastic models presented in papers B–E of the thesis. The above-mentioned preliminary research has since lead to first a project and presently a master’s thesis study of such a time series model.

Regarding the interaction between the natural gas and electricity systems, further research has been conducted on the second half of the feedback loop discussed in Section 3.2.4, i.e. how changes in natural gas consumption due to local CHP operating on market terms may affect the prices of natural gas (if at all) and how this in turn may influence the optimal operational patterns for gas-fired local CHP plants. The results, along with analyses of several other issues involving the Danish natural gas system may be found in the report [46].

Thus the topics presented in the thesis have helped both initiate, diversify, and extend research on modelling of distributed generation on market terms and the consequences thereof.
Appendix A

Two wind power prognosis criteria and regulating power costs

A.1 Abstract

The objective of the present work is to investigate the consequences of the choice of criterion in short-term wind power prognosis. This is done by investigating the consequences of choice of objective function in relation to the estimation of the power curve that is applied in the prognoses. Basically, the choice is between focusing on predicting the energy content of the wind and focusing on the cost of buying regulating power to compensate for the prognosis errors. It will be shown that it can be expected that the two power curves thus estimated will differ, and that therefore also the hourly wind power production predicted will differ. In turn this will influence the operation and economics of the system.

The consequences of this are illustrated by application to the integration of wind power in the Danish parts of the Nordpool area, using recent data. Using a regression analysis the prices of regulating power will be estimated. Then the two mentioned power curves may be estimated using wind speed production from the numerical weather prediction model from the Danish Meteorological
Institute and the corresponding short term prognoses of wind power will be elaborated. From wind power production measurement the errors may be calculated. Combining this information it is possible to find the consequences of inconsequent use of prognosis criterion, i.e., using one criterion in estimating the power curve and another in assessing the quality of the prognosis.

A.2 Introduction

Associated with the introduction of wind power into the electricity system are two activities, making short-term wind power prognoses and neutralising the prognosis errors. The key point in the present paper is that the way the cost (in a general sense) of neutralising the prognosis errors is determined will have consequences for how short-term wind power prognoses should be made. Or, in other words, it will have consequences for the choice of criterion for evaluation of short-term wind power prognoses.

In the present work the objective is to investigate the consequences of the choice of criterion in short-term wind power prognoses. More specifically, the focus is on the consequences in the choice of objective function in relation to the estimation of the power curve applied in the prognosis. The criteria considered are presented in Section A.3 together with a brief description of the background for the problem. Section A.4 presents an analysis of the regulating prices, in both eastern and western Denmark throughout the year 2002. This analysis is then used as the base of one of the two types of criteria investigated in the case study given in Section A.5, using wind speed measurements and wind speed production data from Taastrup in eastern Denmark.

A.3 Wind power prognosis and criteria

In Denmark, the system operators (Eltra [40] in western Denmark and Elkraft System [40] in eastern Denmark) make a wind power prognosis each morning and make bids based on this prognosis to the Nordic electricity spot market, Nord Pool [86], for each of the 24 hours of the following day. The bidding procedure closes at noon and hourly market crosses are formed, determining the quantities of electricity traded each hour of the following day, as well as the associated price.

Prognosis errors on wind power production are a significant source of deviation from the spot market plan, and when taking into account the potential amount
of wind power available, particularly in western Denmark, this can make quite an impact on the regulating costs and prices in the system, as they are precisely the costs that the system operator must meet to smooth out imbalances in the system. See e.g. Holttinen et al. [59] for a description and analysis in relation to this.

It is therefore in the interest of the system operator, and others, to have as good a prognosis as possible available when bidding to the spot market, in order to avoid having to pay for either up or down regulation in the system. However, ‘good’ is a term often hard to define precisely, as several qualities may be desirable, but in practice hard to obtain simultaneously.

When evaluating the quality of wind power prognoses, typical criteria include minimising the root mean square error, the mean absolute error, and the mean error. All these criteria are well based in statistical theory for prognoses.

The choice of criterion for a wind power prognosis naturally depends on the type of decision making the prognosis is to be the base for. When a system operator needs to evaluate the quality of a wind power prognosis, three criteria, as stated in [74], arise naturally:

- The prognosis value of the wind power production should be close to the average of the realised values.
- The sum of deviations between prognosis values and realised values should be small.
- The prognosis should result in low cost of the consequences of prognosis errors.

The first criterion corresponds to minimising the sum of squared deviations (i.e., the mean or average value), and the second to minimising the sum of absolute deviations (i.e., the median). As for the final criteria, it concerns the cases when the cost of wind power prognosis errors is asymmetric, i.e., specified with a cost $c^+$ when the realised value of wind power production lies above the prognosis and a cost $c^-$ when the realised value lies below the prognosis, where $c^+ \neq c^-$. This would naturally relate to the regulating costs where the up and down regulating costs are $c^-$ and $c^+$, respectively.

This may be stated mathematically in the following manner. Given the observations $y_j, j \in J$, and letting $\beta$ denote the prognosis value, the objective would be to minimise
Two wind power prognosis criteria and regulating power costs

<table>
<thead>
<tr>
<th>Interval (m/s)</th>
<th>2-3 m/s</th>
<th>4-5 m/s</th>
<th>5-6 m/s</th>
<th>7-8 m/s</th>
<th>8-9 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.65</td>
<td>3.88</td>
<td>4.67</td>
<td>6.21</td>
<td>7.28</td>
</tr>
<tr>
<td>Median</td>
<td>2.50</td>
<td>3.80</td>
<td>4.60</td>
<td>6.20</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Table A.1: Means and medians of the illustrated intervals.

- $\sum_{j \in J} (y_j - \beta)^2$ for the first criterion,
- $\sum_{j \in J} |y_j - \beta|$ for the second criterion, and
- $\sum_{j \in J^-} c^- (\beta - y_j) + \sum_{j \in J^+} c^+ (\beta - y_j)$ in the third criterion,

where the subset $J$ of observations divides into three subsets such that $y_j < \beta$ for $J^- (\beta)$, $y_j = \beta$ for $J^0 (\beta)$ and $y_j > \beta$ for $J^+ (\beta)$. Note, that the second criterion is in fact a special case of the third criterion, with $c^+ = c^- = 1$.

In this paper, the focus lies on the first and third criteria, henceforth referred to as the quadratic and the cost $(c^+ : c^-)$ criterion, respectively.

If the distribution of the prognosis errors were symmetric, and $c^+ = c^-$ then there would be no difference between the two criteria. However, as illustrated in Figure A.1 and Table A.1, the distribution for the observation site (Taastrup in eastern Denmark) seems to be right skewed, i.e., the mean value is larger than the median. A contributing factor to this skewness is that there is a natural lower limit on errors in the meteorological forecast for wind speed, as wind speeds cannot be negative. The significance of this will become clear when considering the power curves based on the different criteria in Section A.5.

The data used here consists of wind measurements from Taastrup in eastern Denmark, from April 9 to December 31, 2002, provided by Elkraft System and the hourly wind speed prognosis for Abed (the location closest to Taastrup) from the Danish Meteorological Institute (DMI) [30] during the same period.

The wind power prognosis system considered here is built primarily upon two sets of time series: wind speed forecasts, supplied by DMI, and local measurements of wind power production. The combination of these two time series for a specific location is the base of the wind power prognosis in the manner illustrated in Figure A.2.

The wind speed prognosis supplied by DMI is generated by the model HIRLAM (High Resolution Limited Area Model), which every 6 hours gives a forecast for the following 48 hours.

For a more detailed description of how the prognosis is made, see [74]. A brief
Figure A.1: Measured wind speed for various prognosis intervals, Taastrup.

Figure A.2: The structure of a wind power prognosis for a given location.
Two wind power prognosis criteria and regulating power costs

introduction to short-term wind power prediction may be found in [65] and a more thorough overview may be found in [51].

A.4 Analysis of regulating power costs

When determining the values for \( c^+ \) and \( c^- \) in the cost criterion, it is necessary to look at the regulating prices for the relevant price area. In this case, as Taastrup is the location considered, the regulating prices for eastern Denmark during 2002 are used. Here the average down and up regulating prices are 47.50 DKK/MWh and 45.69 DKK/MWh, respectively. The regulating prices as well as the spot prices for the entire year are illustrated in Figure A.3. In this figure, the spot prices are given as the upper curve and regulating prices are given by the lower curve where the down regulating prices are negative and the up regulating prices are positive. The regulating prices indicate the price difference relative to the spot price.

The relationship between the average up and down regulating prices in eastern Denmark is 1:1.04, which corresponds to \( c^+ = 1.04 \) and \( c^- = 1 \), and which, in turn, makes the cost criterion in this case practically identical to the absolute value criterion. Recall, that \( c^+ \) is the cost of regulating down, as the realised value in this case lies above the prognosis value and the system operator must
pay providers of regulating power to produce. Conversely, $c^-$ is the cost of regulating up, as the realised value then lies below the prognosis value. The behaviour of the spot and regulating prices are illustrated in larger detail in Figure A.4.

The up and down regulating prices in western Denmark are illustrated in Figure A.5, and the average prices during 2002 were 59.27 DKK/MWh (up regulation) and 101.94 DKK/MWh (down regulation), corresponding to $c^+ = 1.81$ and $c^- = 1$, respectively.

It should be noted that in this paper it is assumed that variations in wind power are the main cause of regulation in the system. In reality this need not be the case, as regulation is incurred by general imbalance in the system of which wind power variations are not necessarily the only contributing factor. However, in the western Danish system the share of wind power capacity is large (more than 2000 MW out of a total of 6600 MW installed capacity) and so, consequently, may the variations in wind power production also be; therefore wind power is in fact the major contributing factor with regards to imbalance in the system. This makes an analysis of the western Danish system particularly relevant in relation to wind power prognoses and certainly an avenue to be investigated further using the tools presented in [74] and here.

The probabilities of regulating in eastern Denmark, 2002, are illustrated in
Figure A.5: Spot and regulating prices in western Denmark, 2002.

Figure A.6.

Figure A.7 illustrates that although the probabilities in Figure A.6 indicate that the amount of small regulating prices (between 0 and 100 DKK/MWh) are predominant, the impact to the system of larger, although more seldom, regulating prices is significant.

A.5 Numerical results

Using the data presented in the previous sections, i.e., wind power production data and measured wind speeds from Taastrup, and regulating prices from eastern Denmark, power curves for the quadratic criteria and the cost (1.04:1) criteria are estimated and shown in Figure A.8. For comparison, the cost (1.81:1) criteria relating to western Danish regulating prices is also estimated and shown in Figure A.8. One ought however be careful when concluding anything based on this latter estimation, as the location from which production data and measured wind speeds are obtained after all is situated in eastern Denmark. The estimation was performed by discretising the wind speed prognosis values into intervals of 1 m/s length. For each interval an optimal value was found with respect to both the quadratic and the cost criteria functions specified in Section A.3. Also shown in the figure is the distribution of energy (the pale blue or
Figure A.6: Probabilities of regulating up and down for different regulating price intervals, eastern Denmark, 2002. Observe that the price scale has a finer division for prices less than 200 DKK/MWh.

Figure A.7: Distribution of cost of regulating one MWh every hour throughout the year (eastern Denmark, 2002).
Figure A.8: Estimated power curves and distribution of wind power, Taastrup.

bell-shaped curve). Note that the major part of the energy is produced at wind speeds (between 0 and approximately 10 m/s) where the power curve is upwards curving.

Note, that the power curve for the quadratic criterion intersects both with the cost (1.01:1) and the cost (1.81:1) criteria. The intersection with the cost (1.04:1) criterion occurs around a forecasted wind speed value of 9 m/s, whereas the intersection with the cost (1.81:1) criterion does not occur until at a forecasted wind speed value of 12 m/s. In other words, at low forecasted wind speeds prognoses based on either cost criterion have a tendency to underestimate the production in comparison to a prognosis based on the quadratic criterion. At high forecasted wind speeds, the situation is the reverse.

The right skewness of the observations could be the reason that the power curves for the cost criteria lie below the power curve for the quadratic criterion. These are the wind speeds for which the power curves are upwards curving on Figure A.8. A similar observation was made in [74] based on data from Nybølle in eastern Denmark.

The consequences of using a specific criterion when estimating the power curve for prognosis purposes are illustrated in Figure A.9.

Firstly, estimating the power curve using the quadratic (or average value) criterion incurs the consequences for the criteria indicated on the left vertical line.
The value where this line intersects the x-axis is the sum of squares (181662) and the difference between the sum of the prognosis values and the observed values is naught. Next point on the vertical line is the value of the cost (1.04:1) criterion (corresponding to the eastern Danish regulating prices), which is 27307. Lastly, the cost (1.81:1) criterion, which corresponds to the western Danish regulating prices, attains a value of 37614.

When the power curve is estimated using the cost (1.04:1) criterion (i.e., eastern Danish regulating prices – practically corresponding to the absolute value criterion), the consequences are indicated on the second vertical line. The quadratic consequence is 190146 where the second line intersects the x-axis, a higher value than when the power curve is estimated using the quadratic criterion. The difference between the sum of the prognosis values and the observations has declined into the negative (−6245), which indicates that the prognosis on the average underestimates the observed values of energy. The cost (1.81:1) criterion practically maintains its value at 26141.

Finally, and this was done mostly as an exercise, the last vertical line indicates the consequences of estimating the power curve using the cost (1.81:1) criterion. The sum of squares is 215895 where the line intersects the x-axis, and the difference between the prognosis values and the observed values has decreased further (to −15749), indicating that the prognosis now underestimates the observed energy values to an even larger degree than the cost (1.04:1) criterion.
Two wind power prognosis criteria and regulating power costs

does. The very same criterion rises a little in this last case, attaining a value of 27429, and lastly the cost (1.81:1) criterion continues its decline, attaining the value 31837.

Once again, note that this last criterion has mostly been added for the sake of curiosity and, being based on the regulating prices of western Denmark, perhaps does not pertain sensibly to this particular location (Taastrup, eastern Denmark).

Be that as it may, it is still clear from the values mentioned in the previous paragraphs and from Figure A.9, that the criteria are mutually conflicting, regardless of whether one wishes to take the cost (1.81:1) criterion into account or not. But, as mentioned in Section A.3, both the quadratic and the cost criterion (as well as the absolute criterion, which is in fact implicitly considered in the cost (1.04:1) criterion) are relevant to the system operator. Thus, it might be prudent to consider finding the ‘best’ prognosis estimate a multi-criteria problem.

A.6 Concluding remarks

The paper has dealt with criteria in wind power prognosis. In particular two criteria have been considered, viz., the average value criterion and the cost criterion, related to forecasting with the objective of minimising the error in terms of energy and minimising the cost of acquisition of regulating power to compensate forecast errors, respectively.

The regulating prices for eastern and western Denmark were presented and used as a basis for the two criteria considered in this paper. It was shown that the power curves estimated differ according to the criterion that is selected, and the consequences in terms of prognosis error consequences were presented.

The analysis confirms that making short term wind power prognosis should be considered a multi-criteria problem.
Modelling Danish local CHP on market conditions

B.1 Abstract

In Denmark, the development of local combined heat and power (CHP) plants has been characterised by large growth throughout the nineties, based in part on government subsidies in the form of feed-in tariffs. Simultaneously, there has been a significant growth of wind power, particularly in the western Danish system. As both the power produced by the local CHPs and the wind power are prioritised, the production of these types of power is occasionally sufficient to meet the total demand in the system, causing the market price to drop dramatically, sometimes even to zero-level.

In line with the liberalisation process of the energy sectors of the EU countries, it is however anticipated that Danish local CHP are to begin operating on market conditions within the year 2005. This means that the income that the local CHPs previously gained from selling electricity at the feed-in tariff is replaced in part by income gained from selling electricity on the Nordic spot market, Nord Pool. Thus, the production quantities of the local CHPs will depend on the market price.

This paper analyses the new situation. This is done by creating a model for the
Modelling Danish local CHP on market conditions

supply function of a local CHP, which takes into account the local heat demand as well as technical factors such as heat storage facilities and production unit characteristics. Based on an adaptive prognosis for electricity spot prices, bids for the spot market are made in accordance with the rules of the Nord Pool 24-hour cycle.

The paper will discuss the consequences of acting in a liberalised market for a given CHP plant, based on the abovementioned bottom-up model. The key assumption determining the bottom line is the electricity spot price. The formation of the spot price in the Nordic area depends heavily upon the state of the water reservoirs in Norway and Sweden. For this reason, the analysis is undertaken as a parametric study of the electricity spot price.

Keywords: local CHP, liberalisation process, market conditions, Denmark, spot market.

B.2 Introduction

In Denmark, the development of local combined heat and power (CHP) plants has been characterised by large growth throughout the nineties (cf. Figure B.1), based in part on government subsidies in the form of feed-in tariffs. In 2003, local CHP production in western Denmark constituted almost a quarter of the total production [40]. Simultaneously, there has been a significant growth of wind power, particularly in the western Danish system. As both the power produced by the local CHPs and the wind power are prioritised, the production of these types of power is occasionally sufficient to meet the total demand in the system, causing the market price to drop dramatically, sometimes even to zero-level.

At present, Danish local CHP plants trade electricity based on a three stage feed-in tariff given by a high price in peak load hours, a slightly lower price during high load hours, and a low price during low load hours (see Figure B.2).

Further, the plants are obligated to cover the heat demand by CHP production. Only if there is insufficient capacity on the CHP units, may boilers be employed to meet demand. The electricity produced on the CHP units is then sold according to the abovementioned three stage tariff.

According to new legislation [80], however, all plants larger than 10 MW must operate completely on market conditions by January 1, 2005; all plants between 5 and 10 MW have an additional two year respite before they also must operate
Figure B.1: Local CHP growth during the nineties.

Figure B.2: The three stage tariff, according to which the local CHP plants sell their electricity at present.
on market conditions; and plants smaller than 5 MW have a five year respite before their situation is to be re-evaluated.

The most significant difference regarding production planning on the local CHP plants with the new legislation, compared to the three stage tariff, is that prices are no longer known in advance. Thus, the local CHP must make their planning decisions under uncertainty. In addition, it seems reasonable to expect that the requirement mentioned above regarding the fulfilment of the heat demand primarily using the CHP unit will be slackened.

The mathematical models presented in this paper are aimed towards that particular problem, i.e., given a certain expectancy of the prices during the coming day, a production plan (which translates into a bid) is conceived that minimises production cost while meeting the given heat demand.

### B.3 Modelling a local CHP plant

#### B.3.1 The CHP plant

In the following, consider the situation where a local CHP plant supplies heat to the consumers and electricity to the electricity net. The plant consists of a CHP unit (typically a gas-fuelled motor), which produces both heat and power, and a boiler, which only produces heat. Furthermore, the plant is equipped with a heat storage facility. The electricity produced at the plant is sold on the Nord Pool spot market. Electricity production bids must be submitted to this market, at a time where the spot prices are not yet determined.

The purpose of the models developed here is to achieve the least possible expected costs of supplying heat to the heat consumers. The most important element of this is determining how the local CHP should submit bids to Nord Pool.

The electricity-to-heat ratio (in the sequel termed the back pressure value) on the CHP unit is denoted \( c_m \), i.e., 1 MWh heat produced corresponds to \( c_m \) MWh power produced, and the maximal heat production is denoted \( K \). Further, \( V_{\text{max}} \) denotes the limit on the heat storage facility. The production expenses on the plant are \( c_{k^u} \) (DKK/MWh heat) for production on the CHP unit (note, that both heat and power are produced), and \( c^h \) (DKK/MWh heat) for heat from the boiler. Because of the interconnection between heat and power production on the CHP unit, one cannot be priced without considering the other.
B.3 Modelling a local CHP plant

B.3.2 The spot market

The procedure regarding the spot market is as follows. Early in the day prices and volumes are bid to Nord Pool, which then determines the spot price for every hour in the following 24 hour day (from midnight to midnight) based on a joint evaluation of the supply from the electricity producers and the demand of the consumers. In this case, it is assumed that the price which the CHP plant receives for the sale of electricity does not depend on the price or amount of the electricity bid to the market. Thus, the spot price is unknown at the time when the decision of the bidding price of the electricity produced at the CHP plant is made.

However, based on previous observations of the development of the spot price during a normal day at the given time of year, it is possible to make a qualified guess as to what the spot price might be. These guesses are denoted $\pi_s^t$, where $t = 1, \ldots, 24$ indicates the time of day and $s = 1, \ldots, S$ indicates a suitable set of possible developments of the varying circumstances that influence the spot price (such as changes in weather, equipment failure, etc.). A reasonable set of instances of the spot price and associated probabilities $\phi_s$ may be attained by considering data from Nord Pool.

B.3.3 Models

The objective is to minimise the expected net cost by optimising the hourly bid to Nord Pool while meeting heat demand. For a given hour, two possible situations may arise:

1. The heat produced can be utilised. The heat could alternately have been produced on the boiler at the price $c_k$ DKK/MWh heat. The surplus costs of the CHP unit’s power production is therefore $p_1 = (c_{kv} - c^k)/c_m$ DKK/MWh power.

2. The heat produced cannot be utilised. The cost of power production is $p_2 = c_{kv}/c_m$ DKK/MWh power.

Clearly, $p_1 < p_2$. A sketch of the supply curve of the CHP plant for a given hour may be seen in Figure B.3, where $m_1^n$ resp. $m_2^n$ is the heat volume corresponding to the power volume produced when the price is $p_1$ resp. $p_2$.

In Figure B.3(a) the bid price is given as a function of the power bid to the market, and in Figure B.3(b) the bid price is given as a function of the heat ($m^n$)
Modelling Danish local CHP on market conditions

Figure B.3: Supply curve for power production on the CHP unit, (a). (b) shows the corresponding heat amount.

(i.e., the heat volume corresponding to the power volume bid to the market). The volume of power produced and sold depends on the spot price. As there is a direct connection between power and heat volumes via the back pressure value, the corresponding heat volume used \((m^{used}_i)\) may also be said to depend on the spot price. Thus, the (heat) volumes used are

\[
m^{used}_1 = 0 \quad \text{and} \quad m^{used}_2 = 0 \quad \text{if} \quad \pi^s < p_1
\]

\[
m^{used}_1 = m^u_1 \quad \text{and} \quad m^{used}_2 = 0 \quad \text{if} \quad p_1 < \pi^s < p_2
\]

\[
m^{used}_1 = m^u_2 \quad \text{and} \quad m^{used}_2 = m^u_2 \quad \text{if} \quad p_2 < \pi^s
\]

where \(m^u_1\) and \(m^u_2\) are the heat volumes corresponding to the power volumes bid at price \(p_1\) and \(p_2\), respectively. Any missing heat is produced on the boiler. As a help variable, define the binary indicator \(\delta\) as

\[
\delta_{1t}^s = \begin{cases} 
1, & \pi^s < p_1 \\
0, & p_1 < \pi^s 
\end{cases} \quad \text{and} \quad \delta_{2t}^s = \begin{cases} 
1, & p_2 < \pi^s \\
0, & \pi^s < p_2 
\end{cases}
\]

See Section B.9 for a list of symbols.

B.3.3.1 Model with simple storage

The heat demand at hour \(t\), \(d_t\), is assumed deterministic. A simple storage facility is considered in this model. Here, 'simple' indicates that neither the maximal nor the minimal capacity limits of the heat storage are taken into account. There are no costs associated with retrieving heat from the heat storage or depositing heat in it. The problem is considered for single day (i.e., \(T = 24\)) and a mathematical model of the problem may be written as follows:
The objective is to minimise the expected cost (which is compensated by sale of electricity) subject to the constraints that the total amount of heat corresponding to the power bid does not exceed capacity, \((B.2)\); that the total heat production during each hour satisfies the heat demand during that hour, \((B.3)\); and volumes cannot be negative, \((B.4)\). Note that the models in the paper use the heat production as variables. Electricity production may be derived from the heat volumes using the electricity-to-heat ratio.

Exchanging the equality sign in constraint \((B.3)\) for an inequality \((\geq)\) allows heat cooling, i.e., surplus heat may be cooled off at no extra expenses. In this case, when the expected price is particularly high, it is possible to produce additional electricity for sale although the heat demand is already met.

Note, that this model presupposes that it is permissible to avoid production on the CHP unit when prices are low and cover the heat demand solely by boiler production (cf. Section \(B.2\)).

\[ \min \quad \sum_{t=1}^{T} \sum_{s=1}^{S} \phi^s \left( (m^u_{1t} \delta^s_{1t} + m^u_{2t} \delta^s_{2t}) c^{kv} + m^s_{kt} c^k \right) \]

\[ -\pi^s c^m \left( m^u_{1t} \delta^s_{1t} + m^u_{2t} \delta^s_{2t} \right) \]

\[ \text{s.t.} \quad m^u_{1t} + m^u_{2t} \leq K, \quad \forall t \quad (B.2) \]

\[ \sum_{t=1}^{T} m^u_{1t} \delta^1_{s} + m^u_{2t} \delta^2_{s} + m^s_{kt} = \sum_{t=1}^{T} d_t, \quad \forall s \quad (B.3) \]

\[ m^u_{1t}, m^u_{2t}, m^s_{kt} \geq 0, \quad \forall s, t \quad (B.4) \]

B.3.3.2 Model with specified storage

The constraint \((B.3)\) expresses that the heat production over a certain period \((1, \ldots, T)\) must equal the heat demand during that period. This is intuitively correct and permits a shift in time between production and consumption. However, in practice the storage capacity is limited and this should be reflected in the model.

Letting \(V^s_t\) denote the heat storage contents at the beginning of hour \(t\) under
scenario $s$, a mathematical model of the problem may be given as follows:

$$\begin{align*}
\min & \sum_{t=1}^{T} \sum_{s=1}^{S} \phi^s \left( (m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s) c^k + m_{kt}^c c^k \\ & - \pi^s c_m \left( m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s \right) \right) \\
\text{s.t.} & \quad m_{1t}^u + m_{2t}^u \leq K, \quad \forall t \tag{B.5} \\
& \quad V_{t+1}^s = V_t^s + m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s + m_{kt}^c - d_t, \quad \forall s, t = 1, \ldots, T-1 \tag{B.6} \\
& \quad V_t^s = V_T^s + m_{1T}^u \delta_{1T}^s + m_{2T}^u \delta_{2T}^s + m_{kT}^c - d_T, \quad \forall s \tag{B.7} \\
& \quad 0 \leq V_t^s \leq V_{\max}, \quad \forall s, t \tag{B.8} \\
& \quad m_{1t}^u, m_{2t}^u, m_{kt}^c \geq 0, \quad \forall s, t \tag{B.9}
\end{align*}$$

Once again, the equality signs in constraints (B.7) and (B.8) indicate that heat cooling is not allowed. However, the equality sign in constraint (B.8) may be replaced by $\leq$ if heat may be retained for the next 24-hour period. To allow cooling, the equality sign in constraint (B.7) must be replaced with by $\leq$.

### B.4 Analysis

#### B.4.1 Lagrangian relaxation

Two versions of the model (B.1) are considered in this section: without cooling (equality in constraint (B.3)), and with cooling (inequality in constraint (B.3)).

#### B.4.1.1 With cooling

The dual Lagrangian problem can be written as

$$\begin{align*}
\max & \quad f(\lambda^s) \\
\text{s.t.} & \quad \lambda^s \geq 0
\end{align*}$$

where the multiplier $\lambda^s$ is associated with constraint (B.3) and where
\[ f(\lambda^s) = \lambda^s \sum_{t=1}^{T} d_t + \min_{m \geq 0} \left\{ \sum_{t=1}^{T} \sum_{s=1}^{S} \phi^s \left( (m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s) c^{kv} + m_{kt}^s c^k \right) \right. \\
- \pi^s_t c \left( m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s \right) \right\} \] 

\[ \sum_{s=1}^{S} \lambda^s \left( \sum_{t=1}^{T} m_{1t}^u + m_{2t}^u + m_{kt}^s \right) \}

(B.11)

Note, that this objective function is unchanged when constraint (B.3) has an equality sign (i.e., when cooling is not allowed). The first term of the objective function is constant and therefore has no influence on the optimisation. The second term may be rewritten by merging the coefficients of the decision variables (i.e., the \( m \)’s):

\[ \min_{m \geq 0} \left\{ \sum_{t=1}^{T} m_{1t}^u \left[ \sum_{s=1}^{S} \phi^s \left( c^{kv} - \pi^s_t c_m \right) - \lambda^s \right] \right\} \\
+ \sum_{t=1}^{T} m_{2t}^u \left[ \sum_{s=1}^{S} \phi^s \left( c^{kv} - \pi^s_t c_m \right) - \lambda^s \right] + \sum_{t=1}^{T} \sum_{s=1}^{S} m_{kt}^s \left( \phi^s c^k - \lambda^s \right) \}

(B.12)

Thus, the variables may be optimised separately, reforming the problem to:

For each of the \( T \) \( m_{1t}^u \): minimise

\[ m_{1t}^u \left[ \sum_{s=1}^{S} \delta_{1t}^s \left( \phi^s \left( c^{kv} - \pi^s_t c_m \right) - \lambda^s \right) \right]. \] (B.13)

For each of the \( T \) \( m_{2t}^u \): minimise

\[ m_{2t}^u \left[ \sum_{s=1}^{S} \delta_{2t}^s \left( \phi^s \left( c^{kv} - \pi^s_t c_m \right) - \lambda^s \right) \right]. \] (B.14)

For each of the \( T \times S \) \( m_{kt}^s \): minimise

\[ m_{kt}^s \left( \phi^s c^k - \lambda^s \right). \] (B.15)

The coefficients of \( m_{1t}^u \) and \( m_{2t}^u \) are of particular interest. They may be interpreted in the following way: \([\phi^s (c^{kv} - \pi^s_t c_m)]\) is the expected profit from sale of
power in hour $t$ – however, the $\delta$ with which the expression is multiplied indicates that power is not sold unless the spot price is acceptable. Furthermore, $\lambda^s$ may be interpreted as the profit from “sales” of heat to a heat storage facility in hour $t$ (or what a fictive manager of the storage facility is willing to pay for heat, cf. Section B.5.5).

When heat cooling is permitted, only the capacity of the CHP unit limits the production, which makes it a reasonable move to turn the inequality in (B.2) into an equality. This may be argued as follows:

When the spot price is high ($\pi^s_t > p_2$) for but one of the instances, full capacity must be bid, i.e., $m^n_1t + m^u_2t = K$; otherwise a possible profit would be wasted. Should the spot price be in or under the medium level ($\pi^s_t < p_2$) for all instances of the spot price, the value of $m^u_2t$ is of no consequence. Thus, $m^u_2t$ may be chosen so that $m^n_1t + m^u_2t = K$, regardless of the value of $m^n_1t$.

If all instances of the spot price are in the low level ($\pi^s_t < p_1$), nothing is sold, wherefore the values of both $m^n_1t$ and $m^u_2t$ are of no consequence. Here it is also permissible to demand $m^n_1t + m^u_2t = K$ without influencing the optimal solution decisively. Based on this argumentation the inequality sign in (B.2) is replaced by an equality sign in model (B.1) from hereon in this section.

Based on this argumentation, constraint (B.2) makes it possible to write $m^u_2t$ as an expression of $m^n_1t$ and $K$: $m^u_2t = K - m^n_1t$. Thus, the two terms containing $m^n_1t$ and $m^u_2t$ may be written jointly as

$$m^n_1t \left\{ \sum_{s=1}^{S} (\delta^s_1t - \delta^s_2t) \left[ \phi^s (c^{kv} - \pi^s_t c_{m}) - \lambda^s \right] \right\}$$ \hspace{1cm} (B.16)

where the term concerning $K$ is disregarded, as it becomes constant and therefore has no influence on the optimisation.

Let $\alpha_{1t}$ denote the coefficient for $m^n_1t$ in (B.16). As the expression is to be minimised, it is clear how $m^n_1t$ should be determined depending on the value of $\alpha_{1t}$. In the case of high or low spot price ($\pi^s_t > p_2$ and $\pi^s_t < p_1$ respectively), $\delta^s_1t - \delta^s_2t$ becomes 0 (and consequently, $\alpha_{1t} = 0$). Therefore, these instances have no influence on the determination of $\alpha_{1t}$. This corresponds with the fact that in these cases the result is already given regardless of the value of $m^n_1t$: at the high price everything is sold and at the low price nothing is sold.

From the above it may be noted that when cooling is allowed it does not matter how much above the highest or below the lowest level the spot price is, only the medium level is interesting. At the medium price (i.e., $p_1 < \pi^s_t < p_2$), $\delta^s_1t - \delta^s_2t$ becomes 1 and so these are the cases which truly determine the value of $\alpha_{1t}$.
Three possibilities exist:

1. $\alpha_{1t} > 0 \Rightarrow m_{1t} = 0$,
2. $\alpha_{1t} < 0 \Rightarrow m_{1t} = K$,
3. $\alpha_{1t} = 0 \Rightarrow$ all values of $\alpha_{1t}$ between 0 and $K$ minimise the Lagrangian function.

The coefficient $\alpha$ indicates that for given $\lambda^*$'s the solution is such that the hours with negative $\alpha$ values bid full capacity at the low price. Thus it is, indirectly, the values of the $\lambda^*$'s that determine how many hours in which to bid full capacity to the market.

Considering historic average daily spot prices, as illustrated in Figure B.4, an obvious daily pattern emerges.

The hypothesis may be postulated that only the shape of the curve has influence on the optimal bidding, and that the general level of the curve has none. However, it may be shown with a simple example that when working with the models defined in this paper, the general price level is not without importance.

Consider two consecutive hours, let $S = 2$, and assume that both scenarios occur with equal probability, i.e., $\phi^1 = \phi^2 = 0.5$. Letting the costs be given as
$c^{k_u} = 150$ DKK/MWh and $c^k = 105$ DKK/MWh, and defining $c_m = 0.5$, yields $p_1 = 90$ DKK/MWh and $p_2 = 300$ DKK/MWh. Let the spot prices be for hour 1: $\pi^1_1 = 70$, $\pi^1_2 = 110$; and for hour 2: $\pi^2_1 = 130$, $\pi^2_2 = 40$. For the sake of simplicity, it is assumed that only 1 MWh heat is produced during the entire period. Considering the two hours separately yields:

**Hour 1**: In this case, $\pi^1_1 < p_1$ which means $\delta^1_{11} = \delta^1_{21} = 0$, and $p_1 < \pi^2_1 < p_2$ which means $\delta^2_{11} = 1$ and $\delta^2_{21} = 0$. The expected cost of producing in hour 1 is thus

$$0.5 \left( 0 \cdot 150 + 1 \cdot 150 - 70 \cdot 0.5 \cdot 0 \right) + 0.5 \left( 1 \cdot 150 + 0 \cdot 150 - 110 \cdot 0.5 \cdot 1 \right) = 100$$ DKK/MWh.

**Hour 2**: In this case, $p_1 < \pi^1_2 < p_2$ which means $\delta^1_{12} = 1$ and $\delta^1_{22} = 0$, and $\pi^2_2 < p_1$ which means $\delta^2_{12} = 0$ and $\delta^2_{22} = 0$. The expected cost of producing in hour 2 is thus

$$0.5 \left( 1 \cdot 150 + 0 \cdot 150 - 130 \cdot 0.5 \cdot 0 \right) + 0.5 \left( 0 \cdot 150 + 1 \cdot 150 - 40 \cdot 0.5 \cdot 1 \right) = 95$$ DKK/MWh.

It is desirable to place production in the hour with least cost, i.e., hour 2. However, when adjusting the level of the spot prices upwards with 100 DKK/MWh, the situation changes. Once again considering the two hours separately yields:

**Hour 1**: In this case, $p_1 < \pi^1_1 < p_2$ which means $\delta^1_{11} = 1$ and $\delta^1_{21} = 0$, and $p_1 < \pi^2_1 < p_2$ which means $\delta^2_{11} = 1$ and $\delta^2_{21} = 0$. The expected cost of producing in hour 1 is thus

$$0.5 \left( 1 \cdot 150 + 0 \cdot 150 - 170 \cdot 0.5 \cdot 1 \right) + 0.5 \left( 1 \cdot 150 + 0 \cdot 150 - 210 \cdot 0.5 \cdot 1 \right) = 55$$ DKK/MWh.

**Hour 2**: In this case, $p_1 < \pi^1_2 < p_2$ which means $\delta^1_{12} = 1$ and $\delta^1_{22} = 0$, and $p_1 < \pi^2_2 < p_2$ which means $\delta^2_{12} = 1$ and $\delta^2_{22} = 0$. The expected cost of producing in hour 2 is thus

$$0.5 \left( 1 \cdot 150 + 0 \cdot 150 - 230 \cdot 0.5 \cdot 1 \right) + 0.5 \left( 1 \cdot 150 + 0 \cdot 150 - 140 \cdot 0.5 \cdot 1 \right) = 57.5$$ DKK/MWh.

These results lead to the conclusion that production should be placed in the first hour, which is a direct contradiction to the conclusion at the original spot price level.
B.4 Analysis

B.4.1.2 Without cooling

In this case, constraint (B.3) stands as is, i.e. as an equality. Contrarily, the capacity constraint for the CHP unit, (B.2), cannot be an equality, as it is only permissible to produce the heat which is demanded and thus not to cool off any excess heat. Therefore, this constraint should be included in the Lagrangian function. Associating the multiplier $\mu_t$ with constraint (B.2), the dual Lagrangian problem becomes

$$\max \ f (\lambda^s, \mu_t)$$
$$s.t. \ \lambda^s \text{ unlimited}$$
$$\mu_t \leq 0$$

where

$$f (\lambda^s) = \mu_t K + \lambda^s \sum_{t=1}^{T} d_t + \min_{m \geq 0} \left\{ \sum_{t=1}^{T} \sum_{s=1}^{S} \phi^s \left( (m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s) c^{kv} + m_{kt}^s c^k - \pi_t c_m \right) - \sum_{t=1}^{T} \mu_t (m_{1t}^u + m_{2t}^t) ight. \right.$$  
$$\left. - \sum_{s=1}^{S} \lambda^s \left( \sum_{t=1}^{T} m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s + m_{kt}^s \right) \right\}$$  

(B.17)

Only the minimising term of (B.17) has any influence on the optimisation, as the two first terms are constant. Once again, the coefficients of the decision variables are merged and then become:

For each of the $T \ m_{1t}^u$ : minimise

$$-\mu_t + m_{1t}^u \left[ \sum_{s=1}^{S} \delta_{1t}^s \left( \phi^s \left( c^{kv} - \pi_t c_m \right) - \lambda^s \right) \right].$$  

(B.18)

For each of the $T \ m_{2t}^u$ : minimise

$$-\mu_t + m_{2t}^u \left[ \sum_{s=1}^{S} \delta_{2t}^s \left( \phi^s \left( c^{kv} - \pi_t c_m \right) - \lambda^s \right) \right].$$  

(B.19)

For each of the $T \times S \ m_{kt}^s$ : minimise

$$m_{kt}^s \left( \phi^s c^k - \lambda^s \right).$$  

(B.20)
In contrast to the case where cooling is permitted, the coefficients of the two $m^u$’s cannot be merged. Let $\alpha_1^t$ denote the coefficient for $m_1^u$ and $\alpha_2^t$ the coefficient for $m_2^u$. For both coefficients the expression to the right of the summation may be interpreted as the expected value of the profit (as the heat is assumed to be 'sold' at the price $\lambda^s$). Note, that contrary to the case where cooling is allowed, in this case also high spot prices (i.e., higher than $p_2$) will influence the determination of the $\alpha$’s. It should also be noted that it could happen that full capacity would not be bid to the market, as $\alpha_1^t$ and $\alpha_2^t$ may have opposite signs. In the case of opposite signs there is no way of telling which $\alpha$ is positive and which is negative.

Also, note that it becomes more difficult to find a good solution – or even to guarantee a feasible one – as equality constraints (cooling not allowed) impose limitations on the model that are far less of an issue with inequality constraints (cooling allowed).

B.4.2 Shadow prices of the heat storage

Shadow prices may be interpreted as the marginal cost of production. When explaining why the shadow price has a certain value, one must ascertain which time period this marginal production takes place when specified heat storage is included, i.e., model (B.5). The marginal costs in those periods define the shadow price. The shadow prices can be related to the production patterns and costs. They may sometimes be deduced analytically, or calculated in an optimisation algorithm as a part of the optimisation.

Note, that the heat shadow prices, which may be calculated when the bid to the spot market is made, are not the same as the shadow prices that may be calculated when the sale to the spot market is known. There is a shadow price of the former kind for every hour, but one for every spot price scenario of the latter kind. The difference is, that the shadow prices calculated at the time of bidding relate to the expected marginal cost, whereas the other shadow prices relate to a specific spot price scenario (and thus to a specific instance of production).

The relation between these two sets of shadow prices is that the shadow prices calculated at the time of bidding are equal to the expected value (the weighted average) of the shadow prices calculated when the sales are known. Therefore, the numerical values of the latter shadow prices are relatively small. To obtain values comparable with the production costs, the values associated with a given instance of the spot price may be divided by the probability of the instance.

When cooling is allowed, once the heat shadow price relating to constraint (B.3)
is known, the optimal bid is given trivially by \((B.13)-(B.15)\). However, in practice the heat shadow prices are not deduced until the bid is already made.

**B.4.3 Derivation of optimal bidding**

Based on the previous analysis some conclusions may be made relative to the nature of the optimal bids for the spot market.

Consider the first model \((B.1)\) with simple storage. Only the case with cooling is considered here, the case without cooling may be treated similarly. Assume for the moment that the heat shadow price is known. Then \((B.13)-(B.15)\) specify the optimal solution expressed in heat production terms (save for the indeterminate case also described in Section \(B.4.1.1\)), and from this the bidding may in turn be derived essentially as follows: the electricity volume corresponding to the heat volume \(m_{1t}^{u}\) is bid at the price \(p_1\), and the electricity volume corresponding to the heat volume \(m_{2t}^{v}\) is bid at the price \(p_2\) as described in Section \(B.3.3\). Further details are given in Section \(B.4.1\). Intuitively, the bidding should be concentrated on the hours with the highest expected spot prices, and presumably this may make good sense in practice. However, as shown by the counterexample in Section \(B.4.1\) this bidding strategy is not truly optimal.

For the case with a specified storage the essentially same observations may be made. The primary difference from the case with simple storage is that now the heat shadow prices may differ between the hours. Such difference indicates that one of the storage bounds is reached or may be expected to be reached with a certain probability.

In practice the heat shadow prices are not known, and must be found by a numerical method. The models have been implemented in the modelling language \textsc{gams} [92] and the results are given in the sequel.

**B.5 Numerical examples**

To illustrate the models some numerical examples are presented in this section. For a single day, four different situations are considered (with and without cooling, with and without limited storage) associated with four different variations, totally, of the models \((B.1)\) and \((B.5)\).

In the following, let A denote \((B.1)\); let B denote \((B.5)\); let C denote \((B.1)\)
with cooling; and let D denote (B.5) with cooling. Five scenarios are considered represented by the spot price in western Denmark January 1–5, 2003, each with equal probability ($\phi^s = 0.2$ for $s = 1, \ldots, 5$). A maximal capacity of 4 MW for the boiler is introduced. Furthermore, $c_m = 0.5$, $c^{kv} = 150$ DKK/MWh, $c^k = 105$ DKK/MWh, $K = 5$ MWh/h and the time index $t$ runs from 1 through 24. In cases B and D, where storage is limited, the storage limit is $V_{\max} = 15$ MW, and the initial storage reserve is $V_1 = 10$ MW. The heat demand $dt$ is given as 1.5 MW for hours 1–8 and 24, and 2.5 MW for hours 9–23. Running the models with these data yields the optimal values given in Table B.1. It comes as no great surprise, that better values are attained for the models with cooling (C and D) than the ones without cooling (A and B). Also, the model with unlimited storage is more effective than the one with limited storage both when cooling is allowed and when it is not. These result makes good sense mathematically, as the freedom granted by inequality constraints (cooling allowed) enables the models to attain better solutions than with equality constraints (cooling not allowed).

In the following, the four cases are considered more closely for scenarios 2 and 5.

### B.5.1 Case A

This is the case of unlimited storage and no cooling (i.e., model (B.1)). Cooling is not allowed, and so no more heat may be produced than is demanded. As there are no storage limitations, the production may be scheduled where the best result is attained.

Figure B.5(a) shows, for each of the 24 hours, the heat production on the CHP unit and the boiler under price scenario 2. Additionally, the spot price is depicted. Primarily full capacity is bid except in period 14; here the bid is structured in such a way as to ensure that the total heat production equals the total heat consumption. In comparison, Figure B.5(b) depicts the same for price scenario 5. Here, the pattern is clear: when the price is high, full capacity is bid, and nothing otherwise.
B.5.2 Case B

In this case there are storage limitations but no cooling allowed (i.e., model (B.5)), which limits the optimal production pattern. Because of the storage limitations, the bid pattern is more complicated than in case A (see Figure B.6(a) and B.6(b)) both in scenario 2 and 5.

In scenario 2, with the limitations imposed, there is now boiler production both in the beginning and at the end of the day. Furthermore, it is not possible to take advantage of the high price in hour 20, as the storage is filled to capacity and cooling the heat is not permissible. In scenario 5, the pattern is also more
complicated than in case A and the production on the CHP unit is constrained both by the fact that cooling is not allowed and by the storage limitations. The latter phenomenon is clearly seen in the way the storage limit is reached in hours 3–14 and again in hours 21–22.

### B.5.3 Case C

Case C is the model (B.1) with cooling, i.e. inequality in constraint (B.3). In scenario 2, Figure B.7(a) shows that as much as possible, i.e., up to the maximal capacity on the CHP unit, is produced at the high prices. This is possible, as there are no storage limitations, which therefore allows production during any hour in the period. Furthermore, the lack of storage limitations means that boiler production may be avoided.

Note, that in scenario 2 0.5 MWh is produced in hour 10, even though it seems more reasonable to produce in hour 16, as the spot price in those two hours is 200.02 and 206.79 DKK/MWh, respectively. However, the production in hours 10 and 16 is not merely based on the spot price of the relevant scenario but rather on the jointly weighted spot price (cf. $\alpha_{1t}$ in Section B.4.1). The difference between the production pattern in this case and case A (see Figure B.5) is due to the fact that the bid with regard to all five scenarios differs, where cooling is a deciding factor as to when the production is bid.

In scenario 5 (Figure B.7(b)), apart from the maximal CHP production, there is also boiler production in the beginning of the day, due to the extremely high spot price. There is also maximal CHP production later in the period but here
B.5 Numerical examples

Figure B.8: Scenarios 2 and 5 for case D.

the spot price is not high enough to warrant boiler production as well.

B.5.4 Case D

Once again limited storage, but now cooling is permitted. The limits to the storage facility incur that production is no longer merely scheduled when prices are high. This explains the boiler production both in the beginning and the end of the period in scenario 2, in order to meet demand (see Figure B.8(a)).

Further, there is production despite a very low price in hour 21, as the storage limit has already been reached and producing earlier in order to save production for later hours is no longer possible. The situation in scenario 5 (Figure B.8(b)) is basically the same as in case B, except that maximal capacity is bid in one extra hour (hour 20), which cooling permits.

B.5.5 Shadow prices for the heat storage

In case A, considering scenario 2, the CHP unit is the marginal production unit in hour 14 (see Figure B.5(a)), but due to the ‘unlimited’ storage it is also marginal in all other hours. The marginal production price equals profits of power sales subtracted from the production costs: $150 - 0.5 \cdot 226 = 37$ DKK/MWh.

The same applies to case B (see Figure B.9(a)), though it should be noted that
the shadow prices of the storage facility are generally higher than in case D (cf. Figure B.9(a) and B.9(b), respectively). The explanation is that in case B cooling is not allowed, wherefore production is more expensive.

In case D, Figure B.8(a) shows that there is boiler production in hour 1. This means that boiler production is marginal in this hour. However, due to the storage facility, the boiler is also the marginal unit in hours 1–7, as the storage facility may save production in this time period. Because the storage limit is reached at the beginning of hour 8, production from hours 1 through 7 cannot be saved for later. The heat shadow prices in hours 1 through 7 are all 105 DKK/MWh, which is precisely the given production cost on the boiler.

The storage reaches its lower limit again in the beginning of period 11 (cf. Figure B.9(b)). The shadow price in periods 8 through 10 is 50 DKK/MWh, as seen in Figure B.9(b). Figure B.8(a) shows that the marginal unit in period 10 is the CHP unit (it produces between minimum and maximum levels). The electricity spot price in this hour is 200 DKK/MWh. The marginal cost of production in period 10 is then $150 - 0.5 \cdot 200 = 50$ DKK/MWh, which is precisely what may be read on the Figure B.9(b) as the heat shadow price in hours 8–10. Similarly, the heat shadow price in hours 22–24 is 105 DKK/MWh, which is once again the boiler production cost, and Figure B.8(a) shows that the boiler is indeed the marginal unit in period 22. The overall pattern is that when the storage reaches its lower limit, the heat shadow prices fall, and when the storage reaches its upper limit, the heat shadow prices rise.

For hours 11–20 there is an extra level to the interpretation. Recall that the shadow price may be interpreted as the marginal cost of production: if, e.g., 1
Wh extra must be produced and the cost of such a production is \( x \) DKK, then the shadow price is \( x \) DKK/Wh, or \( 10^6 \cdot x \) DKK/MWh. Similarly, if 1 Wh less must be produced, lowering the costs with \( y \) DKK, then the shadow price is \( y \) DKK/Wh, or \( 10^6 \cdot y \) DKK/MWh. However, it is not a given that \( x \) and \( y \) are the same value, e.g. if the marginal unit is not well defined.

In such cases it may therefore hold that one unit is marginal if more is produced while another unit, with different cost, is marginal if less is produced. An example of this may be found in hours 11–20. If less needs to be produced in an hour between 11 and 20, it will be in hour 14; here, the marginal costs are \( 150 - 0.5 \cdot 226 = 37 \) DKK/MWh. If more needs to be produced in an hour between 11 and 20, it cannot be done on the CHP unit period 20, as it is already producing its maximum in that hour, whereas it is possible in hour 15, in which the marginal costs are \( 150 - 0.5 \cdot 214 = 43 \) DKK/MWh. Figure B.9(b) shows the heat shadow price 37 DKK/MWh, but as argued it might as well have shown 43 DKK/MWh.

### B.6 Simulation of an entire year

The simulations in the following section are all based on the model with specified heat storage and no cooling, (B.5).

A simple spot price prognosis is utilised for the model. The scenarios used in the simulation of a given day are based on historical data, which are divided into two types: weekdays and weekend days. The reasoning behind this division is that when a prognosis is made for a given day, only (historical) days of the same type are considered. The prognosis (i.e., the scenarios to be used in the model) is constructed in the following way.

First, the number of scenarios \( (N) \) is chosen. Then, all but one of the scenarios are defined as the \( N - 1 \) previous days of the same type as the given day. Finally, an \( N \)’th scenario is constructed which resembles the daily spot price profile of the year (cf. Figure B.4) only shifted upwards so the spot price in all hours is above \( p_2 \), ensuring that a bid is made even if all other scenarios are below \( p_1 \).

For the purpose of the simulations, an actual local CHP plant has been used as point of reference. The data is given in Table B.2.

The heat demand for an entire year is illustrated in Figure B.10.

The model is analysed using the western Danish spot prices from 2001–2002,
Table B.2: Parameters for the simulation.

<table>
<thead>
<tr>
<th>$c_m$</th>
<th>$c_{kv}$</th>
<th>$c^k$</th>
<th>$V_{max}$</th>
<th>$p_1$</th>
<th>$p_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.848</td>
<td>345</td>
<td>488</td>
<td>23.2</td>
<td>168.24</td>
<td>575.12</td>
</tr>
<tr>
<td>DKK/MWh</td>
<td>DKK/MWh</td>
<td>MWh/h</td>
<td>DKK/MWh</td>
<td>DKK/MWh</td>
<td></td>
</tr>
</tbody>
</table>

Figure B.10: Heat demand for a year.
Figure B.11: Daily means of the spot price in western Denmark, 2001–2002.

As a base of comparison, the optimal bids are calculated given that the prices are known in advance and the cost of these optimal bids is calculated. This base is termed full information (FI). It is then examined how the expected cost diverges from FI while varying the number of spot price scenarios included in the prognosis as well as the general level of the spot prices.

The structure of the simulations where the spot price is not known in advance is as follows. First, the prognosis is constructed as described above. Once the scenarios have been determined, the model is run for the given day and the results are noted. Then, the ultimate heat storage contents are set as the initial heat storage content for the next day and the process is repeated until an entire year has been simulated.

B.6.1 Number of scenarios in the prognosis

In this section it is analysed what the effect is of including different numbers of scenarios in the prognoses. Using, as mentioned, the western Danish spot price for 2001 and 2002 yields the results depicted in Figure B.12.

Initially, there is for both years a falling trend in the deviation from FI the
more scenarios are included in the prognosis. However, the trend actually reverses when a large number of scenarios are included. Furthermore, the increase in run time is significant as the amount of scenarios becomes large, so it is clearly imperative not to include unnecessarily many scenarios in the prognosis. In contrast, one must include sufficiently many scenarios to get a reasonable representation of the future.

For 2001, the number of scenarios in the prognosis has no great influence on the deviation from FI (see Figure B.12). The explanation for this phenomenon is probably that the level of the spot prices was relatively constant in 2001 (see Figure B.11) and therefore there is not much gained by looking back three or fifteen days when constructing a prognosis.

For 2002, the deviation is generally larger. This may be explained by the fact that the prices towards the end of that year are quite volatile in their daily variation, which influences the quality of the prognosis. This trend also explains the increase in the deviation when including 25 scenarios in the prognosis, as the increasing trend during the final months is not ‘caught’ by the prognosis when too much history is included.

Figure B.12: Deviation from FI as a consequence of the number of scenarios in the prognosis.
B.6 Simulation of an entire year

B.6.2 Level of the spot price

In Section B.4.3 an example was given which showed that the level of the spot price does influence the results. In this section it is shown how different spot price levels influence the deviation from FI for 2001 and 2002. The different levels are obtained by adding a given price (positive or negative) to every hour of the historical spot prices, thus shifting the level up or down in accordance with the addition. The deviation from the FI case for both 2001 and 2002 is shown in Figure B.13.

When the prices are very low there is actually no deviation from the FI case, as the heat demand (which is known in advance) is covered by boiler production. The constant shape of the curves at very high spot price levels is due to the fact that once the level is reached where all prices are higher than $p_1$, the same strategy is employed regardless of how high the prices are above $p_1$ and thus the error is the same for all higher levels. Once again, the deviation is greater for 2002 than 2001 for all scenarios but the explanation given in Section B.6.1 still holds, i.e., the large variations in the spot price in the last months of 2002 (cf. Figure B.11) influence the quality of the prognosis. As a curiosity it may be noted that the total cost of fulfilling the heat demand in the 2002 FI case is 4439848.25 DKK and the maximal deviation is 38018.68 DKK, corresponding only to almost 1% of the total cost.

A study has also been made of the case where cooling is allowed (see Figure B.14).

The influence of the uncertain spot prices is clearly different than when cooling
is not allowed (cf. Figure B.14). In particular, the large uncertainty for very high spot price levels that was evident when cooling is not allowed all but disappears when cooling is permitted. The explanation is simple: once the prices reach a sufficiently high level, it is desirable to produce as much as possible in order to take advantage of this fact, as any excess heat produced as a consequence of this may simply be cooled off.

B.7 Concluding remarks

In this paper, mathematical models to handle day-to-day bidding to in the spot market have been defined. This has been done from the point of view of a local CHP plant with heat storage. The analysis of these models included an examination of the shadow prices on the heat storage. Numerical examples to illustrate that the functionality of the models was reasonable were given for four variations: with and without heat cooling allowed, and with and without a specified heat storage.

A simple type of prognosis was constructed and the model with specified storage and no cooling was run consecutively for two separate years (2001 and 2002) using the actual spot prices from these years as basis for the prognoses. The influence of the number of scenarios included in the prognosis as well as the level of the spot prices was examined both with and without cooling.

Without cooling, the number of scenarios in the prognosis mostly affected the results for 2002, due to the fact that the prices during the final months of that
year exhibited much larger variation as well as an increasing trend compared to the prices in 2001.

When spot price levels were low, the deviation from the full information (FI) case was quite small, as most production was supplied by the boiler. At very high levels, the deviation evened out, for once the prices reach a certain level the bidding strategy is unchanged for all levels above it.

With cooling, the shape of the deviation curves changed drastically, both for 2001 and 2002. The large uncertainty for high spot price levels all but disappeared, due to the fact that once the prices reach a sufficiently high level, it is desirable to produce as much as possible in order to take advantage of this fact, as any excess heat produced as a consequence of this may simply be cooled off without extra cost.

There are several avenues open for expansion of the models presented in this paper. One of the more evident is incorporating start-stop characteristics. However, certain numerical problems are associated with this, as the model evolves from a standard linear programming model to an integer programming model with all the difficulties inherit therein. Another avenue is the inclusion of the regulating market which in practice offers the plants the opportunity of additional income by selling or offering to withhold production to compensate for general imbalances in the system. Further, the model could be expanded to include additional CHP units and boilers.

The simulations may also be expanded by examining the influence of the heat demand. One might expect that if it is small and cooling is not allowed, fewer hours of production may be bid to the spot market, which means that it becomes of greater importance to place the bids in the 'right’ hours, i.e., when the spot price turns out to be high. This may be taken one step further by modelling the heat demand as stochastic.

Further, it is not always expedient to have a daily circular constraint on the heat storage, i.e., that the storage contents at the end of a day should equal the storage contents at the beginning of the same day. At times, typically over a weekend where the spot prices generally are lower than on regular weekdays, it would be useful to expand the constraint to run over e.g. 96 hours rather than just 24. This enables the plant to utilise the heat storage to the fullest extent over a weekend, postponing production on the CHP unit till prices once again reached their weekday level.

These issues are currently being investigated in ongoing projects.
B.8 Acknowledgement

This project from which this paper has arisen was supported by Eltra [40] under the PSO programme.

B.9 List of symbols

<table>
<thead>
<tr>
<th>Index</th>
<th>Interval</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>${1, \ldots, T}$</td>
<td>Time (hours)</td>
</tr>
<tr>
<td>$s$</td>
<td>${1, \ldots, S}$</td>
<td>Stochastic instance (scenario)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_m$</td>
<td></td>
<td>Back pressure value</td>
</tr>
<tr>
<td>$c_k^{kv}, c_k^{k}$</td>
<td>DKK/MWh$_{\text{heat}}$</td>
<td>Production costs for CHP unit and boiler, resp.</td>
</tr>
<tr>
<td>$\pi_t^s$</td>
<td>DKK/MWh$_{\text{power}}$</td>
<td>Spot price at time $t$ for scenario $s$</td>
</tr>
<tr>
<td>$p_1, p_2$</td>
<td>DKK/MWh$_{\text{power}}$</td>
<td>Level 1 and 2 prices, respectively</td>
</tr>
<tr>
<td>$m_{1t}^s, m_{2t}^s$</td>
<td>MWh$_{\text{heat}}$</td>
<td>Heat produced on the CHP unit at price $p_1$ resp. $p_2$ at time $t$</td>
</tr>
<tr>
<td>$m_{kt}^s$</td>
<td>MWh$_{\text{heat}}$</td>
<td>Heat produced on the boiler at time $t$, scenario $s$</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td></td>
<td>Probability of scenario $s$ occurring ($\sum_{s=1}^{S} \phi_s = 1$)</td>
</tr>
<tr>
<td>$\delta_{1t}^s, \delta_{2t}^s$</td>
<td></td>
<td>Indicators for price level</td>
</tr>
<tr>
<td>$K$</td>
<td>MW</td>
<td>Maximal capacity on the CHP unit</td>
</tr>
<tr>
<td>$d_t$</td>
<td>MW</td>
<td>Heat demand at time $t$</td>
</tr>
<tr>
<td>$V_t^s$</td>
<td>MWh</td>
<td>Heat storage reserve at the beginning of hour $t$, scenario $s$</td>
</tr>
<tr>
<td>$V_{\text{max}}$</td>
<td>MWh</td>
<td>Heat storage limit</td>
</tr>
</tbody>
</table>
C.1 Abstract

Local CHP development in Denmark has during the 90’s been characterised by large growth primarily due to government subsidies in the form of feed-in tariffs. In line with the liberalisation process in the EU, Danish local CHPs of a certain size must operate on market terms from 2005.

This paper presents a stochastic unit commitment model for a single local CHP plant (consisting of CHP unit, boiler, and heat storage facility) which takes into account varying spot prices. Further, additional technology is implemented in the model in the form of an immersion heater.

Simulations are conducted using the spot prices of the years 2001–2003, both with and without the immersion heater included in the model, and the results are compared to the full information case.

Index Terms— combined heat and power, Denmark, market conditions, power
C.2 Introduction

In Denmark, the development of local combined heat and power (CHP) plants has been characterised by large growth throughout the nineties (cf. Figure C.1), based in part on government subsidies in the form of feed-in tariffs. In 2003, local CHP production in western Denmark constituted almost a quarter of the total electricity production [40]. Simultaneously, there has been a significant growth of wind power, particularly in the western Danish system. As both the power produced by the local CHPs and the wind power are prioritised, the production of these types of power is occasionally sufficient to meet the total demand in the system, causing the market price of electricity to drop dramatically, sometimes even to zero-level.

In line with the liberalisation process of the energy sectors of the EU countries, present Danish legislation states that local CHP are to begin operating on market conditions in the year 2005 [80]. This means that the income that the local CHPs previously gained from selling electricity at the feed-in tariff (cf. Figure C.2) is replaced in part by income gained from selling electricity on the Nordic (Denmark, Sweden, Norway, Finland) spot market, Nord Pool [86]. In addition
a subsidy independent of production level will be obtained. Thus, the electricity
production quantities of the local CHPs will depend on the market price.

Recent work has considered the problem of optimal bidding from an overall point
of view [104], derived conditions for the existence of an optimal supply function
while modelling competition using an appropriate probability distribution [6],
or produced offer stacks for a single generator based on dynamic programming
[73].

In this paper, the situation is considered from the point of view of a local CHP
plant, which must make bids to the Nordic electricity exchange in accordance
with the rules of the Nord Pool 24-hour cycle. Thus, bids are made at a time
where spot prices are unknown and this fact must be taken into account when
modelling, resulting in the stochastic unit commitment model presented. As
the model only considers a local CHP plant, it is reasonable to assume that the
plant is price taking in the market and thus has no direct influence on the spot
price. To illustrate the workings of the model, simulation results are included
C.3 Uncertainties when bidding on the spot market

Each day before noon the local CHP plant bids electricity prices and volumes to the Nordic electricity exchange, Nord Pool [86]. The plant is assumed to be a price taker on the market and the hourly spot prices for the following day are, as mentioned above, unknown at the time of the bidding. The model presented in this paper considers the problem taking into account various spot price scenarios, \( s \), where \( \pi_t^s, t = 1, ..., T, s = 1, ..., S \) indicates the spot price for hour \( t \) in scenario \( s \). A spot price scenario consists of a series of \( T \) spot prices (e.g. based on historical data), where \( T \) typically is a multiple of 24. Each scenario \( s \) occurs with a probability represented by the non-negative parameter \( \phi^s \). In this paper, historical data from Nord Pool was used (see Section C.6).

In the simple model presented in the previous paper [90], a two-level price structure was introduced. The prices \( p_1 \) resp. \( p_2 \) indicated the surplus cost of producing electricity when the heat produced simultaneously could be utilised resp. when it could not (note, that \( p_1 < p_2 \)). When spot prices were expected to be below \( p_1 \) nothing was bid to the spot market; were the spot prices expected to be above \( p_2 \) full capacity was bid; and were the spot prices expected to be between \( p_1 \) and \( p_2 \) the model would determine a suitable bid depending on the heat demand.

This simple approach may be less than expedient in certain cases. For instance, consider the case illustrated in Figure C.3: here the two-price model enforces a shut-down in the intermediate period with low prices (below \( p_1 \)) where it might be more desirable to continue production undeterred throughout the whole nine-hour period, e.g. if the start-up cost of the CHP unit is higher than the cost of producing during the low price period.

Alternatively, in order to handle situations as the example given above, define price levels for bids that adapt to the hourly spot prices. Consider a situation with e.g. five different spot price scenarios for a given hour, where \( \pi^1 < \pi^5 < \pi^3 < \pi^2 < \pi^4 \). In that case, six ordering levels are needed: one level below all spot prices (level \( o = 1 \)), one level above all spot prices (level \( o = 6 \)), as well as four intermediate levels (levels \( o = 2, ..., o = 5 \)). The parameter \( \delta^s_{ot} \) is used to keep track of the order of the spot prices \( \pi^s_t \) for each hour, and is defined as

\[
\delta^s_{ot} = \begin{cases} 
1, & \text{if scenario } s \text{ has order } o \text{ in hour } t \\
0, & \text{else}
\end{cases}
\]

where \( t = 1, ..., T, s = 1, ..., S, \) and \( o = 1, ..., S + 1 \).
C.3 Uncertainties when bidding on the spot market

Each hourly price level has an associated bid volume, \( m_{o,t} \), measured in MWh_{heat}. Figure C.4 illustrates the case with five ordered spot prices for a single hour.

The level below the lowest spot price \( \pi^1 \) (i.e. the first expected spot price for the hour in question) has order 1. This level is too low for the plant manager to willingly bid any volume (i.e. lower than the marginal production cost on the CHP unit). Once the price exceeds the spot price \( \pi^1 \) and enters level 2, the plant manager is willing to bid the volume \( A \). When the price exceeds the fifth expected spot price, \( \pi^5 \), and reaches spot price level 3, the plant manager is willing to bid the total volume \( B \). The full capacity, \( K_{kv} \), of the CHP unit is not bid before the second expected spot price, \( \pi^2 \), is exceeded (i.e. level 5 is reached).

Note, that

- \( \delta^s_{1t} = 1 \) for all \( s \) and \( t \), as all spot price scenarios have at least order 1;

- \( \delta^s_{S+1,t} = 0 \) for all \( s \) and \( t \), as no spot price scenario will ever have order \( S + 1 \).
C.4 Unit commitment model

The model presented in this paper considers a local CHP plant with a power generation unit (in the following termed the CHP unit), which generates heat as a by-product, a boiler, which produces heat only and a heat storage facility. A basic model which handles bidding to the spot market while taking into account constraints regarding production and minimising costs was presented in [90]. It is this basic model which has been expanded upon in this paper by including a unit commitment aspect and startup costs for the CHP unit as well as introducing a new technical tool: an immersion heater, which uses electricity (when prices are low) to produce heat and thus may be seen as a way of storing electricity.

Most Danish local CHP plants started out as heat plants and still have a local heat demand that must be met. The heat demand for period $t$ is included in the model as $d_t$. The electricity-to-heat ratio for the CHP unit (in the following termed the back pressure value) is denoted $c_m$, and indicates that 1 MWh heat is produced when producing $c_m$ MWh power.

In the following, $c_{kv}$, $c_{start}$, and $c_k$ denote, in turn, production costs for the CHP unit, starting costs for the CHP unit, and production costs for the heat
boiler. Further, $K_{kv}$ and $K_k$ represent the maximum production capacity for the CHP unit and the boiler, respectively. Finally, $V^s_t$ is the volume of heat in the heat storage facility at the beginning of period $t$ in scenario $s$ and $V_{\text{max}}$ is the maximum capacity of the heat storage.

Firstly, it must be ensured that the heat produced meets the demand in all periods, which the following constraints handle.

\begin{align*}
V^s_{t+1} &= V^s_t + \sum_o m_{ot} \delta^s_{ot} + m^s_{kt} - d_t, \quad (C.1) \\
\forall s, t &= 1, \ldots, T - 1 \\
V^s_1 &= V^s_T + \sum_o m_{oT} \delta^s_{oT} + m^s_{kT} - d_T, \forall s \quad (C.2)
\end{align*}

The constraints state that the amount of heat in the storage at the beginning of the following period should equal the heat in the storage at the beginning of the present period, as well as the heat produced on the CHP unit and the boiler, once heat demand has been met. Now, let $m_{\text{min}}$ denote the minimum production on the CHP unit. Thus, production in every hour must be either 0 or at least $m_{\text{min}}$. To ensure this, the binary variable $z^s_t$ is introduced, which is 1 when there is production on the CHP unit and 0 otherwise. Thus the production capacity constraints for the CHP unit may be written as

\begin{equation}
z^s_t m_{\text{min}} \leq \sum_o m_{ot} \delta^s_{ot} \leq z^s_t K_{kv}, \forall s, t \quad (C.3)
\end{equation}

The non-negative variable $v^s_t$ indicates whether the CHP unit was started during period $t$ in scenario $s$ and is ensured binary by the optimisation and the constraint

\begin{equation}
v^s_t \geq z^s_t - z^s_{t-1}, \forall s, t = 2, \ldots, T \quad (C.4)
\end{equation}

In order to handle the initial period, the binary parameter $z_{\text{init}}$ is defined, indicating whether the CHP unit was running in the period immediately prior to the time considered. This leads to the inclusion of the constraint

\begin{equation}
v^s_1 = z^s_1 - z_{\text{init}}, \forall s \quad (C.5)
\end{equation}
Starting and stopping characteristics may be implemented similarly for the boiler but is of less interest and thus left out in this paper.

Finally, the capacity restrictions for both the boiler and the heat storage facility must be respected, which leads to the following constraints:

\begin{align*}
0 & \leq m_{skt} \leq K_k, \quad \forall s, t \quad (C.6) \\
0 & \leq V_{st}^s \leq V_{\text{max}}, \quad \forall s, t \quad (C.7) \\
z_{st} & \in \{0, 1\}, m_{ot}, v_{st}^s \geq 0, \quad \forall o, s, t \quad (C.8)
\end{align*}

The problem, prior to introducing the immersion heater, now amounts to minimising the sum of the cost of expected production and startup costs while subtracting expected profits from electricity sales, i.e. minimising the expression:

\[\sum_{s,t} \phi^s \left\{ c_{kv} \sum_o m_{ot} \delta_{ot}^s + m_{skt} c_k^k + \epsilon_{\text{start}} v_{st}^s - \pi_{st}^s c_m \sum_o m_{ot} \delta_{ot}^s \right\}\]

subject to the constraints (C.1)–(C.8).
The output produced by the model is a production plan in terms of heat volumes for both the CHP unit and the boiler. This is then translated to an electricity supply function via the electricity-to-heat ratio, $c_m$, for the CHP unit. The size of the model (C.1)–(C.9), which thus may be categorised as a stochastic integer LP problem, depends on several factors such as the specifications of the plant to which it is applied, the number of spot price scenarios included, as well as the time horizon for which the model is run.

### C.5 Immersion heater

In line with the new legislation it may become permissible to use an immersion heater as a way of producing heat for the heat storage facility by use of electricity. This would primarily be in the case where electricity prices are particularly low (read: lower than heat production costs on the boiler and/or the CHP unit). Thus the model can contain up to three different heat production technologies: combined heat and power unit, heat boiler, and immersion heater. The interconnection between these technologies is illustrated in Figure C.6 where the marginal costs of the technologies are depicted (note that start-up costs are disregarded).
The figure may be interpreted as follows: When the price of electricity is low, it is cheap to produce heat using the immersion heater and expensive to produce heat using the CHP unit, and vice versa when the price of electricity is high (in which case heat production on the CHP unit is compensated by the high price attained for electricity produced simultaneously). The cost of producing heat on the boiler is constant, regardless of the price of electricity. Similarly, fuel prices are assumed constant.

The heat production pattern distributed on the three technologies is therefore simple to determine. Production is placed on the immersion heater when the electricity price is low. If the boiler cost is sufficiently low, the boiler takes over heat production resulting in a medium level price (the horizontal bold line on Figure C.6), whereafter the CHP unit takes over production once the electricity price is sufficiently high. In some cases the boiler cost may be so high that no boiler production takes place and the demanded heat is produced solely by the immersion heater and the CHP unit (in such cases, the horizontal boiler cost lies above the cross between the costs of the immersion heater and the CHP unit).

It should be obvious that the combination of the three technologies will have a stabilising effect on the electricity spot prices. For instance, consider the case where all western Danish local CHP plants (which constitute nearly 25% of the total electricity production in that area, cf. Section C.2) have immersion heaters or similar technological means for electricity-to-heat conversion. If spot prices are very high they are all interested in producing electricity for sale on the market, and if prices are low they all buy electricity to convert to heat. When such a large percentage of the producers act in the manner described it must invariably stabilise the spot prices by helping to eliminate price spikes and extreme low price periods.

The modelling of the immersion heater is analogous to that of the CHP unit. Let $c_{ot}$ represent the volume of electricity purchase offered at the spot price $\pi_s^t$ with order $o$. Note, that the binary used to indicate whether electricity is bought is $(1-\delta_{ot})$ and thus the reverse of the indicator for electricity production. The electricity-to-heat conversion rate is termed $c_e$. For the sake of simplicity, it is assumed that $c_e = 1$, i.e. there is direct conversion from electricity to heat, however any other electricity-to-heat conversion technology, e.g. a heat pump, may be used – the difference is simply a question of changing the size of the conversion rate $c_e$. Due to the nature of $\delta$, it holds that

- $(1 - \delta_{st}) = 0, \forall s, t$ as it is impossible to buy electricity cheaper than the lowest spot price scenario;
- $(1 - \delta_{S+1}^s) = 1, \forall s, t$, i.e. for all scenarios the offer to buy is accepted.
C.6 Simulations

The cost of buying electricity for heat production must of course be deducted in the objective function and capacity constraints for the immersion heater must be included in the model. Further, the amount of heat produced by the immersion heater must be included in the storage constraints (C.1) and (C.2). Revising the model in accordance with these statements implies minimising the expression

$$\sum_{s,t} \phi_s \left\{ (c^{kv} - \pi^{st}_t c_m) \sum_o m_{ot} \delta_{ot}^s + m_{kt} c^k \right\}$$

subject to the storage and capacity constraints

$$V_{t+1}^s = V_t^s + \sum_o m_{ot} \delta_{ot}^s + m_{kt} - d_t$$

$$(C.11)$$

$$V_1^s = V_T^s + \sum_o m_{oT} \delta_{oT}^s + m_{kT} - d_T$$

$$(C.12)$$

$$0 \leq \sum_o e_{ot} (1 - \delta_{ot}^s) \leq e_{max}, \quad \forall s, t$$

$$(C.13)$$

as well as the constraints (C.3)–(C.8). Note, that because it is assumed that the conversion rate of the immersion heater is $c_e = 1$, the volume $\sum_o e_{ot} (1 - \delta_{ot}^s)$ may both symbolise a heat or an electricity volume, according to necessity.

C.6 Simulations

This paper illustrates the above mentioned characteristics by including simulation results achieved by applying the models (C.1)–(C.9) (without immersion heater), and (C.3)–(C.8) and (C.10)–(C.13) (with immersion heater) using data for a local CHP plant located in western Denmark. The plant has a boiler capacity of $K_k = 23.2$ MWh, heat capacity of $K_{kv} = 2.392$ MWh and an electricity capacity of 2.028 MWh on the CHP unit, and the capacity of the heat storage facility is $V_{max} = 25$ MWh. The total heat demand over a year is approximately 15 GWh with clear seasonal variations (i.e. high in winter, low in summer).

The spot price scenarios used were based on the actual western Danish spot prices of 2001–2003. The daily mean spot prices for the years simulated are shown in Figure C.7.
In the Nordic system hydro power accounts for about half of the electricity pro-
duction hence the electricity prices will depend heavily on the amount of hydro
power available. The year 2001 was fairly normal with no extreme oscillations.
The typical spot prices were in the range 150 DKK/MWh to 300 DKK/MWh,
with typical daily and weekly variations, and with no clear trend from beginning
to end of the year.

The year 2002 was a dry year in the Nord Pool area, i.e. less hydro power was
available than usual, which caused the general rising trend in the prices towards
the end of the year, beginning at the level of 2001, and ending at a level around
400 DKK/MWh to 600 DKK/MWh.

In the beginning of 2003, the last effects of the dry year 2002 could still be seen,
whereafter the prices stabilised at the level of approximately 250 DKK/MWh
during the spring. However, over the summer prices fluctuated significantly with
an extreme peak occurring in September. Finally, towards the end of the year,
prices began to fall as spring was approached with its expected large inflow.

Thus, the three years display marked individual characteristics concerning the
general trend of the price level within a year. However, the daily and weekly
variations remained more stable, although price spikes (the highest was 4430
DKK/MWh – nearly 18 times the typical level) were observed as previously
noted.
The spot price scenarios in the models (C.1)–(C.9) (without immersion heater), and (C.3)–(C.8) and (C.10)–(C.13) (with immersion heater) were constructed in the following way. For any given day the set of scenarios consisted of the spot prices of $N$ previous days (with equal probability) as well as an artificial high price scenario with a small probability. The latter ensured that some bid was made in the unlikely event that the $N$ previous days all consisted of exceptionally low spot prices. Further, the spot price level was varied by adding (or subtracting) some positive amount, e.g. 50 DKK/MWh, from all the yearly spot prices. This variation of the spot price level is justified by the aforementioned characteristics of the simulated years (although the ranges selected for the variations is unrealistically large, this is to display more clearly the asymptotic behaviour of the results).

Simulating both with and without the immersion heater (IH) for each year, yields six cases in total, which were implemented using GAMS [92]. The various simulation cases are named in Table C.1.

All cases were simulated with $N = 3$, $N = 5$, and $N = 7$. The results were in each case compared to the case where the prices were known in advance (the full information case). This yielded the differences depicted in Figures C.8, C.9, and C.10, respectively. All three figures have been deliberately truncated so the differences for the levels below +500 DKK/MWh may be seen clearly. The values for the two truncated spot price levels, +500 DKK/MWh and +2000 DKK/MWh, were approximately 7 million DKK and 25 million DKK, respectively, for all the tested instances of $N$.

One interesting aspect, which is evident in all three figures, is that for each of the simulated years the cases in which the immersion heater is included (B, D, F) have smaller errors than the corresponding cases without the immersion heater (A, C, E), the single exception being for $N = 7$ where cases C and D (2002) cross paths around the level +150 DKK/MWh. From the point of view of a single CHP plant the presence of an immersion heater (or similar technology) thus has a stabilising effect on the total cost. This goes well in conjunction with the statement from Section C.5 that the relationship between CHP production and heat production via conversion technologies should stabilise the market prices somewhat in the long run. The size of the error compared to the total cost varies
Figure C.8: Difference between the full information case and the stochastic case after simulation of cases A–F with varying spot price levels and $N = 3$.

Figure C.9: Difference between the full information case and the stochastic case after simulation of cases A–F with varying spot price levels and $N = 5$. 
C.7 Conclusions

A stochastic unit commitment model for a single local CHP plant consisting of a CHP production unit, a boiler, and a heat storage facility was presented in the paper. The model takes into account varying spot prices as well as starting costs on the CHP unit. The possibility of including extra technology in the form of an immersion heater or similar technology which transforms electricity to heat was also implemented in the model.

Simulations were conducted for the years 2001–2003 using actual data from a western Danish local CHP plant and the spot prices for western Denmark during that period. The simulations considered the situation both with and without

Figure C.10: Difference between the full information case and the stochastic case after simulation of cases A–F with varying spot price levels and \( N = 7 \).

significantly from year to year in the middle range cases, i.e. the interval ±150 DKK/MWh. For \( N = 5 \) at the neutral spot price level (+0 DKK/KWh) it is 4% with IH and 8.5% without in 2001, 15% with IH and 14% without in 2002, and 23% with IH and 27% without in 2003. This confirms the analysis of the different characteristics in the three years, cf. the descriptions on page 5. 2001 was the 'standard' year, 2002 had the rising trend but was otherwise fairly stable, and 2003 had several volatile periods with significant price spikes.
the inclusion of the immersion heater.

From the simulation results it was seen that the inclusion of the immersion heater helped reduce uncertainty, as the difference between the simulated cases and the full information case dropped with up to more than 4% of the total cost. Additionally, the characteristics of the three years simulated described early in Section C.6 were represented in the way the size of the error compared to the total cost differed from year to year.

An avenue for further research would be to consider more than one plant in the model. One perspective that arises from such an avenue is the possibility of looking at the entire western Danish local CHP system, i.e. by aggregating the model using available data of the distribution of different CHP technologies and their characteristics.
Appendix D

A Partial Load Model for a Local Combined Heat and Power Plant

D.1 Abstract

This paper considers a local combined heat and power (CHP) plant faced with bidding into the spot market while taking into account assorted physical limitations. In contrast to classic unit commitment problems there is the added difficulty of coping with a power production unit which also produces heat to be utilised, local heat demand, a heat only production unit, and a heat storage facility.

As prices are unknown at the time of production planning, a simple stochastic model is implemented to construct an optimal plan under uncertainty. In the model, the CHP unit is segmented and with it all the parameters (cost, electricity to heat ratio, etc.) connected to it. This enables taking into account varying efficiency and production costs as well as enabling the handling of pollutant emissions, the extent of which depend in various ways on the level at which production takes place.
A case study is conducted using data from a typical local CHP plant and the years 2003 through 2006 are simulated to assess the accuracy of the stochastic model compared to the deterministic case.

D.2 Nomenclature

The notation used throughout the paper is stated below.

- $b_t^s$: boiler production under scenario $s$, in hour $t$
- $b_{\text{min}}$: minimum production capacity on the heat boiler
- $c_e^n$: cost of emissions of type $e$
- $c_m^g$: electricity to heat ratio, segment $g$
- $c_g^{kv}$: production cost for the CHP unit, segment $g$
- $c_k^h$: production cost for the heat boiler
- $c_g^{\text{start}}$: start-up cost for the CHP unit
- $c_k^{\text{start}}$: start-up cost for the heat boiler
- $\delta_{st}$: equals 1 if scenario $s$ has order $o$ in hour $t$, else 0
- $d_t$: heat demand in hour $t$
- $K_h$: production capacity of the heat boiler
- $K_g^{kv}$: heat production capacity of the CHP unit, segment $g$
- $m_{\text{min}}^g$: minimum heat production capacity of the CHP unit, segment $g$
- $m_{ot}^g$: heat production on the CHP unit, segment $g$, order $o$, hour $t$
- $V_t^s$: volume of the heat storage at the beginning of hour $t$, scenario $s$
- $V_{\text{max}}$: maximum capacity of the heat storage
- $v_t^s$: start-up variable for the CHP unit, scenario $s$, hour $t$
- $w_t^s$: start-up variable for the heat boiler, scenario $s$, hour $t$
- $y_t^s$: binary variable indicating production on the heat boiler, scenario $s$, hour $t$
- $y_{\text{init}}$: binary parameter indicating whether there was production on the heat boiler prior to hour 1
$z_t^s$ binary variable indicating production on the CHP unit, scenario $s$, hour $t$

$z_{init}$ binary parameter indicating whether there was production on the CHP unit prior to hour 1

D.3 Introduction

Local combined heat and power (CHP) plants constitute a not insignificant share of the power production in Denmark, particularly the western part of the country. Since January 1st, 2005 these plants have been required to act on market terms, i.e. sell the power produced on the spot market (or bilaterally), rather than at the fixed feed-in tariff previously used. Many versions of the unit commitment problem have been discussed in the literature, from the so-called economic dispatch (the task of dispatching the entire system at least cost given a certain demand) to optimal bidding to an electricity spot market in various shades. However, CHP is rarely touched upon and then mainly from the system point of view.

Local CHP production in Denmark has, in recent years, undergone several changes. Perhaps most significant was the transition from unloading production at the mentioned feed-in tariff to selling it on market terms, preferably at Nord Pool\textsuperscript{1} [80]. In connection herewith, the plants must plan their production and their bids to the power market at a time when the sales prices for electricity are yet unknown.

Most Danish local CHP plants originated as heat plants whose raison d’être was servicing a local district heating network. The sale of electricity is primarily seen by the plant operators as a way of financially compensating for the additional expenses of electricity production compared to pure heat production, the latter of which the plants are often contractually bound to supply to the local network. The local CHP plants are all thermal, with fuel types including (but not limited to) waste, bio gas, and straw but a large majority are fuelled by natural gas [29]. They often also possess a heat storage facility (typically a large hot water tank) and/or a purely heat producing unit. The local CHP plants range in size from less than one hundred kW to nearly 100 MW and the gas-fired units are particularly characterised by their rapid commitment ability.

The unit commitment problem in energy production planning typically involves scheduling generators in order to meet system demand while taking into account the physical constraints connected herewith and minimising operating costs (see

\textsuperscript{1}The Nordic power exchange [86].
e.g. [10] and [101]). The scheduling of production incorporates binary variables which indicate whether the production unit is operating and the problem is thus a mixed integer problem. For more detailed overviews of work performed on the problem, see [57], [83], and [93].

In line with the deregulation of various electricity markets in recent years, e.g. in Australia, New Zealand, Spain, and the Nordic market, other types of unit commitment problems have arisen, namely those that consider the problem from the point of view of power generating companies acting on the market. This has resulted in several papers on how production companies with a portfolio of several large production units should bid to the market. The approach varies: considering imperfect information ([6], [103]), taking risk aversity into account [91], the response of a thermal unit to an electricity spot market [9] (this is similar to the approach in the present paper, except that prices are known at the time of planning and the unit is not a CHP), considering an oligopolistic production company [25], and, in contrast, veiwing the production company as a price-taker ([26]).

Alternatively, the problem of unknown prices at the time of planning has been managed by utilising a time series model fitted to historical electricity prices to forecast the prices of the bidding day, [27], [75]. Applying this to the Nordic power system would probably require a transfer function model (see e.g. [14]), as the Nordic spot prices depend heavily upon the level of the Norwegian and Swedish water reserviors and thus on precipitation.

Combined heat and power is not often taken into consideration but is of particular interest in the Danish system due to the large penetration of said technology, especially in the western part of the country. Typically, when CHP is considered, the economic dispatch problem is still prevalent [22], [54], [100], [16], though probabilistic production simulation of power systems with CHP may also be found [66], [99].

The present paper employs a different perspective than the above-mentioned works: it examines CHP (acting on the Nordic market\(^2\)) but concentrates on a single small plant - in this respect a production company is considered. However, as it is a local CHP that is considered and not a large production company which produces only electricity, the perspective is somewhat skewed in comparison to the mentioned papers. Here, the production of the CHP plant is examined in terms of heat production: the primary concern of the local CHP plant is meeting the heat demand. Simultaneously, the plant manager must take into account that at the time of bidding (12-36 hours prior to the time of operation),

\(^2\)The Nordic electricity spot market, Nord Pool ([86]), is a day-ahead market with hourly prices. Submitted bids are for an hour at a time and given no later than noon 12-36 hours prior to the day of operation.
electricity spot prices are unknown. Note, that the bid is made for the entire 24 hours of the coming day of operation at the same time. If prices are expected to be low (i.e. less than the cost of production), the CHP plant may satisfy heat demand in that period by extracting heat from the storage facility or utilising the heat only production unit. When high prices are expected, clearly it is advantageous to bid electricity from the CHP production unit, provided there is room in the storage facility for the heat produced which is not fed directly into the local network. Preliminary work on this subject may be found in [89], [90]. Further, this paper takes into account the possibility of segmenting the load on the CHP unit, as several factors vary according to the load level, e.g. the production efficiency of the unit and amount of pollutants emitted during production. Segmenting the load may be understood in the manner that the load variable is divided into different levels and the characteristica (i.e. the parameters) that pertain to the load variable may thus be varied depending on the production level of the CHP unit. For instance, production efficiency is typically higher when producing at full load than producing at minimum load and often more pollutants are emitted at lesser load levels than at higher levels.

The paper is organised as follows. The unit commitment problem with uncertainties is formulated in Section D.4; in Section D.5 a case study is conducted using data from [39]; and finally Section D.6 concludes upon the work and suggests avenues for further research.

D.4 Problem formulation

In the present paper a model is presented, elaborating on previous work ([89]), which alleviates planning difficulties for a single local CHP plant, consisting of a combined heat and power generating unit, a heat boiler, and a heat storage facility. The model, which is a two-stage stochastic programming model, creates a day-ahead production schedule for the plant that minimises production costs, start-up costs, and cost of emissions, taking into account restrictions regarding production capacity of both the CHP unit and the heat boiler; start-up constraints for the CHP unit and the heat boiler; capacity limitations of the heat storage facility; meeting heat demand; while minimising production expenses, including starting costs for both production units. The first stage contains the bidding decisions and the second stage comprises the economic dispatch. The variables pertaining to the various production units all relate to or designate heat volumes and may be converted to electricity volumes by the electricity-to-heat ratio. The planning is done under uncertainty, in the respect that a number of price scenarios each weighted with a probability are taken into consideration in the objective function. Each scenario consists of a set of 24 hourly electricity
spot prices.

There are, however, other factors that may influence planning besides the expected sales price of the electricity generated, such as emission of environmental pollutants that are generated during power production, or the fact that production may be more efficient, and thus more economical, at certain load levels. These concerns are also taken into account in the present model. In this paper a typical representative of local Danish CHP plants is considered, i.e. a gas-fired unit of roughly 10 MW. As gas-fired units typically have rapid commitment abilities, disregarding ramp rates in the model is not unreasonable. Also, the plant is considered price-taker in the market, as it is but one of many such units which participate, thus the perfect market assumption is in fact made.

D.4.1 Decision structure

The problem is considered over a period of time $T = \{1, \ldots, T\}$, here 24 hours, which is the relevant amount of hours for bidding to the day-ahead spot market. The spot price scenarios, each of which consists of a set of 24 hourly spot prices, are designated $\pi^s$, where $s \in S = \{1, \ldots, S\}$.

In the simple linear model\textsuperscript{3} for a local CHP presented in a previous paper [90], a two-level price structure was introduced. The prices $p_1$ resp. $p_2$ indicated the surplus cost of producing electricity when the heat produced simultaneously could be utilised resp. when it could not (note, that $p_1 < p_2$). When spot prices were expected to be below $p_1$ nothing was bid to the spot market; were the spot prices expected to be above $p_2$ full capacity was bid; and were the spot prices expected to be between $p_1$ and $p_2$ the model would determine a suitable bid depending on the heat demand. In this model, also, start-up costs were not taken into consideration.

This simple approach may be less than expedient in certain cases. For instance, consider the case illustrated in Figure D.1: here the simple two-price model enforces a shut-down in the intermediate period with low prices (below $p_1$) where it might be more desirable to continue production undeterred throughout the whole nine-hour period, e.g. if the start-up cost of the CHP unit is higher than the cost of producing during the low price period.

Alternatively, in order to handle situations as the example given above, define price levels for bids that adapt to the hourly spot prices. Consider a situation with e.g. five different spot price scenarios for a given hour, where $\pi^1 < \pi^5 <$

\textsuperscript{3}Which did not include starting costs.
$\pi^3 < \pi^2 < \pi^4$. In that case, six ordering levels are needed: one level below all spot prices (level $o = 1$), one level above all spot prices (level $o = 6$), as well as four intermediate levels (levels $o = 2, \ldots, o = 5$). The parameter $\delta_{ot}^s$ is used to keep track of the order of the spot prices $\pi_t^s$ for each hour, and is defined as

$$\delta_{ot}^s = \begin{cases} 1, & \text{if scenario } s \text{ has order } o \text{ in hour } t \\ 0, & \text{else} \end{cases},$$

where $t \in T$, $s \in S$, and $o \in O = \{1, \ldots, S + 1\}$.

Each hourly price level has an associated bid volume, $m_{ot}$, measured in MWh$_{heat}$. Figure D.2 illustrates the case with five ordered spot prices for a single hour.

The level below the lowest spot price $\pi^1$ (i.e. the first expected spot price for the hour in question) has order 1. This level is too low for the plant manager to willingly bid any volume (i.e. lower than the marginal production cost on the CHP unit). Once the price exceeds the spot price $\pi^1$ and enters level 2, the plant manager is willing to bid the volume $A$. When the price exceeds the fifth expected spot price, $\pi^5$, and reaches spot price level 3, the plant manager is willing to bid the total volume $B$. The full capacity, $K_{kv}$, of the CHP unit is not bid before the second expected spot price, $\pi^2$, is exceeded (i.e. level 5 is reached).
Figure D.2: Ordering of spot prices for a single hour with associated total bid volumes.

Note, that

- \( \delta_{s,t}^s = 1 \) for all \( s \) and \( t \), as all spot price scenarios have at least order 1;
- \( \delta_{s+1,t}^s = 0 \) for all \( s \) and \( t \), as no spot price scenario will ever have order \( S + 1 \).

### D.4.2 Segmenting the load

By segmenting the load variable \( (m_{olt}) \) and the electricity-to-heat ratio \( (c_m) \) as well as the heat production cost \( (c_{kv}) \) of the CHP unit using the index \( g \in G = \{1, ..., G\} \), it is possible to take into account that power production efficiency is not necessarily linear within the capacity limits of the CHP generator. The segmenting enables the representation of the mentioned variable and parameters as piecewise linear functions. Total production, or load, is assumed continuously varying between the minimum of segment 1 and the maximum of segment \( G \) (which equals the installed production capacity of the unit). Thus maximum for one segment equals minimum for the next and segment \( g \) needs to be fully loaded before any production can take place in segment \( g+1 \). The total production on the CHP unit is the sum of the production on the individual segments.
When thus segmenting the production cost on the CHP unit it is important to bear in mind the hazards involved in doing so. In the case study (Section D.5) it is assumed that the marginal cost of production on the CHP unit (MCP) is constant on each segment and, so to say, decreasing as a function of the load which allows for convexity of the objective function (see Section D.4.3). Nevertheless, as soon as the MCP changes character, there is the risk of non-convexity of the total cost as a function of the load. For instance if the MCP is linear on each segment with slopes of opposite signs or merely if one of the intermittent segments had higher MCP than the surrounding segments it would have consequences for the convexity of the total cost.

D.4.3 Objective function

Let $b^s_t$ denote the production on the heat boiler for period $t$ under scenario $s$, $c^k$ the production cost of the same unit, $v^s_t$ and $w^s_t$ are non-negative variables that indicate whether a start-up of the CHP unit resp. the boiler has taken place, and finally $c^k_{\text{start}}$ and $c^k_{\text{start}}$ are the start-up costs for the CHP unit resp. the boiler. The costs considered in the model are the production costs of the CHP generator and the boiler, as well as the starting costs for both units.

$$\sum_{o \in \mathcal{O}} \sum_{g \in \mathcal{G}} c^k_g m^g_{ot} \delta^s_{ot} + b^s_t c^k + c^k_{\text{start}} v^s_t + c^k_{\text{start}} w^s_t$$ (D.1)

Emissions from a production unit naturally depends on the load. However, the efficiency of the unit often varies depending on the load as well, i.e. emissions may be relatively low at full load compared to half load. Including an emission cost in the objective function is relatively simple. Letting $n^g_e$ indicate emission of type $e \in \mathcal{E} = \{1, \ldots, E\}$ for and $c^e_n$ the associated cost, emission expences may be considered in the model by including the term

$$\sum_{e \in \mathcal{E}} c^e_n \sum_{o \in \mathcal{O}} \sum_{g \in \mathcal{G}} n^g_e m^g_{ot} \delta^s_{ot}$$ (D.2)

in the objective function. Thus, one may consider both environmental and economical consequences of whether production takes place under full or partial load. Income from sales of electricity produced on the CHP unit enters into the objective function as the term

$$\pi^s_t \sum_{o \in \mathcal{O}} \sum_{g \in \mathcal{G}} c^g_m m^g_{ot} \delta^s_{ot},$$ (D.3)

where $\pi^s_t$ is the expected spot price for period $t$ under scenario $s$. 
Letting $\phi^s$ denote the probability for scenario $s$, the above terms may be incorporated into an optimisation model, the objective of which is to minimise

$$\sum_{s \in S} \sum_{t \in T} \phi^s \left\{ (D.1) + (D.2) - (D.3) \right\}$$

subject to constraints given in the following paragraph.

### D.4.4 Constraints

The maximum load capacity of a segment equals the difference between the upper and lower bounds of the segment. The minimum load capacity of segment 1 is given by $m_{g_{\min}}^g, g \in \{1\}$ and is zero for the remaining segments. The production limits on the CHP unit, i.e. production must remain between between the capacity limits of each segment during any given period, may be stated as

$$z_{g,t}^s m_{g_{\min}}^g \leq \sum_{o \in O} m_{g,ot}^g \delta_{ot}^s \leq z_{g,t}^s K_{kv}^g, \; g \in G, \; s \in S, \; t \in T.$$ (D.5)

where $\sum_{o} m_{g,ot}^g \delta_{ot}^s$ is the heat production on the CHP unit at time $t$; $z_{g,t}^s$ is a binary variable indicating whether or not the unit is producing in segment $g$ and during period $t$, under scenario $s$; and $K_{kv}^g$ is the heat production capacity of the CHP unit in segment $g$. The structure of the constraint for a given segment $g$ ($g \neq 1$) is illustrated in Figure D.3.

In this case, $m_{g,t}^g = A$, $m_{g,t}^g = B - A$, and $m_{g,t}^g = K_{kv}^g - B$, where $K_{kv}^g$ is the maximum capacity of the CHP unit. This is in accordance with the constraint, as $A + (B - A) + (K_{kv}^g - B) = K_{kv}^g$ and $m_{g_{\min}}^g < A$.

Because of the 'sequential additivity' of the segments and because the cost is not assumed convex, a constraint is needed to ensure that if the CHP unit is producing in the segment following the present, then the present segment must be fully loaded, i.e.

$$\sum_{o \in O} m_{g,ot}^g \delta_{ot}^s \geq z_{t-s}^{s,g+1}(K_{kv}^g - m_{g_{\min}}^g), \; g \in G, \; s \in S, \; t \in T.$$ (D.6)

A similar constraint is introduced, linking the segments directly. If the CHP unit is running in the following segment, it must also be running in the present segment. This translates to

$$z_{g,t}^s \geq z_{t-s}^{s,g+1}, \; g \in G \setminus \{G\}, \; s \in S, \; t \in T.$$ (D.7)
Finally, a constraint that guarantees connectivity between start-up and production is needed. It ensures that a start-up is not planned in a given hour (i.e. $v_s^t = 1$) unless production is bid in the first segment of the hour in question (here, $v_s^t$ indicates whether the CHP unit has been started during period $t$ under scenario $s$). Conversely, if there is production, it should be at least minimum production. This results in the constraint

$$m_{\text{min}}^1 v_s^t \leq \sum_{o \in O} m_o^1 \delta_{ot}^s, \quad s \in S, t \in T.$$  \hspace{1cm} (D.8)

There are several additional constraints that concern the physical aspects of the problem. Firstly, heat demand must be met. Let $V_s^t$ denote the volume of the heat storage at the beginning of period $t$ under scenario $s$ and $d_t$ the heat demand in period $t$. Ensuring that the total amount of heat demanded equals the heat produced on both CHP unit and boiler plus any heat extracted from the storage facility during a given hour is accomplished by the constraints

$$V_{t+1} = V_t + \sum_{o \in O} \sum_{g \in G} m_o^g \delta_{ot}^s + b_s^t - d_t, \quad s \in S, t \in T \setminus \{T\},$$  \hspace{1cm} (D.9)

$$V_1 = V_T + \sum_{o \in O} \sum_{g \in G} m_o^g \delta_{oT}^s + b_{ST}^t - d_T, \quad s \in S.$$  \hspace{1cm} (D.10)
Constraints (D.9) ensure that the amount of heat produced when producing electricity for sale on the spot market does not exceed the free space in the storage once heat demand has been met. Fixing the initial volume of the storage strictly between limits then guarantees (by constraints (D.10)) that the storage is neither completely empty nor full to capacity at the end of the time interval considered. The equality sign in both (D.9) and (D.10) signifies that excess heat may not be cooled off.

Further, the non-negative variable \( v^s_t \), which indicates whether the CHP unit was started during period \( t \) under scenario \( s \), is ensured binary by the optimisation and the constraint

\[
v^s_t \geq z^s_g - z^s_{g-1}, \ g \in \{1\}, s \in \mathcal{S}, t \in T \setminus \{1\}. \tag{D.11}
\]

The initial condition

\[
v^s_1 = z^s_g - z_{\text{init}}, \ g \in \{1\}, s \in \mathcal{S} \tag{D.12}
\]

handles the first period, where \( z_{\text{init}} \) is a parameter which indicates whether the CHP unit was running in the period immediately prior to the time considered.

As for the boiler, the production limitations may be stated as

\[
y^s_t b_{\text{min}} \leq b^s_t \leq y^s_t K_b, \ s \in \mathcal{S}, t \in T, \tag{D.13}
\]

where, similar to the CHP unit, \( y^s_t \) is a binary variable indicating whether the boiler is in operation during period \( t \) under scenario \( s \); \( b^s_t \) is the heat produced on the boiler during period \( t \), scenario \( s \); \( b_{\text{min}}^s \) is the minimum production permitted on the boiler; and \( K_b \) is the production capacity of the boiler. The start-up variable \( w^s_t \) is controlled by the constraints

\[
w^s_t \geq y^s_t - y^s_{t-1}, \ s \in \mathcal{S}, t \in T \setminus \{1\}, \tag{D.14}
\]
\[
w^s_1 = y^s_1 - y_{\text{init}}, \ s \in \mathcal{S}, \tag{D.15}
\]

where \( y_{\text{init}} \) indicates whether production was taking place on the boiler in the period prior to the time considered. Finally, the variable limits must be included:

\[
0 \leq V^s_t \leq V_{\text{max}}, \ s \in \mathcal{S}, t \in T \tag{D.16}
\]
\[
z^s_g, y^s_t \in \{0, 1\}, \ g \in \mathcal{G}, s \in \mathcal{S}, t \in T \tag{D.17}
\]
\[
0 \leq m^g_{ot}, b^s_t, v^s_t, w^s_t, \ g \in \mathcal{G}, o \in \mathcal{O}, s \in \mathcal{S}, t \in T. \tag{D.18}
\]
D.5 Case study

In the following section a case study is conducted using data from [39], where several factors are varied depending on the load. Simulations are carried out for the years 2003-2006 (the latter only partially, according to available data) in order to compare the stochastic model described in Section D.4 with the perfect information case (in which prices are known in advance). The calculations were carried out on a Dual Core 2.01 GHz CPU using CPLEX 10.0.1 under GAMS\textsuperscript{4} [92].

D.5.1 Basic data

Several parameters are associated with a given segment: minimum and maximum production capacities, production costs, electricity to heat ratios, and emission of pollutants. This latter may of course be divided into several types of emission, which again may depend in various ways upon the load. The typical pollutants emitted from a gas-fired engine are CO, NO\textsubscript{x} and UHC (Unburned Hydro Carbons), which therefore are the ones considered in the case study. The emission of CO is initially high but decreases significantly in the second production segment to a lower level held fairly steadily throughout the remaining production segments. NO\textsubscript{x} emission levels are initially intermediate, increase until the third production segment before dropping to a low level in the final two segments. Finally, the emission of UHC decreases from an intermediate to a low level in the second segment and remains stable for the remaining production segments.

The local CHP plant considered in the case study has a CHP unit with a capacity of $K_{Cu}^E = 10.981$ MW heat, the capacity of the heat boiler is $K_k = 9.8$ MW, and the capacity of the storage facility is $V_{max} = 150$ MW. The CHP unit has five segments ($G = \{1, \ldots, 5\}$), to each of which the parameters listed above have a value associated. These values may be seen in Table D.1. The emission values are given in Table D.2. The heat demand is low in summer, high in winter, and gradually decreasing and increasing during spring and autumn, respectively. Additionally, there are typical diurnal variations (higher demand in the day than during the night).

Simulations were carried out for the years 2003-2005 as well as for the part of

\textsuperscript{4}The General Algebraic Modelling System is a high-level modelling language for mathematical programming problems. It allows for relatively simple definition of a given model and then accesses a solver (such as CPLEX) in order to produce a solution. The main advantage of GAMS is that even significant changes in the model are relatively easy to implement and do not necessitate changes in some associated solution algorithm.
Table D.1: Parameter values for the segmented parameters.

<table>
<thead>
<tr>
<th>g</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{min}$</td>
<td>0.64</td>
<td>0.71</td>
<td>0.75</td>
<td>0.77</td>
<td>0.82</td>
</tr>
<tr>
<td>$m_{min}^g$ [MWh]</td>
<td>0.400</td>
<td>2.875</td>
<td>5.750</td>
<td>8.625</td>
<td>9.803</td>
</tr>
<tr>
<td>$K_{kv}^g$ [MWh]</td>
<td>2.875</td>
<td>5.750</td>
<td>8.625</td>
<td>9.803</td>
<td>10.981</td>
</tr>
<tr>
<td>$c_{kv}^g$ [DKK/MWh]</td>
<td>185</td>
<td>174</td>
<td>163</td>
<td>152</td>
<td>141</td>
</tr>
</tbody>
</table>

Table D.2: Emission values for three emission types considered.

<table>
<thead>
<tr>
<th>g</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO [g/MWh]</td>
<td>12</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>NO$_x$ [g/MWh]</td>
<td>4</td>
<td>10</td>
<td>12</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>UHC [g/MWh]</td>
<td>3</td>
<td>1</td>
<td>0.4</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

the year 2006 where data was available (from January 1st to September 29th) using the western Danish spot prices. The average daily prices of the mentioned years are depicted in for 2003-4 in Figure D.4 and for 2005-6 in Figure D.5.

Hydro power accounts for roughly half of the power production in the Nordic system and thus the available volume primarily in the Norwegian and Swedish reservoirs heavily influence the spot price. Years are categorised as ‘dry’, ‘wet’ and ‘normal’ as a consequence of the mentioned dependence. In dry years where precipitation has been sparse the spot price will typically increase (sometimes dramatically) towards the end of the year and the pattern may even be visible in the beginning of the following year. This was the case in the winter of 2002-2003 where aftereffects of the dry year 2002 may be seen in the beginning of 2003 (Figure D.4). In contrast, the year 2004 was very normal with only a slight downward trend at the very end of the year (Figure D.4).

In 2005 an increase in the level of the prices may be observed in the beginning of the year (Figure D.5). The level stabilises around 300 DKK/MWh which is an increase of around 100 DKK/MWh compared to the two previous years, cf. Figure D.4, and the level persists in 2006 so far with a few fluctuations and perhaps even a hint of increasing. Furthermore, the trend so far of the water levels in the reservoirs indicates that 2006 will also be a dry year and, as may be seen in Figure D.5, the spot prices have already begun an additional slightly increasing trend from the end of the summer. It would also seem that the volatility of the prices has increased in 2005 and 2006. In conclusion, the years simulated depict a wide array price behaviour and should serve to illustrate various traits in the model.

The spot price scenarios in the stochastic model described in Section D.4 each
Figure D.4: Daily average spot prices in the western Danish price area, 2003-2004.

Figure D.5: Daily average spot prices in the western Danish price area, 2005-2006. Note the difference in scale from Fig. D.4.
Figure D.6: Value of perfect information for the model with one segment, $N = 5$.

consists of a set of 24 hourly electricity spot prices and were constructed in the following manner. The number $N$ of scenarios was chosen. For any given day the spot prices of the previous $N$ days of the same type$^5$ were given equal probability. To ensure bidding in cases where the prices of the previous days were low, an artificially high set of spot prices were included but given a comparatively small probability. The artificial scenario was based on the average daily profile of 2002 vertically translated to a level above the cost of producing electricity on the CHP unit.

D.5.2 Simulation results

Simulations were carried out both for a segmented model with five segments and, as comparison, for the same model with only a single segment. In the latter case all segment-related parameters are set to the value of segment 5 in the original model (with the exception of the minimum production value which is set to the value of segment 1 in the original model). Simulations of both the segmented and non-segmented models were further performed with 5, 10, and 20 scenarios included. The value of perfect information (VPI), i.e. the absolute difference between the stochastic and the deterministic solutions, are depicted in Figures D.6 through D.11 for the four years in question.

$^5$ The type distinguishes between weekdays and weekend days.
Figure D.7: Value of perfect information for the model with 5 segments, $N = 5$.

Figure D.8: Value of perfect information for the model with one segment, $N = 10$. 
Figure D.9: Value of perfect information for the model with 5 segments, $N = 10$.

Figure D.10: Value of perfect information for the model with one segment, $N = 20$. 

A Partial Load Model for a Local Combined Heat and Power Plant
Some general observations may be made of the simulations using the segmented models (Figures D.7, D.9, and D.11). As may be expected, the VPI is least in the year 2004 where the spot prices were stable at the same level with no large oscillations the entire year. The VPI is somewhat larger in 2003, a year with several price spikes and a decreasing trend in the beginning of the year and it would seem that there is a delayed effect of these specific characteristica. The VPI relatively quickly establishes stability in the period of large oscillations in the spot prices (see Figure D.4). However this ability turns into a disadvantage once the spot prices stabilise wherefore the level of the VPI increases dramatically. The same effect is visible in the latter half of the 2003, as the error increases in the time following a period of price spikes where the spot prices have stabilised somewhat.

The tendency mentioned regarding the VPI in the 2003 simulations is somewhat repeated in the years 2005 and 2006. When there is a stable price trend, the VPI adjusts after a small delay but abrupt changes in the spot price pattern usually affects larger errors. The effect of the single peak in 2005 is slightly different than the peaks in 2003. In 2005 the single spot price peak affects a similarly singular peak in the VPI. This may be explained by the fact that the prices around the peak behave with relative stability (cf. Figure D.5) in comparison to the volatile behaviour of the spot prices surrounding the peaks in both the first and latter halves of 2003 (cf. Figure D.4).
The difference between the segmented and non-segmented models is also clearly visible when comparing Figures D.6, D.8, and D.10 to Figures D.7, D.9, and D.11. Including the possibility of segmenting the load yields a greater flexibility in the model, resulting generally in a lower VPI for the segmented models in all four simulated years. The tendency is particularly visible during the winter half of the year (Jan-Mar and Oct-Dec). Increasing the number of scenarios from 5 to 10 results in a decrease the VPI in approximately 28% of the days in the year 2003, 29% of the days in the year 2004, 41% in 2005, and 41% 2006 in comparison to the five scenario cases. Similarly, increasing in the number of scenarios from 10 to 20 results in a decrease in the VPI in approximately 30%, 23%, 38%, and 35% of the days in the year 2003, 2004, 2005, and 2006, respectively, in comparison to the ten scenario cases. However, the improvements are not quantitatively very great as may be seen from the Figures and the change incurs a dramatic increase in computation time, especially for the segmented model which takes several hours to solve compared to 10-15 minutes for the five scenario non-segmented model, so in the case of an actual implementation of such a model the plant manager must weigh accuracy against time limitations.

D.6 Conclusions

A model was presented which considered the problem of constructing a production plan for a local CHP plant in order to bid into the day-ahead market while meeting heat demand. The model took into consideration the possibility of segmented production upon which several parameters (such as production cost, emissions, electricity to heat ratio) depended. Thus it was made possible to prioritise concerns regarding regulation of environmental pollutants. In the present model, emissions were penalised in the objective function. Another way of including environmental concerns would have been to include constraints which stated that emissions must be within certain limits. In that case one might still keep the penalty term in the objective function according to priorities.

Simulations were undertaken, running the model throughout the years 2003-2005 and the part of 2006 were data at present was available. The stochastic model was compared with a deterministic version, where prices were fully known in advance and the value of perfect information (VPI) was examined. Further, the segmented model was compared to a model with only a single segment considering once again the difference between the stochastic and deterministic solutions. The results indicated that using segmentation yielded a flexibility in

---

6Recall that bidding must be done before noon on the Nord Pool exchange and in order to obtain the best precision the heat demand should be updated as late as possible before running the model.
the model which reduced the VPI, particularly during the winter half of the simulated years. Increasing the number of scenarios in the model from 5 to 10 and from 10 to 20 also provided a reduction of the VPI for both the segmented and non-segmented cases. However, the increased number of scenarios brought about significantly longer computational times, especially for the segmented model. In a real life case the use of many scenarios may be restricted, at the cost of greater accuracy, due to time limitations.

With the current prognosis approach in the model, the tendency was that while price trends were stable, the VPI was small. However, sudden changes in trends affected a greater VPI but the effect would peter out as the new price trend became stable. Taking into consideration the simplicity of the prognosis method utilised in the stochastic model, the performance must be deemed acceptable. However, a further avenue of research could be to attempt to describe the Nordic spot price fluctuations using a time series model, use the predictions from such a model as the spot price input in the deterministic version of the presented stochastic model and see how it compares to the stochastic method of anticipating prices based on historic data presented above. Using a time series model would enable taking into account factors such as expected temperature, precipitation, and wind.

Another issue is the deterministic nature of the heat demand in the present model. Whenever a model such as the present is to be utilised in reality issues of data arise. It would be useful, when optimising the production costs for a given local CHP plant, to have e.g. a fitted time series model for the local heat demand which the plant provides for (see also [98]).
Appendix E

Local CHP Plants between the Natural Gas and Electricity Systems

E.1 Abstract

Local combined heat and power (CHP) plants in Denmark constitute an important part of the national energy conversion capacity. In particular they supply a large share of the district heating networks with heat. At the same time they are important consumers as seen from the gas network system, and they contribute significantly to the electricity production. CHP is, together with the wind power, the almost exclusive distributed generation in Denmark.

This paper deals with the CHP as intermediary between the natural gas system and the electricity system. In particular, the relationship between the peak hour characteristics of the electricity and gas systems will be investigated. The point is here that the two systems will tend to have peak demand during the same hours. This is the typical situation, since load is high during the same hours of the day and of the year. Moreover, the random variations in the load will be correlated between the systems, because both demands in part depend on the climate.
The analysis in the paper is based on a numerical model which simulates the operation of a CHP plant with heat storage. The conditions for the operation of the plant are assumed to be consistent with the conditions that are expected to be in force in Denmark during 2005, where a large part of the local CHP plants will change from being paid for electricity production according to a feed-in tariff, to a situation where the electricity is to be sold on market conditions. The results will highlight the CHP plant as the link between three energy supply systems, viz., district heating, gas and electricity.

E.2 Introduction

In Denmark, three energy systems form a very interesting and interconnected structure. Natural gas has become a key primary source of energy for both electricity generation and, both directly and indirectly, for heating purposes. The electricity and district heating systems meet in combined heat and power (CHP) generation facilities, of which most are natural gas fired, at locations all over the country.

Denmark has since 1982 extracted increasing amounts of natural gas from the North Sea of which much is used for electricity and heat generation purposes. Electricity generation by local CHP facilities commenced around 1990 and has, due to government subsidies, steadily increased ever since, only slightly tapering off in recent years. Local CHP capacity increased from roughly 200 MW in 1990 to nearly 2.5 GW in 2000. Most local CHPs started out as pure heat production facilities, constructed for the sole purpose of meeting the local heat demand. As they have been converted to combined heat and power generation they are thus direct connection points of the three energy systems.

This paper considers precisely this interconnection, and attempts to illustrate the consequences of recent legislation which imposes that local CHP facilities must now operate on market terms when unloading generated electricity. In Sections E.3 and E.4 the three energy systems are described. Section E.5 gives some background on the technology types considered. Section E.6 describes the modelling procedure and in Section E.7 simulations are presented which illustrate the situation. Finally, Section E.8 contains discussions and conclusions.
E.3 The Natural Gas System

The Danish supply of natural gas originates in the off-shore oil and gas fields in the North Sea. Two high pressure pipelines extend along the sea bed and make landfall in Jutland. They meet at the Nybro gas treatment plant near the western coast of Denmark (see Figure E.1) where up to 24 million cubic metres (energy content roughly equal to 1000 TJ) of gas can be treated daily. From Nybro two 30 inch transmission lines extend across Jutland towards the major junction at Egtved.

From here one connection goes south to the Danish-German border at Ellund. Another goes north to the gas storage facility at Lille Torup and terminates in the city of Aalborg. Finally a transmission line runs all the way East across the country, passing Odense and crossing both “Belts” to arrive on the outskirts of Copenhagen near Karlslunde. From here one line proceeds to the Stenlille storage facility while others produced to supply the area of Greater Copenhagen and the northern parts of Zealand and ultimately a transmission line crosses Öresund to supply our Swedish neighbours.

Most of these major transmission lines are 20–30 inches in diameter and scaled to a maximum pressure of 80 bars. At no place in the transmission network is the pressure allowed to descend below 42 bars in order to secure adequate pressure.
at the final delivery locations. Along the transmission network are metering and regulation stations (M/R stations). From here, natural gas is pulled from the transmission system into the underlying distribution networks. Here the responsibility for network operation is also passed from the transmission system operator Gastra [40] to one of the four distribution system operators.

These operators, along with the storage system operator, are public companies responsible for providing the basic services of natural gas supply. Each company has an economic structure for financing their operations. They develop products for capacity and volumetric throughput in the system and provide balancing services. The model used in this article is a reflection of present and previous structures with regard to the services available to the gas transporter.

E.4 The Electricity and District Heating Systems

E.4.1 The Electricity System

The Danish electricity system is at present separated into two systems by the Great Belt between Funen and Zealand. The western and eastern regions each have their own independent transmission system operator (TSO) – Eltra and Elkraft-System, respectively – yet recent legislation [1] has decreed that the two unite with Gastra into a single TSO (to be called EnergiNet.dk [40]) with responsibility for the entire Danish electricity and natural gas system.

There are international connections from the western system to Germany, Norway, and Sweden, and from the eastern system to Sweden and Germany, but the two Danish systems are not (yet) interconnected.

The majority of the electricity production stems from either large central plants, wind power, or local CHP plants of sizes ranging from a few kW to around 400 MW. The change in the composition of the national electricity production capacity from 1983 to 2003 is shown in Figure E.2.

E.4.2 The District Heating System

In Denmark, heat demand varies considerably over the year, with minimum during the months of July and August and peaking from December to February
with significant diurnal and weekly variations.

The heat produced stems from large central CHP plants, local CHP plants, district heating plants, and private producers, as illustrated in Figure E.3.

As may be seen from the figure, the majority of the heat production takes place at the large central CHP plants. However, the share of production handled by district heating has decreased significantly as many of these facilities over time have been converted to local CHP plants.

### E.4.3 Local Combined Heat and Power Plants

Combined heat and power plants began to emerge in Denmark in the late 1980’s and attributing to government policies and subsidies the amount of facilities grew steadily during the 1990’s. The principal idea was that heat production units that supplied the district heating systems around the country should be replaced by CHP facilities where heat was merely a fortunate by-product of electricity generation. Most were also equipped with heat storage facilities, which to a certain degree made possible the production of electricity even when the heat demand was satisfied.
Figure E.3: Composition of heat production from 1980 to 2003 (source: Danish Energy Authority [29])

Until 2005, electricity produced at local CHPs was prioritised and sold according to a feed-in tariff, the so-called three stage tariff (see Figure E.4).

New legislation [80] states, however, that after January 1st 2005, all facilities with capacities larger than 10 MW must operate on market terms, which implies selling the electricity they produce on regular electricity markets, e.g. the Nordic electricity exchange, Nord Pool [86] where prices fluctuate considerably.

E.5 Combined Heat and Power Technologies

There are several key factors which define the local CHP plants. First there is the location: All electricity producing generators not located at one of the 19 central plant locations are by definition local. In a broad sense there are four categories of thermal CHP technologies, which are described in the following subsections.
E.5 Combined Heat and Power Technologies

E.5.1 Engine-driven Plants

Engine-driven plants are the most common form of local CHP. Basically a combustion engine powers an electricity generator to produce electricity. Heat recovery systems enable the use of heat from cooling water, lubricants and exhaust in the district heating network. Engines are generally fired by either natural gas or diesel. These plants account for about 85% of the number of installed plants and 45% of the electricity generation capacity in Denmark [38].

E.5.2 Steam Turbines

A steam turbine is basically a boiler which heats water, thus forming steam. This steam is pressurised over the length of a turbine, which in turn drives a generator. Traditionally exit steam has been cooled by an intake of seawater, but by replacing this with a district heating condenser, the heat is transferred to the district heating system. Steam turbines can either be back pressure units, which supply electricity and heat at a constant ratio, or extraction units. The latter gives added flexibility as it may vary from full back pressure mode to full condensing mode. Steam turbines with extraction capability combines the seawater condenser with the district heating condenser to get the desired ratio between electricity and heat. Approximately 6% of local CHP units in Denmark are steam turbines, but being generally large units they cover near
20\% of installed electricity generation capacity [38].

E.5.3 Gas Turbines (single cycle)

A gas turbine can be coupled with a generator to generate electricity. Again heat can be recovered from exhaust gas. The advantage of gas turbines is that heat can be recovered without reducing the electricity production efficiency. However gas turbines have low regulatory ability, in that efficiency is drastically reduced when operating below nominal power [38].

E.5.4 Combined Cycle Technology

The combined cycle technology encompasses the combination of a gas turbine and a steam turbine. Instead of the steam turbine being supplied with steam from a boiler, it is driven by the high pressure exhaust from the gas turbine. Combined cycle facilities are usually of a rather large size. The combination of single cycle steam turbines and combined cycle facilities account for 6\% of facilities in Denmark and an electricity generation capacity share of 35\% [38].

E.5.5 Included Technologies

Three technologies were included in the simulations namely gas engines, single-cycle steam turbines and combined cycle facilities. Traditional steam turbines were omitted as there are only a few of these which use natural gas as primary fuel. They are however often used in conjunction with municipal waste furnaces, but these are due to technical and environmental considerations less likely to be put on market terms, at least not in the same form.

E.6 Modelling the Interconnected Systems

The degree of interdependence of the three systems increases with the gradual opening of the energy markets. Previously, local CHPs sold electricity at a fixed feed-in tariff, but from January 2005 this has changed for all local CHPs larger than 10 MW, and from January 2007 also for facilities larger than 5 MW [80]. The dependency of CHPs on the developments in the electricity market
also resonates in the natural gas system. Local CHPs are major consumers of natural gas, and the future will demonstrate exactly how much this will influence the natural gas system, both in terms of daily operations and with regard to long-term security of supply.

In modelling the operation of individual plants first according to the feed-in tariff and old rules for natural gas [48] and secondly according to expectations of the development in market price and according to newer rules [49] and tariff structures of the natural gas system, it is possible to extract certain tendencies which follow from the liberalisation process, and specifically how this will affect the natural gas system.

E.6.1 Single Plant Model

The single plant model was created in connection with the project *Optimal drift af prioriterede anlæg* [39]. It is a linear programming (LP) model, which considers the situation from the point-of-view of a single gas-fired CHP plant. It is assumed that the plant consists of a CHP unit, which produces both electricity and heat; a boiler (also gas-fired), which only produces heat; and a heat storage facility.

The aim of the model is to minimise the production costs of the facility (including fuel costs), while selling electricity on the electricity spot market, and making sure that the local heat demand is met. The model takes into account the uncertainties inherent in the fact that spot prices are unknown at the time of bidding. Using historical data for days of the same type as the one for which the bid is made, the model creates a prognosis for the spot prices on the day of bidding and uses this in the optimisation.

In connection with the present paper, a second version of the model has been created that models the possibility of selling the electricity according to the three stage tariff. This model version is inherently deterministic, as the three stage tariff is well-known in advance of the production planning.

Both versions of the model are implemented using the GAMS modelling language [92].
E.6.2 Aggregated Model

Two types of simulations are run using the abovementioned model. One simulation is run for a number of relevant facility types using historical data with regard to the three stage tariff, heat demand, as well as rules and tariffs for natural gas transmission/distribution. This gives an indication of how historical load with respect to natural gas is likely to have been in the simulated time period, for the technology in question.

Using data describing the location, type, and capacity of gas-fired CHP facilities in Denmark results from simulations conducted with the model described in the previous section are aggregated multiplicatively to a number of network nodes matching the M/R stations of the natural gas transmission system. Hence an hourly load curve is generated for about 50 locations in Denmark.

This gas consumption profile is compared with realised total gas throughput on the M/R stations, and used to extract the portion of total delivered which was likely not used for heat and power generation in CHP facilities. This portion of gas is hereafter referred as residual consumption. It is assumed that from the year 2003 to the present there has been no change in residual consumption.

Hereafter, a second simulation is performed, this time with post liberalisation cost structures and using the spot market price for CHP electricity delivery. Results are once again aggregated according to the capacity index, added to the residual gas consumption, and compared with historic values. It is now possible to see whether the new structures with respect to cogeneration and natural gas supply serve to instigate increased or decreased total natural gas consumption. From this it is possible to evaluate the impact of the new liberalised structures may have for the natural gas transmission network.

E.7 Simulation Results

Three simulations have been performed using old rules and tariffs, and three using new rules and a model for calculating expected market price based on historical data. Each simulation consists of four one-week samples. All gas data is based on 2003 loads kindly supplied by Gastra A/S [40]. Results were generated for individual areas M/R station level. Thus station-by-station load variations can be observed.

For clarity and overview only national aggregated results are presented here. It
is prudent to comment, however that the results vary from station to station depending on the presence of local CHP capacity adjacent to the individual stations, as would be expected. Thus while nationally aggregated results may not give rise to any major concerns, a thorough review of results station by station show that some places are more affected than others by the introduction of market terms.

Figure E.5–Figure E.8 illustrate the historical natural gas consumption and the simulated consumption for local CHPs based on spot prices and on the three stage tariff. The unit is MWh equivalents (MWh e) which is the heat content of the natural gas calculated in MWh.

In January (Figure E.5) the results of the spot price simulation correspond nicely to the natural gas consumption. The dips in the graph are due to occasional expectations of low spot prices. The tariff simulation, however, is surprisingly volatile. This may be due to an overestimation of the available capacity. Also, the stability of the spot price consumption is due to the conservatism against the uncertainty inherent in the spot market. Interestingly enough, volatile and uncertain prices make stable consumption profiles.

For April (Figure E.6), the tariff simulations are more reasonable with fewer extreme peaks in consumption. Note also that April is considered ‘summer’ and thus there is no evening peak in the tariff. The consequence is clearly seen when compared to January (Figure E.5), as the evening consumption peak has
decreased significantly. There is an increasing stability in the spot consumption which follows the total consumption.

In August (Figure E.7) heat demand is low, thus in the tariff case consumption only occurs at peak prices, and in the spot price case (where prices are generally low) consumption is stabilised by the possibility of storing heat. It is interesting to note that there is a slight delay between the tariff consumption peaks and the historical outtake at the M/R station. This indicates an average delay in the distribution system of approximately 1.5–2 hours. Note the large difference in scale compared to January (Figure E.5), which may be explained by the large variations in heat demand from summer to winter.

The pattern in October (Figure E.8) is likewise reasonable for both spot price and tariff simulations. The fact that the graphs are so level may be explained by the use of weekly capacity products in the natural gas transmission system. If, as is often the case, the local CHP purchases yearly transmission capacity products, the scale of these would be defined more by the high heat demand in winter, which would give a greater degree of freedom for the remaining year.

In general the amount of natural gas consumed is approximately identical for both the tariff and spot price cases. This indicates there is no notable change in the amount of electricity generated under the spot market setting compared to the tariff setting.
Figure E.7: Historical and simulated consumption – August 2003.

Figure E.8: Historical and simulated consumption – October 2003.
E.8 Discussion and Conclusion

It is clear from the simulation results that the production patterns of local CHPs will change as a result of the new conditions for their operation. When using cautious prediction of the spot prices the consequence is a stabilising effect on the consumption of natural gas. Unsurprisingly, the largest variations in consumption occurred in January as there was an opportunity of increased production due to the high heat demand.

The spot price model leads to greater stability in the consumption profile. This may be attributed to the use of short-term capacity products and caution in the face of uncertainty. The analysis does not support any existing concerns that the inclusion of local CHP to the spot market should serve to destabilise the security of the natural gas supply.

The simulations also indicate that there are no notable changes in electricity production when considering the spot market setting compared to the tariff setting.

There are issues of symmetry of both information and of response at work here. It is unlikely that all plants of similar types will operate exactly according to the same patterns. In reality every plant is different and this will force out some of the symmetry in the results. In addition there is the assumption that all local CPHs act unilaterally. In practice local CPHs pool their effort with regard to analysis and planning in responding to market developments. This increases symmetry of information but decreases symmetry of response by coordination between plants.

One unfortunate drawback of the applied approach is that there is a lack of feedback to the electricity market. Local CHPs are assumed to be price-takers, yet their relative market share and potential for symmetric response is likely to impact market movements. As electricity prices were fixed in the simulation according to historical values, the amount of electricity generated by local CHPs changed. Likewise it would have been interesting to see, how feedback to the natural gas market would have affected prices, and especially how this again would have altered the state of the natural gas supply system. Developing tools for this type of research, where the market influences the state of the supply system, which in turn affects the market development, is definitely an area for future research.
Appendix F

**Power Reserve Management by Two-Stage Stochastic Programming**

F.1 Abstract

Although bilateral trades and spot markets aim to balance power supply and demand, real-time imbalances may still occur due to non-anticipated supply and demand behaviour. The real-time balance is the responsibility of the power system operator and is achieved by means of regulation purchased in a corresponding market. To ensure sufficient regulation in the market, the system operator has the possibility of reserving regulating power in advance. As reserves are however purchased prior to actual operation reserve decisions are naturally subject to supply and demand uncertainty. In contrast, regulation decisions can be deferred until uncertainty has been observed and the system is operating. In the present paper this is formalized by formulating the regulating reserve management problem as a two-stage stochastic program. A case study that concerns the regulating reserve management problem of the system operator of western Denmark is discussed in detail.

*Keywords:* Power reserves; uncertainty modelling; stochastic programming
F.2 Introduction

In a power system that comprises several participants on both the supply and the demand side, it is the task of the system operator to balance production and consumption by means of regulation. To do the balancing, regulation reserve management becomes important. Previous studies indicate that power optimisation problems can be handled successfully by mathematical programming. With this in mind, we present an application of stochastic programming to a power reserve management problem.

Very often power optimisation problems involve uncertainty and to deal with the uncertainty, stochastic programming may come into play. In the literature, power production planning and power operation problems within stochastic programming have attracted considerable attention. Examples on hydro-thermal power production planning are [17], [35] and [52]. Both problems are two-stage stochastic programs. Whereas [17] and [52] seek to find a unit commitment plan, [35] seeks a schedule that can be compensated for in the future. Hydro-power production planning problems within multi-stage stochastic programming can be found in [44], [60] and [84]. Whereas [44] and [60] handle water scheduling through space and time respectively, [84] considers unit commitment. Hydro-thermal and purely thermal power operation problems are handled in [13], [35] and [53] using multi-stage stochastic programming. Recently, also simultaneous optimisation of power production and physical trading have appeared in stochastic programming. [64] presents a multi-stage stochastic program in which all stages allow for spot market disposals and purchases. [78] and [45] incorporates spot market bidding in a two-stage stochastic program. For another power trading problem within stochastic programming, which involves more than one market, see also [102].

To our knowledge, prior work on power reserve management is limited. However, the authors of [104] formulate a stochastic optimisation problem for the coordination of bidding strategies in day-ahead and reserves markets. In contrast to the present problem, the problem is formulated from the perspective of the supplier, which explains the link between day-ahead and reserve market exchange that is caused by production capacity limits. Nevertheless, the problem has some similarities to the problem of this paper in that both volumes and market prices are determined within the corresponding model. In the same spirit, the authors of [107] determine pricing and procurement of reserves in a power market. A stochastic model based on social welfare maximization allows for a so-called capacity-reliability analysis that relates the available reserve capacity to the probability of reserves shortage.

In the present paper we address the problem of regulating reserve management
faced by the power system operator. The problem arises in the process of maintaining the balance between power demand and supply. The system operator corrects imbalances by regulation procured in the regulating market and sufficient amounts are not necessarily available unless reserved in advance. The major challenge of reserving regulation is that of uncertain supply and demand. To handle this uncertainty, the power reserve management problem will be analyzed by means of stochastic programming.

F.3 Power reserves

The project grew out of a collaboration with the former Eltra\footnote{now a part of the overall Danish power and gas system operator Energinet.dk [40]}, which is the power system operator of western Denmark. Due to decentralization of the power generation and deregulation of the power markets, many procedures either have been modified recently or will be within the near future. In particular, Eltra have made plans to improve the model on which power reserve management is based, which makes reserves a topic of current interest.

It is necessary to distinguish between different types of reserves

(i) Automatic regulation reserves: Reserves that cover imbalances from the time of appearance until a regulation bid is activated. The reserves are provided by running plants capable of adjusting upwards and consumers capable of adjusting downwards. Activation begins automatically within two to three minutes.

(ii) Manual regulation reserves: Reserves in the form of regulation resources that suppliers are obligated to sell in the regulating market. Activated manually within 10 minutes.

(iii) Running and available plants: Reserves for ensuring supply in spite of transmission lines or units falling out. Consist of available plants that can be started, running plants that can adjust upwards and consumers that can adjust downwards. Running and available plants plants are activated either automatically or manually.

(iv) Emergency start plants: Reserves reestablishing the system in case of blackout.

This paper considers manual regulation reserves. The reason for considering such reserves should be clear from the following discussion. In a typical power
system the operator is responsible for balancing supply and demand. Prior to
operation, predicted supply and demand are balanced by resources provided by
the bilateral trades and the spot market trades. Supply and demand however
often differ from the predictions and imbalances still occur when the system is
operating. To overcome such real-time imbalances, the system operator compens-
sates suppliers and consumers for adjusting production and demand accordingly.
The imbalances are covered by so-called regulation resources provided by the
regulating market, which is established by system operators for the purpose of
real-time power trading. Suppliers either increase or decrease power production
above or below the amounts committed through the bilateral contracts and the
spot market contracts and offer the adjustments to the regulating market as
so-called up- and down-regulation, respectively. Consumers offer power demand
adjustments to the regulating market in a similar fashion. The system operator
purchases up regulation in the case of excess demand and down regulation in the
case of excess supply. In some cases, the amounts of resources provided by the
regulating market are insufficient to fully cover imbalances. This may happen if

(i) Imbalances are substantial due to extreme supply and demand behaviour,
unforeseen weather changes leading to unpredicted wind production or
non-anticipated heat demand etc.

(ii) Spot market prices are sufficiently high to prevent market participants
from saving resources for trading in the less secure regulating market.

(ii) Considerable failures occur during transmission, e.g. important transmis-
sion lines fall out.

To ensure that sufficient amounts of resources are available even when facing the
above situations, regulation can be reserved prior to trading in the regulating
market. The system operator may agree with reserve suppliers for the right to
purchase regulation. This is done by activating a reserve bid that obligates the
supplier to bid an amount of regulation into the regulating market. The right
is a type of option as the system operator obtains the possibility of purchasing
regulation in the regulating market but is not forced to do so. As a result, the
system operator faces the trade-off between purchasing regulation at the market
price only, thereby risking insufficiency of resources, and paying both the market
price and an additional fixed price to ensure regulation is available.

When maintaining the power balance and managing regulating reserves the
system operator must consider the uncertainties of the power system. Sources of
uncertainty include supply and demand as well as regulating prices and volumes.
As system imbalances are caused by demand and supply the focus is chosen to
be supply and demand uncertainty. The major problem of managing regulating
reserves is that reserves must be purchased prior to balancing. If reserve capacity turns out to be insufficient additional capacity must be procured elsewhere, often at a considerable price, or the system simply breaks down. On the other hand, reserves constitute serious costs which makes excess reserve capacity unwanted. Stochastic programming provides a tool for determining reserve levels that takes the non-anticipated supply and demand behaviour into account.

As already explained, different power markets come into play in the correction of imbalances. To fully understand the daily work of the system operator, consider the following time schedule for acting in these markets. The system operator purchases reserves for a longer time period at the regulating reserve market (in western Denmark, a formal market has not yet been established). Currently, the length of this period is one month or longer though a reduction to 24 hours is planned. The remaining actions concern a 24-hour operation day. By noon bids must be submitted to the spot market (in the Danish case, the Nordic market Nord Pool [86]). Having balanced predicted supply and demand, activated bids are announced by 14:00. Finally, from 24:00 to 24:00 actual supply and demand imbalances are continuously corrected by trading in the real-time regulating market (in this case, still a local market, although the integration in the Nordic market is planned).

The paper is organized as follows. The power reserve management problem is presented in Section 3. Section 4 explains how uncertainty affects reserve management and the problem is stated as a two-stage stochastic program. By assuming a discrete distribution of the random data, the problem is transformed into a large-scale mathematical program that is solved by a specially designed solution procedure in Section 5. A specific instance of the power reserve management problem is addressed using data from the power system operator of western Denmark, Eltra, and computational results are reported in Section 6.

F.4 The power reserve management problem

A given planning horizon is considered. In practice, the power balance should be maintained at every time point but to facilitate computations the time is discretised. The regulation bids to the regulating marked have duration of a number of full hours. Accordingly, the planning horizon is discretised into hourly time intervals and the finite number of such intervals are denoted by $T = \{1, \ldots, T\}$. The planning horizon for purchasing regulating reserves may range from one to several months. In western Denmark some regulating reserve contracts have long durations, whereas some contracts are traded regularly on a monthly basis. Regarding the case study, the planning horizon is chosen to
be one month, i.e. $T = 24 \times 30 = 720$. However, to increase the flexibility of the system it is intended to reduce it as is the case for eastern Denmark that trades regulating reserve contracts on a daily basis.

F.4.1 Reserves

According to the present situation, the system operator of western Denmark purchases regulation reserves mostly locally. Although not fully formalized, regulation reserves are traded on a monthly auction or as individual contracts of a longer duration. A regulation reserve bid consists of an offer period, a volume to be available throughout the offer period, a variable price that applies to the portion of the volume that is activated and a fixed price for activation of the bid. We assume for the application that regulation reserves are traded only on a monthly auction so that the offer period of a regulation reserve bid is always one month. The system operator pays the variable price for the actual amount of regulation used and the fixed price for the availability of regulation.

Regulation reserves are modelled as follows. Regulation divides into up- or down-regulation, and so do regulation reserves. Thus, the superscripts $up$ and $do$ are used. The indices $\mathcal{I} = \{1, \ldots, I\}$ are adopted to represent different reserve bids. The variables $\delta_{i}^{up}, \delta_{i}^{do} \in \{0, 1\}, i \in \mathcal{I}$ indicate whether the reserve bids are activated or not and the prices for activation are denoted $c_{i}^{up}, c_{i}^{do}, i \in \mathcal{I}$. Then fixed regulation reserve purchase costs compute as

$$\sum_{i \in \mathcal{I}} (c_{i}^{up} \delta_{i}^{up} + c_{i}^{do} \delta_{i}^{do})$$

F.4.2 Regulation

The regulating market serves as a tool for the system operators to balance power supply and demand during operation. A larger power system may share a common regulating market. The Nordic system operators, the Swedish Svenska Kraftnät, the Norwegian Statnett and the Danish Energinet.dk, have established such a common market, in which western Denmark was the last part of the system to be integrated in January 2006.

The suppliers to the regulating market are power balance providers that submit bids to the market. Regulation bids divide into up-regulation or down-regulation bids. Upward regulation make consumers decrease demand or suppliers increase production (system operators 'buys' power) and downward regulation
The power reserve management problem

make suppliers decrease production or consumers increase demand (system operators 'sells' power). A regulation bid consists of an offer period, a price and a volume. The offer period may be a number of full hours. During the offer period the volume is constant whereas the price may vary between hours. We however assume that regulation bids have an offer period of only one hour and thus both the volume and the price is constant. Generally, the up-regulation price is specified as the system spot price (assuming no grid congestion) and a raise, i.e. the up-regulation price is always above the system price. Similarly, the down-regulation price is calculated as the system price (assuming no grid congestion) and a deduction, i.e. the down-regulation price is always below the system price. Prices are usually given as positive numbers unless the system operator sells up-regulation or buys down-regulation. We assume prices are always positive.

Only recently the regulating market of western Denmark has begun to restructure. western Denmark trades regulation mostly locally although a full integration to the Nordic regulating market is on its way. Before July 2006, the regulating market of western Denmark was a pay-as-bid market, whereas now the general rule is to use local marginal prices as market prices. Nordic marginal prices should be fully in use by January 2008.

If regulation is purchased outside western Denmark, transmission capacity limits may apply. Such limits are due to physical limitations or political agreements. Since, for the current application, regulation is mostly purchased locally, we have however omitted transmission capacity limits.

Regulation comprises purchases in the regulating market that have been and have not been reserved in advance. In the case of direct purchases, \( \mathcal{J} = \{I + 1, \ldots, I + J\} \) are included to index different bids. Volumes are denoted \( \tilde{q}^{up}_{it}, \tilde{q}^{do}_{it}, i \in \mathcal{J}, t \in T \) and corresponding prices are denoted \( \tilde{p}^{up}_{it}, \tilde{p}^{do}_{it}, i \in \mathcal{J}, t \in T \).

In the case of reserved purchases, recall that the indices \( \mathcal{I} = \{1, \ldots, I\} \) are included to represent different bids. Volumes are denoted \( \tilde{q}^{up}_{i}, \tilde{q}^{do}_{i}, i \in \mathcal{I} \). We assume all reserved purchases will be available at the regulating market, that is, there is no failure of supply. Note that whereas for direct purchases bids are time dependent, for reserved purchases bids are time independent. Corresponding prices are denoted \( \tilde{p}^{up}_{it}, \tilde{p}^{do}_{it}, i \in \mathcal{I}, t \in T \). For direct purchases prices can vary freely, whereas for reserved purchases prices should stay between limits that are agreed upon when reserving regulation. A bid is not necessarily activated completely. Actual purchases are represented by the variables \( q^{up}_{it}, q^{up}_{it} \in \mathbb{R}_+, i \in \mathcal{I} \cup \mathcal{J}, t \in T \).
F.4.2.1 Pay-as-bid pricing

Costs of purchasing regulation, whether reserved or direct, consist of up-regulation expenses and down-regulation income

\[ \sum_{t \in T} \sum_{i \in \mathcal{I} \cup \mathcal{J}} (\bar{p}_{it}^{up} q_{it}^{up} - \bar{p}_{it}^{do} q_{it}^{do}) \]

The following bounds concern reserved purchases

\[ q_{it}^{up} \leq \tilde{q}_{it}^{up}, \quad q_{it}^{do} \leq \tilde{q}_{it}^{do}, \quad i \in \mathcal{I}, t \in \mathcal{T} \quad (F.1) \]

whereas direct purchases must submit to the bounds

\[ q_{it}^{up} \leq \tilde{q}_{it}^{up}, \quad q_{it}^{do} \leq \tilde{q}_{it}^{do}, \quad i \in \mathcal{J}, t \in \mathcal{T} \quad (F.2) \]

F.4.2.2 Marginal pricing

For each hour the marginal price is determined as the price of the last bid that has been activated in the direction the system is regulated. That is, if the system is up-regulated, the marginal price is the highest price of the up-regulation bids that have been activated. Likewise, if the system is down-regulated, the marginal price is the lowest price of the down-regulation bids that have been activated. The variables \( \gamma_{it}^{up}, \gamma_{it}^{do} \in \{0, 1\}, i \in \mathcal{I} \cup \mathcal{J} \) indicate whether the regulation bids are activated or not for both reserved and direct regulation. Moreover, the variables \( p_{it}^{up}, p_{it}^{do} \in \mathbb{R}_+^{n_1} \) represent the marginal prices. In the case of up-regulation, the marginal price is \( p_{it}^{up} = \max \{ \tilde{p}_{it}^{up}, \gamma_{it}^{up} : i \in \mathcal{I} \cup \mathcal{J} \} \) and in the case of down-regulation, the marginal price is \( p_{it}^{do} = \max \{ \tilde{p}_{it}^{do}, \gamma_{it}^{do} : i \in \mathcal{I} \cup \mathcal{J} \} \).

Costs of purchasing reserved and direct regulation amount to

\[ \sum_{t \in T} (p_{it}^{up} \sum_{i \in \mathcal{I} \cup \mathcal{J}} q_{it}^{up} - p_{it}^{do} \sum_{i \in \mathcal{I} \cup \mathcal{J}} q_{it}^{do}) \quad (F.3) \]

Evidently, (F.3) is nonlinear. In order to be consistent with a mixed-integer linear formulation, the variables \( \rho_{it}^{up}, \rho_{it}^{do} \in \mathbb{R}_+^{n_1}, t \in \mathcal{T} \) can be introduced and (F.3) can be replaced by

\[ \sum_{t \in T} (\rho_{it}^{up} - \rho_{it}^{do}) \]

and
F.4 The power reserve management problem

\[ \rho_{it}^{up} \geq \bar{p}_{it}^{up} \sum_{i \in I \cup J} q_{it}^{up} - M(1 - \delta_{it}^{up}), \quad i \in I \cup J, t \in T \]  
\[ \rho_{it}^{do} \leq \bar{p}_{it}^{do} \sum_{i \in I \cup J} q_{it}^{do} + M(1 - \delta_{it}^{do}), \quad i \in I \cup J, t \in T \]  

(F.4) \hspace{1cm} (F.5)

with

\[ M = \max \left\{ \bar{p}_{it}^{up} \left( \sum_{i \in J} \bar{q}_{it}^{up} + \sum_{i \in I} \bar{q}_{it}^{up} \right), \bar{p}_{it}^{do} \left( \sum_{i \in J} \bar{q}_{it}^{do} + \sum_{i \in I} \bar{q}_{it}^{do} \right) : i \in I \cup J, t \in T \right\} \]

For both reserved and direct purchases, the volumes activated cannot exceed the volumes bid

\[ q_{it}^{up} \leq \bar{q}_{it}^{up} \gamma_{it}^{up}, \quad q_{it}^{do} \leq \bar{q}_{it}^{do} \gamma_{it}^{up}, \quad i \in I \cup J, t \in T \]  
\[ (F.6) \]

Reserved regulation cannot be activated unless the regulation reserve bids are activated

\[ \sum_{t \in T} \gamma_{it}^{up} \leq \delta_{i}^{up}, \quad \sum_{t \in T} \gamma_{it}^{do} \leq \delta_{i}^{do}, \quad i \in I \]  
\[ (F.7) \]

F.4.3 Balancing

For the system operator to balance power, demand and supply imbalances are considered during operation. If the actual demand exceeds actual supply, the system operator purchases up-regulation, if the actual supply exceeds actual demand, the system operator purchases down-regulation. Imbalances between demand and supply is usually fully covered by regulation purchased directly or reserved in advance. In the case of insufficient regulation however excess demand and supply may occur. Excess demand and supply may result in irregular in- and out-flows from abroad that are penalized hardly. Such in- and out-flows, however, can be avoided by forcing consumers to decrease demand, by forcing power plants to shut down generating units or by stopping wind turbines, in which cases severe costs must be paid. Let the variables \( \varepsilon_{it}^{up}, \varepsilon_{it}^{do}, t \in T \) denote
excess demand and supply. If $b_t^{up}, b_t^{do}, t \in T$ denote penalty costs, excess demand and supply give rise to the following costs

$$\sum_{t \in T} (b_t^{up} e_t^{up} + b_t^{do} e_t^{do})$$  \hspace{1cm} (F.8)

The power balance constraints are the following

$$\sum_{i \in I \cup J} (q_i^{up} - q_i^{do}) + e_t^{up} - e_t^{do} = D_t - S_t, \ t \in T$$  \hspace{1cm} (F.9)

where $D_t, S_t, t \in T$ denote demand and supply. Note that supply include central production, decentral production and wind production as well as import and demand consists of national consumption and export.

### F.4.4 Market integration

When western Denmark is fully integrated in the Nordic regulating market, regulation will no longer be purchased mostly locally, but the following situation will apply, as is already the case for eastern Denmark. A power system often forms a part of a larger system and as concerns the present application, the power system of western Denmark is connected to systems of Sweden, Norway and Germany. It should be remarked however that western Denmark is not connected to eastern Denmark. The larger power system can be modelled as a network in which the nodes $\mathcal{N} = \{1, \ldots, N\}$ represent uncongested power systems that exchange power with the neighboring systems. The nodes are connected to the remaining nodes by edges representing transmission lines. A common network operator maintains the balance between supply and demand by purchasing regulating power. Imbalances are covered in part by local purchases and in part by foreign exchange. In the case of foreign exchange, transmission capacity limits may come into play. With grid congestion different price zones are established. We assume no grid congestion so that a common Nordic marginal price applies. Due to high price levels and rather inflexible trading conditions, regulating power exchange with Germany is rare. Thus, in this application, the connection between western Denmark and Germany is ignored. Accordingly, $\mathcal{N} = \{1, 2, 3\}$ where 1 represents western Denmark, 2 Sweden and 3 Norway. The common Nordic network operator is the Norwegian system operator, Statnett.

Divide the regulation bids according to the location of balance provider, that is $\mathcal{I} = \cup_{n \in \mathcal{N}} \mathcal{I}_n$ and $\mathcal{J} = \cup_{n \in \mathcal{N}} \mathcal{J}_n$. Then the regulation reserve management prob-
lem of the common network operator involves only a few changes in modelling. (F.8) and (F.9) are replaced by

\[ \sum_{t \in T} \sum_{n \in \mathcal{N}} (b_{tn}^{up} e_{tn}^{up} + b_{tn}^{do} e_{tn}^{do}) \]

and

\[ \sum_{i \in I_n \cup J_n} (q_{it}^{up} - q_{it}^{do}) + e_{tn}^{up} - e_{tn}^{do} = D_{tn} - S_{tn}, \quad t \in T, n \in \mathcal{N} \] (F.10)

F.5 The stochastic programming problem

The regulation reserve management problem presented in the preceding sections is a deterministic problem. The problem, however, involves incomplete information on some of the data and such uncertainties should be taken into consideration. This can be achieved by means of stochastic programming. Uncertainties concern regulating prices and volumes that can be changed until an hour prior to operation. Moreover, demand and supply uncertainty arises because of failure in supplying, unforeseen weather changes leading to unpredicted wind production or non-anticipated heat demand. As the system imbalances are caused by demand and supply, we assumed that only demand and supply is uncertain. That is, we implicitly assume that regulation prices and volumes are known in advance and cannot be changed.

A stochastic program is based on an alternating process of decisions and information and the most basic one is the two-stage stochastic program. Here, first-stage decisions are made immediately and do not anticipate the future outcome of uncertainty, whereas second-stage decisions are deferred until uncertainty has been disclosed and utilize the additional information. The most obvious optimisation criterion is to minimize the sum of deterministic first-stage costs and expected second-stage costs. For an introduction to stochastic programming, see [12], [63] and [87].

Although information evolves over time and a multi-stage stochastic program could be relevant, we approximate the problem by a two-stage stochastic program. We find this approximation sufficient to capture the interplay between reserves and regulation purchases. Since reserves must be purchased up at least a month in advance, reserve decisions are first-stage decisions. Decisions have to
be made before operation and thus with incomplete knowledge of future supply and demand. On the contrary, regulation bids have an activation period of at most ten minutes and therefore can be purchased very close to operation which makes regulation decisions second-stage decisions. The objective is to minimize reserve costs and expected future regulation and penalty costs.

The uncertain data is represented by a stochastic process on some probability space. To make the problem computationally tractable, we assume a discrete multivariate distribution. Outcomes of uncertainty will be referred to as scenarios indexed by \( S = \{1, \ldots, S\} \) and denoted by \((D_t^i - S_t^i)_{t \in T, s \in S}\). The corresponding probabilities will be denoted by \( \pi^s, s \in S \). First-stage reserve decisions are \( \delta_{up}^j, \delta_{do}^j \in \{0, 1\}, j \in J \), whereas second-stage regulation decisions are indexed \( q_{it}^{up,s}, q_{it}^{do,s}, p_{it}^{up,s}, p_{it}^{do,s} \geq 0, i \in I \cup J, t \in T, s \in S \) etc.

The two-stage stochastic programming formulation of the regulating reserve management is the following problem, depending on whether pay-as-bid or marginal pricing applies. The extension to market integration should be straightforward.

**Pay-as-bid pricing**

\[
\begin{align*}
\min & \quad \sum_{j \in J} (c_{up}^j \delta_{up}^j + c_{do}^j \delta_{do}^j) \\
+ & \quad \sum_{s \in S} \pi^s \sum_{t \in T} \sum_{i \in I} (p_{it}^{up,s} q_{it}^{up,s} - \bar{p}_{it}^{do,s} \bar{q}_{it}^{do,s}) \\
+ & \quad \sum_{s \in S} \pi^s \sum_{t \in T} (b_{it}^{up,s} e_{it}^{up,s} + b_{it}^{do,s} e_{it}^{do,s})
\end{align*}
\]

s.t. \((F.1) - (F.2), (F.9)\)

\( \delta_{up}^j, \delta_{do}^j \in \{0, 1\}, j \in J \)

\( q_{it}^{up,s}, q_{it}^{do,s}, p_{it}^{up,s}, p_{it}^{do,s}, e_{it}^{up,s}, e_{it}^{do,s} \geq 0, i \in I \cup J, t \in T, s \in S \)

**Marginal pricing**

\[
\begin{align*}
\min & \quad \sum_{j \in J} (c_{up}^j \delta_{up}^j + c_{do}^j \delta_{do}^j) \\
+ & \quad \sum_{s \in S} \pi^s \sum_{t \in T} (\rho_{it}^{up,s} - \rho_{it}^{do,s}) \\
+ & \quad \sum_{s \in S} \pi^s \sum_{t \in T} (b_{it}^{up,s} e_{it}^{up,s} + b_{it}^{do,s} e_{it}^{do,s})
\end{align*}
\]
The scenario generation has been kept rather simple. The differences between demand and supply data constitute a time series and, thus, has been analyzed by means of the field. In order to capture the behaviour of demand and supply differences and in particular model the differences as a stochastic process, historical data profiles have been used. In that demand and supply show strong correlations over time, the stochastic process is chosen as an autoregressive process which, to keep things as simple as possible, is chosen to be of order one. The autoregressive stochastic process, cf. \[14\] is the following

\[ D_t - S_t = \phi(D_{t-1} - S_{t-1}) + \epsilon_t, \quad t \in \mathbb{Z} \]  

(F.13)

where \( \{\epsilon_t\}_{t\in\mathbb{Z}} \) is a Gaussian white noise process. Scenarios of demand and supply differences \( \{D_t - S_t\}_{t\in\mathcal{T},s\in\mathcal{S}} \) are generated by sampling from (F.13). To reflect the true probability distribution, a large number of scenarios has been generated by the use of Monte Carlo sampling.

### F.6 Solution procedure

The problems (F.11) and (F.12) can be considered large-scale mixed-integer linear problems solvable by standard software packages or mixed-integer linear stochastic programs amenable to decomposition algorithms such as progressive hedging [69] or dual decomposition [18]. Being able to solve the problems as mixed-integer linear problems is valuable since the approach is very flexible. Adding further linear constraints is uncomplicated. In particular, the approach becomes relevant with constraints that introduce dependencies between hours. However, with the current simplicity of the model, it can be solved by a specially designed procedure that utilizes the structure of the problem. The solution procedure is motivated by current practice of the system operator.

If formalized, the problems (F.11) and (F.12) can be stated as
\[
\min \left\{ c^T \delta + \sum_{s \in S} \pi^s \sum_{t \in T} Q_t^s(\delta), \delta \in \mathbb{B}^n \right\}
\]
\[
Q_t^s(\delta) = \min \{ p^T q | Wq = h_t^s - T\delta, q \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \}
\]

where \( \delta \in \mathbb{B}^n \) represent the first-stage decisions, \( q \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \) represent the second-stage decisions and data vectors and matrices are derived from the problems.

**Procedure F.6.1 Enumerate**

*Step 1 (Initialization)* Let \( \bar{z} = \infty \).

*Step 2 (Enumeration)* Choose a first-stage solution, \( \delta \).

*Step 3 (Evaluation)* Let \( \bar{z} = \min \{ \bar{z}, c^T \delta + \sum_{s \in S} \pi^s \sum_{t \in T} Q_t^s(\delta) \} \), where \( Q_t^s(\delta) \) is calculated as in Procedure F.6.2 for \( t \in T, s \in S \). Return to step 2.

**Procedure F.6.2 Merit order**

*Step 1 (Initialize)* If \( D_t^s - S_t^s > 0 \), the system must be down-regulated. Let

\[
\mathcal{I}^{up} = \{ i \in \mathcal{I} : \delta_{i}^{up} = 1 \}
\]

index the activated reserve bids. Available regulation bids are then indexed by \( \mathcal{I}^{up} \cup \mathcal{J} \).

Likewise, if \( D_t^s - S_t^s < 0 \), the system must be up-regulated. Let

\[
\mathcal{I}^{do} = \{ i \in \mathcal{I} : \delta_{i}^{do} = 1 \}
\]

index the activated reserve bids. Available regulation bids are then indexed by \( \mathcal{I}^{do} \cup \mathcal{J} \).

*Step 2 (Ranking)* In the case of up-regulation, activate (fully unless the imbalance is covered by less) the regulation bid \( (\bar{p}_{it}^{up}, \bar{q}_{it}^{up,s}) \) with the lower price from the bids indexed by \( \mathcal{I}^{up} \cup \mathcal{J} \). Delete the bid from the set \( \mathcal{I}^{up} \cup \mathcal{J} \). If \( \mathcal{I}^{up} \cup \mathcal{J} = \emptyset \), the remaining imbalance is excess demand. If the imbalance is covered, stop. Otherwise, return to Step 2.
In the case of down-regulation, activate (fully unless the imbalance is covered by less) the regulation bid \((\bar{p}_{it}^{do}, \bar{q}_{it}^{do,s})\) with the higher price from the bids indexed by \(I^{do} \cup J\). Delete the bid from the set \(I^{do} \cup J\). If \(I^{do} \cup J = \emptyset\), the remaining imbalance is excess supply. If the imbalance is covered, stop. Otherwise, return to Step 2.

F.7 Computation results

As already stated, the case study concerns the regulating reserve management problem of the western Denmark system operator. The data dates from June 2006, just prior to the transition from pay-as-bid pricing to marginal pricing. Hence, we solve the problem with both pay-as-bid pricing (F.11) and local marginal pricing (F.12). Reserve bids comprise bids to the auction of June as well as individual contracts that may have a longer duration. As the system operator intends to reduce the offer period of reserve bids, we assume that such individual contracts have an offer period of only one month. The reserve bids consist of seven up-regulation bids and one down-regulating bid. The volumes and the fixed prices of the reserve bids are released by Energinet.dk [40]. The variable prices have been randomly generated based on the announced regulating market prices. As regards regulating bids to the market, ten bids have been constructed. Both volumes and prices have been randomly generated based on the regulating market prices and the total amounts of up- and down-regulation bid into the market. From this, the imbalances between demand and supply has been calculated. The data has been provided by Nord Pool [86]. The penalties for excess demand and supply are both set sufficiently high to prevent uncovered imbalances on a regular basis.

With the current data, the problem (F.11) contains eight binary variables and no constraints in the first stage and 53,280 continuous variables and 13,680 constraints in the second stage. The problem (F.12) contains the same number of variables and constraints in the first stage and 54,720 continuous variables and 39,608 constraints in the second stage.

The Procedures F.6.1 and F.6.2 were implemented in C++ and computations were carried out on an Intel Xeon 2.67 GHz processor with 4 GB RAM.

We have solved the problems (F.11) and (F.12) with the Procedures F.6.1 and F.6.2 and listed the results. For a varying number of scenarios, Table F.1 displays the average optimal values and CPU times of ten different runs. Obviously, marginal pricing results in a higher optimal value than pay-as-bid pricing. The first column of Tables F.2 and F.3 shows the total balancing costs divided into
reserves, regulation costs and penalty costs. Recall that regulation costs consist of up-regulation expenses and down-regulation income and costs may therefore be both positive and negative. The second column of Tables F.2 and F.3 gives the total imbalances divided into regulation and excess supply and demand along with the reserved regulation that is available but not necessarily activated. Regulation consists of both up- and down-regulation. All numbers are based on 100 scenarios and are averages of ten different runs. It is clear that both for pay-as-bid pricing and marginal pricing reserves are highly necessary in covering imbalances in an optimal fashion. Finally, Table F.4 lists the reserve bids and indicate activation or not. All ten runs show the same result and indeed support the use of reserves.

To compare the stochastic programming approach to a deterministic approach, we have solved the expected value problem, in which stochastic demand and supply differences have been replaced by their expected values. Moreover, we have computed the results of using the expected value solution (EEV). The average EEVs and CPU times of ten different runs are displayed in Table F.1. As can be shown is always the case, the EEVs exceed the optimal values of the stochastic programs. In Tables F.3 and F.4 the total balancing costs and the total imbalances are divided into reserves, regulation and excess supply and demand and Table F.4 indicate activation or not of the reserve bids. Since imbalances often cancel out on average, no reserve bids are activated in the deterministic case. The result of using the expected value solution however is a need for a larger amount of direct regulation and if not available, larger excess demand and supply. This is indeed reflected in higher regulation costs, much higher penalty costs and thus higher total costs. The percentual values of the stochastic solutions, that is, the percentual saving in costs of using the stochastic solutions rather than the deterministic solutions, are significant as the numbers are in the range of 36–38 percent. In conclusion, stochastic programming has its relevance in the regulation reserve management problem.
Table F.2: Computational results, 100 scenarios, 10 runs.

<table>
<thead>
<tr>
<th></th>
<th>Costs/DKK</th>
<th>Volume/MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay-as-bid Total</td>
<td>4.99e+07</td>
<td>86060.23</td>
</tr>
<tr>
<td>Pricing Reserve</td>
<td>3.46e+06</td>
<td>94320.00</td>
</tr>
<tr>
<td>Regulation</td>
<td>1.32e+06</td>
<td>81545.20</td>
</tr>
<tr>
<td>Excess supply and demand</td>
<td>4.52e+07</td>
<td>4515.03</td>
</tr>
<tr>
<td>Marginal Total</td>
<td>5.22e+07</td>
<td>86060.23</td>
</tr>
<tr>
<td>Pricing Reserve</td>
<td>3.46e+06</td>
<td>94320.00</td>
</tr>
<tr>
<td>Regulation</td>
<td>3.68e+06</td>
<td>81545.20</td>
</tr>
<tr>
<td>Excess supply and demand</td>
<td>4.51e+07</td>
<td>4515.03</td>
</tr>
</tbody>
</table>

Table F.3: Computational results, EVP, 10 runs.

<table>
<thead>
<tr>
<th></th>
<th>Costs/DKK</th>
<th>Volume/MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay-as-bid Total</td>
<td>7.89e+07</td>
<td>86226.71</td>
</tr>
<tr>
<td>Pricing Reserve</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Regulation</td>
<td>1.56e+06</td>
<td>78496.00</td>
</tr>
<tr>
<td>Excess supply and demand</td>
<td>7.73e+07</td>
<td>7730.71</td>
</tr>
<tr>
<td>Marginal Total</td>
<td>8.13e+07</td>
<td>86226.71</td>
</tr>
<tr>
<td>Pricing Reserve</td>
<td>0.9e+00</td>
<td>0</td>
</tr>
<tr>
<td>Regulation</td>
<td>3.94e+06</td>
<td>78496.00</td>
</tr>
<tr>
<td>Excess supply and demand</td>
<td>7.73e+07</td>
<td>7730.71</td>
</tr>
</tbody>
</table>

Table F.4: Computational results, 10 runs.

<table>
<thead>
<tr>
<th></th>
<th>Up regulation</th>
<th>Down regulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price DKK</td>
<td>Volume MW</td>
<td>Activation 100 sce. EVP</td>
</tr>
<tr>
<td>288000</td>
<td>12</td>
<td>yes no</td>
</tr>
<tr>
<td>625000</td>
<td>25</td>
<td>yes no</td>
</tr>
<tr>
<td>384000</td>
<td>16</td>
<td>yes no</td>
</tr>
<tr>
<td>330000</td>
<td>11</td>
<td>yes no</td>
</tr>
<tr>
<td>10298880</td>
<td>298</td>
<td>no no</td>
</tr>
<tr>
<td>714000</td>
<td>21</td>
<td>yes no</td>
</tr>
<tr>
<td>966450</td>
<td>30</td>
<td>yes no</td>
</tr>
</tbody>
</table>
F.8 Further research

It could be argued that regulating reserve management affects spot market trading in that purchasing regulating reserves prevent suppliers from disposing of production in the spot market. As the system operator reserves regulation, less production capacity becomes available for the spot market. We have implicitly assumed that production capacity for the spot market is not seriously affected. The assumption is justified if producers allocate production capacity for the spot market and the regulating market separately. However, it would be valuable to further investigate the matter. For instance, the model of the present paper could be incorporated as a part of a larger model that also includes the spot market.

Acknowledgments

The authors wish to acknowledge gratefully the help of Hans Ravn from the RAM-løse edb and Peter Børre Eriksen, Henning Parbo and Jens Pedersen from the former Eltra, now Energinet.dk.


