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Modeling Demand Response in Electricity Retail Markets as a Stackelberg Game

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Abstract

We model the retail market with dynamic pricing as a Stackelberg game where both retailers (leaders) and flexible consumers (followers) solve an economic cost-minimization problem. The electricity retailer optimizes an economic objective over a daily horizon by setting an hourly price-sequence, which is then communicated to the end-consumers. In turn, on the basis of such price sequence, consumers optimize a utility function that accounts both for energy procurement costs and for the benefit loss resulting from deferring consumption. The game is formulated as a Mathematical Problem with Equilibrium Constraints (MPEC) and cast as a Mixed Integer Linear Program (MILP), which can be solved using off-the-shelf optimization software. In an illustrative example, we consider a retailer associated with both flexible demand and wind power production. Such an example shows the efficiency of dynamic pricing as a way to control the load for minimizing the imbalances due to wind power, assesses the overall economic results for the retailer and the consumers as well as the dynamic properties of consumer flexibility.

Keywords: demand response, bilevel programming, Stackelberg games, wind power, electricity markets

1 Introduction

The increasing political pressure to reduce the environmental impact of electricity generation is causing a massive deployment of production capacity from unpredictable and intermittent renewable energy sources, such as wind and solar. Facilitating the integration of such sources in electricity markets is therefore seen as of primary importance.

Power markets nowadays are still designed according to the principle of *demand-following supply*, which dictates that the energy generation portfolio of the system should be flexible enough to always match the load. Given the non-dispatchable nature of wind and solar power, which cannot guarantee a certain production level in all meteorological conditions, a reliable power system operation needs the backup from dispatchable, conventional sources. Obviously, this fact limits the share of renewable generation capacity that can be integrated in a power system.

One of the key factors for easing the large-scale integration of renewables is demand response. Indeed, its efficient deployment can bring about a shift in power markets, by endowing them with a *supply-following demand* whose flexibility can be exploited to match the variable output of renewable sources. Several initiatives have been proposed in order to involve both large and small consumers in the provision of demand flexibility, including most notably load shedding programmes, time-of-use and real-time tariffs

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for consumers [11]. Although large consumers are already allowed in many European countries to provide demand response, e.g. by participating at power exchanges or at load shedding programmes, the development of initiatives to involve small consumers are still at an experimental stage.

In order to involve small consumers in demand response, many advocate the use of dynamic price signals. Several questions, though, are still unanswered, including quantifying the potential of dynamic pricing for peak-shaving, load-shifting and reduction of imbalance costs, its impact on the social welfare and the redistribution of the welfare surplus to the players involved.

In this work, we model the retail market with dynamic pricing as a Stackelberg game [12] where both retailers (leaders) and flexible consumers (followers) solve an economic cost-minimization problem. The electricity retailer optimizes an economic objective over a daily horizon by setting an hourly price-sequence, which is then communicated to the end-consumers. In turn, on the basis of such a price sequence, consumers optimize a utility function that accounts both for energy procurement costs and for the benefit loss resulting from deferring consumption. We consider that consumers are flexible in their consumption for heating, and model heating dynamics using state-space models similar to [8].

This paper is structured as follows. Section 2 illustrates the conceptual framework from a high-level perspective. The mathematical formulation of such framework is then presented in Section 3. An illustrative example enlightening the main features of the model is introduced in Section 4. Finally, Section 5 concludes the paper.

2 Conceptual framework

This section illustrates the general concept behind the model presented in this paper, which is a Stackelberg (or leader-follower) game. The structure of such a game is represented as a block diagram in Figure 1. A leader, in this case the retailer, minimizes its objective function $f(\pi, l_1, l_2, \dots, l_n)$. For the

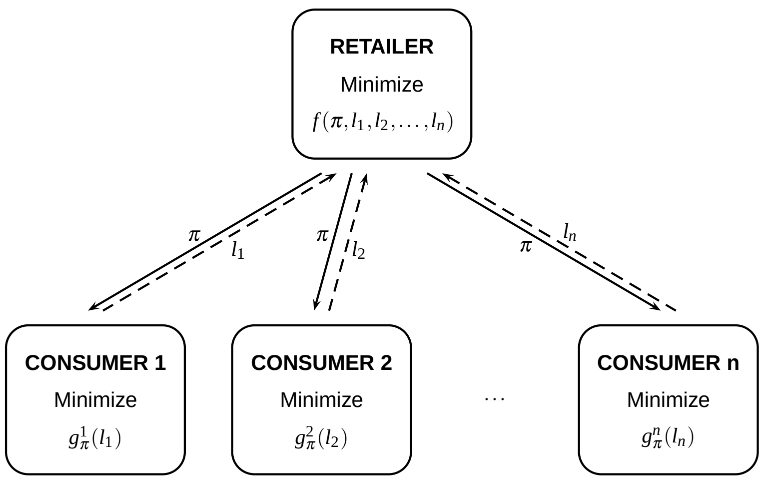


Figure 1: *Conceptual framework for modeling demand response as a bilevel program. Solid arrows indicate exchange of information, while dashed arrows represent inferences on the consumer behavior*

sake of generality of this section we will leave the definition of the objective function, which is most likely of economical nature, to Section 3.2. As the notation tells explicitly, though, such objective function directly depends on the price π, which is a decision variable of the leader. Furthermore, the objective function f depends on the demand $l_i, i = 1, 2, \dots, n$ from the n consumers (followers) associated to the leader.

Obviously, the consumptions l_i are ultimately decisions of the consumers. In a demand response framework with variable price, consumers would have to solve optimization problems where the minimization of the cost of electricity procurement is weighted against the loss of comfort implied by possible anticipations or delays in energy consumption. For example considering the electricity consumption for heating, the objective function $g_\pi^i(l_i)$ for consumer i would sum the cost of electricity to a penalty for the deviation of the indoor temperature from a certain reference [3]. Such an optimization problem includes the consumption l_i as decision variable, and is parameterized in the price π decided by the retailer. This problem would then be solved directly by the consumer's smart appliances, once the price signal π is broadcast by the leader.

The solid lines in Figure 1 indicate that there is a direct information exchange when the price schedule is sent from the leader to the follower. The dashed lines indicate that the followers do not directly communicate their consumption schedule to the leader. Nevertheless, the leader can infer the followers' consumption schedule given the price signal π on the basis of a model of the consumers.

As a final remark in this section, we point out that the leader of the Stackelberg game need not necessarily be a retailer, but any market entity setting the dynamic consumer price. In different demand response frameworks, the leader could be the Transmission System Operator (TSO), a Distribution System Operator (DSO) or an aggregator of consumers [9]. Obviously, such leaders would have different objective functions, but the bilevel structure of the problem would remain practically unchanged.

3 Mathematical formulation

The Stackelberg game sketched in Section 2 can be formulated rigorously in the framework of *Mathematical Programs with Equilibrium Constraints* (MPEC) [6]. Such games have a bilevel structure where one or more (lower-level) optimization problems are nested in another (upper-level) one. This can be formulated as

$$
\begin{aligned}
\text{Min.}\quad & f(\pi, l_1, l_2, \dots, l_n)\,, && \text{(1a)}\\
\text{s.t.}\quad & h(\pi, l_1, l_2, \dots, l_n) \le 0\,, && \text{(1b)}\\
& l_1 \in \arg\min \left\{ g_\pi^1(l_1) \text{ s.t. } m_\pi^1(l_1) \le 0 \right\}, && \text{(1c)}\\
& l_2 \in \arg\min \left\{ g_\pi^2(l_2) \text{ s.t. } m_\pi^2(l_2) \le 0 \right\}, && \text{(1d)}\\
& \qquad\qquad\vdots \\
& l_n \in \arg\min \left\{ g_\pi^n(l_n) \text{ s.t. } m_\pi^n(l_n) \le 0 \right\}. && \text{(1e)}
\end{aligned}
$$

Notice that this formulation employs the same notation as Figure 1. The lower-level problems are represented by (1c)–(1e), which include the feasibility constraints $m_\pi^i(l_i) \le 0$, besides the objective functions $g_\pi^i(l_i)$. The upper-level problem consists in the minimization of the objective function $f(\pi, l_1, l_2, \dots, l_n)$ in (1a) subject to the feasibility constraint (1b), and further constrained by the optimality of the lower-level problems.

Despite its clarity, formulation (1) cannot be translated directly into a computationally manageable optimization problem, owing to the nested optimization of the lower-level problems in (1c)–(1e). Fortunately though, such optimization problems can be replaced by their Karush-Kuhn-Tucker (KKT) conditions under reasonably mild assumptions. Indeed, KKT conditions are necessary and sufficient for optimality if the lower-level problems are convex and their constraints satisfy some regularity conditions [5]. If this holds, the bilevel problem (1) can be reformulated as the following single-level program

$$
\begin{aligned}
\text{Min.}\quad & f(\pi, l_1, l_2, \dots, l_n)\,, && \text{(2a)}\\
\text{s.t.}\quad & h(\pi, l_1, l_2, \dots, l_n) \le 0\,, && \text{(2b)}\\
& \text{KKT conditions of problem for Consumer } 1\,, && \text{(2c)}\\
& \text{KKT conditions of problem for Consumer } 2\,, && \text{(2d)}\\
& \qquad\qquad\vdots \\
& \text{KKT conditions of problem for Consumer n}\,. && \text{(2e)}
\end{aligned}
$$

In the remainder of this section, we will formulate problem (2) explicitly and deal with the nonlinearities in the KKT conditions.

3.1 Lower-level problem

As previously mentioned, we focus on the flexibility of the load due to heating. Therefore, the consumer (lower-level) problem is the optimal scheduling of electricity consumption for heating. Similarly to [8], we model the heat dynamics of buildings using state space models [7]. Furthermore, the objective function is defined as the sum over all the T time periods considered in the optimization horizon of the cost of purchasing electricity plus a quadratic penalty for deviations of the temperature from a certain reference fixed a priori. This results in the following problem

$$\begin{aligned}
\text{Min.} \quad & g_\pi(l) = \sum_{t=1}^{T} c(x_{1,t} - \bar{x}_{1,t})^2 + \pi_t l_t \,, && && \text{(3a)}\\
\text{s.t.} \quad & x_{1,t} = a_{11}x_{1,t-1} + a_{12}x_{2,t-1} && : \lambda_{1,t} \qquad t = 1, \dots, T\,, && \text{(3b)}\\
& x_{2,t} = a_{22}x_{2,t-1} + bl_t && : \lambda_{2,t} \qquad t = 1, \dots, T\,, && \text{(3c)}\\
& l_t \geq 0 && : \mu_t \qquad t = 1, \dots, T\,. && \text{(3d)}
\end{aligned}$$

The objective function in (3a) comprises two terms: a quadratic penalty for deviations of the indoor temperature (i.e. the first state $x_{1,t}$) from the reference $\bar{x}_{1,t}$, multiplied by the parameter c, and the cost of purchasing electricity $\pi_t l_t$. The objective function is parameterized in the decision variable π_t of the upper-level problem. Constraints (3b) and (3c) are the state updates of the model for heat dynamics. The second state $x_{2,t}$ is determined in (3c) as a function of its value at the previous step and the electricity consumption. In turn, the indoor temperature $x_{1,t}$ depends in (3b) on the previous values of x_1 and x_2, but not directly on the consumption. Finally (3d) enforces the nonnegativity of the consumption. In principle, an upper bound could be imposed as well, but this is not necessary in this case since the tuning of the parameters discourages too high consumption levels. We finally point out that the symbols after the colons in (3b)–(3d) are the dual variables associated with the relative constraints.

We remark that the optimization model (3) is akin to the one in [3]. An important difference, though, is the quadratic penalty for deviations from a point reference, rather than the penalty for deviations out of a reference band in [3]. The latter objective function would indeed result in a degenerate lower-level problem with multiple solutions. As a consequence, there would be a multiplicity of Stackelberg solutions [4], while the optimization model (2) would only determine (one of) the strong Stackelberg solution(s) [6].

Since the optimization problem (3) is a convex minimization problem with linear constraints, the KKT conditions are necessary and sufficient for optimality [5]. Therefore, problem (3) is equivalent to the following set of conditions

$$\begin{aligned}
& 2c(x_{1,t} - \bar{x}_{1,t}) + \lambda_{1,t} - a_{11}\lambda_{1,t+1} = 0 && t = 1, 2, \dots, T-1\,, && \text{(4a)}\\
& 2c(x_{1,T} - \bar{x}_{1,T}) + \lambda_{1,T} = 0\,, && && \text{(4b)}\\
& \lambda_{2,t} - a_{12}\lambda_{1,t+1} - a_{22}\lambda_{2,t+1} = 0 && t = 1, 2, \dots, T-1\,, && \text{(4c)}\\
& \lambda_{2,T} = 0\,, && && \text{(4d)}\\
& \pi_t - b\lambda_{2,t} - \mu_t = 0 && t = 1, \dots, T\,, && \text{(4e)}\\
& x_{1,t} = a_{11}x_{1,t-1} + a_{12}x_{2,t-1} && t = 1, \dots, T\,, && \text{(4f)}\\
& x_{2,t} = a_{22}x_{2,t-1} + bl_t && t = 1, \dots, T\,, && \text{(4g)}\\
& 0 \leq \mu_t \perp l_t \geq 0 && t = 1, \dots, T\,. && \text{(4h)}
\end{aligned}$$

Such conditions include the stationarity conditions (4a)–(4e) for the Lagrangian of problem (3) taken with respect to $x_{1,t}$ ($t = 1, 2, \dots, T-1$), $x_{1,T}$, $x_{2,t}$ ($t = 1, 2, \dots, T-1$), $x_{2,T}$ and l_t ($t = 1, 2, \dots, T$), respectively. Furthermore, the set (4) includes the constraints of the primal and of the dual of problem (3), as well as the complementarity conditions (4h) relative to the inequality constraints (3d). The latter condition implies that both l_t and μ_t are nonnegative and at least one of them is zero at any time.

Although (4h) is a nonlinear constraint, it can be equivalently recast as the following set of mixed-integer linear constraints [2]

$$l_t \geq 0 \qquad t = 1, 2, \ldots, T\,, \tag{5a}$$
$$l_t \leq i_t M_l \qquad t = 1, 2, \ldots, T\,, \tag{5b}$$
$$\mu_t \geq 0 \qquad t = 1, 2, \ldots, T\,, \tag{5c}$$
$$\mu_t \leq (1 - i_t) M_\mu \qquad t = 1, 2, \ldots, T\,, \tag{5d}$$
$$i_t \in \{0, 1\} \qquad t = 1, 2, \ldots, T\,, \tag{5e}$$

where M_l and M_μ are “large enough” constants, which ensure that constraints (5b) and (5d) are never binding when the right-hand side is different from 0. Notice that with this reformulation, the nonlinearity of (4h) has been traded with the integrality of the conditions (5).

The values used for the parameters in problem (3) are listed in Table 1. Although such values are

Parameter	Value	Unit
c	0.003	€°C^{-2}
$\bar{x}_{1,t}$	$22 + 2\cos\left(\frac{3}{4}\pi + \frac{2\pi t}{24}\right)$	°C
a_{11}	0.9	-
a_{12}	0.07	-
a_{22}	0.95	-
b	2	°C kW^{-1}h^{-1}

Table 1: *Values of the parameters in the consumer problem* (3)

arbitrary in this work, they are chosen so that they produce realistic results. Our focus here is on the properties on the model, therefore we refer the reader interested in a realistic estimation of models for heat dynamics to [8]. The temperature reference is chosen so that it has a daily period with its peak during the afternoon. The c constant is chosen so that the results are sensible in a realistic range of electricity prices. In principle, one could imagine that the producers of heating systems would provide a number of such realistic constants to be chosen by the consumer.

An example of the interaction between prices and consumer behavior can be seen in Figure 2. This figure

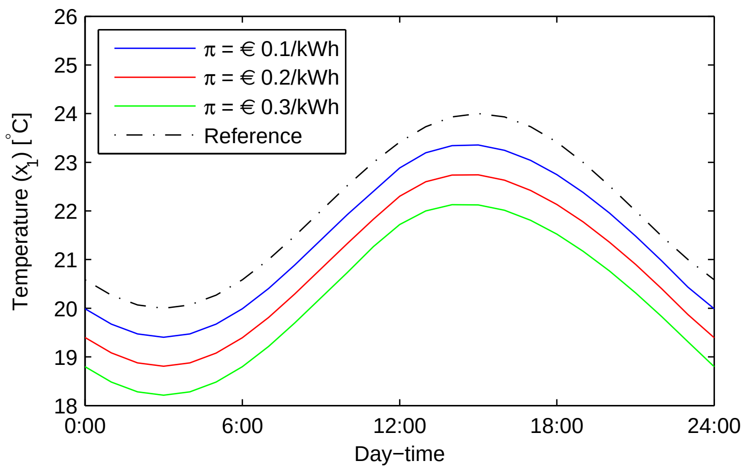


Figure 2: *Reference following for the consumer problem at difference price levels*

illustrates the evolution of the indoor temperature $x_{1,t}$ during an entire day when the consumer receives a constant price schedule. The results for three different price levels are shown: $\pi_t =$€ 0.1/kWh, $\pi_t =$€ 0.2/kWh and $\pi_t =$€ 0.3/kWh. As one can notice, the heating system never follows precisely the schedule, but there is always a negative difference. This is due to the fact that the cost of electricity is positive, while the penalty for deviations is quadratic, the combination of which makes it optimal to follow the reference at a certain distance from below. Furthermore, if the price is increased the consumer is less reluctant to deviate from the reference and accepts a lower temperature.

3.2 Upper-level problem

While the structure of the lower-level problem is well defined—the minimization of a cost function for the consumer based on the heat dynamics of buildings—different configurations of the upper-level problem could be thought of, depending on whom the leader of the Stackelberg game is.

In this work, we analyze the case of a retailer equipped with a wind power production facility and a flexible load. To keep the analysis of the problem as simple as possible, we consider only one consumer, although in principle more consumers could be included. It is assumed that the retailer participates at a spot market, and that it has to provide the market operator with a schedule for the hourly load consumption during the following day with a certain advance in time. This is currently the case in NordPool, the Scandinavian electricity market, where retailers have to purchase every day at noon their expected consumption for each hour of the following day.

For the sake of simplicity, we focus on the use of demand response for reducing the imbalance costs due to the deviations of wind power production from its day-ahead forecast. We assume that the day-ahead schedule s_t for power delivery at hour t of the following day is given by the difference of the day-ahead prognoses of wind out-turn $\widehat{w}_t$ and of consumption $\widehat{l}_t$

$$s_t = \widehat{w}_t - \widehat{l}_t \,. \tag{6}$$

We remark that s_t could be both positive or negative, indicating a power delivery to or withdrawal from the grid, respectively. As far as the prognosis for wind power production is concerned, we assume without loss of generality that the retailer uses point forecasts issued before the time of bidding on the spot market. This bidding strategy is still rather common among wind power producers [1]. Besides, the consumption forecast can be determined by solving the consumer problem (3) with a constant price signal. In this work, we consider the range of prices between €0.1/kWh and €0.3/kWh. Therefore, a reasonable choice when determining the day-ahead load prognosis is a constant price signal of $\widehat{\pi}_t =$€0.2/kWh, $\forall t$. This gives the retailer room for adjusting the consumer price later in both directions.

At the real-time stage, the net power withdrawal or delivery will differ from the schedule s_t, owing to the unpredictability of wind power production. Since the level of uncertainty decreases with shorter lead times, we can expect that the forecasts available one hour ahead are less uncertain than the day-ahead ones. Therefore, the retailer has the possibility of smoothing out the forecast errors by sending the consumers a price signal that encourages them to absorb such deviations.

In stochastic programming, it is customary to represent uncertainty with a discrete number of scenarios. We will index the scenarios using the subscript ω. Each scenario is associated with its probability p_ω and with a specific realization of wind power production $w_{t\omega}$, which is the only stochastic variable we consider here. Figure 3 shows 10 scenarios for wind power out-turn with a 36-hour horizon, along with the corresponding day-ahead point forecast. Both point forecasts and scenarios are generated using time series models following the method proposed in [10]. As one can see in the plot, the hour-ahead information is in the form of a scenario fan, i.e. the first predictions coincide for all the scenarios and represents the current (known) wind power production. This implies that there is perfect information on the current wind power out-turn. In practical situations where decisions are not made exactly in real-time, though very close, the scenarios would differ also at the first time period.

For every scenario ω and time-period t, the net deviation from the schedule is given by

$$d_{t\omega} = w_{t\omega} - l_t - s_t \,, \tag{7}$$

where l_t is the actual consumption from the consumers associated with the retailer. In many electricity markets, deviations from the day-ahead schedule are penalized and in general unwanted, as they require the system operators to take costly corrective measures (e.g. the activation of power reserves). Resembling

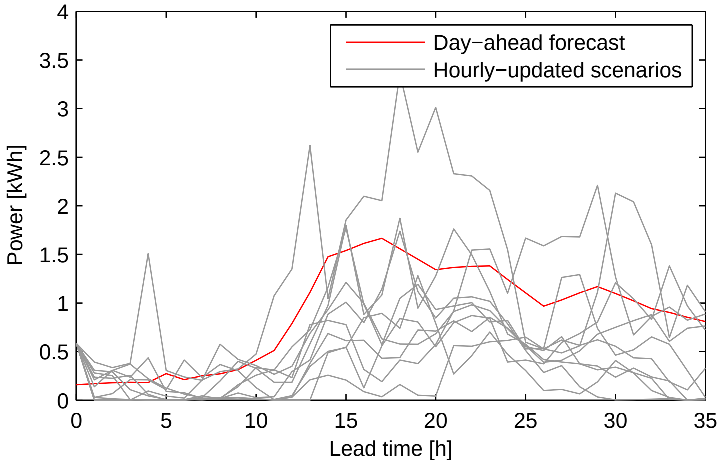


Figure 3: *Example of day-ahead forecast compared to the relative hour-ahead scenarios*

the operation of a virtual power plant, we aim at minimizing the absolute value of the deviation from the day-ahead schedule (or power imbalance). In order to include the absolute value in a linear optimization model, we split the imbalance into its positive and negative parts, d_t^+ and d_t^- respectively, which are defined as follows

$$d_t^+ = \begin{cases} d_t, & d_t \geq 0\,, \\ 0, & d_t < 0\,, \end{cases} \qquad d_t^- = \begin{cases} 0, & d_t \geq 0\,, \\ d_t, & d_t < 0\,. \end{cases} \tag{8}$$

Notice that if the objective function is the minimization of the absolute value of the deviations, it is not necessary to enforce the piecewise definition (8). Indeed, it is easy to verify that, for any optimization problem including the following

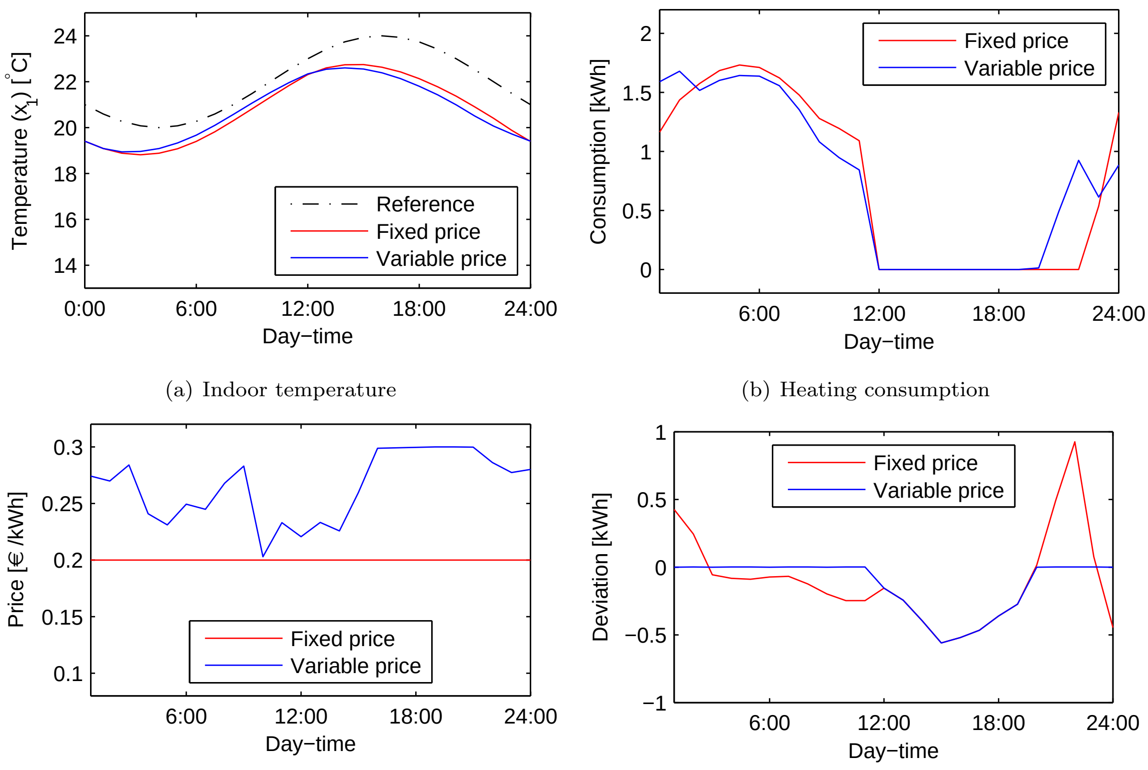
$$\begin{aligned}
\text{Min.}\ \ & d_{t\omega}^+ - d_{t\omega}^-\,, && \text{(9a)}\\
\text{s.t.}\ \ & d_{t\omega}^+ \geq w_{t\omega} - l_t - s_t\,, && \text{(9b)}\\
& d_{t\omega}^- \leq w_{t\omega} - l_t - s_t\,, && \text{(9c)}\\
& d_{t\omega}^+ \geq 0, d_{t\omega}^- \leq 0 && \text{(9d)}
\end{aligned}$$

the optimum is unchanged if (9b), (9c) and (9d) are replaced by the piecewise definitions (8).

The final formulation of the problem is then readily given by

$$\begin{aligned}
\text{Min.}\ \ & \sum_{\omega=1}^{N_\omega} p_\omega \left(\sum_{t=1}^{T} d_{t\omega}^+ - d_{t\omega}^- \right) && && \text{(10a)}\\
\text{s.t.}\ \ & d_{t\omega}^+ \geq w_{t\omega} - l_t - s_t && \forall t, \omega && \text{(10b)}\\
& d_{t\omega}^- \leq w_{t\omega} - l_t - s_t && \forall t, \omega && \text{(10c)}\\
& d_{t\omega}^+ \geq 0, d_{t\omega}^- \leq 0 && \forall t, \omega && \text{(10d)}\\
& 0.1 \leq \pi_t \leq 0.3 && \forall t && \text{(10e)}\\
& \{(4a)\text{–}(4g), (5a)\text{–}(5e)\}\ \text{ for every consumer}
\end{aligned}$$

The objective function (10a) is the expectation of the sum over all the T time periods in the horizon of the absolute value of power imbalance. Compared to the constraints of model (9), the optimization problem (10) includes (10e), which enforces that the price is never higher than €0.3/kWh nor lower than €0.1/kWh. This ensures that the consumer will not be asked to give up too much comfort for balancing the operation of the virtual power plant. Finally, the KKT conditions guaranteeing optimality of the lower-level problem are also included. We remark that problem (10) is a MILP, which can be solved by employing off-the-shelf optimization software.



(a) Indoor temperature (b) Heating consumption

(c) Consumer price (d) Net deviation

Figure 4: *Dynamics of relevant quantities during hourly simulations spanning one day with rolling horizon*

After the optimization problem (10) is solved, the retailer sends the optimal price signal π_t to the consumers for all the time periods included in the horizon. The process is then repeated iteratively rolling the optimization horizon forward. As a result, only the first price in the signal is actually charged to the consumer at any iteration of this rolling process.

4 Illustrative example

This section illustrates an example where the model presented in Section 3 is simulated over one day. In total 24 optimizations are run, one for every hour of the day, employing a rolling horizon. At each hour the wind power scenarios are updated for the entire horizon, and the initial conditions are set to the corresponding output values at the previous step of the procedure.

Figure 4 illustrates the dynamics of relevant variables during the simulation. The reader should keep in mind that only realized values are used to produce these plots, i.e. the evolutions are obtained by concatenating the first value of each variable at each step of the rolling procedure. The indoor temperature for the consumer is shown in Figure 4(a). As one can see, the temperature dynamics with variable price is similar to the one with fixed price. In general the difference between the two dynamics is rather small, and appears to consist in a small time lag. Figures 4(b) and 4(d) deserve to be analyzed jointly. The former one illustrates the dynamics of the consumption in the variable price and the fixed price cases, while the latter shows the power imbalance at each time step. It should be noticed that the power imbalance in the fixed price case is entirely caused by the deviation of wind power production with respect to the day-ahead forecast. By comparing Figures 4(b) and 4(d), we notice that the differences in consumption are driven by the wind power imbalances. Indeed, such imbalances are absorbed when possible by deviations in the load induced by the variable price. The only period when the wind power imbalances cannot be absorbed is between 12:00 and 19:00, when the consumption is already null (and therefore cannot be reduced any further) and the wind power deviation is negative (underproduction). Finally, we remark that the wind power imbalance (equivalent to the deviation in the fixed price case in Figure 4(d)) is mostly negative during the considered day, i.e. the wind plants are underproducing. To reduce such imbalances, there is a need to cut the consumption. This is achieved by sending a dynamic price signal higher than the fixed price, see Figure 4(c).

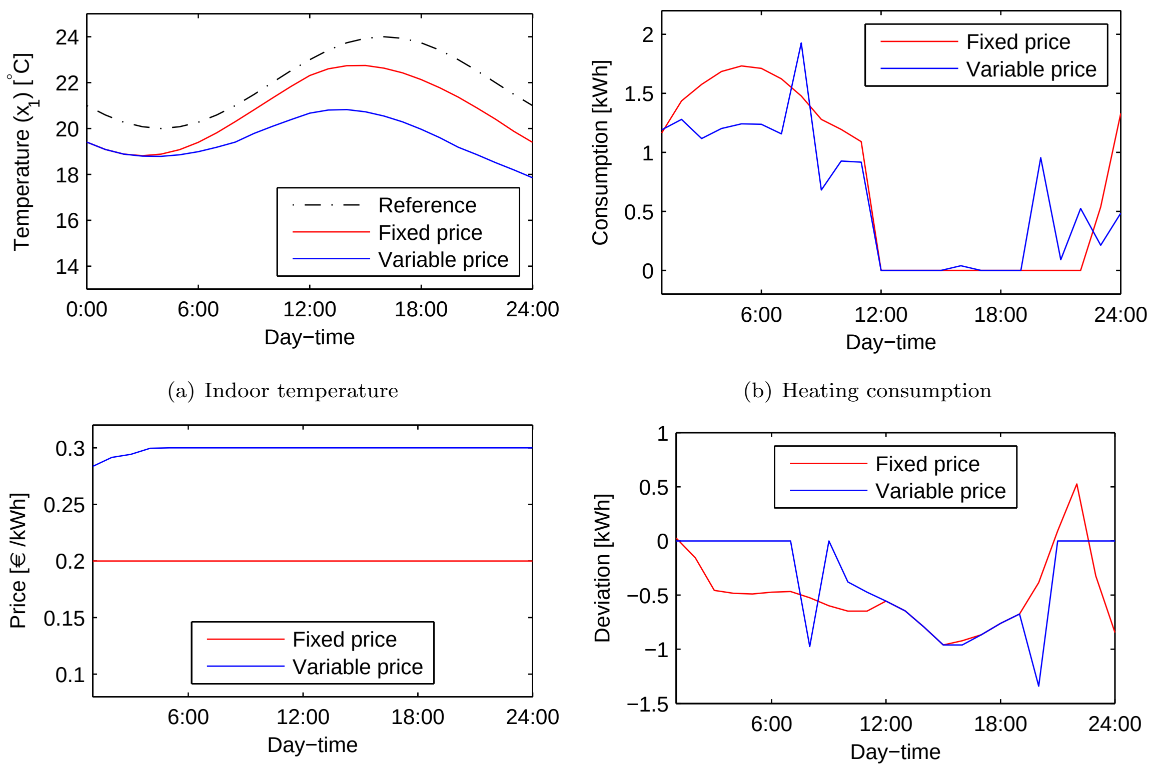
Overall results are included in Table 2. As one can see, the use of dynamic pricing allows the retailer

	Unit	Fixed price	Variable price
Expected total imbalance	kWh	6.79	2.98
Consumer cost	€	3.57	4.77
Consumer discomfort	(€)	0.11	0.13

Table 2: *Overall results of hourly simulations spanning one day with rolling horizon*

to cut over half of the power imbalances it would incur using a fixed price. These cost savings, though, are not passed on to the consumers. Indeed, owing to the higher prices, they are charged a total cost larger by roughly one third in the case of dynamic pricing. In general, one would expect that the wind power forecasts are unbiased, i.e. that the error in the long run has zero mean. As a result, the hours where the consumers are charged higher prices should be equally likely than hours where the price is lower. Nevertheless, there is a need to further reward consumer flexibility in order to encourage their participation in demand response programs. Finally, the third row in Table 2 shows the consumer discomfort, i.e. the sum of the quadratic penalties in the objective function (3a). As one could expect, there is a slight increase when dynamic pricing is used, indicating that the consumer is giving up a marginal fraction of her/his comfort for ensuring the balance of the system.

The dynamics of a similar simulation are shown in Figure 5. In this example, a biased day-ahead forecast



(a) Indoor temperature

(b) Heating consumption

(c) Consumer price

(d) Net deviation

Figure 5: *Dynamics of relevant quantities during hourly simulations spanning one day. The wind power forecasts are biased and overestimate the actual production*

of wind power production is used. Such bias is introduced by adding 0.4 kWh to the point forecast, which constantly overestimates the wind out-turn as a result. This can be seen in the red line in Figure 5(d), which is negative except for the 22nd hour of the day. Owing to the constant negative imbalance of wind power production, the dynamic price rapidly shoots up to the upper limit (€0.3 /kWh). The system is working under considerable stress: for example, the indoor temperature with dynamic pricing in Figure 5(a) is roughly 2°C lower compared to the temperature with fixed price. The quadratic penalty in the objective function makes it difficult to deviate further from the reference temperature. In other words, the “storage” capacity of the heating system is fully used. As a consequence, as one can see in Figure 5(d), the system can hardly absorb imbalances after the 8th hour in the simulation.

5 Conclusion

In this work, we introduced a bilevel framework for modeling demand response. We consider consumer flexibility for heating, and set up a lower-level problem where consumers solve an economic cost minimization problem, once they are given a price schedule as input. Then, such a problem is incorporated as a set of Karush-Kuhn-Tucker equilibrium conditions in an upper-level problem. The framework is general, and different upper-level problems can be thought of considering different market players.

We describe an illustrative example where the upper-level problem is the one of a retailer equipped with a wind power production facility, whose objective is the minimization of the trading imbalances owing to the wind uncertainties. This can be thought of as a virtual power plant operational problem.

The results of the example show that dynamic pricing provides an effective signal for smoothing out the retailer’s trading imbalances. The benefits for the consumers, though, are not always clear. Indeed, overestimation of wind power production at the day-ahead stage results in higher consumer prices, which result in both increased costs and reduced comfort. Furthermore, the heating flexibility has dynamic properties that makes it akin to an energy storage. When wind power forecasts are constantly biased, the capacity of the consumers to absorb deviations is rapidly exhausted.

Further results, where the retailer’s participation at the day-ahead and real-time markets are optimized simultaneously are available in [13].

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