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Flanking transmission of continuous ground plates

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Abstract

The ground plate, consisting of a concrete plate resiliently separated to the ground by an elastic layer of mineral wool or polystyrene foam plastics, is a common building detail in dwellings, at least in Scandinavia. The separating wall is typically a lightweight construction with high transmission loss. The common acoustical recommendation to the constructors is either to totally separate the ground plate under the separating wall, or to increase the vibration transmission loss by means of extra plate thickness under the separating wall. However, the first recommendation is not always possible for reasons of statics, the second has practical drawbacks in terms of extra costs and long drying times. A solution, studied herein, is to have a continuos plate under the separating wall, and instead increase the thickness of the entire plate. The aim of the paper is to present a prediction model for flanking transmission of continuous ground plates. Hence, the question is what thickness is necessary to yield a reasonable total transmission loss. The model is analytic, using a modal approach. The result is presented in terms of weighted insertion loss as a function of design parameters.

1. Introduction

The intention of this paper is to deal with the sound insulation capacity of a concrete ground slab resting on a resilient material, a Winkler-foundation, in terms of its flanking transmission. The separating direct path between source and receiver room is assumed to be a lightweight wall with high transmission loss. The resilient layer is typically of mineral wool or polystyrene expanded plastics. This is a typical situation in many Scandinavian terrace houses without a cellar.

The common recommendation to the constructors is either to totally separate the ground plate under the separating wall, or to increase the vibration transmission loss by means of extra plate thickness under the separating wall. However, the first recommendation is not always possible for statical reasons, the second has practical drawbacks in terms of extra costs and long drying times. Therefore, a solution, studied herein, is to have a continuous plate under the separating wall, and instead increase the thickness of the entire plate. The question then is what thickness is necessary to yield a reasonable total transmission loss compared to a lightweight wall with given transmission loss.

Gudmundsson [1] studied a fourth case of the same problem, using a thin elastic layer separating the two slabs. He also studied many different cases of vibration transmission loss, due to e.g. point masses and elastic layers. However, in principal only the vibration level difference was studied, not taking the entirely situation including the rooms in consideration.

Thus, the aim of the paper is to present a prediction model for flanking transmission of continuous ground plates. In order to analyze this type of problem, the approach to the problem can often be divided into two main philosophies. In the first philosophy the problem is attacked with an energy average thinking, often combined with empirical knowledge and data. The other philosophy is to attack the problem in great detail, using deterministic thinking and analytical and/or numerical methods. The first philosophy includes Statistical Energy Analysis (SEA), power methods as in EN 12354 [2] and empirical models. The second philosophy includes analytical and numerical methods as modal analysis and the Finite Element Method (FEM). An elaborate literature survey of the available prediction model approaches (suitable for lightweight structures) is presented in [3].

For the present case there are at least two main (hypothetical) drawbacks with a SEA or power-flow type of model. Firstly, it can be assumed that the transmission efficiency cross the junction will be taken as unity, which probably is to high (or $K_{ij} = 0$ dB, which probably is to low). Secondly, the fact that the plate is resting on a elastic foundation, which is a important fact of the model at low frequencies, is not easy to include in a SEA model as it mainly is a resonance phenomenon. There will be no flanking path included from the floor (the plate) to the wall or vice versa. This simplification can be justified in terms of the large mass ratio.

2. Formulation of the problem

The idea is to describe the problem in detail, using modal analysis. The plate is assumed to be simply supported at the boundaries, leading to sine-modes. This assumption is not entirely true, but is a good approximation. Also the two rooms, the source and receiver room, are included in the model. The rooms are described by cosine-modes, corresponding to rigid walls. According to Ljunggren is the actual type of room boundary condition important; the transmission of a ordinary homogenous wall due to resonant response increases by 6 dB at frequencies below the critical frequency is orthogonal partitions is compared with a wall mounted in a baffle [4]. The wall separating the rooms is assumed to be infinitely thin. The time dependency is assumed to be of the form $we^{i\omega t}$, $\omega = 2\pi f$ is the angular frequency and t is the time, and $e^{i\omega t}$ henceforth is suppressed.

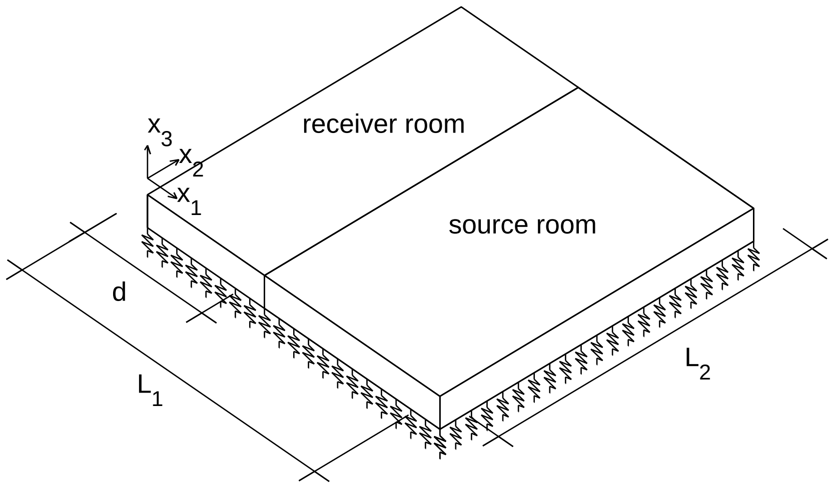

Figure 1: The geometry of the plate.

2.1 Description of the plate and of the rooms

The displacement field of the plate can be described as

$$D\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right)^2 w - m''\omega^2 w = p_e\theta(x_1 - d) + p_t - Kw \tag{1}$$

where $w = w(x_1, x_2)$ is the displacement of the plate, $D = EI'/(1-\mu^2)$ is the bending stiffness of the plate, E is the Young's modulus, $I' = h^3/12$, h is the thickness of the plate, K is the stiffness of the resilient layer, m'' is the mass per unit area, $p_e = p_e(x_1, x_2)$ is the excitation pressure, $p_t = p_t(x_1, x_2)$ is the total reaction pressure of the surrounding fluid and $\theta(x_1)$ is the Heaviside step-function. Equation (1) together with the boundary conditions

$$w|_{\partial S} = 0 \quad \partial^2 w/\partial n^2|_{\partial S} = 0 \tag{2}$$

describes the problem. The plate occupies the area $S \in \{x_1, x_2 | 0 < x_1 < L_1, 0 < x_2 < L_2\}$, ∂S denotes the boundary of the plate. The subareas $S_r \in \{x_1, x_2 | 0 < x_1 < d, 0 < x_2 < L_2\}$ and $S_s \in \{x_1, x_2 | d < x_1 < L_1, 0 < x_2 < L_2\}$, corresponding to the receiver and source room respectively, are also needed. As a consequence of equations (1) and (2) the displacement field can be expanded in terms of sine-modes

$$w(x_1, x_2) = \sum_{n,m=1}^{\infty} \hat{w}_{nm}\varphi_{nm}(x_1, x_2), \tag{3}$$

where the sum with double indexes is a short notation for a double sum, the 'hat'-symbol $\hat{\cdot}$ is used to denote the coefficients of the plate-modes φ_{nm}, and where

$$\varphi_{nm}(x_1, x_2) = \sin(\alpha_n x_1)\sin(\beta_m x_2), \tag{4}$$

where $\alpha_n = \pi n/L_1,\ \beta_m = \pi m/L_2$. These modes fulfills the boundary conditions (2). The derivatives in the governing equation (1) are found to be

$$\frac{\partial^2}{\partial x_1^2}\sum_{n,m=1}^{\infty}\hat{w}_{nm}\varphi_{nm} = -\sum_{n,m=1}^{\infty}\hat{w}_{nm}\alpha_{nm}^2\varphi_{nm}, \tag{5}$$

$$\frac{\partial^2}{\partial x_2^2}\sum_{n,m=1}^{\infty}\hat{w}_{nm}\varphi_{nm} = -\sum_{n,m=1}^{\infty}\hat{w}_{nm}\beta_{nm}^2\varphi_{nm}. \tag{6}$$

The two rooms are the source room (with index s) and the receiver room (with index r), as described in Figure 1. In the source room, the plate is loaded with an exciting pressure p_e and a reaction pressure p_t. In the receiver room is the plate only loaded with the reaction pressure p_t. Thus, the total radiated acoustic (unknown) reaction pressure p_t can be divided into two parts

$$p_t(x_1,x_2) = p_t(x_1,x_2)\theta(x_1-d) + p_t(x_1,x_2)(1-\theta(x_1-d)) \tag{7}$$

This two parts of the reaction pressures can be described in terms of the cosine-modes in the rooms,

$$p_t(x_1,x_2) = \sum_{n,m=0}^{\infty}\check{p}_{nm}^{(t)}\psi_{nm}^{(s)}(x_1,x_2),\ x_1 > d, \tag{8}$$

$$p_t(x_1,x_2) = \sum_{n,m=0}^{\infty}\tilde{p}_{nm}^{(t)}\psi_{nm}^{(r)}(x_1,x_2),\ x_1 < d, \tag{9}$$

where $\check{\cdot}$ and $\tilde{\cdot}$ are used to denote the coefficients for the source and receiver room respectively. The same expansion is also used for the (known) excitation pressure

$$p_e(x_1,x_2) = \sum_{n,m=0}^{\infty}\check{p}_{nm}^{(e)}\psi_{nm}^{(s)}(x_1,x_2). \tag{10}$$

The modes of the rooms are assumed to be

$$\psi_{nm}^{(s)}(x_1,x_2) = \cos(\gamma_n(x_1-d))\cos(\beta_m x_2) \tag{11}$$

$$\psi_{nm}^{(r)}(x_1,x_2) = \cos(\eta_n x_1)\cos(\beta_m x_2) \tag{12}$$

when the x_1x_2-dependency is considered; in the x_3-direction an $\exp(\pm\kappa_{nm}x_3)$ dependency is assumed (where κ_{nm} is imaginary for travelling waves), corresponding to a totally absorbing roof. In equations (11-12) the wavenumbers are $\gamma_n = \pi n/(L_1-d),\ \eta_n = \pi n/d$ and β_m was introduced above. The acoustic pressures p_t and p_e is both assumed to fulfill the Helmholtz equation.

2.2 The excitation pressure and the fluid reaction

The excitation pressure is described in terms of the source-room-modes in equation (10). A diffuse field is assumed in the source-room, and the choice of the excitation pressure coefficients hence are

$$\check{p}_{nm}^{(e)} = \begin{cases} \mathrm{e}^{\mathrm{i}2\pi\theta_{nm}} & \text{if} \quad k^2 \geq \gamma_n^2+\beta_m^2 \\ 0 & \text{if} \quad k^2 < \gamma_n^2+\beta_m^2 \end{cases} \tag{13}$$

where θ_{nm} is a uniformly distributed stochastic variable $\in [0, 1]$, so that every possible mode is excited, and $k = \omega/c$ is the wavenumber in the air.

The excitation pressure has to be expressed in terms of the plate-modes. Thus, rewrite the excitation as

$$p_e(x_1, x_2)\theta(x_1 - d) = \sum_{n,m=1}^{\infty} \hat{p}_{nm}^{(e)}\varphi_{nm}(x_1, x_2) \tag{14}$$

The connection between $\hat{p}_{nm}^{(e)}$ and $\check{p}_{nm}^{(e)}$ is found by using (10) in (14), multiplying the result by $\varphi_{pq}(x_1, x_2)$ and then integrate over the plate. The result reads, if using the notations

$$\int_S \psi_{nm}\varphi_{pq}\mathrm{d}S = \Omega_{n,p}^{(1)}\Omega_{m,q}^{(2)}, \quad \int_S \varphi_{nm}\varphi_{pq}\mathrm{d}S = \frac{L_1 L_2}{4}\delta_{np}\delta_{mq} \tag{15}$$

where the first equation can be given as closed expressions, and in the second integral δ_{np} is the Kronecker delta,

$$\hat{p}_{pq}^{(e)} = \frac{4}{L_1 L_2}\sum_{n,m=0}^{\infty} \check{p}_{nm}^{(e)}\Omega_{n,p}^{(1)}\Omega_{m,q}^{(2)} \tag{16}$$

which is a quadratic form.

The fluid loading is to be included in the model. The acoustic pressure field in the source and receiver room satisfies the Helmholtz equation. The plate vibration field is connected to the acoustic pressure field via the boundary condition

$$\left.\frac{\partial(p_t + p_e\theta(x_1 - d))}{\partial x_3}\right|_{x_3=0} = -\omega^2\rho w, \tag{17}$$

where the pressures are described in terms of the plate-modes φ_{nm}. Thus, $p_e\theta(x_1 - d)$ is found as (14) but with each term multiplied with a factor $\exp(\kappa_{nm}x_3)$, and the corresponding expression for p_t where each term multiplied with a factor $\exp(-\kappa_{nm}x_3)$, and where $\kappa_{nm} = (\alpha_n^2 + \beta_m^2 - k^2)^{1/2}$. The coefficients $\hat{p}_{nm}^{(t)}$ is now to be determined. Thus, apply the derivatives so that the coefficients are found as

$$\hat{p}_{nm}^{(t)} = \omega^2\rho\hat{w}_{nm}/\kappa_{nm} + \hat{p}_{nm}^{(e)}. \tag{18}$$

2.3 Solution of displacement field and examination of the power

Apply equations (3), (5-6), (14), (16) and (18) on the governing equation (1). This yields, for the coefficient of the displacement field,

$$\hat{w}_{nm} = \frac{2\hat{p}_{nm}^{(e)}}{D(\alpha_{nm}^2 + \beta_{nm}^2)^2 - m''\omega^2 - \omega^2\rho/\kappa_{nm} + K}, \tag{19}$$

and the displacement field can be calculated with (3) for each choice of $\check{p}_{nm}^{(e)}$.

The transmission coefficient τ is the quotient real power (denoted with a superscript $\Re$) radiated from the plate to the receiver room over real power excitating the plate from the source room, $\tau = \Pi_t^{\Re}/\Pi_e^{\Re}$. The complex excitating power is

$$\Pi_e = \frac{1}{2}\int_{S_s} p_e v_e^* \mathrm{d}S, \tag{20}$$

where v_e is the velocity in the x_3-direction due to the excitation pressure p_e. Using the notations $\kappa_{nm}^{(s)} = (\gamma_n^2 + \beta_m^2 - k^2)^{1/2}$, and $\varepsilon_n = 1$ if $n > 0$ and $\varepsilon_n = 0$ if $n = 0$, equation (20) can after some manipulations be written

$$\Pi_e = \frac{(L_1 - d)L_2}{\mathrm{i}8\omega\rho} \sum_{n,m=0}^{\infty} \varepsilon_n \varepsilon_m \kappa_{nm}^{(s)*} |\breve{p}_{nm}^{(e)}|^2. \tag{21}$$

In the same way, the complex power transmitted to the receiver room is

$$\Pi_t = \frac{1}{2} \int_{S_r} p_t v_t^* \mathrm{d}S, \tag{22}$$

where the pressure p_t and the velocity v_t is to be evaluated in the receiver room modes $\psi_{nm}^{(r)}$, equation (9). Equation (22) can after some manipulations be written

$$\Pi_t = \frac{-\mathrm{i}\omega^3 \rho d L_2}{8} \sum_{n,m=0}^{\infty} \varepsilon_n \varepsilon_m |\tilde{w}_{nm}|^2 / \kappa_{nm}^{(r)}. \tag{23}$$

where $\kappa_{nm}^{(r)} = (\eta_n^2 + \beta_m^2 - k^2)^{1/2}$.

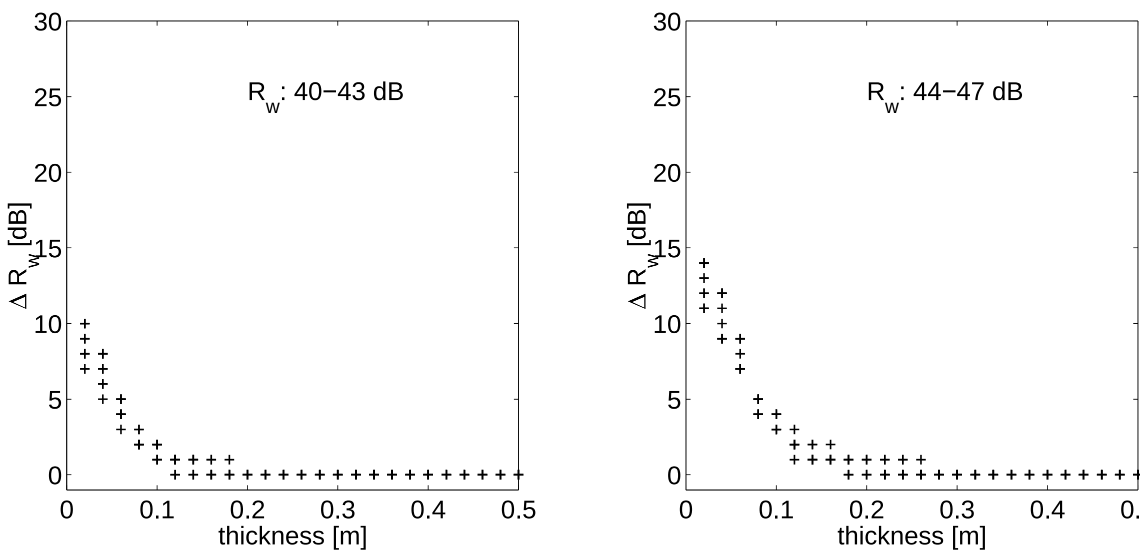

Figure 2: Reduction of the weighted sound transmission index R_w as a function of the thickness of the slab. Left $R_w : 40 - 43$ dB, right $R_w : 44 - 47$ dB.

3. Numerical evaluation and results

The transmission via the separating wall is included from laboratory measurements. The wall data used can be found in [5]. The walls are divided in groups of 4 dB gap in R_w with eight walls in each grope, as seen in Table 1. Each choice of slab thickness yields a sound reduction index for the flanking path. This path is added to the direct path of each wall, giving a total sound reduction index that is compared to the direct path acording to $\Delta R_w = R_{d,w} - R_{tot,w}$ dB, where $R_{d,w}$ is the sound reduction index for the direct path and $R_{tot,w}$ is the sound reduction index for the total construction (the direct

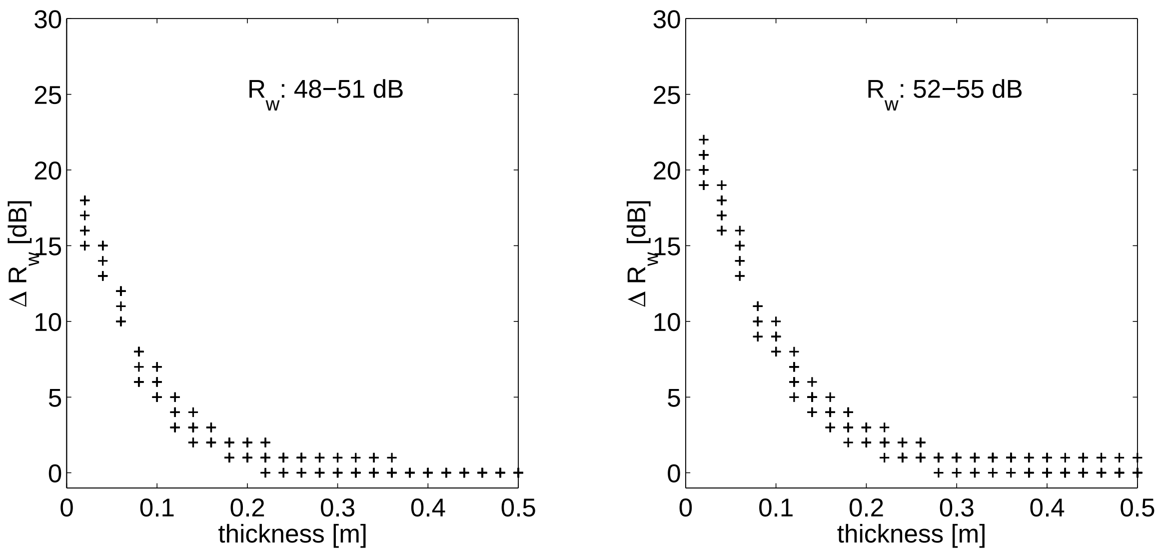

Figure 3: Reduction of the weighted sound transmission index R_w as a function of the thickness of the slab. Left $R_w : 48 - 51$ dB, right $R_w : 52 - 55$ dB.

R_w dB group	Wall 1	Wall 2	Wall 3	Wall 4	Wall 5	Wall 6	Wall 7	Wall 8
$40 - 43$	41	42	43	42	43	43	40	41
$44 - 47$	47	47	47	46	44	45	44	45
$48 - 51$	48	49	51	51	51	48	50	49
$52 - 55$	54	54	53	53	54	55	52	52
$56 - 59$	57	58	59	59	56	56	58	59
$60 - 63$	60	63	63	61	63	63	62	61

Table 1: The weighted sound reduction index R_w for the different walls used in the evaluation.

path and the flanking path). Thus, a positive ΔR_w means a declining performance of the construction.

In the numerical calculations the following data has been used: The geometry parameters where chosen to be $L_1 = 7.123$ m, $L_2 = 6$ m and $d = 3.390$ m, roughly corresponding to two living rooms. The data for the air was $c_0 = 340$ m/s and $\rho_0 c_0 = 400$ kg m^{-2}s^{-1}, and a small portion of damping was introduced to the air in order to avoid numerical problems, $\eta_{air} = 0.001$. The Young's modulus for the plate was $E = 2.6 \cdot 10^{10}$ N/m^2, Poisson's ration is $\mu = 0.3$ and the density is $\rho = 2300\ kg/m^3$. And a small portion of damping was introduced to the Young's modulus, $\eta_E = 0.01$. The resilient layer is assumed to be of polystyrene expanded plastic with Young's modulus $E_{res} = 9 \cdot 10^6$ N/m^2, damping $\eta_{res} = 0.01$, and thickness $h_{res} = 0.1$ m, and the stiffness is calculated as $K = E_{res}/h_{res}$. The thickness of the plate was varied in the calculations from 2 cm to 50 cm. The frequency region is chosen to be from 10 Hz to 2300 Hz. For higher frequencies a mass-law is assumed.

The result is presented in terms of weighted insertion loss as a function of the slab thickness in Figure 2 to 4.

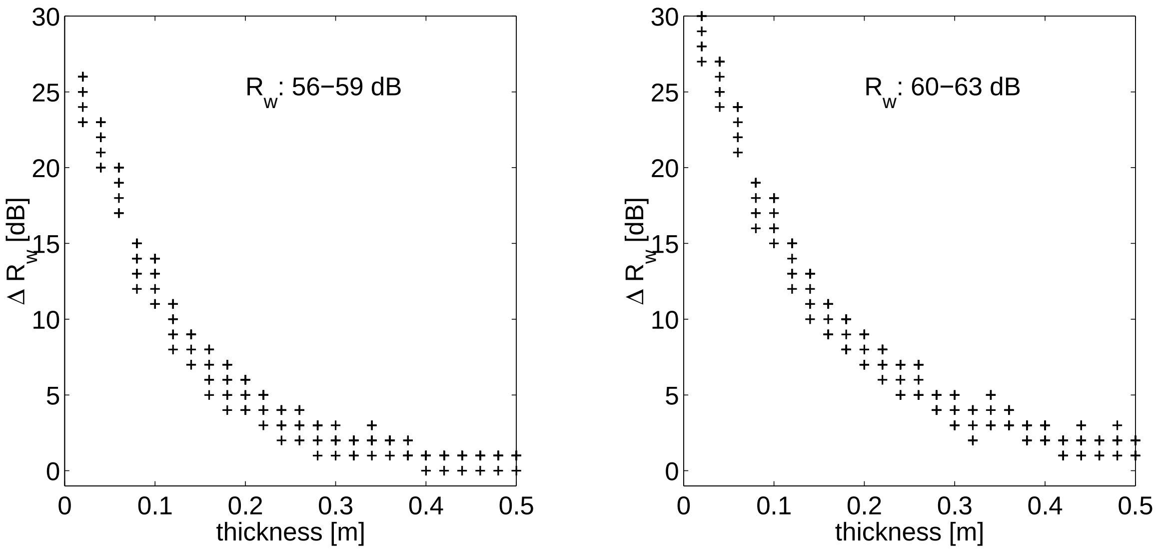

Figure 4: Reduction of the weighted sound transmission index R_w as a function of the thickness of the slab. Left $R_w : 56 - 59$ dB, right $R_w : 60 - 63$ dB.

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