Wake decay constant for the infinite wind turbine array
Application of asymptotic speed deficit concept to existing engineering wake model

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Application of asymptotic speed deficit concept to existing engineering wake model

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Application of asymptotic speed deficit concept to existing engineering wake model

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• Asymptotic speed deficit of the “WAsP Park” model
• Adjustment of WAsP Park model
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Very large wind farms:

- Standard wake models seem to underpredict wake effects.

Recent investigations by Sten Frandsen [1, 2]:

- The reason is the lack of accounting for the effect a large wind farm may have on the atmospheric boundary layer, e.g. by modifying the vertical wind profile.

- In some way the effect of an extended wind farm resembles that of a change in surface roughness: increased equivalent roughness length.

Idea:

- While more detailed models are underway [3], modify the existing WAsP Park engineering wind farm wake model to take this boundary-layer effect into account.

When should a wind farm be considered as large/infinite?

(Hand drawing illustrating the initial idea)
Asymptotic speed deficit from boundary layer considerations (2)

BL-Limited infinite wind farm

Geostrophic wind speed $U(z|z > H) = G$

Roughness $z_{00}$

Friction velocity $u_0$

Hub height shear $t = \rho C_i U_h^2$

Jump in friction velocity at hub-height due to rotor thrust: $\rho (u_{\text{eff}})^2 = \rho (u_i)^2 + t$

Approximation: homogeneously distributed thrust $c_t$

$c_t = \pi/8 \frac{C_t}{(s_r s_f)}$, $t = \rho C_t U_h^2$

$s_r$ and $s_f$: dimensionless* WTG-distances (along- and across-wind) *by $D_{\text{rotor}}$

$Z<h$: profile according to ground surface friction velocity $u_i$ / roughness $z_0$.

$Z>h$: profile according to increased friction velocity $u_{\text{eff}}(= u_{i0})$ / roughness $z_{0\text{eff}} (=z_{00})$.

Equivalent, effective surface roughness: $z_{0\text{eff}}^2 = h_H \cdot \exp \left( -\kappa / \sqrt{c_i + \left( \kappa / \ln(h_H / z_0) \right)^2} \right)$

Wake Decay for the Infinite Wind Turbine Array [5]
Asymptotic speed deficit from boundary layer considerations (3)

Approximate geostrophic drag-law

\[ G \approx \frac{u_s}{\kappa} \left( \ln \left( \frac{G}{f z_0} \right) - A_s \right) \]

General hub-height wind speed:

\[ U(h) = \frac{G}{1 + \left( \ln \left( \frac{G}{h \cdot f} - A_s \right) i \right)} \]

Free flow: \( i_0 = \frac{1}{\ln \frac{h}{z_0}} \)  
Flow over wind farm: \( i_{Tot} = \sqrt{i_0^2 + i_{add}^2}, \quad i_{add} = \frac{c_i}{\kappa} \)

Relative speed deficit \( \varepsilon: \)

\[ 1 - \varepsilon = \frac{1 + \ln \left( \frac{G}{h \cdot f'} \right) i_0}{1 + \ln \left( \frac{G}{h \cdot f'} \right) i_{Tot}} \]

Wake Decay for the Infinite Wind Turbine Array [6]
Asymptotic speed deficit from boundary layer considerations (3)

Comparison with wind farm (Horns Rev):
\[ s_r \approx s_f \approx 7, \ h = 80\text{m}, \ D_R = 60\text{ m} \]

Wake deficit about 50% of the BL-limiting value.
Horns Rev wind farm NOT “infinite”.

![Graph showing speed ratio vs. free wind speed](image)

<table>
<thead>
<tr>
<th>Horns Rev</th>
<th>Power density (W/m²) [4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance for severe wake interference (kwake = 0.075)</td>
<td>Horns Rev 2MW turb’s (observed)</td>
</tr>
<tr>
<td>7.5 km</td>
<td>Entire North Sea 5 MW turb’s (Frandsen – BL-limited)</td>
</tr>
<tr>
<td>Actual extension</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>1.8</td>
</tr>
</tbody>
</table>

Wake Evolution and speed deficit [5,6]

Speed deficit from single wake:

\[ \delta V_{01}^{\text{(type)}} = U_0 \left( 1 - \sqrt{1 - C_t} \right) \left( \frac{D_0}{D_0 + 2kX_{01}} \right)^2 \frac{A_{\text{(type)overlap}}}{A_i^{(R)}}, \quad \text{(type)} = \text{"dir."}, \text{ "ref."} \]

Resulting speed deficit at a downwind turbine:

\[ \delta V_{\text{turb}}^2 = \sum_{i \in \text{upw.turb}'s} \left( (\delta V_{i,\text{turb}}^{\text{(dir.)}})^2 + (\delta V_{i,\text{turb}}^{\text{(ref.)}})^2 \right) \]


Wake Decay for the Infinite Wind Turbine Array [8]
Asymptotic speed deficit of the “WAsP Park” model

Speed deficit for a turbine in an infinite wind farm

Speed deficit the same for all turbines, thus also the turbine thrusts.

Infinite (convergent!!) sum:

\[
(\delta V)^2 = \left( U_{\text{upwind}} \varepsilon_0 \right)^2 \sum_{j=1}^{\infty} N(s_j) \varepsilon_w(x_j)^2; \quad \varepsilon_w(x) = \left( \frac{D_R}{D_R + 2kx} \right)^2; \quad \varepsilon_0 = \left( 1 - \sqrt{1 - C_t} \right)
\]

\(x_j\): Distance to upwind turbine row \(j\). \(N(x_j)\): number of turbines row \(j\) throwing wake on the rotor in focus. 

\(U_{\text{upwind}}\): Wind speed immediately upwind of a turbine

The infinite sum may be approximated by an infinite integral - a simple function \(G\):

\[
\frac{\delta V}{U_{\text{upwind}} \varepsilon_0} = G_{\text{park}}(k; s_r, s_f, h/D_R, C_t)
\]

Since \(U_{\text{upwind}} = U_w = U_{\text{free}} - \delta V\):

\[
\frac{\delta V}{U_{\text{Free}}} = \varepsilon_w = \frac{\varepsilon_w^{\text{app}}}{1 + \varepsilon_w^{\text{app}}}; \quad \varepsilon_w^{\text{app}} = \varepsilon_0 G_{\text{park}}(\text{layout}; k)
\]

Wake Decay for the Infinite Wind Turbine Array [9]
Adjustment of the “WAsP Park” model

Adjustment to match the BL-based asymptotic speed deficit

For “deep” positions the wake expansion coefficient $k$ of the Park Model is modified to approach the BL-based asymptotic speed deficit value $k_{\text{inf}}$:

$$
\delta V_{\text{infin.park}} (k_{\text{inf}}; [s_r, s_f, h, C_t]) = \delta V_{\text{BL-based}} (s_r, s_f, h, C_t)
$$

The $k$-change applies when a wake overlaps with a downwind rotor (to both wakes involved), using a relaxation factor $F_{\text{relax}}$:

$$
k_{j+1}^{\text{adj}} = k_j^{\text{adj}} + (k_{\text{inf}}^{\text{adj}} - k_j^{\text{adj}}) \frac{A_{\text{overlap}}}{A_w} F_{\text{relax}}
$$

The change of the wake expansion coefficient is indicated.

Model parameters used in the following (based on Horns Rev data):

- $k_{\text{initial}} = 0.075$ (recommended value for onshore!)
- $F_{\text{relax}} = 0.2$
Comparative wind farm predictions: Horns Rev (1)

Turbines: 2MW, $D_R = 80m$, $H_{hub} = 60m$
Layout: $s_r = s_f = 7$

Wake Decay for the Infinite Wind Turbine Array [11]
Comparative wind farm predictions: Horns Rev (2)

Wind direction: 270° +/- 3°
Wind speed: 8.5 m/s +/- 0.5 m/s

Wake Decay for the Infinite Wind Turbine Array [12]
Comparative wind farm predictions: Horns Rev (3)

Wind direction:
- 222° +/- 3°
- 222° +/- 3°

Wind speed:
- 8.5 m/s +/- 0.5 m/s
- 12.0 m/s +/- 0.5 m/s

Wake Decay for the Infinite Wind Turbine Array [13]
Comparative wind farm predictions: Nysted (1)

Turbines: 2.33 MW, \(D_R = 82\text{m}, H_{\text{hub}} = 69\text{m}\)
Layout: \(s_r = 10.6, s_f = 5.9\)

Wake Decay for the Infinite Wind Turbine Array [14]
Comparative wind farm predictions: Nysted(2)

Wind direction: 278° +/- 2.5°
Wind speed: 10.0 m/s +/- 0.5 m/s

Wind direction: 263° +/- 2.5°
Wind speed: 10.2 m/s +/- 0.5 m/s

Wake Decay for the Infinite Wind Turbine Array [15]
Conclusions

- The adjustment of the wake expansion coefficient towards a value matching the BL-limited asymptotic speed deficit seems a valuable engineering approach.
- A value for the wake expansion coefficient close to that normally used for onshore locations seems reasonable in this approach also for off-shore wind farms.
- The model (relaxation factor) needs to be fine-tuned in order not to produce over estimations.
- The model needs to be tested on situations with wake effects between neighboring wind farms.