Factors Affecting Coriolis Flowmeter Accuracy, Precision, and Robustness

PhD Thesis

(\bar{E}I\bar{v}^\prime)'' = q - \rho A\ddot{v}

\sqrt{17} \quad \sum_{\chi^2}

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Stephanie Enz
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Factors Affecting Coriolis Flowmeter Accuracy, Precision, and Robustness

by

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Preface

This thesis is submitted in partial fulfilment of the requirements for obtaining the degree of PhD at the Technical University of Denmark (DTU). The work was carried out at the Department of Mechanical Engineering, Section Solid Mechanics, Technical University of Denmark and at Siemens A/S, Flow Instruments (SFI) in the period August 2007 to July 2010. The PhD project was funded by the Danish Agency of Science, Technology and Innovation and SFI.

First, I would like to thank my supervisor associate professor, dr.techn. Jon Juel Thomsen for his helpful advice and scientific guidance during my work, for always encouraging me to go on and for his outstanding support and for many valuable discussions during the project.

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Special thanks to my colleagues at SFI for three challenging and inspiring years, in particular Rune Ravn, Max Hansen, Jan Zangenberg and Uwe Hermann; and also Jens Parkum, now Danfoss A/S, and Nicolai Mathiasen.

I would like to express my gratitude to my colleagues and fellow Ph.D. students at FAM for helpful discussions and suggestions as well as for creating a friendly, humorous and inspiring working environment at FAM.

Last, but not least, I would like to thank my friends and family for being there during the hard times and the times of celebration.

Kgs. Lyngby, 31 July 2010

Stephanie Enz
Resumé (in Danish)

Faktorer der påvirker Coriolis flowmåler nøjagtighed, præcision og robusthed


En analytisk model af et vibrerende rør, hvor der strømmer en væske, er formulert. Denne model er anvendt til at analysere effekten af en pulserende strømning, unøjagtigt monterede aktuatorer, samt effekten af asymmetrisk placerede sensorer på faseskiftet. Dette har ført til hypotesen, at nøjagtighed af CFM påvirkes af unøjagtigt monterede aktuatorer og pulserende strømninger, mens præcisionen kan påvirkes af asymmetrisk placerede sensorer og unøjagtigt monterede aktuatorer. En pulserende væskestrøm kan ligeledes have en indflydelse på flowmålerens robusthed, hvis utilisгрerede inducerede svingninger af røret ikke opdages og kontrolleres.

En numerisk finite element og finite volume (FE/FV) model af et vibrerende væskefyldt rør er udviklet for at undersøge effekten af ikke-ideelle hastighedsprofiler af væsken. Dette synes at påvirke CFM nøjagtighed og præcision.

Eksperimentelle undersøgelser udført for at validere teoretisk baserede forudsigelser vedrørende strukturelle ikke-idealiteter i røret og temperaturen- dringer af omgivelserne understøtter, at nøjagtighed og præcision af CFM påvirkes af disse ikke-idealiteter.

De præsenterede numeriske simulationer er forholdsvis ressourcekrævende og begrænset til de valgte parametre og betingelser. Til gengæld muliggør de matematiske modeller et langt bredere parameter studie, som effektivt giver et direkte indblik i de styrende faktorer af faseskift, hvorpå generelle konklusioner for mere komplicerede systemer kan etableres. Eksperimenter med en kommersiel CFM understøtter de undersøgte teoretisk baserede hypotese fremført i denne afhandling, og demonstrerer hermed, hvordan de analytiske modeller kan øge forståelsen og forudsiges de strukturelle egenskaber af reelle systemer som CFM. Yderligere validering af flere af de analytiske forudsigelser anbefales, da disse modeller synes at være mere værdifulde for praktiske anvendelser end ressourcekrævende, detaljerede, numeriske FE/FV simuleringer.
Abstract

Factors affecting Coriolis flowmeter accuracy, precision and robustness

The purpose of the work presented in the thesis is to gain new knowledge on the dynamic behaviour of fluid-conveying pipes with focus on axial shifts in vibration phase caused by fluid flow and various imperfections. This is of relevance for the general understanding of elastic wave propagation, and of particular interest for manufacturers of devices exploiting phase shifts as a mean of measuring, e.g., fluid mass flow and density as is the case in Coriolis flowmeters (CFMs). The imperfections investigated using analytical, numerical and experimental methods are: Pulsating fluids, imperfectly mounted actuators, asymmetrically located detectors, non-ideal fluid velocity profiles, pipe imperfections and ambient temperature changes.

A simplified mathematical model of a vibrating fluid-conveying pipe is formulated to study the effect of pulsating fluids, imperfect actuator and detector positions on phase shift. It is found that CFM accuracy may be affected by imperfectly mounted actuators and fluid pulsations. CFM precision could be influenced by imperfectly mounted actuators and asymmetrically located motion detectors. CFM robustness may be affected by fluid pulsations, if they induce unforeseen pipe motions that are unnoticed and uncontrolled.

A numerical finite element and finite volume (FE/FV) model of a vibrating fluid-conveying pipe is employed to investigate the effect of imperfect fluid velocity profiles, and it appears that they do affect CFM accuracy and precision.

Performed experimental investigations support analytically based hypothesis regarding pipe imperfections and ambient temperature, i.e. CFM accuracy and precision could be influenced by non-ideally distributed damping and pipe mass, as well as by ambient temperature changes.

The presented numerical simulations are highly resource demanding and limited to the particular parameters and conditions simulated. The established simplified mathematical models yield analytical results, which offer direct insight into the involved effects and allow general conclusions for more complicated systems. Experiments with a commercial CFM support the investigated analytically based hypotheses, demonstrating how simple mathematical models can aid understanding and predicting the behaviour of real systems such as CFMs. Further experimental testing of the analytically based predictions is recommended, since simple analytical predictions appear to be more valuable for real applications than resource-demanding detailed numerical FE/FV simulations.
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Chapter 1
Introduction

A fluid’s flow rate can be measured by exploiting different physical principles. Various types of flowmeters are therefore available, e.g., electromagnetic, Coriolis, ultrasonic, thermal and vortex flowmeters.

The work of this thesis is motivated by the suspicion that Coriolis flowmeters (CFM) are vulnerable to imperfections related to: Periodic pulsations in the flow, non-ideal detector and actuator locations, imperfect fluid velocity profiles, imperfect material properties, i.e. mass and damping, and ambient temperature changes. The aim of this thesis is to analyse these imperfect conditions by employing analytical, numerical and experimental methods, yielding knowledge of if and how these imperfections could affect CFMs.

The following introduction gives a short description of the CFM technology and an overview of the current state of the art regarding the investigated imperfections.

1.1 Coriolis flowmeter technology

An early description of a flowmeter based on the Coriolis effect can be found in [1]. Nowadays, CFMs exploit the same physical principle and are employed in different industries (Tab. 1.1). An introductive survey of the CFM technology can be found in [3], and a review of the last decades scientific work regarding the CFM technology

<table>
<thead>
<tr>
<th>Industry</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and beverage</td>
<td>Brewing: Measurement of flow density, CO₂ injection, dosing of hop extract. Food: Measurement of flow and density, content dosing of yoghurt, fruit or oil. Production: Batching, dosing, filling</td>
</tr>
<tr>
<td>Oil and gas</td>
<td>Metering skids, pipeline transfer, CBG/LPG dispensers, bulk loading, density measurement</td>
</tr>
<tr>
<td>Paper and pulp</td>
<td>Measurement of paperstock, pulp, additives, bleaches, colourants</td>
</tr>
<tr>
<td>Water and waste water</td>
<td>Flocculant dosing, sludge flow and density measurement</td>
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<tr>
<td>Pharmaceutical</td>
<td>Batching, dosing, filling, solvent extraction</td>
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<td>Chemical</td>
<td>Measurement of concentration and density, batching to reactors</td>
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<td>Mining</td>
<td>Measurement of abrasive sludge and slurries</td>
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</table>
is given in [4].

The CFM working principle is based on fluid-conveying pipes driven at resonance: If there is no fluid flow (cf. Fig. 1.1(a)), resonant excitation (symbolised by $F_d$) will force the pipe to vibrate in its fundamental symmetric mode. This mode is also called drive mode with the corresponding frequency denoted drive frequency $\omega_d$. The actual drive frequency depends on the size of the flowmeter, and ranges from 80 to 1000 vibrations per second. The excited vibration amplitudes are too small to be seen, but can be felt by touching the flowmeter pipes. Once a fluid is flowing through the pipe, Coriolis forces are developed due to directional changes in the moving fluid induced by the pipe vibrations: The fluid mass $m$ (cf. Fig. 1.1(b)) is flowing at a given velocity $v$ through a vibrating pipe. The fluid velocity is assumed to be constant across any cross-section of the pipe perpendicular to the pipe axis. The fluid mass will experience an angular momentum $\Omega$ due to the resonant pipe vibrations of the fundamental symmetric mode, with the angular momentum changing magnitude as the fluid mass moves from location $r$ along the pipe to $r + \delta r$. Due to the Coriolis acceleration the pipe will then experience Coriolis forces $F_c$ that are [3, 5]:

$$F_c = -2 m \Omega \times v.$$  \hspace{1cm} (1.1)

The direction of the Coriolis forces is determined by the direction of $\Omega$ and $v$, since they appear as a cross-product in (1.1). Following the right-hand rule (cf. Fig. 1.1(c)), one sees that the two pipe halves experience equal but opposite directed forces $F_c$ (cf. Fig. 1.1(b)). These forces induce small antisymmetric motions, so that the pipe is also vibrating in a twisted mode (exaggerated in Fig. 1.1(d)), which

![Fig. 1.1: Working principle of a Coriolis flowmeter. (a) Vibration mode in case of zero fluid flow resonantly excited by force $F_d$. (b) Fluid mass $m$ flowing at velocity $v$ through vibrating pipe from location $r$ to $r + \delta r$ giving rise to Coriolis forces $F_c = -2 m \Omega \times v$. (c) Right-hand-rule. (d) Exaggerated torsion of vibrating pipe due to equal but opposite directed Coriolis forces $F_c$.](image-url)
1.1 Coriolis flowmeter technology

typically corresponds to the first antisymmetric mode and which is denoted the Coriolis mode with the corresponding frequency being the Coriolis frequency $\omega_c$. The motions of the twisted mode are imposed on the pipe’s driven motion. The combined motion is a travelling wave, meaning that different points of the pipe axis do not cross the equilibrium plane simultaneously. The resulting time shifts $\Delta t_0$ in zero-crossing between transversely vibrating points along the pipe can be related to an axial shift in vibration phase $\Delta \Psi$ by the drive frequency $\omega_d$ [6, 7, 8]:

$$\Delta \Psi = \omega_d \Delta t_0.$$  \hspace{1cm} (1.2)

CFMs exploit this by measuring the phase shift, which, under certain ideal circumstances [9], can be assumed to be proportional to mass flow $\dot{m}$:

$$\dot{m} \propto \Delta \Psi,$$  \hspace{1cm} (1.3)

and independent of other factors. However, in reality the phase shift could be influenced by a vast variety of imperfections, such as flow pulsations, non-ideal boundary conditions, nonlinearity, and uneven distribution of mass, stiffness and damping, some of which may then be reflected as apparent changes in mass flow, yielding erroneous flowmeter readings.

CFMs are generally made of two parts: a transmitter and a sensor. A representative installation of a CFM can be seen in Fig. 1.2. The transmitter is a device made of hardware and software, e.g. forcing the flowmeter pipes to vibrate, signal processing data received from the vibrating pipes in order to calculate phase shift and mass flow, and showing the measured mass flow on a display. The sensor (hereafter denoted CFM) consists of the vibrating pipes with attached actuator.

![Fig. 1.2: Transmitter and sensor from Siemens A/S.](image)

![Fig. 1.3: Simplified drawing of CFM with two measuring pipes.](image)
Chapter 1 Introduction

and detectors. Figure 1.3 shows a schematic of a CFM made of two bended, parallel pipes. Commercial CFMs are available in sizes from 1.5 mm to 200 mm in outer pipe diameter [3, 4, 10], with flow measurement ranges from 0.001 to 2 × 10⁶ kg/h [4]. Industrial CFM designs employ either single or dual pipe configurations. The materials used for such pipes are typically stainless or Hastelloy steel, titanium or zirconium [3, 4] depending on the application of the CFM. The forced pipe vibrations are maintained by an electrodynamic actuator mounted midpipe on the pipes and driven in a feedback-loop to ensure the correct resonant excitation [3]. At least two symmetrically located detectors are used to measure the pipe displacement at the inlet and outlet section of the flowmeter. The actuator and detectors consist of a magnet attached to one pipe and a pickoff coil attached to the other pipe. Electrodynamic detectors are used since the magnet moving in the magnetic field will generate a sinusoidal voltage output signal, from which the time-shift in zero-crossing between two symmetrically located points can be measured and the phase shifts estimated.

Single pipe designs consist of a straight or fairly straight pipe. The advantage of single straight pipe meters is, e.g., the compactness of the design, the easy inspectability, cleaning, self-draining, sanitary, a low pressure drop, and the uncesssarity of flow splitters. The disadvantages include, e.g., pipe constrainments, meter-mounting effects, and limited liquid temperature ranges, since high temperature differences between the measuring pipe and housing can increase the axial forces on the pipe and lead to pipe damages, which particularly straight pipes are prone to as they experience the largest axial forces [4].

Dual pipe configurations could be made of (a) two parallel and straight or bended pipes requiring flow splitters, or (b) one pipe bended to form a double loop which does not requiring flow splitters. Bended pipe designs are preferred when metering fluids with high temperatures, and they are said to be immune to meter-mounting variations. A disadvantage of bended pipe configurations is the burdensome cleaning and inspection. The measuring system must be sensitive to even small disturbances since the oscillation amplitudes are generally only a few micrometers. To protect this system from the influence of the environment, e.g. external vibrations, the system has to be balanced. Since it can be challenging to find a proper balancing mechanism for single pipe configurations, dual pipe configurations might be preferred [4].

1.2 Factors affecting Coriolis flowmeter accuracy, precision and robustness

Measurement devices should be accurate, precise and robust. The accuracy of a measurement system is the degree of closeness of the measurement to its true value. The precision is the degree to which repeated measurements under unchanged conditions give the same results. Robustness is defined to be the ability of a system to cope with (unpredictable) variations in its operating environment with minimal
1.2 Factors affecting Coriolis flowmeter accuracy, precision and robustness

damage, alteration or loss of functionality. For CFM manufacturers in particular it is of interest to know, which factors could be contributing to the lack of zero-shift stability observed with some industrial CFMs. A lack of zero-shift stability will yield meter readings under (supposedly) constant mass flow, e.g. even in case of zero mass flow, and thus influence CFM accuracy and precision. Four factors, suspected to influence CFM accuracy, precision and robustness, are investigated in this thesis: 1) Flow pulsations, 2) asymmetrical actuator and detector positions, 3) imperfect fluid velocity profiles, and 4) structural non-uniformities. Regarding these factors, a review of the known literature will be given in the following, as well as a description of the contribution of this thesis.

1.2.1 Flow pulsations (Thesis paper [P1]-[P3])

How do perturbations related to the fluid flow influence the dynamic behaviour of fluid-conveying pipes? A pulsating flow, e.g. caused by gear, piston or peristaltic pump in the pipe system or fast valve openings and closings, can generate severe vibrations in a pipe system [11]. For CFMs this is of relevance, since flow pulsations will cause pulsating Coriolis forces, induce unforeseen forced oscillations of the measuring pipe(s), and could result in erroneous flowmeter readings.

The study in [12] investigated a straight pipe filled with an oscillating fluid and without any external harmonic excitation. The results indicate that a pulsating flow in a fluid-conveying pipe can cause parametric resonance. The dynamics of a pipe conveying fluid with steady velocity and harmonically varying flow velocity and boundary conditions other than simply-supported are investigated in [13]. The paper states, that the equation of motion derived in [12] is erroneous, since it neglects the axial movements of the pipe due to an axial acceleration of the fluid associated with the imposed velocity perturbations. An investigation of parametric and combination resonances of a continuous flexible cantilevered pipe due to a pulsating flow with clamped pipe ends is presented in [14]. It is concluded, that parametric and combination resonances are possible, if the flow velocity in the pipe is harmonically perturbed. A straight pipe conveying fluid with external harmonic excitation is investigated by [15]. Plug flow is assumed and both pipe ends are fixed. A coupled non-linear equation of motion for the longitudinal and transverse displacements is presented, where the motion is coupled via the non-linear terms. The natural frequencies are computed from the linearized equations, as well as the time histories for the displacements. A simple CFM, comprising a straight pipe, rigidly built-in at its ends and conveying a pulsating flow is considered in [6]. The study involves a simple, but not systematic, perturbation analysis, i.e. the order of magnitude of neglected terms is not recorded. Effects of pulsating flow on the detector signals are investigated. For pulsating flow the detector signals are shown to contain components involving at least four frequencies, whereas detector signals in the steady flow case contain components at the drive frequency and Coriolis frequency. Flow pulsation frequencies, which might cause problems, are identified to be the sum and
difference of the drive and Coriolis frequency. In this case an infinite amplitude motion corresponding to the second mode frequency is excited by the flow pulsation. According to [16], problems arise with CFMs, when the frequency of the flow pulsations coincides with one of the resonance frequencies of the meter. The problems are most severe when the frequency is equal to the Coriolis frequency.

The performance of CFMs under small flows, which are pulsating, is tested experimentally in [17] using two U-tube CFMs. It is shown that CFMs are susceptible towards excitation of oscillations, i.e. pulsating fluid flows. Small amplitudes of the pulsation at resonance are shown to be sufficient to disturb the flow meter operation. The experimental response of CFMs has also been investigated in [7]. The tests show that the used CFM gives erroneous results for flow pulsations at the Coriolis frequency and at a frequency being the difference between the drive and the Coriolis frequency, which confirms the findings in [6]. The effect of external vibrations on CFMs has been experimentally studied in [18]. Errors, induced by vibrations at the Coriolis frequency, are identified to be due to the algorithm implemented for determining the phase shift.

A procedure for modelling pulsating flow in CFMs using the finite element code ANSYS is presented in [19]. It is concluded that flow measurement errors in the presence of pulsating flow are due to the signal processing and not due to the basic meter calibration. The dynamic response of CFMs to flow pulsations is investigated [20] using a straight tube meter, finite element simulations of flow tubes and experiments with commercially available flowmeters. The tested flowmeters show the presence of sensor signal noise at the Coriolis frequency in the measured response.

The results of previous studies are valuable, but lack a systematic perturbation analysis to fully uncover the importance of the involved parameters. This thesis offers a systematic perturbation analysis of a vibrating fluid-conveying pipe yielding analytical expressions for how axial shifts in vibration phase are affected by flow pulsation. This is to answer the question: Could CFM accuracy, robustness and precision be influenced by a pulsating fluid?

1.2.2 Asymmetric actuator and detector positions (Thesis paper [P4])

CFM vibrations are resonantly driven by an electromagnetic actuator. During the assembly of commercial CFMs, the actuator is individually mounted midpipe on the flowmeter pipe. The detectors, used to meter the motions of the flowmeter pipes, are located near the antinodes of the pipes’ Coriolis mode. Manufacturing variations as well as improper handling of the CFM might change the detector positions.

Studies regarding the electromagnetic actuator and detectors either focus on how they affect the phase shift as added masses [21, 22, 23], or investigate measurement resonance-control systems and algorithms controlling the feedback loop to ensure the correct resonant excitation of the fluid-conveying pipe [24, 25, 26, 27, 28]. No studies are available investigating the effect of deviations from ideal detector and actuator positions. The generally applicable technique [29] is employed to derive a
simple analytical expression for the phase shift depending on the actuator position. This will be used to create hypotheses for commercial CFMs by answering the question: Does a non-midpipe excitation or an asymmetrical pipe motion detection influence the phase shift measurement when employing vibrating fluid-conveying pipes?

1.2.3 Non-ideal fluid velocity profiles (Thesis paper [P5]-[P7])

Do internal flow conditions, such as imperfect fluid velocity profiles, affect the dynamic behaviour of fluid-conveying pipes? This is relevant to know, e.g., when exploiting flow-induced oscillations of pipes to determine the fluids mass flow or density, as done with CFMs.

The weight vector approach is a tool enabling the prediction of velocity profile effects in flowmeters [P7]. The concept of the weight vector is presented in [30] and [31]. For electromagnetic flowmeters, this approach is extensively applied and experimentally verified, see e.g. [32, 33, 34, 35, 36, 37, 38]. The extensive work done by J. Hemp has evidently had an impact on the development of the weight-vector approach for electromagnetic and Coriolis flowmeters: He initially worked with the theory for electromagnetic flowmeters [32, 33], and has in the recent years focused on the application of the same theory for CFMs. The basic weight vector theory for CFMs has been described in [39], presenting a technique for developing an analytical expression for the weight vector. A first application of the same technique for CFMs is presented in [40] and [41]. The former shows the derivation of the weight vector theory for CFMs, whereas the latter presents the calculation of the CFM sensitivity if the effect of fluid viscosity is to be taken into account. More recent studies are published in [42, 43, 44].

The effect of flow conditions on a CFM having a straight and slender measuring pipe is investigated in [45]. Computational fluid dynamics (CFD) analyses were done to study the flow in measuring pipes using different inlet conditions. Varying Reynolds numbers, frequencies of vibration and different dimensions of the measuring pipe are investigated. The employed numerical model does actually not allow the determination of the pipe’s mode shape due to fluid forces. To overcome this deficiency, it is suggested to couple a solid dynamics analysis of the pipe to the CFD analysis. Velocity profile effects for straight pipes have been investigated in [46]. Velocity profile effects are estimated using CFD analyses. Results were presented for shell-type CFMs, which showed flowmeter sensitivity loss due to velocity profile effects. A review of the known literature and open questions regarding velocity profile effects in Coriolis mass flowmeters is presented in [47]. The presentation covers, e.g., effects due to disturbed inlet flow effects and the dependency on the Reynolds number. The presented modelling concepts combine analytical models and numerical methods, i.e. the weight vector approach and CFD analyses. It is stated, that inconvenient velocity profiles result in disturbed flow conditions within the flowmeter. However, it is concluded that the magnitudes of velocity profile
effects depend on many constructional and operational parameters of the CFMs. The presented studies are limited to straight pipe configurations, and questions remain regarding the understanding of velocity profile effects in CFMs with curved measuring tubes, being frequently employed in commercial configurations.

This thesis offers investigation of how internal flow conditions influence the dynamics of straight and bended fluid-conveying pipes. Flow related disturbances, i.e. velocity profile effects, are of particularly interest, since they are occurring in real CFM applications.

1.2.4 Structural non-uniformities (Thesis paper [P8])

How do non-uniform material properties influence the dynamic behaviour of fluid conveying pipes? Knowing this is of relevance for CFM applications, where structural properties of flowmeter pipes, i.e. damping, stiffness and mass, may change yielding, e.g., altered energy dissipation, inaccurate and imprecise measurements and drifting zero-shifts.

For lightly damped structures, such as CFMs, damping is of special interest, especially when it is fluctuating in time. Based on the work by Thomsen and Dahl [29], which shows that asymmetric rotational damping at the pipe supports could lead to a phase shift similar to the one due to the mass flow, the investigations are extended to the effects of non-uniform damping along the pipe. Straight CFMs are investigated in [48] using a lumped-parameter model to explain processes causing density and mass flow reading errors, stating that these errors could be caused by asymmetric damping of the flowmeter’s measuring pipe. Even though damping changes are not large, they could lead to significant errors. Reasons for asymmetric damping may be two-phase gas-fluid flows where bubbles are neither small nor finely dispersed in a viscous liquid. The ability of an aerated flow to cause asymmetric damping is also mentioned in [49]. A need of further theoretical work regarding this topic is expressed [48]. Even though the work in [29] does not enable investigations of the error dependency on, e.g., bubble size or distribution in a two-phase fluid, it can be used to validate the statement in [48] that error measurements of CFM seem to be due to the damping not being symmetric along the measuring pipe.

The stiffness of the flowmeter pipe is in reality non-uniform. Additional stiffness may be introduced by the signal pick-ups. The stiffness could locally be changed during the manufacturing of the frequently employed bended pipe configurations, e.g. U- or Ω-shaped pipes, where plastic deformation and the chosen geometry can give rise to alterations of the stiffness. Analyses of the effects of non-uniform stiffness are not apparent from the CFM literature. However, some work is done and presented in [50], analysing the free-vibration of non-uniform beams to determine formulas for predicting the fundamental natural frequencies of these beams.

The signal pick-ups and actuator are in many CFMs installed directly on the measuring pipe, and can therefore be seen as added masses. This has initiated investigations on how size and placement of theses affect the phase shift [21, 22, 23].
From the results of a simple model of a straight and slender measuring tube, it is concluded that added-mass effects depend on fluid density [23]. It is also concluded that even small added masses can lead to non-negligible variations of the phase shift. A model of an elastic beam, solved by perturbation methods, computer algebra and finite element modelling, is used to predict the effect of the signal pick-up masses is presented in [22]. It is also concluded that if the proportionality factor between the phase shift and mass flow is not independent of the fluid properties, it cannot be used as a calibration constant. It is concluded, that the calibration constant of CFMs becomes fluid-density-dependent, if the masses of the pick-ups are not negligible compared to the pipe mass or if the pick-ups are located in certain regions of the pipe. Based the results presented in [21], it is concluded that the calibration constant is only weakly dependent on the concentrated mass at the middle of a fluid-conveying pipe employed for symmetric excitation of vibration, introducing no relevant fluid property dependency. However, it is also stated, that the opposite applies, when the excited mode is antisymmetric.

A simplified mathematical model, as the one in [29], and a systematic perturbation analysis have been used to investigate the effects of the above described non-uniform distributions analytically [51, 52], and hypotheses have been created for real CFM. In this thesis experimental methods are employed to test these hypotheses using a commercial CFM from Siemens A/S, Flow Instruments (SFI).

1.3 This thesis

An Industrial PhD project is a company focused PhD project. The purpose of the IndustrialPhD program by the Danish Agency of Science, Technology and Innovation is to educate scientists with an insight into the commercial aspects of research and development, increase R&D and innovative capacity in private companies, and build networks disseminating knowledge between universities and private companies. The eight papers [P1]-[P8] at the end of this thesis are directed to a scientific audience and reflect the scientific part of the author’s PhD. The last paper [P9] is directed to the R&D department at SFI. To reflect the author’s insight in the commercial aspects of R&D, this thesis is written as a summary of nine papers, focusing on the information necessary for a research and development engineer to understand the results of this work. Details regarding the calculations are deliberately omitted, since they can be found in the thesis papers.

The first paper “Predicting phase shift of elastic waves in pipes due to fluid flow and imperfections” [P1], coauthored with J.J. Thomsen, J. Dahl and N. Fuglede, presents analytical and experimental studies of different imperfections possibly influencing the zero-shift of Coriolis flowmeters. This paper was a contribution to “The Sixteenth International Congress on Sound and Vibration (ICSV16)”. The initial study of the effect of fluid pulsations is presented in “Predicting phase shift effects for vibrating pipes conveying pulsating fluid” [P2], as a contribution to the “7th EUROMECH Solid Mechanics Conference (ESMC2009)”. This short paper
Chapter 1 Introduction

is coauthored with J.J. Thomsen. The study presented in [P2] is continued yielding “Predicting phase shift effects for vibrating fluid-conveying pipes due to Coriolis forces and fluid pulsation” [P3]. This journal paper is also coauthored with J.J. Thomsen and has been submitted for journal publication.

In “Effect of asymmetric actuator and detector position on Coriolis flowmeter and measured phase shift” [P4] a combined analytical and numerical analysis is carried out to investigate the effect of manufacturing inaccuracies. The paper has been accepted for journal publication by Flow Measurement and Instrumentation.

A preliminary study of the fluid-related effects on Coriolis flowmeters, employing different computational methods via fluid-structure-interaction, is presented in “Dynamics of fluid-conveying pipes: Effect of velocity profiles” [P5]. This short paper is coauthored with J.J. Thomsen and contributed to the “XXII International Congress of Theoretical and Applied Mechanics (ICTAM2008)”. The work was continued and the results are presented in the journal paper “Dynamics of fluid-conveying pipes: Numerical investigation of velocity profile effects” [P6], which has been submitted for journal publication. In “Assessment of the applicability of the weight vector theory for Coriolis flowmeters” [P7] the state of the art literature regarding the weight vector theory is reviewed, its applicability discussed and its vulnerability pointed out. This short paper was a contribution to “XIX IMEKO World Congress”.

The paper “Experimental investigation of zero phase shift effects for Coriolis flowmeters due to pipe imperfections” [P8] presents the results of an experimental study employing a commercial CFM, testing hypotheses based on a simple model of a non-ideal CFM pipe. This paper has been coauthored with J.J. Thomsen and S. Neumeyer and has been accepted for journal publication by Flow Measurement and Instrumentation.

The technical report “Review of sources inducing zero-shift stability problems” [P9] reviews and summarises some of the work carried out by J.J. Thomsen, J. Dahl, N. Fuglede and S. Enz relevant for SFI.

The outline of this thesis is as follows: Chapter 2 is devoted to the effect of pulsating flows on CFMs. Investigations regarding the effect of an asymmetric excitation and pipe motion detection can be found in Chapter 3. Chapter 4 covers the effects of non-ideal fluid velocity profiles on CFMs. Chapter 5 contains the results of experimental investigations regarding the effects of non-uniform material properties on CFMs. Finally, a discussion of the results, the concluding remarks and future perspectives are given in Chapter 6.
Chapter 2
Effect of pulsating flows on Coriolis flowmeters

This chapter is related to the papers [P1]-[P3]. It contains the investigation of the
effect of pulsating flows on vibrating fluid-conveying pipes. The analytical work for
a simplified model will be summarised and results will be given yielding practical
hypothesis for real CFMs.

2.1 Analytical model and solution process

The considered model is that of a simply supported, straight, fluid-conveying pipe
(Fig. 2.1). Using simply supported or clamped-clamped boundary conditions leads
in the end to the same hypotheses. Clamped-clamped boundary conditions would
however yield less transparent results due to elaborate mode shapes, even though
they are closer to installations in real CFMs. The investigated effect is assumed
to be basically similar for more complicated geometries employed in CFMs, e.g.
bended and/or dual pipe configurations with attached detectors and actuators. The
pipe is assumed to convey a fluid with the longitudinal fluid velocity \( v(t) \), which
is a function of time \( t \), having a harmonic pulsation around the mean velocity \( v_0 \),
defined by the pulsation amplitude \( q \in [0;1] \) and pulsation frequency \( \omega_f \), i.e.:

\[
v(t) = \varepsilon v_0 (1 + q \cos(\varepsilon \omega_f t))
\]  

(2.1)

A book-keeping parameter \( \varepsilon \) is introduced to mark terms that are assumed small,
i.e. \( \varepsilon < < 1 \). Equation (2.1) implies that the (non-dimensional) mean fluid velocity
\( v_0 \) is small compared to unity, i.e. the (dimensional) mean fluid speed is small
compared to the characteristic wave speed \( \hat{c} l \). Displacement pumps would cause
slowly-pulsating fluids being described by (2.1) when \( s = 1 \). Valves would typically
cause non-slow fluid pulsations being defined by \( s = 0 \). From a practical point
of view, high-frequency pulsations \( (s = -1) \) are uninteresting to investigate, since

![Fig. 2.1: Simply supported pipe conveying pulsating fluid.](image-url)
common displacement pumps and valves employed in CFM applications would not
excite pulsations in this frequency range.

The equation of motion governing transverse pipe motions \(u(x, t)\) is derived from
expressions for the kinetic and potential energies employing Hamilton’s principle
\[53, 54\] yielding:

\[
\ddot{u} + \dddot{u} + \varepsilon c \dot{u} + \varepsilon \alpha (2 \dot{v} u' + v^2 u'' + \dot{v} u') = -\varepsilon p_a \delta(x - x_p) \cos(\Omega_p t + \phi_0),
\]
(2.2)

the boundary conditions for a simply-supported pipe are:

\[
u(0, t) = u''(0, t) = u(1, t) = u''(1, t) = 0,
\]
(2.3)

and \(c\) is the pipe’s damping, \(p_a\) the amplitude of a time-harmonic excitation force
applied at \(x = x_p\) having frequency \(\Omega_p\) and phase \(\phi_0\), \(x \in [0; 1]\) the axial
coordinate and \(\delta(x)\) Dirac’s delta function. Differentiation with respect to space \(x\)
and time \(t\) is denoted (‘) and (‘). All parameters and variables in (2.1) - (2.3) are
non-dimensionalised, see [P3] for further details. The unperturbed linear natural
frequencies \(\omega_0j\) and corresponding mode shape functions \(\varphi_{0j}\) of the pipe described
by (2.2)-(2.3) are:

\[
\omega_0j = (j\pi)^2, \quad \varphi_{0j} = \sqrt{2} \sin(j\pi x), \quad j = 1, 2, \ldots
\]
(2.4)

The first three terms in (2.2) represent, respectively, the transverse inertia of
the pipe and fluid, the flexural stiffness of the pipe and the effect of pipe damping.
The fourth and fifth term representing Coriolis and centrifugal forces, respectively,
are due to the fluid flowing at speed \(v\) through a pipe segment with instantaneous
curvature \(1/u''\), causing pipe rotations at angular velocity \(u'\). Terms similar to these
occur in many other studies regarding fluid-conveying pipes, e.g. \[55, 56\]. However
(2.2) differs from the corresponding equation in e.g. \[57\] by the presence of the sixth
term \(\dot{v} u'\) representing the effect of time-varying flow speed.

The following assumptions have been used when developing (2.2) - (2.3): The pipe
is considered to be a slender beam made of a linear elastic material with small
damping, uniform cross section, and uniform mass and stiffness distribution. Pipe
deformations are assumed to occur only in the transverse direction and rotations of
the pipe are assumed small. Shear deformation, longitudinal inertia and rotatory
inertia is neglected. The external forcing amplitude \(p_a\) is small, since the pipe is
driven at resonance. The fluid has a homogeneous density and is inviscid, incom-
pressible, filling out the entire inner cross-section area and perfectly coupled to the
motion of the pipe. It is assumed that the pipe’s nominal deformations do not alter
significantly by the effect of internal fluid pressure and frictional drag.

The method of multiple scales is used to solve (2.2) approximately for two types of
fluid pulsations, “slow” and “non-slow”, following the generally applicable technique
established in [29]. The pipe response will be calculated yielding analytical predic-
tions for lateral pipe vibrations (Sec. 2.2-2.3). The equation of motion (2.2) is also
solved by pure numerical analysis using a Galerkin expansion with the purpose of
2.2 Slow fluid pulsations

In this case the fluid velocity is considered to oscillate much slower than the primary transverse pipe drive, i.e. $\omega_f << \Omega_p \approx \omega_01$. The two-mode approximate pipe response $u(x,t)$ becomes:

$$u(x,t) = a_{01} \left[ \varphi_{01} \cos(\Omega_p t + \eta_{01}) + \varepsilon \frac{16\alpha v_0 \omega_01 \varphi_{02}}{3(\omega_02 - \omega_01)^2} \left[ 1 + q \cos(\omega_f t) \right] \sin(\Omega_p t + \eta_{01}) \right]$$

$$- \varepsilon \frac{p_a \varphi_{02}(x_p)}{\omega_02 - \omega_01} \cos(\Omega_p t + \phi_0),$$

with the stationary amplitude $a_{01}$ being:

$$a_{01} = \frac{2\kappa_1}{c}, \quad \text{with} \quad \kappa_1 = \frac{p_a \varphi_{01}(x_p)}{2\omega_01}.$$  \hfill (2.6)

Terms of order $O(\varepsilon^2)$ (and smaller) and mode contributions higher than the second (i.e. $\varphi_{0j}$ with $j > 2$) are being ignored in (2.5). The slowly varying resonant phase functions $\eta_{01}$ can be found in [P3]. Equation (2.5) predicts, that the pipe will mainly vibrate in its fundamental symmetric mode $\varphi_{01}$, the corresponding term is of order $O(\varepsilon^0)$, which is not small. It appears, that on top of the motion of the fundamental mode, there will be small additional antisymmetric motions of the second mode $\varphi_{02}$, caused by the mean unidirectional mass flow (term involving $\alpha v_0$), the pulsating mass flow (term involving $\alpha v_0 q$), and the external asymmetric excitation (term involving $p_a$), all of order $O(\varepsilon^1)$ and therefore small in magnitude.

The part of the pipe response related to the mean and pulsating part of the mass flow appears to be phase-shifted $90^\circ$ with respect vibrations of the resonantly excited fundamental mode $\varphi_{01}$. The resulting motion is a traveling wave, so that different points of the pipe axis do not cross the equilibrium line $u(x,t) = 0$ simultaneously. CFMs “measure” the resulting time shift $\Delta t_0$ in zero crossing between two pipe axis points, or the corresponding phase shift $\Delta \Psi$, which under certain ideal circumstances can be considered proportional to mass flow and thus be interpreted as a measure of mass flow. The analytical prediction (2.5) is tested against results by pure numerical analysis, showing good agreement; see App. A for further details.
2.3 Non-slow fluid pulsations

Fluid pulsation frequencies $\omega_f$ could induce different types of resonances. For CFM applications the following are of particular interest, since they could be caused by common pumps and valves: $\omega_f \approx 2\omega_{01}$ (primary parametric resonance) and $\omega_f \approx \omega_{02} - \omega_{01}$ (first combination resonance), with $\omega_{01}$ and $\omega_{02}$ being the first and the second natural frequency of the pipe. Non-resonant fluid pulsations are also considered. Higher resonances are mathematically possible, but since they are not caused by typical pumps and valves employed in common CFM applications, they are not investigated.

2.3.1 Primary parametric resonance ($\omega_f \approx 2\omega_{01}$)

Principle parametric resonance implies that a small parametric excitation can produce a large system response [58], when the excitation frequency (here the pulsation frequency $\omega_f$) is near twice a natural frequency of the system (here the fundamental frequency $\omega_{01}$). The approximate pipe response $u(x,t)$ turns out to be:

$$
\begin{align*}
  u(x,t) &= a_01 \left\{ \varphi_{01} + \varepsilon \left[ \sum_{\omega_{02}}^{\omega_{01}} q_{\omega_{02}} \left[ \frac{3\omega_{01} - \omega_{01}}{\omega_{02}^2 - \omega_{01}^2} + \frac{3\omega_{01} + \omega_{01}}{\omega_{02}^2 - 9\omega_{01}^2} \right] \sin(\omega_{01}t) \right] \cos(\Omega_p t + \eta_{01}) \\
  &\quad + \varepsilon \left[ \sum_{\omega_{02}}^{\omega_{01}} q_{\omega_{02}} \left[ \frac{3\omega_{01} - \omega_{01}}{\omega_{02}^2 - \omega_{01}^2} + \frac{3\omega_{01} + \omega_{01}}{\omega_{02}^2 - 9\omega_{01}^2} \right] q \cos(\omega_{01}t) \right] \sin(\Omega_p t + \eta_{01}) \\
  &\quad - \varepsilon p_u \varphi_{02} \varphi_{02}(x_p) \cos(\Omega_p t + \phi_0) \right\},
\end{align*}
$$

with the stationary amplitude $a_01$ being given by (2.6). It appears from (2.7), that the pipe vibrates in its fundamental symmetric mode $\varphi_{01}$. In addition there are small antisymmetric motions (terms involving $\varphi_{02}$) due to mean and unsteady mass flow and external excitation. One sees (cf. (2.5) for slow fluid pulsations) that an additional motion is induced, i.e. the term involving $\sin(\omega_{01}t)$. The factor multiplying $\cos(\omega_{01}t)$ is changed compared to the equivalent one in (2.5) and predicted to depend also on the pulsation frequency $\omega_f$. Equation (2.6) predicts, that principle parametric resonance will here not lead to (in theory) infinite amplitude motions, such as is otherwise usually the case with parametric resonance when there are no nonlinearities to limit the response [58]. To understand the lack of parametric resonance of the fundamental mode $\varphi_{01}$ one may consider the parametric excitation terms in the equation of motion (2.2): the fifth term in (2.2) is negligibly small under the relevant conditions, while the fourth and sixth term are proportional to the pipe slope $u'$ or angular velocity $\dot{u}'$. Thus, with dominantly symmetric (wrt. $x = 1/2$) vibrations, the small transverse forces induced by the fluid average out to zero over the pipe length, rather than amplifying motions of $\varphi_{01}$. The comparison between the analytical approximation (2.7) and results by pure numerical analysis, show good agreement, see App. A.
2.3 Non-slow fluid pulsations

2.3.2 First combination resonance ($\omega_f \approx \omega_{02} - \omega_{01}$)

In case of first combination resonance, fluid pulsations with the frequency $\omega_f \approx \omega_{02} - \omega_{01}$ are predicted to excite pipe motions at the drive frequency, i.e. $\Omega_p = \omega_{01}$, but also the Coriolis frequency, i.e. $\omega_{02}$. The approximated pipe response $u(x,t)$ is:

\[
u(x,t) = ao1 \left\{ \begin{array}{c}
\varphi_{01} - \varepsilon \frac{2(\omega_{01} - \frac{1}{2} \omega_f) \alpha v_0 q \varphi_{02}}{3\omega_{01}(\omega_{02} - \omega_{01})} \sin(\omega_f t) \cos(\omega_{01} t + \psi_{01}) \\
+ \varepsilon \alpha v_0 \varphi_{02} \left[ \frac{16\omega_{01}}{3(\omega_{02}^2 - \omega_{01}^2)} + \frac{(2\omega_{01} - \omega_f)q}{3\omega_{01}(\omega_{02} - \omega_{01})} \cos(\omega_f t) \right] \sin(\omega_{01} t + \psi_{01}) \\
+ ao2 \left\{ \begin{array}{c}
\varphi_{02} + \varepsilon \frac{2(\omega_{02} + \frac{1}{2} \omega_f) \alpha v_0 q \varphi_{01}}{3\omega_{02}(\omega_{01} - \omega_{02})} \sin(\omega_f t) \cos(\omega_{02} t + \psi_{02}) \\
+ \varepsilon \alpha v_0 \varphi_{01} \left[ \frac{16\omega_{02}}{3(\omega_{02}^2 - \omega_{01}^2)} + \frac{(2\omega_{02} + \omega_f)q}{3\omega_{02}(\omega_{01} - \omega_{02})} \cos(\omega_f t) \right] \sin(\omega_{02} t + \psi_{02}) \\
- \varepsilon p_o \varphi_{02} \varphi_{02}(x) \frac{\omega_{02}^2 - \omega_{01}^2}{\cos(\Omega_p t + \phi_0)}. \end{array} \right. \\
\end{array} \right. \right\} \right. (2.8)
\]

Stationary solutions to (2.8) have constant amplitudes $a_01$ and $a_02$:

\[
\begin{align*}
a_01 &= \frac{\kappa_1}{\sqrt{\frac{1}{4} \sigma^2 + \frac{\kappa_{23}(\frac{1}{4} \sigma^2 + \kappa_{23})}{\kappa_1^2 + \sigma^2}}}, \\
a_02 &= \frac{\kappa_1}{\sqrt{\kappa_2^2 + \sigma^2 \left( \frac{\kappa_2}{2 \kappa_3} + \frac{1}{4} \sigma^2 + \sigma^2 \right)}}),
\end{align*}
\]

with $\kappa_1$ given by (2.6), and:

\[
\begin{align*}
\kappa_2 &= \frac{4(\omega_{02} - \frac{1}{2} \omega_f) \alpha v_0 q}{3\omega_{01}}, \\
\kappa_3 &= \frac{4(\omega_{01} + \frac{1}{2} \omega_f) \alpha v_0 q}{3\omega_{02}}.
\end{align*}
\]

In (2.9), $\sigma$ is a parameter indicating the closeness of the pulsating frequency $\omega_f$ to $\omega_{02} - \omega_{01}$, i.e. $\omega_f = \omega_{02} - \omega_{01} + \sigma$. The phases $\psi_{01}$ and $\psi_{02}$ in (2.8) can be found in [P3]. It appears from (2.8) that the pipe vibrates mainly in its fundamental symmetric mode $\varphi_{01}$ at the drive frequency $\omega_{01}$ (term involving $a_01 \varphi_{01} \cos(\omega_{01} t + \psi_{01})$), and its antisymmetric mode $\varphi_{02}$ at the Coriolis frequency $\omega_{02}$ (term involving $a_02 \varphi_{02} \cos(\omega_{02} t + \psi_{02})$), with both terms being of order $O(\varepsilon^0)$, i.e. non-small, and therefore considered to dominate the pipe vibrations. The resulting pipe motion is therefore predicted to be a combination of the symmetric and antisymmetric mode. Terms involving the mean mass flow $\alpha v_0$, unsteady mass flow $\alpha q v_0$ and external excitation amplitude $p_a$ are small and will contribute with small additional motions of the antisymmetric and symmetric type. The amplitude of motions due to the unsteady mass flow depends on the drive, Coriolis and pulsation frequency, and differs from corresponding terms in (2.5) and (2.7). Good agreement is shown between the analytical approximation (2.8) and results by pure numerical analysis, see App. A.
2.3.3 Non-resonant case ($\omega_f$ away from $2\omega_{01}$ and $\omega_{02} - \omega_{01}$)

Since the fluid pulsation frequency $\omega_f$ is away from $2\omega_{01}$ and $\omega_{02} - \omega_{01}$, only the external forcing is causing resonant pipe vibrations, and the pipe response $u(x,t)$ is predicted to be:

$$u(x,t) = a_{01} \left\{ \left[ \varphi_{01} - \frac{16\alpha v_0 \varphi_{02}}{3(\omega_{01} - \omega_f)^2 - \omega_{02}^2} \sin(\omega_f t) \right] \cos(\Omega_p t + \eta_{01}) + \frac{16\alpha v_0 \varphi_{02}}{3(\omega_{02}^2 - \omega_{01}^2)} + \frac{8\alpha v_0 \varphi_{02}}{3(\omega_{01} - \omega_f)^2 - \omega_{02}^2} \cos(\omega_f t) \right\} \sin(\Omega_p t + \eta_{01}) - \varepsilon p_a \varphi_{02} \varphi_{02}(x_p) \cos(\Omega_p t + \phi_0),$$

with the stationary amplitude $a_{01}$ being given by (2.6). The pipe is predicted to vibrate mainly in its fundamental symmetric mode $\varphi_{01}$. On top of this there will be small additional motions of the antisymmetric mode $\varphi_{02}$. Terms involving the mean mass flow $\alpha v_0$ and the external forcing amplitude $p_a$ are similar to those in, e.g., (2.5). Terms involving the unsteady mass flow $\alpha qv_0$ differ from those previously seen (2.7) and (2.8).

2.4 Filtered pipe displacement and phase shift

Equations (2.5)-(2.11) predict, that the detector signals sampled by a CFM will contain frequency components additional to the drive frequency $\Omega_p$, when the conveyed fluid has a mean and an unsteady velocity component, the latter pulsating with frequency $\omega_f$. In CFMs one would typically employ narrowband signal filtering to remove data at frequencies other than the drive frequency $\Omega_p$ being only interested in knowing the correct short term mean mass flow, averaged over several drive oscillation periods $2\pi/\Omega_p$. The filtered pipe displacements are obtained as the parts of (2.5)-(2.11) oscillating at the drive frequency $\Omega_p$, i.e. an ideal narrowband signal filter has been employed which does not allow other frequencies than the drive frequency to pass through; see [P3] for further details:

$$\hat{u}(x,t) = a_{01} \left\{ \varphi_{01} \cos(\Omega_p t + \eta_{01}) + \varepsilon \frac{16\alpha v_0 \varphi_{02}}{3(\omega_{02}^2 - \omega_{01}^2)} \sin(\Omega_p t + \eta_{01}) \right\} - \varepsilon p_a \varphi_{02} \varphi_{02}(x_p) \cos(\Omega_p t + \phi_0).$$

After employing that the CFM excitation is usually applied midpipe, i.e. at $x_p = \frac{1}{2}$, and the application of trigonometric identities, (2.12) yields an analytical expression for the difference in phase $\Delta \hat{\Psi}$ between two detectors, located symmetrically around midpipe, i.e. $x_{1,2} = \frac{1}{2} \pm \Delta x$, $\Delta x \epsilon ]0; \frac{1}{2}[$:

$$\Delta \hat{\Psi}(x) = s(\Delta x) \alpha v_0,$$
2.5 Hypotheses for CFMs

with the flowmeter sensitivity \( s(\Delta x) \), being a factor of proportionality between the phase shift and mass flow, given by:

\[
s(\Delta x) = \frac{64\omega_{01}}{3(\omega_{02}^2 - \omega_{01}^2)} \sin(\pi \Delta x).
\]  

Equation (2.14) is a known result [3], i.e. that the flowmeter sensitivity increases with nearness of the natural frequency of the fundamental symmetric mode \( \omega_{01} \) to that of the antisymmetric second mode \( \omega_{02} \).

From (2.5)-(2.11) it can be seen that fluid pulsations are predicted to give rise to vibrations at frequencies \( \Omega_p \pm \omega_f \). Since the bandwidth \( \Delta \omega_{pb} \) of real narrowband filters is small (i.e. \( \Delta \omega_{pb} = O(\varepsilon) \)), but finite, other frequencies than \( \Omega_p \) could be in the pass band if \( \omega_f < \frac{\varepsilon \Delta \omega_{pb}}{2} \). In case of non-slow fluid pulsations, the order of magnitude of the pulsation frequencies (i.e. \( \omega_f = O(1) \)) implies, that the side frequencies \( \Omega_p \pm \omega_f \) will be in the stopband of the narrowband filter, since \( \omega_f > \frac{\varepsilon \Delta \omega_{pb}}{2} \), and therefore be filtered away. CFMs are therefore in case of non-slow fluid pulsations predicted to measure only the mean mass flow \( \alpha v_0 \), with the mean phase shifts predicted by (2.13). The pulsation frequencies of slowly pulsating fluids are small, i.e. \( \omega_f = O(\varepsilon) \). This can give rise to vibrations at frequencies \( \Omega_p \pm \varepsilon \omega_f \), that could pass through the narrowband filter, if \( \varepsilon \omega_f \leq \frac{\varepsilon \Delta \omega_{pb}}{2} \). In this case, CFMs are predicted to measure unsteady phase shifts \( \Delta \Psi(x, t) \):

\[
\Delta \Psi(x, t) = s(\Delta x) \alpha v(t),
\]  

with the sensitivity \( s(\Delta x) \) given by (2.13). Equation (2.15) predicts CFMs to measure the correct unsteady mass flow \( \alpha v(t) \) if the fluid is pulsating slowly.

2.5 Hypotheses for CFMs

The results of the simplified model have lead to hypotheses regarding the accuracy, precision and robustness of CFMs. These hypotheses have been tested against results by pure numerical analysis using a Galerkin expansion.

2.5.1 Accuracy and precision

The analytical prediction for the phase shift (2.13) yields the following hypothesis regarding CFM accuracy: CFMs should meter the correct mean mass flow \( \alpha v_0 \) in case of pulsating fluids, if the detector signals are narrowband filtered, so that they only contain oscillations at the drive frequency \( \Omega_p \).

CFM precision could, e.g., be influenced by the following two factors: Zero-shifts (i.e. phase shifts even in case of zero mass flow) or sensitivity changes. Fluid pulsations are predicted not to yield zero-shifts, as the phase shifts depend on the presence of non-zero mass flow, cf. (2.13). The sensitivity appears not to depend on fluid related quantities, cf. (2.14), it should therefore not be influenced by the
presence of a pulsating fluid.

Figure 2.2 shows the phase shift as function of mean mass flow $\alpha v_0$, comparing the results in case of non-pulsating fluid flow ($q = 0$) based on analytical approximation (solid line) and numerical simulations (symbol marker ◦). The analytical phase shifts are calculated using (2.13). The phase shifts based on numerical simulations are calculated using pipe displacement data at two symmetrically located pipe points, i.e. the antinodes of the antisymmetric mode $\varphi_{02}$ with $x_{1,2} = \frac{1}{2} \pm \frac{1}{4}$, from which the time shifts $\Delta t_0$ in zero-crossing are calculated, which then can be related to the phase shift $\Delta \Psi$ by the drive frequency $\Omega_p$ [6, 7, 8]:

$$\Delta \Psi = \Omega_p \Delta t_0.$$  \hspace{1cm} (2.16)

For all considered cases of non-slow fluid pulsations the displacement data contains components at other frequencies than the drive frequencies, cf. the analytical predictions (2.7), (2.8) and (2.11), yielding elastic waves at different frequencies propagating at different speeds, each with its own time shift. Since the time shift of the combined wave is thus not well defined, (2.16) cannot be straightforward used to calculate the phase shift in case of non-slow fluid pulsations. A narrowband-filtering of the data is necessary to extract oscillations at the drive frequency $\Omega_p$; this was done using a 4th order Butterworth filter. From the filtered data the times shifts in zero-crossing are calculated. Employing (2.16) yields the results depicted with symbol markers ◦, □, ⊿ in Fig. 2.2. It appears that the analytical prediction agrees remarkably well with the numerical simulations, implying that the simple expressions give about the same accuracy as the numerical solution. This applies for both slow and non-slow fluid pulsation. Small discrepancies are seen only for large mass

Fig. 2.2: Phase shift $\Delta \Psi$ as a function of mass flow $\alpha v_0$. No fluid pulsation ($q = 0$): Numerical simulation (◦) and perturbation analysis (2.13) (solid line). Fluid pulsations with frequencies: $\omega_f = 0.001 \ll \omega_{01}$ (◦), $\omega_f = 19.654 \approx 2 \omega_{01}$ (□), $\omega_f = 29.614 \approx \omega_{02} - \omega_{01}$ (○), all from numerical simulations. Parameters: $\alpha = 0.3$, $q = 0.001$, $p_a = 0.001$, $x_p = \frac{1}{2}$, $c = 0.01$, $\Delta x = \frac{1}{4}$, and $\Delta x_p = \frac{1}{2}$. 
2.5 Hypotheses for CFMs

flow. This reflects the decrease in accuracy of the analytical approximation as parameters assumed small, here \( \alpha v_0 \), increase. The results shown in Fig. 2.2 support the hypothesis, that CFM should capture the correct mean phase shift \( \alpha v_0 \), when employing proper filtering of the data sampled by the detectors. The results support also the hypothesis, that CFM precision, influenced by zero-shifts and sensitivity changes, is not affected by fluid pulsations. Zero-shifts are not induced, cf. Fig. 2.2.

The sensitivity appears not to be changed, as this would show as a change in line slope.

2.5.2 Robustness

Fluid pulsations, that are slow, i.e. \( \omega_f < \omega_01 \), or non-slow but not of the combination resonance type, i.e. \( \omega_f \) is away from \( \omega_{02} - \omega_{01} \), are predicted to cause dominating pipe vibrations of the first symmetric mode \( \varphi_{01} \), with the stationary amplitude \( a_{01} \) as given by (2.6). Equation (2.6) predicts the amplitude \( a_{01} \) to be independent of fluid related quantities, e.g. velocity \( v_0 \), pulsation amplitude \( q \) or frequency \( \omega_f \). For the fluid pulsations it applies for, (2.6) predicts that these pulsations do not necessarily influence CFM robustness, since they cannot cause vibration amplitudes, which cannot be controlled by adjusting the amplitude of the external force \( p_a \). Figure 2.3 shows the vibration amplitude \( a_{01} \) as a function of time, when the fluid is slowly pulsating, i.e. \( \omega_f << \omega_{01} \). The results for the analytical prediction are calculated using (2.6), with parameters as given in the figure legend. The same parameters are also used in the numerical simulations. Figure 2.3 exemplifies the general observations made for fluid pulsations that (2.6) apply for: It can be seen that the numerical vibration amplitude is increasing until it reaches a constant value. This constant value is the same as the analytical prediction supporting the prediction by (2.6).

Fluid pulsations, that are close to primary combination resonance, i.e. \( \omega_f \approx \omega_{02} - \omega_{01} \), are predicted to induce pipe motions of both the symmetric (\( \varphi_{01} \)) and

![Fig. 2.3: Vibration amplitude \( a_{01} \) as function of time for slow fluid pulsation, \( \omega_f = 0.001 << \omega_{01} \) from numerical simulation (S) and stationary analytical prediction (A). Parameters: \( v_0 = 0.1 \), and \( q = 0.001 \), other parameters as for Fig. 2.2.](image-url)
of the antisymmetric mode ($\varphi_{02}$), with the dominating stationary amplitudes being given by (2.9). Figure 2.4(a) shows the domination stationary vibration amplitudes for small pulsation amplitude $qv_0 = 0.0008$. As appears the vibration amplitudes converge to stationary values predicted analytically by (2.9). The vibration amplitude ratio $\frac{a_{02}}{a_{01}} = 0.035$ implying that motions of the antisymmetric mode will be small compared to those of the symmetric mode. For fluid pulsations close to sharp combination resonance, i.e. $\sigma \to 0$, (2.9) predicts that the dominating amplitudes will generally decrease for increasing fluid pulsation amplitude $qv_0$. This appears also from the results of numerical simulations, details can be found in [P3]; though below a certain pulsation amplitude (vanishing as $c^2 \to 0$), the amplitude of the second mode increases with $qv_0$. Equation (2.9) has, among other things, also lead to the hypothesis, that fluid pulsations of the sharp combination resonance type induce vibration amplitudes of the antisymmetric mode $\varphi_{02}$ that could be of the same order of magnitude as those for the driven resonant mode $\varphi_{01}$. This seems to hold for the case depicted in Fig. 2.4(b), which shows the dominating vibration amplitudes in case of combination resonance for pulsation amplitude $qv_0 = 0.015$ obtained from numerical simulations, for parameters as given in the figure caption. The ratio between the dominating amplitudes is $\frac{a_{02}}{a_{01}} = 0.72$ indicating both amplitudes to be of the same order of magnitude. Recall, that the amplitude ratio for the case shown in Fig. 2.4(a) is 0.035 for pulsation amplitude $qv_0 = 0.0008$. Depending on the magnitude of the pulsation amplitude $qv_0$, the stationary amplitude $a_{02}$ of the vibrations of the antisymmetric mode may even, according to (2.9), outgrow those of the symmetric mode $a_{01}$. This is illustrated in Fig. 2.5(a), which shows the dominating stationary vibration amplitudes $a_{01}$ and $a_{02}$ predicted by (2.9) for increasing pulsation amplitude $qv_0$ with parameters as given in the figure caption. In CFMs the forcing amplitude $p_a$ is feedback-controlled, i.e. when detecting a decreasing amplitude $a_{01}$ of the symmetric mode for increasing pulsation amplitude $qv_0$, the control schemes would increase the forcing amplitude $p_a$ to maximise $a_{01}$. How-

![Diagram](image_url)

**Fig. 2.4:** Dominating vibration amplitudes $a_{01}$ and $a_{02}$ as function of time for $\omega_f = 29.6104 \approx \omega_{02} - \omega_{01}$ from numerical simulation. Parameters: (a) $v_0 = 0.4$ and $q = 0.002$, and (b) $v_0 = 0.3$ and $q = 0.05$, other parameters as for Fig. 2.2.
However, (2.9) predicts that this would also increase the amplitude of the antisymmetric modes $a_{02}$. This is illustrated by Fig. 2.5(b) showing the dominating stationary vibration amplitudes in case of feedback amplitude control, i.e. for adjusted $p_a$ where $a_{01}$ takes the constant value $a_{01,c}$ for all values of $qv_0$. For CFMs this could yield the following consequences: Certain control schemes in the feedback-algorithm could leave increased vibrations of the antisymmetric modes unnoticed, e.g. if the measured signal is taken simply as the mean (would be zero) of measured motions at symmetrically located measurement coils, or if the feedback signal is based on measured signals narrowband-filtered at the drive frequency $\Omega_p$. This leads to the hypothesis, that fluid pulsations of the sharp combination resonance type could affect CFM robustness by exciting unsupervised antisymmetric modes of vibration. If large or persisting, these vibrations could cause fatigue failure of the CFM.

2.6 Conclusion

The particular combination of simple modeling and perturbation analysis has been employed to study the effect of fluid pulsations on CFM accuracy, precision and robustness.

Typically employed narrowband filtering of the detector signals is predicted to yield the measurement of the correct mean phase shifts in case of pulsating fluid flow. For the particular case of slowly pulsating fluids it is also predicted, that narrowband filters with a small, but finite, passband width could let vibrations at frequencies other than the drive frequency through. The resulting unsteady phase shifts should be correctly metered by the CFM, yielding correct measurements of the unsteady mass flow.

Narrowband filtering could leave unwanted vibrations in certain modes undetected and therefore uncontrolled: In particular those, which are of the combination resonance type, could influence CFM robustness.
The validation of the hypotheses is based on numerical modeling only. The simple analytical expressions, both for lateral pipe displacements and phase shifts, would be even more valuable for application, if their predictions were validated experimentally employing real CFMs. The validity of analytically predicted unsteady phase shifts could, e.g., be tested by bypassing the filter in a real CFM removing data at frequencies other than the drive frequency. This should also enable the experimental validation of the analytically predicted pipe responses. The literature offers suggestions for how an experimental setup for testing the above stated hypotheses could be set up employing real CFMs and pumps, valves and pistons.
Chapter 3
Effect of asymmetric external excitation and pipe motion detection on Coriolis flowmeters

This chapter is a summary of the work presented in [P4]. Manufacturing uncertainties and improper handling of the CFM might result in the actuator and detectors being asymmetrically located. The effect of these asymmetries on vibrating fluid-conveying pipes and axial shift in vibration phase is investigated using simplified analytical models. Practical hypotheses for CFMs will be tested numerically.

3.1 Analytical model and its solutions

The straight, fluid-conveying, simply supported pipe driven by an asymmetrically located force in Fig. 3.1 is described by the equation of motion, obtained using Hamilton’s principle:

$$\ddot{u} + u'''' + \varepsilon c \dot{u} + \varepsilon \alpha (2v\dot{u} + v^2 u'') = -\varepsilon p_a \delta(x - x_p) \cos(\Omega_p t), \quad (3.1)$$

governing the transverse pipe motions $u(x, t)$, with boundary conditions (2.3), as well as the same non-dimensionalised parameters, variables and assumptions as for (2.2). Equation (3.1) is solved to predict the phase shift $\Delta \Psi$ between transversely vibrating points along the pipe. Paper [P4] presents details of the solution of (3.1), which follows the approach described by [29]. The result is a simple analytical expression describing lateral pipe vibrations at any point and time:

$$u(x, t) = a_{01} \varphi_{01} \cos(\Omega_p t + \eta_{01}) + \frac{16 \alpha v \omega_{01} a_{01} \varphi_{02}}{3(\omega_{02}^2 - \omega_{01}^2)} \sin(\Omega_p t + \eta_{01})$$

$$+ \frac{p_a \varphi_{02} \varphi_{02}(x_p)}{\omega_{02}^2 - \omega_{01}^2} \cos(\Omega_p t), \quad (3.2)$$

![Fig. 3.1: Schematic of simply supported, fluid-conveying pipe with asymmetric external excitation and pipe motion detectors.](image)
with \( a_{01} \) and \( \eta_{01} \) being, respectively, the resonant amplitude and phase. From (3.2) it is seen that the pipe basically vibrates in its fundamental symmetric mode \( \varphi_{01} \), with small additional antisymmetric motions of the second mode \( \varphi_{02} \) added on top of the symmetric motion. Two relevant antisymmetric causes, which could contribute to an axial shift in vibration phase, can be identified: (a) a non-zero mass flow \( \alpha v \), and (b) external antisymmetric forcing at \( x = x_p \) with amplitude \( p_a \) and frequency \( \Omega_p \).

### 3.2 Asymmetrical external excitation

During the assembly of Coriolis flowmeters, the deviation from the ideal actuator position is minimised as much as possible. But, non-ideally positioned actuators do occur and small deviations from the ideal position of the actuator can be described by:

\[
x_p = \frac{1}{2} \pm \varepsilon \Delta x_p,
\]

with \( x_p = \frac{1}{2} \) being the ideal actuator position, \( \varepsilon \) a bookkeeping parameter indicating the smallness of the deviation, and \( \Delta x_p \) the deviation from the ideal actuator location. Equation (3.2) leads, after the application of trigonometric identities, to an analytical expression for the difference in phase between two symmetric detectors at \( x_{1,2} = \frac{1}{2} \pm \Delta x_d \):

\[
\Delta \Psi = \frac{4\omega_{01}}{\omega_{02}^2 - \omega_{01}^2} \left[ \frac{16}{3} \alpha v \mp 2\pi \Delta x_p \right] \sin(\pi \Delta x_d),
\]

see [P4] for further details regarding the calculations. The expression for the phase shift (3.4) has been Taylor expanded and only terms with the two highest orders of magnitude are kept. If the actuator is placed at its ideal position, i.e. \( \Delta x_p = 0 \), the second term will be zero, and hence there will only be a phase shift due to mass flow. However, if the actuator is not exactly placed midpipe, i.e. \( \Delta x_p \neq 0 \), there will be a contribution to the phase shift caused by the asymmetrical excitation. This term is denoted zero-shift, since it will also be present in case of zero mass flow, i.e. \( \alpha v = 0 \). During the initial meter calibration one can compensate for the presence of this zero-shift. However, since this zero-shift depends on the pipe’s damping \( c \), which might fluctuate over time, e.g., due to temperature changes, bobbles in the flow or wear, the zero-shift will also fluctuate and might show as changes in the metered mass flow. This leads to the hypothesis that asymmetrical forcing combined with fluctuating pipe damping could be a factor contributing to lack of zero-shift stability observed with some industrial CFMs.

The theoretical approximation (3.4) is tested against the solution of a full numerical model obtained using a standard Galerkin approach, see [P4] for details regarding the numerical model. Figure 3.2 illustrates the variation of the phase shift \( \Delta \Psi \) with increasing mass flow \( \alpha v \) for five levels of excitation asymmetry, i.e. \( \Delta x_p = \{ -0.1, -0.05, 0, 0.05, 0.1 \} \), with the pipes’ damping ratio \( \zeta \) being (a) 0.05%,
3.2 Asymmetrical external excitation

(b) 0.2%, and (c) 0.5%, and the other parameters as given in the figure legend. A damping ratio of 0.05% is typical for CFM pipes. The middle line corresponds in all cases to the ideal pipe excitation, i.e. $\Delta x_p = 0$. The analytical approximation (3.4) is seen to agree very well with the numerical solution. Small deviations between the two solutions can be identified for high mass flow, i.e. $\alpha v = 0.1$. This implies that the analytical approximation has approximately the same accuracy as the numerical solution. However, it provides much more insight into if and how the imperfection “asymmetric actuator” affects the phase shift. Figure 3.2 depicts how an asymmetry in the pipe excitation will cause a phase shift even at zero fluid flow. It can be seen, that the magnitude of the zero shift depends strongly on the magnitude of the pipe damping ratio.

![Graphs showing phase shift for varying mass flow](image)

**Fig. 3.2:** Effect of asymmetrical external forcing ($x_p = \frac{1}{4} + \Delta x_p$) on phase shift $\Delta \Psi$ for varying mass flow $\alpha v$, obtained by analytical approximation (3.4) (symbol marker) and by numerical solution (line marker) for different ratios $\zeta$ a) 0.05%, b) 0.2%, c) 0.5%. From top to bottom the symbols and lines show different levels of force location asymmetry $\Delta x_p = \{-0.1, -0.05, 0, 0.05, 0.1\}$. Other parameters: $\Delta x_d = \frac{1}{4}$, $\alpha = 0.3$, $p_a = 0.001$, $N = 7$. 
3.3 Asymmetrical detector position

Manufacturing variations or improper handling of the CFM, e.g. dropping it accidentally on the ground, might change the detector positions, which are usually located symmetrically and near the antinodes of the flowmeter’s Coriolis mode. Straight pipe configurations, like the one in Fig. 3.1, have the antinodes of this mode in \( x_{1,2} = \frac{1}{2} \pm \frac{1}{4} \). Asymmetric detector positions are described by:

\[
\begin{align*}
    x_1 &= \frac{1}{2} - \Delta x_d + \varepsilon \Delta \hat{x}_1, \\
    x_2 &= \frac{1}{2} + \Delta x_d + \varepsilon \Delta \hat{x}_2,
\end{align*}
\]

with \( \Delta x_d \in [0; \frac{1}{2}] \) being the symmetric offset from the midpipe position \( x_{1,2} = \frac{1}{2} \), and \( \Delta \hat{x}_{1,2} \) a small deviation, indicated by the book-keeping parameter \( \varepsilon \), from the symmetric offset. Following the procedure described in [P4] one obtains, after Taylor expanding and keeping only terms of the two highest orders of magnitude, an analytical prediction for the phase shift between two asymmetrically located detectors:

\[
\Delta \Psi = \frac{32 \omega_0 \alpha v_0}{3(\omega_{02}^2 - \omega_{01}^2)} \left[ 2 \sin(\pi \Delta x_d) + \pi (\Delta \hat{x}_2 - \Delta \hat{x}_1) \cos(\pi \Delta x_d) \right].
\]

The first term in (3.7) represents the phase shift due to mass flow measured between symmetrically located detectors. The second term represents an additional phase shift due to an asymmetry in the detector position. The effect of the asymmetry is seen to be largest for detector positions near midpipe and vanishes monotonously as the detectors are located near the supports. Equation (3.7) yields the following hypothesis: Asymmetrically positioned detectors are predicted not to give rise to zero-shifts, as the additional phase shift depends on the mass flow. However, they could change the flowmeter’s sensitivity and therefore lead to erroneous measurements.

The analytical prediction is tested against the solution of a full numerical model. Figure 3.3 depicts the variation of the phase shift \( \Delta \Psi \) for different levels of detector position asymmetry with parameters as given in the figure legend. To obtain Fig.(a) only one detector is moved from its “ideal” position, whereas both detectors have been moved to arrive at Fig.(b). The comparison between the results based on the analytical prediction and the full numerical model shows, that both agree rather well, with only minor discrepancies for larger mass flow, i.e. \( \alpha v = 0.1 \). The changing slope of the lines in Fig. 3.3, representing different degrees of asymmetry, supports, that the sensitivity of CFMs could be affected when the detectors are asymmetrically located.

3.4 Conclusion

It has been demonstrated how asymmetrically positioned actuators and detectors influence the phase shift of vibrating fluid-conveying pipes. It is predicted, that asym-
3.4 Conclusion

metrical excitation combined with fluctuating pipe damping induces phase shifts, even in case of zero mass flow. This could be a factor contributing to the lack of zero-shift stability seen with some commercial CFMs. Asymmetrically located detectors give rise to phase shifts, which could change the sensitivity of CFMs and therefore lead to erroneous flowmeter readings. It is concluded, that non-ideal actuator and detector positions could influence CFM accuracy and precision.

The presented simple expressions and the hereof following hypotheses, which are assumed to be basically similar for more complicated geometries, would be even more valuable for applications if their predictions were tested experimentally with CFMs. Laboratory experiments are therefore highly recommended to follow up on the presented analytical investigations.

Fig. 3.3: Effect of asymmetrical detector position \((x_{1,2} = \frac{1}{2} \pm \Delta x_d + \Delta x_{1,2})\) on phase shift \(\Delta \Psi\) for varying mass flow \(\alpha v\), obtained by analytical approximation (3.7) (symbol marker) and numerical solution (line marker). (a) From top to bottom: \(x_1 = 1/4 + \Delta x_1\) with \(\Delta x_1 = \{-0.1, -0.05, 0\}\) and \(x_2 = 3/4\). (b) From top to bottom: \(x_1 = 1/4 + \Delta x_1\) and \(x_2 = 3/4 + \Delta x_2\) with \(\Delta x_1 = \{-0.025, 0, 0.025\}\) and \(\Delta x_2 = \{0.025, 0, -0.025\}\). Employed parameters: \(x_p = \frac{1}{2}\). Other parameters as for Fig. 3.2.
Chapter 3  Effect of asymmetric external excitation and pipe motion detection on Coriolis flowmeters
Chapter 4
Effect of non-ideal fluid velocity profiles on Coriolis flowmeters

This chapter is related to the work in [P5]-[P7]. The main results of purely numerical models and simulations will be summarised. This yields hypotheses for how non-ideal fluid velocity profiles could influence CFM accuracy and precision.

4.1 Computational model

Figure 4.1(a) shows a single, straight, resonantly vibrating, fluid-conveying pipe with clamped-clamped boundary conditions. A similar pipe, though with bended segments, is shown in Fig. 4.1(b). Specifications of the considered geometries, material data and fluid properties can be found in App. B. These two pipe configurations are investigated using two numerical models: One of the pipe, solved with ANSYS’
finite element solver, and another one for the fluid inside of the pipe, solved by CFX’s finite volume solver. Two-way fluid-structure interaction (FSI) is utilised, which can readily be set up in the two commercial programmes ANSYS and CFX under the framework of ANSYS WORKBENCH. The thesis paper [P6] presents details regarding the coupled numerical simulations.

When designing CFMs one typically assumes plug-flow, i.e. a constant fluid velocity across any cross-section of the pipe perpendicular to the pipe axis and no boundary layer adjacent to the inner pipe wall. But “real” fluid flow through a CFM installed in a piping system is either fully developed or disturbed. The first type of flow is illustrated in Fig. 4.2(a) and can be described by the power-law equation [59]. Velocity profiles in a disturbed flow are exemplified in Figs. 4.2(b)-(c) and would in practical applications, like the one shown in Fig. 1.2, be caused by bended pipe components installed upstream of the flowmeter. To arrive at Figs. 4.2(b)-(c) numerical studies of bended pipes have been conducted prior to the FSI simulations. The velocity profiles shown in Figs. 4.2(a)-(c) have been employed in the FSI simulations for peak fluid velocities up to 10 m/s. The conducted numerical simulations allow a straightforward investigation of these “realistic” flows, which has not yet been achieved by analytical methods.

4.2 Results

The numerically investigated pipes vibrate - like CFM pipes - in a resonantly driven symmetric mode, with a small overlay of vibrations of an antisymmetric mode excited by fluid flow with varying velocity profiles. The combined pipe motion is a travelling wave leading to time shifts $\Delta t_0$ in zero-crossing between different axis points. The time-harmonic nodal displacement data of two symmetrically located nodes is employed to determined the time shift $\Delta t_0$. The considered nodes are located in the antinodes of the pipe’s first antisymmetric mode; this corresponds to a location typically used in commercial CFMs designs. Equation (2.16) is used to relate the calculated time shift $\Delta t_0$ to the corresponding axial shift in vibration phase $\Delta \Psi$. The results are in the following given in terms of phase shift $\Delta \Psi$ as a

![Fig. 4.2: Examples of fluid velocity profiles applied as inlet conditions of the fluid domain.](image-url)

(a) Fully developed profile with maximum velocity $v_{max} = 4.89$ m/s, (b) disturbed velocity profile with $v_{max} = 5.75$ m/s, (c) disturbed velocity profile with $v_{max} = 9.93$ m/s.
4.2 Results

function of fluid mass flow $\dot{m}$. The mass flow $\dot{m}$ is calculated using [59]:

$$\dot{m} = \rho_f A_f v_{max},$$  \hspace{1cm} (4.1)

with $\rho_f$ being the fluid density, $A_f$ the cross-sectional area of the fluid domain, and $v_{max}$ the peak fluid velocity encountered at the inlet of the fluid domain.

4.2.1 Straight pipe configuration

Figure 4.3 shows the phase shift $\Delta \Psi$ as function of mass flow $\dot{m}$, obtained from numerical investigations of a straight pipe conveying fluid flows with either fully-developed (□) or disturbed (○) velocity profiles applied as inlet boundary condition.

It readily appears, that fluid flow with disturbed velocity profiles induces smaller phase shifts than fluid flow with fully-developed profiles. The relationship between phase shift and mass flow seems for both cases to be linear, but the factor of proportionality (hereafter denoted sensitivity) is not the same throughout the considered mass flow range. The different slopes of the trendlines for the two cases indicate that the sensitivity depends on the velocity profile of the fluid flowing through the pipe. The effect appears to be most significant for mass flows smaller than 1 kg/s. The work presented in [45, 60] predicts that sensitivity changes might occur when the flow switches from turbulent to laminar. The results depicted in Fig. 4.3 are due to turbulent flow with Reynolds numbers $Re = 1.4 \times 10^4 \ldots 14 \times 10^4$. In addition to the previous findings, this implies that sensitivity changes might also occur while

![Fig. 4.3: Phase shift $\Delta \Psi$ as function of mass flow $\dot{m}$. Flow through straight pipe with fully-developed (□) and disturbed (○) velocity profiles applied at the inlet of the fluid domain. Data from numerical simulations (symbol markers) connected by trendlines.](image-url)
the flow is turbulent and not just during the transition from laminar to turbulent flow.

The results shown in Fig. 4.3 yield the hypothesis that CFM sensitivity could be fluid velocity profile dependent. This may influence the accuracy and precision of CFMs, since the flow conditions under which the meter is calibrated to determine the sensitivity will typically differ from the conditions encountered in service. In practical applications this can yield erroneous flowmeter readings.

4.2.2 Bended pipe configuration

Figure 4.4 shows the phase shift $\Delta \Psi$ as function of mass flow $\dot{m}$, obtained from the numerical investigation of a bended pipe conveying fluid flows with either fully-developed (□) or disturbed (○) velocity profiles. Due to the bended segments of the investigated pipe geometry, disturbances will be induced, while the fluid is flowing through the pipe. This means, that the fluid flow will not remain fully-developed, even though a fully-developed flow, like the one shown in Fig. 4.2(a), is applied at the inlet of the fluid domain. The investigation of bended pipes is of particular relevance for CFM manufacturers, since bended pipes configurations are frequently employed in CFM designs.

It appears that there is a linear relationship between phase shift and mass flow for some parts of the measurement range. As for the straight pipe dealt with in the previous section, it does not seem that the sensitivity is the same throughout the entire mass flow range. For the considered bended pipe there is no clear trend on

![Graph showing phase shift as function of mass flow for bended pipe configuration](image-url)

**Fig. 4.4:** Phase shift $\Delta \Psi$ as function of mass flow $\dot{m}$. Flow through bended pipe with fully-developed (□) and disturbed (○) velocity profiles applied at the inlet of the fluid domain. Data from numerical simulations (symbol markers) connected by trendlines.
how disturbed velocity profiles applied at the inlet of the fluid domain influence the phase shift. For mass flow of 0.8 kg/s and smaller, the phase shift was apparently reduced, whereas it was increased for mass flows larger than 1 kg/s. The results imply that varying fluid velocity profiles for similar mass flow induce different phase shifts. To the author's knowledge there are no other studies available where the effect of velocity profiles on bended pipe configurations is investigated.

This leads to the following hypothesis: The sensitivity of CFMs utilising vibrations of bended pipes could be fluid velocity profile dependent. For practical applications employing these CFMs to measure mass flow, this could yield erroneous flowmeter readings, since the CFM accuracy and precision could be affected by the imperfect fluid velocity profiles.

4.3 Conclusion

Numerical simulations using two-way FSI have been employed to study the effect of non-ideal fluid velocity profiles. This resulted in the hypothesis, that CFM accuracy and precision could be influenced by velocity profile effects, since the flowmeter sensitivity appears to depend on the fluid velocity profile.

The presented results are limited to the particular parameters and conditions investigated. Since the magnitude of velocity profile effects on commercial CFM might depend on constructional and operational parameters, additional numerical investigations are necessary to test if other configurations yield the same hypothesis. The established numerical models can readily be extended to test other geometries and conditions than the ones presented in this work. However, there is a risk that the computational times may prohibit the investigation of parameter dependencies in any depth, since the conducted simulations are highly resource-demanding in terms of the analyst’s time and computational resources. This should in particular be remembered when trying to incorporate two-way FSI analyses of more complex geometries, e.g. real CFM designs, in industrial product development processes.

The literature [7, 18, 20] provides descriptions of test rigs employed to investigate the dynamic behaviour of real CFMs. These examples could inspire an experimental setup for testing the effect of velocity profiles on commercial CFMs, with the purpose of validating the above stated hypothesis, since it is so far only based on numerical simulations.

Experimental observations, made during the calibration of commercial CFMs with bended pipes, imply that also real configurations experience a decrease in flowmeter sensitivity for small mass flows [61]. This observation is however limited to one particular flowmeter design and the source for this particular decrease appears not to be known [61]. Since there appears to be correlation between the computationally based hypotheses and experimental observations, a systematic experimental investigation is recommended to be conducted.

In future investigations of velocity profile effects, one should also consider to employ analytical pipe models and an approximate analysis, like in Chap. 2 and 3, [P1],
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[P3], [P4] and [29], since this type of model can provide a more direct insight into the parameters at play and help to increase the benefit of numerical simulations.

It is shown in [P7], that the weight vector theory for CFMs holds a significant potential for predicting effects of velocity profiles on CFMs. But it is also shown that this theory is vulnerable, since it has not yet been shown that it can be used to investigate realistic pipe designs and boundary conditions. Numerical models and simulations employing two-way FSI as described in [P6] can help to overcome these problems, since they allow a straightforward investigation of the dynamic behaviour of vibrating fluid-conveying pipes.
Chapter 5
Effect of structural non-uniformities on Coriolis flowmeters

This chapter is a summary of thesis paper [P8]. The previous chapters have shown how analytical and numerical methods can be employed to investigate effects influencing the dynamic behaviour of vibrating fluid-conveying pipes. This chapter deals with an experimental investigation of phase shift effects due to pipe imperfections. The analytical work, which has motivated the experimental investigations, will be summarised, and the experimental setup and procedures will be described. The experimental results will be presented and discussed.

5.1 Analytical prediction

Figure 5.1 shows a single, straight, vibrating, fluid-conveying, simple supported and imperfect pipe. Imperfections considered in the analytical model in [51] and [52] include: Non-uniform damping, mass and stiffness distributions, uniform but not proportional damping, weak non-linearities, as well as two types of boundary conditions, i.e. simple supported and clamped-clamped, the later not being depicted in Fig. 5.1. Transverse motions of the pipe are described by the equation of motion [51, 52]:

\[
\ddot{u} + u'''' + \varepsilon \left( \alpha_p(x)\ddot{u} + [EI(x)u'']'' + \alpha_f[v^2u'' + 2v\dot{u}'] - \mu^2 \left[ \eta + \frac{1}{2} \int_0^1 (u')^2 dx \right] u'' \\
+ \gamma u^2 + L_k[\ddot{u}] + L_c[\dot{u}] + \beta f(\dot{u}) \right) = \varepsilon p \delta(x - x_p) \cos(\Omega pt). \tag{5.1}
\]

In (5.1) \( \alpha_p \) is the variation in distributed pipe mass relative to total pipe mass, \( \alpha_f \) the ratio of fluid mass to total pipe mass, \( EI(x) \) the axially varying flexural
Chapter 5 Effect of structural non-uniformities on Coriolis flowmeters

stiffness, \( \mu \) the pipe slenderness ratio, \( \eta \) a measure of the axial pipe deformation, \( \gamma \) the coefficient of asymmetric stiffness wrt. \( u = 0 \), \( L_k = k_u(x) + \frac{d}{dx}[k_\theta(x) \frac{d}{dx}] \) and \( L_c = c_u(x) + \frac{d}{dx}[c_\theta(x) \frac{d}{dx}] \) linear spatial differential operators describing, respectively, arbitrarily distributed damping and stiffness, \( k_u, \theta \) and \( c_u, \theta \) functions describing the axial distribution of, respectively, stiffness (additional to that of the pipe itself) and viscous damping per unit length, with subscript \( u/\theta \) indicating transverse / rotational distributions that may be non-uniform and discontinuous, \( \beta \) the coefficient of generalised damping, and \( f(\dot{u}) \) a velocity depended (possible non-linear) generalised damping function.

Equation (5.1) is solved approximately following the technique presented in [29], yielding an analytical prediction for the phase shift \( \Delta \Psi \) between transversely vibrating pipe points. Details regarding this can be found in [51, 52]. The difference in phase \( \Delta \Psi \) for an imperfect pipe between the transverse motion of two pipe points \( x_1, x_2 \) that are positions symmetrically around the middle of the flowmeter pipe, i.e. \( x_1, x_2 = \frac{1}{2} \pm \Delta x, \Delta x \in [0; \frac{1}{2}] \), is [51, 52]:

\[
\Delta \Psi = \frac{2\varepsilon \varphi \omega \varphi}{\varphi((\varphi^2 - \varphi^2)} \left( 2\alpha f v \int_0^1 \varphi \varphi' dx + \int_0^1 \varphi L_c \varphi dx \right),
\]

(5.2)

with \( \varphi \) and \( \omega \), \( j = 1, 2 \) being, respectively, the mode shapes and natural frequencies for the fluid filled pipe with either of two sets of boundary conditions (simple supported or clamped).

Equation (5.2) leads to the following hypotheses for CFMs, that have been tested experimentally:

**H1** Non-uniformly distributed damping may cause phase shifts, which are inseparable from the phase shift caused by mass flow \( \alpha f v \), i.e. if the integral with \( L_c \) does not vanish. The integral vanishes for symmetrical damping distributions, but not for asymmetric distributions, which can cause phase shifts in proportion to their asymmetry. The effect of asymmetrical damping is predicted to be of the same order of magnitude as the effect of mass flow and be present even at zero fluid flow.

**H2** Imperfections expressed by generally non-uniform perturbations of the pipes mass do not affect the measured phase shift, at least to the order of approximation employed. For practical applications a possible effect could be overshadowed by the effects of other imperfections.

**H3** Temperature may change pipe natural frequencies (by a change in fluid density) as well a damping properties, and thus affect phase shift.

Equation (5.2) predicts also, that:

**H4** The linear meter sensitivity, i.e. the factor of proportionality between phase shift \( \Delta \Psi \) and mass flow \( \alpha f v \) employed by CFM manufacturers, increases with a smaller difference between \( \omega_2 \) and \( \omega_1 \).
5.2 Experimental setup

H5 Mass flow $\alpha_f v$ will induce phase shifts, increasing proportionally with mass flow.

H6 Uniformly distributed generalized damping is predicted not to have an effect on the measured phase shift to the order of accuracy used in the approximation.

H7 Imperfections due to generally non-uniform perturbations of the pipes stiffness should not affect the measured phase shift, i.e. to the order of approximation employed.

Hypotheses H4-H7 have not been tested by the experiments conducted at DTU. Data provided by SFI could however be used to validate H5. H4 is a well-known result, cf. [3].

5.2 Experimental setup

Laboratory measurements have been conducted employing a commercial CFM with two U-shaped pipes. Damping ratios for modes of interest were estimated using B&K PULSE modal analysis in combination with standard impulse hammer testing and averaged frequency response functions. The build-in CFM software and electronics were used for metering the phase shift measured by the CFM. The experimental setup for investigating H1 and H2 is depicted in Fig. 5.2. Two setups have been employed to investigate the effect of damping: Neodymium magnets and a copper plate (Fig. (a)) for contactless asymmetrical energy dissipation exploiting the principle of

![Diagram](image)

Fig. 5.2: Experimental setup for determining resonance frequencies, damping ratios and phase shift for an industrial Coriolis flowmeter in case of (a) asymmetrical damping, (b) symmetrical damping and (c) external added masses.
Chapter 5 Effect of structural non-uniformities on Coriolis flowmeters

eddy current damping, and damping gel tape inducing both symmetrical and asymmetrical damping (Fig. (b)). The effect of added mass was investigated by gluing small masses to the flowmeter pipe using bee-wax (Fig. (c)). To investigate H3, a heat gun was used to heat up the flowmeter pipes, see [P8] for schematic of the test setup, and neither gel tape nor magnets and the copper plate were influencing or attached to the flowmeters pipe during these experiments. The temperature of the room in which the experiments were conducted was 20-21°C. The background noise on data logged by the CFM is measured to be three orders of magnitude smaller than the phase shifts due to mass flow, see App. C for details.

To evaluate whether the phase shift induced by a given imperfection is significant, the phase shifts due to mass flow should be known. Since the employed experimental setup does not allow actual mass flow through the CFM, experimental data provided by Siemens A/S is used to determine the phase shifts induced by mass flow. The results are depicted in Fig. 5.3. A linear relationship between mass flow and the corresponding phase shift is identified, supporting H5. Two key-performance indicators are taken from Fig. 5.3: The phase shift $\Delta \Psi_{max} = 1.1 \times 10^{-3}$ rad caused by full rate mass flow, and the zero-shift requirement $\Delta \Psi_0 = 4.7 \times 10^{-7}$ rad, which indicates how much the metered phase shift is allowed to fluctuate in case of zero mass flow.

5.3 Distributed damping (H1)

Figure 5.4 shows the measured phase shift $\Delta \Psi$ as function of the flowmeter pipes’ damping ratio $\zeta$, with damping being asymmetrically, external and contact free applied using neodymium magnets, and the damping ratios $\zeta$ being measured for the drive mode. Employing up to eight magnets and a varying distance to the flowmeter’s pipe enables the investigation of the damping range with a high resolution. The experimental results presented in Fig. 5.4 support H1. One can identify a linear relationship between applied damping and induced phase shift. It appears, that

![Fig. 5.3: Phase shift as function of mass flow based on experimental data.](image-url)
asymmetrically applied damping has increased the pipe damping from the base value $\zeta = 0.064\%$ up to $\zeta = 0.164\%$, and induced phase shifts up to $\Delta \Psi = 1.02 \times 10^{-3}$ rad. The phase shift due to the maximum level of applied asymmetric damping is about $1/10$ of the full rate phase shift $\Delta \Psi_{\text{max}}$. In a practical application the phase shifts due to asymmetric damping would be inseparable from those caused by mass flow and might be mistaken as a change in mass flow, unless either the mass flow or damping asymmetry is known, or known to be constant in time. Even phase shifts induced by small asymmetric damping, e.g. $\zeta = 0.07\%$, are much bigger than the zero-shift requirement $\Delta \Psi_0$ of the tested flowmeter. A present zero-shift could be removed by a proper initial meter calibration. However, damping could change subsequent to the meter calibration, e.g. due to wear, air bubbles in the fluid, lubrication, or temperature changes. A fluctuating damping distribution could therefore be a factor contributing to the lack of zero-shift stability occasionally observed with industrial CFM.

Figure 5.5 shows the effect of asymmetrical damping on the measured phase shift $\Delta \Psi$ when using gel tape instead of neodymium magnets. The gel tape was initially intended to absorb surface strains near the nodes of the flowmeters Coriolis mode. However, since the largest effect of gel tape on pipe damping was observed when placing it at the Coriolis mode antinodes, i.e. close to the sensor coils (also denoted detectors), the gel tape’s primary effect was to increase air damping. Phase shifts, caused by asymmetric damping induced by attached damping tape, are of the same order of magnitude as those due to mass flow, and they increase (roughly) proportional to the asymmetrically induced damping. Also these results support H1. It appears, that gel tape has increased pipe damping 2-3 times more than the neodymium magnets yielding higher phase shifts (cf. Figs. 5.4 and 5.5). The magnets did however allow for a higher resolution of the damping range. Negative phase shifts were induced when applying the damping close to sensor coil 2 (Fig.
Chapter 5  Effect of structural non-uniformities on Coriolis flowmeters

5.5(b)); this change in the sign of the phase shift is predicted by (5.2), as shown in the appendix to [P8]. If a part of the pipe damping changes slowly in time, this may increase as well as decrease the measured phase shift and thus meter readout, depending on where the damping changes along the flowmeter pipes.

Figure 5.5 illustrates the measured phase shift $\Delta \Psi$ as a function of the pipe damping ratio $\zeta$, when the pipe damping is increased by applying tape as symmetrical as possible wrt. the axial coordinates. The damping tape was attached close to the driver coil (also denoted actuator). The mean phase shift $\bar{\Delta \Psi} = -9.43 \times 10^{-5}$ rad and standard deviation $\sigma(\Delta \Psi) = 5.24 \times 10^{-4}$ rad of the measured phase shifts are

![Figure 5.5](image)

**Fig. 5.5:** Phase shift $\Delta \Psi$ as function of pipe’s damping ratio $\zeta$. Damping applied externally and asymmetrically using damping gel tape. Tape attached either close to sensor coil 1 (a) or 2 (b).

Figure 5.6 illustrates the measured phase shift $\Delta \Psi$ as a function of the pipe damping ratio $\zeta$, when the pipe damping is increased by applying tape as symmetrical as possible wrt. the axial coordinates. The damping tape was attached close to the driver coil.

![Figure 5.6](image)

**Fig. 5.6:** Phase shift $\Delta \Psi$ (symbol markers) as function of pipe damping ratio $\zeta$, mean phase shift $\bar{\Delta \Psi}$ (dashed line), and standard deviation $\sigma(\Delta \Psi)$ (dotted line). Damping applied as symmetric as possible by damping gel tape close to driver coil.
shown by, respectively, the dashed and dotted lines. The observed phase shifts are approximately one order of magnitude smaller than the phase shifts due to actual mass flow (cf. Fig. 5.3), supporting H1, which predicts that phase shifts due to symmetrical damping should give rise to phase shifts being at least one order of magnitude less than the primary effect due to mass flow. Symmetrically increasing damping appears also not to give rise to linearly increasing phase shifts, as seen by comparing Fig. 5.4, 5.5 and 5.6. Asymmetrically applied damping did induce larger phase shift changes than symmetrically applied damping, even though symmetrical damping lead to much higher damping ratios.

5.4 Non-uniform pipe mass perturbations (H2)

Figure 5.7 shows the measured phase shift $\Delta \Psi$ as a function of added mass. The results depicted in Figs. (a)-(e) were obtained by adding up to 9 grams (4% of total pipe mass) in an asymmetrical way, whereas up to 12 grams (5% of total pipe mass) have been attached as symmetrical as possible yielding Fig. (f). Any amount of small added mass, applied either symmetrically or asymmetrically, appears to induce phase shifts, which increase with increasing added mass. The induced phase shifts are however at least two orders of magnitude smaller than those due to mass flow. This supports H2, which states that the effect of added mass should be smaller than that of mass flow. The induced phase shift changes are small, and a check

![Graph showing measured phase shift $\Delta \Psi$ as a function of added mass, asymmetrically (a)-(e) and symmetrically (f). Mass applied: (a) On either side of upper pipe parts only (cf. Fig. 5.2(c)), (b) at either upper or lower parts of both pipes, (c) at both upper and one lower pipe part, (d) diagonally opposite on upper and lower pipe part, (e) on upper and lower part of one pipe, (f) at both upper and lower pipe parts.](image-url)
of the influence of measurement variability (see App. C) supports, that they are indeed caused by the effect of adding mass. For practical applications metering mass flow, the experimental results imply, that a possible effect of added mass would be overshadowed by other effects. It should be noticed, that the phase shifts induced by added mass are larger than the CFM’s zero-shift requirement $\Delta \Psi_0$. This indicates that small amounts of added mass could explain a possible drift of the zero-shift during the CFM’s lifetime caused by, e.g., corrosion, cavitation, or particles attached to the inner CFM pipe walls of an improperly cleaned CFM.

5.5 Temperature (H3)

Figure 5.8(a) shows the phase shifts caused by heat impact from air with temperature of 58°C measured at the heat gun outlet by a thermometer. The grey lines show the data logged by the CFM, and the black lines the running average; in the following we consider only the latter, low pass filtered data. From $t_{s,h} = 290$ s the heat guns was turned on, and turned off again at $t_e = 860$ s. After the heat gun is turned on, the phase shift is seen to increase and fluctuates around a higher value, approximately $1.2 \times 10^{-5}$ rad, corresponding to 0.1% of the full rate phase shift $\Delta \Psi_{\text{max}}$ and hereby supporting H3. The phase shift returns to its original value after the heat gun was turned off, though after an unexplained period of 210 s.

To arrive at Fig. 5.8 (b), the heat gun was exchanged to investigate lower temperature variations. The heat gun, emitting air at temperature 29°C, was turned on at

![Figure 5.8](image-url)

**Fig. 5.8:** Phase shift $\Delta \Psi$ measured by CFM exposed to temperature variations of ambient environment. Gray line: Data measured by CFM. Black line: Running average CFM data (Windowsize: 50 samples). Events: $t_{s,h}$ heat gun started emitting hot air, $t_{s,c}$ heat gun started emitting cold air, $t_e$ heat gun turned off.
5.6 Overshadowing pipe imperfection effects

The temperature was reduced at \( t_{s,c} = 540 \text{s} \) to 22\(^\circ\)C, and the heat gun turned off at \( t_e = 1020 \text{s} \). The measured phase shift increased to \( 7 \times 10^{-6} \text{ rad} \) when the heat gun was turned on, decreased to \(-7 \times 10^{-6} \text{ rad} \) when the temperature was lowered, both corresponding to 0.06\% of the full rate phase shift \( \Delta \Psi_{\text{max}} \) and supporting H3, and returned to the same magnitude as before the heat impact, once the heat gun was turned off. The scattering, or high-frequency variations, of the logged data around the running average increased, when blowing with a heat gun on the flowmeter’s pipes. This is most noticeable in Fig. (b), and it reflects probably noise and air flow induced pipe vibrations. For each of the two cases the measured phase shift returned to its original value, when turning off the heat gun, indicating that the changes in phase shift in fact where caused by a change in temperature. The phase shifts caused by sudden temperature changes are larger than the zero-shift stability requirement \( \Delta \Psi_0 \). Temperature changes, occurring during the flowmeter’s application, could therefore explain a drifting zero-shift. One can here only speculate on the actual link between temperature and phase shift: The positioning of the heat gun causes the two sensor coils to experience different temperatures and amounts of air flow. The sensor coils, which can be seen as small dampers, i.e. pistons inside a cylinder, would then experience asymmetrical changes in the viscosity of air. Thus by blowing air on the flowmeter’s pipes one could have induced asymmetrically distributed damping, resulting in phase shift changes as predicted by H1, which is now supported by experimental results (cf. Sec. 5.3).

5.6 Overshadowing pipe imperfection effects

Figure 5.9(a) shows the measured phase shift \( \Delta \Psi \) as a function of asymmetrically added mass, when adding up to 20 grams to one flowmeter pipe, corresponding to approximately 9\% of the pipes total mass. The analytical prediction (5.2) is valid for small changes in pipe mass, and the measured phase shift appears to be...
little affected by adding small amounts of mass up to 3 grams. Adding more mass increases the measured phase shift significantly, in fact up to the same order of magnitude as those due to mass flow. In real CFM applications, mass changes of that order of magnitude are less likely to be seen, but if they occur, they could be mistaken for a change in mass flow. The pipe damping, measured each time a weight has been added, is looked at to answer the question whether the weights (4-20 g) due to their surface size (see Fig. 5.10) could have acted as air dampers and not just as added mass. From Fig. 5.9(b), which shows measured phase shift $\Delta \Psi$ as a function of pipe damping ratio $\zeta$, it can be seen that adding mass has also increased the pipes damping, in fact up to a factor of five when adding 20 grams to one flowmeter pipe. This asymmetrically induced increase in pipe damping could explain the significant phase shifts induced by large added mass. The effect of added mass, which is predicted to be small as long as the mass is small, could in this case be overshadowed by the effect of asymmetrical damping, offering a possible explanation for the order of magnitude of the phase shifts shown in Fig. 5.9(a).

5.7 Conclusion

It has been demonstrated how experimental methods can be used to test theoretically based hypotheses employing a commercial CFM. Experimental observations support the hypothesis regarding the effect of pipe damping, added pipe mass, and ambient temperature changes on measured phase shift. Asymmetrically applied pipe damping yields phase shifts that could be mistaken for changes in mass flow. Symmetrically applied damping, non-uniform perturbations of pipe mass and temperature changes are seen to have a negligible effect on mass flow measurement.

Some commercial CFMs have occasionally shown a drifting zero shift. Based on the order of magnitude of the experimentally observed phase shifts caused by imperfections related to pipe damping, added mass and temperature changes, these could be imperfections contributing to time-varying phase shifts, even in case of zero mass flow, if the imperfections themselves vary in time.

The experimental tests have shown that simplified mathematical models and approximate analysis are useful in providing direct insight into which imperfections affect phase shift, even for complex systems such as CFMs, and in which manner. It has been shown that some pipe imperfections could lead to incorrect mass flow measurements. Other sources mentioned in Sec. 5.1 for which hypotheses exist, i.e. H6 (uniformly distributed damping) and H7 (non-uniform perturbations of pipe stiffness), are still to be investigated experimentally. After modifications, the experimental setup described in Sec. 5.2 could also be used to test the hypotheses presented in [29]; being that any asymmetric fluctuation in rotational support damping could be mistaken for a change in mass flow, whereas finite rotational stiffness at simple supports or small transverse damping at flexible supports induce phase shifts that are ignorable for typical CFM applications.
Chapter 6
Conclusion

The work presented in this PhD thesis is a contribution to the research field of elastic wave propagation. By investigating the dynamic behaviour of fluid-conveying pipes new knowledge has been gained on how axial shifts in vibration phase are affected by fluid flow and various imperfections. The imperfections investigated using analytical, numerical and experimental methods are: Pulsating fluids, imperfectly mounted actuators, asymmetrically located detectors, non-ideal fluid velocity profiles, pipe imperfections and ambient temperature changes. The acquired knowledge increases the understanding of how these imperfections could influence the accuracy, precision and robustness of measuring devices exploiting phase shifts as means of measuring, and it is therefore of particular interest for Coriolis flowmeter (CFM) manufacturers.

It is shown how a simplified mathematical model of a single, vibrating, fluid-conveying pipe can be used to study the effect of pulsating fluids, imperfectly mounted actuators and asymmetrically located detectors on phase shift. The obtained predictions for simple analytical models offer direct insight into the factors at play, and yield practical hypotheses for real CFMs: 1) CFM accuracy might be affected by imperfectly mounted actuators and fluid pulsations, 2) CFM precision could be influenced by non-ideally located actuators and detectors, and 3) CFM robustness may be affected by fluid pulsations, if unnoticed and uncontrolled pipe motions are induced. Representative examples have been tested against pure numerical solutions, and good agreement was demonstrated.

A numerical finite element and finite volume (FE/FV) model of a vibrating fluid-conveying pipe has been employed to investigate the effect of imperfect fluid velocity profiles. Based on simple pipe configurations consisting of single, vibrating fluid-conveying pipes with straight and/or bended pipe segments it is predicted that imperfect velocity profiles could affect CFM accuracy and precision. It is also concluded that detailed computational models of real CFMs, incorporating fluid-structure interaction between the pipe(s) and the conveyed fluid, can be readily set up in the commercial programmes ANSYS and CFX following the procedures described in this thesis.

The results of the conducted experiments with a commercial CFM support the theoretically based hypotheses regarding pipe imperfections, i.e. CFM accuracy and precision could be influenced by non-ideally distributed damping and pipe mass as well as by ambient temperature changes. The employed experimental set up allowed the study of the effect of various imperfections on phase shift measured by the CFM, even though there was no actual fluid flow through the meter. This is due to the fact that the investigated imperfections induce zero-shifts, i.e. phase shifts even at
zero fluid flow. For CFM manufactures it is of relevance to know, which imperfections could induce zero-shifts, since some commercial CFMs have occasionally metered zero-shifts. Based on the findings presented in this thesis, the following imperfections could contribute to the occurrence of zero-shifts: Pipe damping changes (either symmetrically or asymmetrically), non-uniformly added pipe mass, ambient temperature changes, and asymmetrically located detectors. The hypothesis regarding the latter is only based on analytical predictions and has not yet been tested experimentally.

The employed analytical, numerical and experiment methods whose results have been presented in this thesis should not stand alone, but be combined with each other to fully uncover their individual potentials:

The presented analytical models ignore substantial features of real CFMs, which typically have two curved pipes with mounted detectors and actuators and clamped rather than simple supports. Based on the assumption that the involved effects are basically similar for more complicated systems, the established simplified mathematical models for single straight pipes have lead to analytical results yielding general conclusions for real CFMs, since they offer direct insight into the effects at play. Experiments with a commercial CFM validated theoretically based hypotheses; hereby supporting that even a simple mathematical model can aid understanding and predict the behaviour of real systems with more complicated geometries. Further experimental testing of the analytically based predictions regarding pulsating flow and non-ideal actuator and detector positions is recommended, since simple analytical predictions - from a researcher’s point of view - appear to be more valuable for real applications than resource-demanding detailed numerical FE/FV models. The work presented in [7, 18, 20, 62] offers suggestions for how a test rig could be set up for testing CFM susceptibility to pulsating fluids. The effect of non-ideal actuator and detector positions could readily be tested with purpose-build imperfect CFMs. An experimental setup like the one described in this thesis could also be used to test another theoretically based hypothesis reported in [29], i.e. that any asymmetry in rotational support damping does cause phase shifts even in case of zero fluid flow.

The conducted simulations employing a numerical FE/FV model are highly resource-demanding in terms of analyst’s time and computational resources, and they are limited to the particular parameters and conditions simulated. Nevertheless, they have the capability of being further employed in a product developing environment for the investigation of different flowmeter designs and their sensitivity to different fluid flow related effects, such as compressible flow, two-phase flow, highly viscous fluids, installation effects or pulsating flow. Despite the best efforts of software developers, commercial or non-commercial software might contain errors. Experimental testing of the hypotheses presented in this thesis regarding velocity profile effects, and also of the results of possible future simulations investigating other effects, is therefore highly recommended.
CFMs employ vibrating fluid-conveying pipes to measure mass flow. The findings presented in this PhD thesis increase the knowledge and understanding of the dynamic behaviour of these pipes, so that one can now predict if and how certain factors, i.e. manufacturing variations, non-ideal fluid flow and structural changes occurring during the meters lifetime, could influence CFM accuracy, precision and robustness. The obtained knowledge helps, e.g., to understand why some commercial CFMs occasionally measure phase shifts even though there is no actual mass flow. The knowledge is gained from newly established analytical and computational models, which are solved approximately and numerically, as well as laboratory experiments with a commercial CFM. The established methods of analysis allow a systematical investigation of the involved effects, and can readily be extended to investigate factors whose influence is not yet fully understood.
References


REFERENCES


Appendix A
Numerical testing of analytically predicted pipe response

Analytical predictions for the approximated pipe response have been tested against results by pure numerical analysis. Details of the numerical solution of the equation of motion (2.2) can be found in [P3]. Representative results will be shown. Other fluid velocities $v_0$ and fluid pulsation amplitudes $q$, than the ones shown in the figure captions, give similar results wrt. the accuracy of the tested analytical predictions.

A.1 Slow fluid pulsations $\omega_f << \omega_{01}$

Figure A.1 depicts the lateral pipe displacement $u(x, t)$ in case of a slowly pulsating fluid ($\omega_f = 0.0001 << \omega_{01}$) determined using numerical simulation (lines) and the analytical approximation (2.5) (symbol markers) at points located in the antinodes of the antisymmetric mode $\varphi_{02}$, i.e. $x_1 = \frac{1}{4}$ (Fig. (a)) and $x_2 = \frac{3}{4}$ (Fig. (b)), representing common detector locations employed in CFMs. The results are calculated using the parameters given in the figure legend. The analytical approximation agrees rather accurately with the numerical simulation implying that the simple analytical expression (2.5) gives about the same accuracy as the numerical solution.

![Figure A.1](image_url)

**Fig. A.1:** Displacement of two symmetrically located points along a fluid-conveying simply supported pipe determined using numerical simulation (lines) and a perturbation method (symbol markers) at pipe axis points (a) $x_1 = \frac{1}{4}$ and (b) $x_2 = \frac{3}{4}$. Parameters: $v_0 = 0.3$, $q = 0.003$, $\omega_f = 0.0001 << \omega_{01}$, $\alpha = 0.3$, $p_a = 0.001$, $x_p = \frac{1}{2}$, $c = 0.01$, and $\Delta x_p = \frac{1}{2}$.
A.2 Non-slow fluid pulsations

A.2.1 Principle parametric resonance $\omega_f \approx 2\omega_0$

Figure A.2 shows the lateral pipe displacement $u(x, t)$ in case of a non-slowly pulsating fluid with $\omega_f = 19.7106 \approx 2\omega_0$ determined using numerical simulation (lines) and the analytical approximation (2.7) (symbol markers) at $x_1 = \frac{1}{4}$ (Fig. (a)) and $x_2 = \frac{3}{4}$ (Fig. (b)). The results show good agreement. Figure A.3 exemplifies the

Fig. A.2: Displacement of two symmetrically located points along a fluid-conveying simply supported pipe determined using numerical simulation (lines) and a perturbation method (symbol markers) at pipe axis points (a) $x_1 = \frac{1}{4}$ and (b) $x_2 = \frac{3}{4}$. Parameters: $\omega_f = 19.7106 \approx 2\omega_0$, other parameters as for Fig. A.1.

Fig. A.3: Displacement of two symmetrically located points along a fluid-conveying simply supported pipe determined using numerical simulation (lines) and a perturbation method (symbol markers) at pipe axis points (a) $x_1 = \frac{1}{4}$ and (b) $x_2 = \frac{3}{4}$. Parameters: $v_0 = 0.9, q = 0.001, \omega_f = 19.4807 \approx 2\omega_0$, other parameters as for Fig. A.1.
A.2 Non-slow fluid pulsations

Accuracy of the analytical approximation (2.7) when the assumption of the smallness of the mean fluid velocity, i.e. \( v_0 = \epsilon v_0 \), is not fulfilled. To arrive at Fig. A.3 the mean fluid velocity has been increased to \( v_0 = 0.9 \) with parameters as given in the figure caption. Small discrepancies between the analytical and numerical solution are seen, being an expression for the violation of the underlying assumptions of (2.7). Similar observations are made for the other considered cases of fluid pulsations when the mean fluid velocities are non-small.

A.2.2 Combination resonance \( \omega_f \approx \omega_{02} - \omega_{01} \)

Figure A.4 illustrates the lateral pipe displacement \( u(x, t) \) in case of a non-slowly pulsating fluid with \( \omega_f = 29.6116 \approx \omega_{02} - \omega_{01} \) determined using numerical simulation (lines) and the analytical approximation (2.8) (symbol markers) at \( x_1 = \frac{1}{4} \) (Fig. (a)) and \( x_2 = \frac{3}{4} \) (Fig. (b)). Good agreement between the analytical and numerical results is seen.

![Fig. A.4: Displacement of two symmetrically located points along a fluid-conveying simply supported pipe determined using numerical simulation (lines) and a perturbation method (symbol markers) at pipe axis points (a) \( x_1 = \frac{1}{4} \) and (b) \( x_2 = \frac{3}{4} \). Parameters: \( v_0 = 0.4, q = 0.002, \omega_f = 29.6116 \approx \omega_{02} - \omega_{01} \), other parameters as for Fig. A.1.](image)

A.2.3 Non-resonance \( \omega_f \) away from \( 2\omega_{01} \) and \( \omega_{02} - \omega_{01} \)

Figure A.5 illustrates the lateral pipe displacement \( u(x, t) \) in case of a non-slowly pulsating fluid with \( \omega_f = 15 \) determined using numerical simulation (lines) and the analytical approximation (2.11) (symbol markers) at \( x_1 = \frac{1}{4} \) (Fig. (a)) and \( x_2 = \frac{3}{4} \) (Fig. (b)). Again one can see that there is good agreement between the analytical and numerical results.
Fig. A.5: Displacement of two symmetrically located points along a fluid-conveying simply supported pipe determined using numerical simulation (lines) and a perturbation method (symbol markers) at pipe axis points (a) $x_1 = \frac{1}{4}$ and (b) $x_2 = \frac{3}{4}$. Parameters: $v_0 = 0.5$, $q = 0.001$, $\omega_f = 15$, other parameters as for Fig. A.1.
Appendix B
Pipe configurations and material data for numerical simulations

Two configurations are investigated with purely numerical simulations: A straight and a bended pipe (cf. Fig. 4.1). Table B.1 specifies the investigated geometrical configurations. The investigated pipes are assumed to be made of steel, and they are assumed to convey the fluid water, see Tab. B.2 for utilised material data and fluid properties.

Table B.1: Pipe configurations

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<thead>
<tr>
<th>Straight pipe</th>
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<td>Inner pipe radius $r_i$</td>
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<tr>
<td></td>
<td>Length $L_2 = L_3 = L_4$</td>
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<td></td>
<td>Pipe radii $r_1 = r_2 = r_3 = r_4$</td>
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<td></td>
<td>Pipe angles $\theta_1 = \theta_2 = \theta_3 = \theta_4$</td>
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</table>

Table B.2: Material data

<table>
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<th>Fluid - water</th>
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<td>density $\rho_f$</td>
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<td>dynamic viscosity $\mu$</td>
</tr>
<tr>
<td>damping $c$</td>
<td>kinematic viscosity $\nu$</td>
</tr>
</tbody>
</table>
Appendix B  Pipe configurations and material data for numerical simulations
Appendix C
Measurement variability

This appendix contains confidential experimental data and is due to commercial considerations not available in the published version of the PhD thesis.
Predicting phase shift of elastic waves in pipes due to fluid flow and imperfections.

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PREDICTING PHASE SHIFT OF ELASTIC WAVES IN PIPES DUE TO FLUID FLOW AND IMPERFECTIONS

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Flexural vibrations of a fluid-conveying pipe is investigated, with special consideration to the spatial shift in phase caused by fluid flow and various imperfections, e.g., non-ideal supports, non-uniform stiffness or mass, non-proportional damping, weak nonlinearity, and flow pulsation. This is relevant for understanding wave propagation in elastic media in general, and for the design and trouble-shooting of phase-shift measuring devices such as Coriolis mass flowmeters in particular. A multiple time scaling perturbation analysis is employed for a simple model of a fluid-conveying pipe with imperfections. This leads to simple analytical expressions for the approximate prediction of phase shift, providing direct insight into which imperfections affect phase shift, and how. The analytical predictions are tested against results obtained by pure numerical analysis (Galerkin expansion), showing very good agreement.

1. Introduction

An important quantity characterizing elastic wave propagation is the phase shift in oscillation between any two points of the medium. We present a systematic perturbation approach for deriving analytical expressions that relate phase shift to parameters characterizing vibrating pipes conveying fluid flow and possible imperfections. This is relevant to gain further insight into which factors influence phase shift and how, and for applications such as the design and troubleshooting of Coriolis mass flowmeters, where the primary quantity measured is phase shift.

The vibrations of fluid-conveying pipes have been actively investigated for more than six decades\textsuperscript{1}, and some of this work is motivated by Coriolis flowmetering applications. Of the latter, many publications provide valuable analyses on how mass flow and also imperfections influence phase shift, see e.g. the rather recent overview by Anklin et al\textsuperscript{2}. Some works examine the effects of flexible supports on vibrations, but without consideration to fluid flow\textsuperscript{3}, or with consideration to fluid flow but not to phase shift\textsuperscript{4, 5}. Raszillier and Durst\textsuperscript{6} used a perturbation-like approach to derive analytical expressions for the phase shift for fluid-conveying pipes, but did not consider imperfect supports. Effects of imperfect supports on phase shift were studied recently\textsuperscript{7}, though only numerically. Several studies derive analytical or semi-analytical predictions for phase shifts in the context relevant here. Typically the analytical expressions are for the ideal case\textsuperscript{6}, or given in terms of motions of the pipe, or in terms of parameters requiring numerical solutions of eigenvalue problems\textsuperscript{8, 9}, or the results are not tested against numerical simulation. Kutin and Bajsic\textsuperscript{10} derive analytical phase shift expressions taking into account imperfections in the form of axial force and added mass, and show these to agree with numerical solutions. The approximations are made by linearizing the effects of imperfections near the “perfect” case. This approach is appealing, appears simpler.
than the one described in the present work, and may be used for other imperfections as well. However, it assumes the pipe response to be simply time-harmonic in the drive frequency, and thus cannot be generalized to incorporate, e.g., nonlinearity, flow pulsations, or external disturbances at frequencies other than the drive frequency. Also, it does not provide a systematic and generalizable way of handling imperfections, such as the standard perturbation approach used here.

With the present work we aim to present, exemplify, and test a systematic approach for the derivation of simple approximate analytical expression for phase shifts of pipe vibrations caused by fluid flow and various imperfections, using simplified models of real flowmeters.

Sect. 2 presents the mathematical model of a fluid-conveying pipe, with small imperfections related to boundary supports and pipe uniformity. Sect. 3 summarizes the results of a multiple scales perturbation analysis to predict phase shifts for this model, gives examples of application, and check the approximation accuracy. Sect. 4 presents some results obtained using the same analysis technique for some other types of imperfection: flow pulsation, nonlinearity, generalized damping, and asymmetrically distributed damping.

2. Mathematical model

The model system in Fig. 1 is a straight fluid-conveying pipe. Using beam theory and Newton’s second law, or Hamilton's principle, the equation for transverse pipe motions \( u(x,t) \) can be written:

\[
\ddot{u} + u'''' + \varepsilon \left[ \alpha \left( 2v \dot{u}' + v^2 u'' + \dot{v} u' \right) + L_k u + L_\beta \ddot{u} \right] = \varepsilon p \delta(x - x_p) \cos(\Omega t),
\]

\[
L_k = k_u(x) - \frac{d}{dx} \left( k_\rho(x) \frac{d}{dx} \right), \quad L_\beta = \beta_u(x) - \frac{d}{dx} \left( \beta_\rho(x) \frac{d}{dx} \right),
\]

\[
u + \varepsilon \pi^2 \kappa_a u''' = u'' = 0 \quad \text{for} \ x = 0, \quad u - \varepsilon \pi^2 \kappa_a u'''' = u' = 0 \quad \text{for} \ x = 1,
\]

where \( x \in [0;1] \) is the axial coordinate, \( L_k \) and \( L_\beta \) are linear spatial differential operators for describing arbitrarily distributed damping and (additional) stiffness, (3) describes imperfect boundary conditions at \( x \in \{0,1\} \), and all parameters, variables, and functions are nondimensional:

\[
x = \tilde{x}/l \in [0;1], \quad u = \tilde{u}/l, \quad t = \tilde{t}/l, \quad \tilde{v} = \tilde{v}/l, \quad \rho = \rho/l, \quad \alpha = \rho/l, \quad \kappa_a = 48\pi^2 E\Omega^2/l^3, \quad \beta = \beta/l, \quad \Omega = \tilde{\Omega}/l.
\]

Here a tilde denotes a corresponding physical variable or parameter, time \( t \) is nondimensionalized by a characteristic frequency \( \tilde{\omega} \), the coordinate \( x \) and deformation \( u \) by the pipe length \( l \), flow speed \( v \) by the characteristic propagation speed \( \tilde{\omega} \) of transverse elastic waves, \( \rho A \) is the mass per unit length (subscript \( p/f \) indicating pipe/fluid) and \( EI \) the flexural stiffness per unit length, \( \alpha \) the ratio of fluid mass to total mass, the functions \( k_u(x) \) and \( \beta_u(x) \) describe the axial distribution of stiffness and viscous damping per unit length (subscript \( u/f \) transverse/rotational), \( p \) is the amplitude of the harmonic force at \( x=x_p \) with normalized frequency \( \Omega \), \( \kappa_\alpha \) and \( \kappa_\beta \) are normalized transverse flexibilities of the supports at \( x=0, l \), \( \delta(x) \) is Dirac’s delta function, dots and primes denote differentiation wrt. \( t \) and \( x \), and \( \varepsilon \) is a bookkeeping parameter used for marking terms that are assumed to be small compared to other terms. Of the assumptions underlying (1)-(4), the most important are those of pipe slenderness, small rotations, and incompressible plug flow at a constant or slowly changing speed \( v = \eta(x,t) \) much lower than the characteristic speed of transverse elastic waves.

**Fig. 1.** Model of vibrating fluid-conveying pipe. (a) Ideal system; (b) with some imperfections: small distributed transverse and rotational stiffness and damping, and finitely large transverse support stiffness (i.e. small flexibility).
The first four terms in (1) represent, respectively, transverse inertia of the pipe and fluid, flexural stiffness of the pipe, and Coriolis and centrifugal forces on the pipe due to the fluid flow; Terms similar to these occur in many pipe-flow studies\(^1\)\(^6\), while the fifth term represents the effect of time-varying flow speed. The functions \(k_{a0}\) and \(\beta_{a0}\) can be used to include any kind of small non-uniformity in linear stiffness and dissipation, and even for representing support stiffness and damping (in terms of Dirac delta functions \(\delta(x-x_0)\), \(x_0 \to \{0,1\}\)). Small support flexibility, i.e. very high but not infinite support stiffness, is instead represented by the flexibility coefficients \(k_{ud}\) and \(\kappa_{ud}\) in the boundary conditions (3). We define an "ideal pipe" as having no damping \((\beta_{a0} = \beta_{ud} = 0)\), no inhomogeneity in transverse or flexural stiffness \((k_a = k_o = 0)\), and no transverse flexibility at either support end \((\kappa_{u0} = \kappa_{uo} = 0)\).

3. Analytical prediction of phase shift

3.1 Calculating the response at primary resonance

A solution \(u(x,t)\) to (1)-(3) is needed that holds approximately under the assumptions on small quantities stated above, and for the standard operating condition of a Coriolis flowmeter, i.e. resonant or near-resonant excitation \(\Omega = \omega_0\) of the fundamental symmetric mode \(\phi_{00}(x)\). Here \(\omega_0 = (\pi/\lambda)^2\) and \(\phi_{0j}(x) = \sqrt{2}\sin(j\pi x)\), \(j = 0, 1, \ldots\) are, respectively, the natural frequencies and mode shapes for the unperturbed \((\varepsilon = 0)\) ideal pipe. The solution \(u(x,t)\) can then be used for setting up analytical predictions for the difference \(\Delta \phi\) in vibration phase measured between two symmetric pipe points \(x_1 = \pm \Delta x\), \(\Delta \varepsilon \in [0; 1/4]\). Aiming at a generally applicable technique that can readily be adapted to investigate other kinds of imperfections for fluid-conveying pipes, we use method of multiple scales\(^ {12,13}\), which can readily handle, e.g., nonlinearity. Details of the calculations are given elsewhere\(^1\), while here space limitation allows only a summary:

After substituting a two-timescale expansion \(u(x,t) = u_0(x,T_1,T_2) + \varepsilon u_1(x,T_1,T_2) + O(\varepsilon^2)\) into the partial differential equation of motion (1)-(3) and separating out the equations for \(u_0\) and \(u_1\), we solve for the dominating function \(u_0\) in terms of a series expansion of fundamental mode shapes \(\phi_{00}(x)\). Then a general solution for the first-order correction \(u_1\) can be written in terms of the same mode shapes, but with additional small rigid body corrections, \(\phi_{0j} = \sqrt{2} j^3 (\kappa_{u0}(1-x) - (-1)^j \kappa_{uo} x)\), corresponding to the freedom of movement given by the flexible supports. Solving for \(u_1\) involves a standard Galerkin procedure, which as a by-product gives the solvability condition for \(u_0\). The latter is non-standard due to the flexible supports, i.e. it does not simply express requirement of orthogonality of the inhomogeneous part of the differential equation to solutions of the homogeneous part. To magnitude order \(\varepsilon\) of accuracy, the approximate solution becomes:

\[
u(x,t) = a_{01}(t) \left[ h_j(x) \cos(\Omega t - \eta_0) + h_{\beta}(x) \sin(\Omega t - \eta_0) \right] + \varepsilon \frac{1}{\Delta \xi^2} p \phi_{01}(x) \phi_{01}(x) \cos(\Omega t),\]

where:

\[
h_j(x) = \phi_{0j}(x) + \varepsilon \left[ \phi_{00}(x) + \frac{1}{\Delta \xi^2} (\pi \kappa_2 - k_j) \phi_{02}(x) \right],\]

\[
h_{\beta}(x) = \varepsilon \frac{1}{\Delta \xi^2} \left( \frac{16}{5} \alpha \nu + \beta_2 \right) \left( 1 + \frac{15}{\Delta \xi^2} \kappa_1 \right) \phi_{02}(x) \phi_{02}(x),\]

\[
\kappa_1,2 = \kappa_{u0} \pm \kappa_{uo}, \quad k_j = \frac{1}{\Delta \xi^2} \int \left( k_u \phi_{0j} \phi_{0j} + k_o \phi_{0j} \phi_{0j} \right) dx, \quad \beta_j = \int \left( \beta_u \phi_{0j} \phi_{0j} + \beta_o \phi_{0j} \phi_{0j} \right) dx.\]

This solution includes modal contributions only from the lowest symmetric mode \(\phi_{01}\) and the second antisymmetric mode \(\phi_{02}\), with corresponding rigid body corrections \(\phi_{01}\) and \(\phi_{02}\), and ignores terms of magnitude order \(\varepsilon^2\) and smaller. The amplitude \(a_{01}\) and phase \(\eta_0\) are slowly varying functions of time, and solutions of a pair of modulation equations:

\[
\dot{a}_{01} = -\frac{1}{2} \beta_{a0} a_{01} + \frac{1}{\Delta \xi^2} \phi_{01}(x) p \sin \eta_0, \quad \dot{\eta}_{01} = \Omega - \omega_0 + K_1 + a_{01}^{-1} \frac{1}{\Delta \xi^2} \phi_{01}(x) p \cos \eta_0,\]

where \(K_1 = \frac{1}{2} \alpha \nu + \pi \kappa_1 - \frac{1}{2} k_j\). Eqs. (5)-(8) shows how the response basically (i.e. when \(\varepsilon = 0\)) is a resonant motion of the pipe in its driven, fundamental mode \(\phi_{01}\). On top of this there are small additional motions (those multiplied by \(\varepsilon\)) accounting for the effects of mass flow, external asymmetric forcing, and the various imperfections quantified by \(\kappa_{u0}, \kappa_{uo}, \kappa_2, k_2, \beta_2\). The antisymmetric second mode \(\phi_{02}\) is only excited if there are "asymmetric causes", i.e. mass flow \(\alpha \nu\), or asymmetry in...
damping or stiffness imperfections ($\kappa_2,k_2,\beta_2$), or asymmetric external forcing ($x_p \neq 0$). It also appears that the part of the response related to the flow and the damping ($h_p(x)$) is phase-shifted $90^\circ$ in time wrt. the response related to elastic stiffness ($h_t(x)$). For an ideal pipe one has $h_t = \varphi_0^1(x)$, $h_p = \varepsilon \frac{16}{45\pi} \alpha \varphi_0^2(x)$, i.e. the pipe vibrates in the fundamental symmetric mode $\varphi_0^1$, with a small overlay – proportional to mass flow $\alpha v$ – of vibrations of the antisymmetric second mode. Since the latter vibrates $90^\circ$ out-of-phase wrt. the fundamental mode, the resulting motion $u$ is a traveling wave, i.e. the nodes of the pipe vibration pattern move in time, and different points on the pipe axis do not cross the equilibrium line simultaneously. With Coriolis flowmeters this time shift $\Delta x$ in zero-crossing between two pipe axis points, or the corresponding phase-shift $\Delta \psi = \Omega \Delta x$ is assumed proportional to mass flow $\alpha v$ and thus – after suitable calibration – a measure of mass flow. However, as appears already from (5)-(8), various imperfections may induce time- or phase-shifts in a manner similar to that of mass flow. For example, $\alpha v$ occurs in summation with $\beta_2$, so the effect of mass flow and (asymmetric or non-proportional) damping on phase shift cannot be distinguished.

3.2 Calculating phase shift under mid-pipe sharply resonant excitation

For the typical case of Coriolis flowmeter operation we can assume sharply resonant excitation (in reality this is ensured by feedback excitation), $\Omega = \omega_0^1 = \pi^2-K_1$, and find the stationary vibrations by letting $\dot{a}_0 = \dot{\eta}_0 = 0$ in (9) and solve for $a_0$ and $\eta_0$ analytically. With (5)-(8) one finds:

$$u(x,t) = \frac{\varepsilon P}{x^2} \sqrt{h_t(x)^2 + h_p(x)^2} \sin(\omega_0^1 t - \psi(x)), \quad \psi(x) = \arctan\left(\frac{h_p(x)}{h_t(x)}\right) \approx \frac{h_p(x)}{h_t(x)}$$

(10)

where the latter approximation is due to the smallness of parameters, and $\psi(x)$ is the phase of $u$ at location $x$. The difference $\Delta \psi$ in phase between two symmetric measurement points $x_{1,2} = \frac{x}{2} \pm \Delta x$ is:

$$\Delta \psi = \psi(\frac{x}{2} - \Delta x) - \psi(\frac{x}{2} + \Delta x)$$

Inserting (10) into (11) and Taylor-expanding, we find a simple expression for the phase shift. For the common situation of measuring near the antinodes of the second mode ($\Delta x = \frac{x}{2}$) it becomes:

$$\Delta \psi(\frac{x}{2}) = \frac{32\varepsilon}{45\pi}(1 + r_{1/4}^2 K_1)\left(\alpha v + \frac{1}{16} \beta_2\right); \quad r_{1/4} = 2(1 + \frac{1}{15\pi}) - \frac{1}{6} \approx 1.463. \quad (12)$$

Here the coefficient multiplying $\alpha v$ is the linear meter sensitivity, i.e. the ratio of phase shift to mass flow. Multiplying the sensitivity by $\frac{1}{16} \beta_2$, it appears, one obtains the meter zero shift, i.e. the phase shift measured at zero mass flow. Had $\omega_0^1 = (n\pi)^2$ not already been substituted for the unperturbed natural frequencies, the factor $32\sqrt{2}/45\pi^2$ in (12) would appear instead as $32\sqrt{2}/3(\omega_0^1 - \omega_0^2)$, i.e. the sensitivity grows with the closeness of the drive frequency ($\approx \omega_0^1$) of the fundamental, symmetric mode to the frequency ($\omega_0^2$) of the antisymmetric mode; this observation is well-known in Coriolis flowmeter applications.

3.3 Application examples: Effects of imperfections

Next we illustrate two applications (more are given elsewhere) of the prediction (12) and test its accuracy against numerical solution. For the examples we use a mass ratio of $\alpha = 0.3$ and a mass flow range $\alpha v \in [0; 0.1]$, roughly corresponding to a specific commercial Coriolis flowmeter measuring water flow up to full nominal flow rate. Unless otherwise stated, the pipe damping is taken to be axially uniform with $\beta_2(x) = 0.002$ (corresponding quality factor $Q = \omega_0^1/\beta_2 \approx 5000$ or damping ratio $1/2Q \approx 0.01\%$ for the drive mode $\varphi_0^1$). Purely numerical solutions to (1)-(3), used for testing the accuracy of (12), were obtained using standard Galerkin expansion, using as expansion functions a set of lowest mode shapes satisfying all boundary conditions as detailed elsewhere. The numerical solution converges towards the exact solution as more modes are included; typically only insignificant changes in results were observed by using more than 4-8 modes.
3.3.1 Effect of rotational damping at supports, $\beta(x) \neq 0$

In this case the pipe is ideal except for the presence of rotational damping at the simple supports, i.e. $\beta_r = k_{a0} = k_{u0} = k_{u} = 0$ but $\beta(x) \neq 0$ at $x=0,1$. Specifically we let $\beta(x) = \beta_{a0} \delta(x) + \beta_{u0} \delta(x-1)$, where $\delta(x)$ is Dirac's delta function and $\beta_{a0}$ and $\beta_{u0}$ are the coefficients of linear rotational damping at the left and right pipe support, respectively. Using (12) with (5)-(8) gives the phase shift:

$$\Delta \psi = \frac{32\sqrt{2}}{45\pi} \left( \alpha \nu + \frac{1}{2} \nu^2 \left( \beta_{a0} - \beta_{u0} \right) \right),$$

(13)

which implies the following: a) Small rotational damping at the supports does not necessarily cause phase shift, but any asymmetry in support damping, corresponding to $\beta_{a0} \neq \beta_{u0}$, causes a phase shift proportional to the difference in damping coefficients; b) the phase shift caused by fluid flow is inseparable from the phase shift caused by damping asymmetry, unless either the fluid flow or the damping asymmetry is known, or known to be constant; c) damping asymmetry will cause a phase shift $\Delta \psi \neq 0$ (the zero-shift) even at zero fluid flow ($\alpha v=0$).

Fig. 2 shows the variation of phase shift $\Delta \psi$ with mass flow $\alpha v$, for different values of asymmetry $\beta_{a0} - \beta_{u0}$ in rotational support damping. The middle line is for symmetric damping, the top and bottom lines for an asymmetry ten times the pipe damping $\beta_v$, and the two middle lines for an asymmetry equal to the pipe damping. As appears the analytical approximation (13) (lines) rather accurately agrees with numerical solution (symbol markers). Hence the simple expression (13) gives about the same accuracy as the numerical solution, but provides much more insight.

3.3.2 Effect of transverse flexibility at supports, $\kappa_{a0} + \kappa_{u0} \neq 0$

The pipe is here ideal except for a small flexibility at the supports supposed to be simple, i.e. $\beta_r = \kappa_{a0} = \kappa_{u0} = 0$, but $\kappa_{a0} + \kappa_{u0} \neq 0$. Using (12) with (5)-(8) gives the phase shift:

$$\Delta \psi = \frac{32\sqrt{2}}{45\pi} \left( 1 + r_{1/4} (\kappa_{a0} + \kappa_{u0}) \right) \alpha v,$$

(14)

with $r_{1/4}$ defined in (12). This implies that: a) Transverse support flexibility increases meter sensitivity, but does not affect phase shift at zero mass flow; b) the increase in sensitivity is proportional to the total support flexibility, and is thus present even with purely symmetric support flexibility; c) the effect of transverse support flexibility on phase shift is of second order (proportional to the square product of small terms $\kappa_{a0}, \kappa_{u0}$ and $\alpha v$), and thus of lower order than the effect of mass flow $\alpha v$.

Fig. 3(a) illustrates the variation of phase shift with mass flow for three levels of support flexibility $\kappa_{a0} = \kappa_{u0}$ in the range $10^{-3}$ to $10^{-1}$. The analytical approximation (14) (lines) rather accurately agrees with the numerical solution (symbol markers), with minor discrepancies only seen for the largest support flexibility ($10^{-1}$). Fig. 3(a) also illustrates the general observations made from (14) above, i.e. there is no zero-shift, but an increase in sensitivity with support flexibility. Fig. 3(b) shows how the relative deviation between analytical prediction and numerical analysis decreases as the support flexibility becomes smaller, i.e. as the assumptions for the perturbation analysis is better met. The order of magnitude of the relative error appears to match that of the support flexibility,
e.g. for $\kappa_u = \kappa_{ul} = 10^{-2}$ the deviation has also order of magnitude $10^{-2}$.

4. Other imperfections

The perturbation analysis used can be readily extended to account for other kinds of model imperfection. We give a few results of work in progress along this line.

4.1 Flow pulsation

Pulsating flow is common, e.g. with pumps or periodically operated valves. As for Coriolis flowmeter operation, flow pulsation could affect both accuracy and structural integrity. The effect of flow pulsation on pipe vibration and flowmeter phase shift is currently investigated using the perturbation approach described in Sect. 3. This includes consideration to possible effects of resonant flow, i.e. external, parametric, combination, or internal resonance. With flow pulsation Eq. (1) remains valid, while the assumption made in Sect. 2 of constant or slowly varying flow speed is dropped by letting instead $v = v_0(1 + g \sin(\omega_f t))$, where $v_0$ is the mean flow speed, and $v_0 q$ the amplitude in flow speed pulsation assumed to vary time-harmonically at frequency $\omega_f$. Of the various possible cases of resonance and non-resonance we mention here just primary parametric resonance of the flow pulsation to the pipe, $\omega_f \approx 2 \omega_{01}$. Using the approach already described in Sect. 3, the prediction for the phase shift becomes:

$$\Delta \psi = \frac{v_0 q}{4 \pi} \alpha v_0 \left(1 + \frac{\alpha v_0 q}{16 \alpha^2} \sin(\omega_f t)\right),$$

which has been tested to agree well with results of numerical simulation. According to (15) the time-averaged phase shift is proportional to the mean mass flow $\alpha v_0$, thus a Coriolis flowmeter should measure correct mean flow even with a parametric resonant flow. However, as also appears, the meter reading for the mass flow pulsation amplitude will be a factor $\omega_f/16 \pi^2$ larger than the true value $\alpha v_0 q$. We note that in the limit $\omega_f \to 0$ of slowly varying flow speed $v$, the result (15) coincides with (12), when imperfections other than pulsating flow is neglected.

Other cases are currently investigated, e.g. combination resonances, in particular $\omega_f \approx \omega_{02} - \omega_{01}$. A relevant issue for future investigation is internal resonance, e.g. $\omega_{02} \approx 2 \omega_{01}$ or $\omega_{03} \approx 3 \omega_{01}$. When combined with external resonance and even weak nonlinearity (unavoidable in real meters), internal resonance may allow energy to flow between modes, and thus could cause error in meter readings at certain flow pulsation frequencies.

4.2 Nonlinearity

Effects of weak nonlinearity can be studied with moderate extra effort, using the analytical
approach described in Sect. 3. The model equation of motion (1) is then augmented with cubic and quadratic nonlinearities, modelling nonlinear effects of, e.g., pipe stretching, pre-buckling, damping, and magnetic forces. The perturbation analysis in this case requires solving a single polynomial equation for the stationary resonance vibration amplitude; this amplitude is used to calculate the phase shift via expressions similar to (5) and (11).

4.3 Generalized damping

An analytical phase shift approximation has been calculated\textsuperscript{20} for uniformly distributed damping being a general function of velocity, i.e. with \( L_{\beta} \dot{u} \) in (1) generalized to \( \beta \int f(\dot{u}) \), where \( \beta \) is a magnitude parameter, and \( f \) any integrable function of velocity \( \dot{u} \). The prediction is similar to (12), with the coefficient \( \beta_2 \) computed as in (7), but replacing \( L_{\beta} \phi_{01} \) with \( \xi_1 \), where \( \xi_1(x) = \pi^{-1} \int_{-\infty}^{\infty} f(-\omega_0 \tilde{\alpha}_{01} \phi_{01} \sin \tau) \sin \tau d\tau \) is the Fourier sine coefficient of \( f \) evaluated along the fundamental first-mode velocity response \(-\omega_0 \tilde{\alpha}_{01} \phi_{01} \sin \tau\). Thus the generalization to almost any kind of velocity-dependent damping function involves only simple integration; e.g. quartic damping \( f = \dot{u}^4 \) gives \( \xi_1(x) = -\frac{\pi}{12} \omega_0^2 \tilde{\alpha}_{01}^2 \phi_{01}^2(x) \). An application of this could be to motivate the use of linear damping models by more than just computational convenience: If results change insignificantly with other models, that would be a more adequate reason for linearization or simplification.

4.4 Asymmetrically distributed damping

The damping distribution functions \( \beta_\alpha(x) \) and \( \beta_\beta(x) \) in (2) allows for arbitrarily distributed linear damping. It is found\textsuperscript{20} that the effect of linear distributed damping parallels that of rotational support damping (cf. Sect. 3.3), i.e. the phase shift is proportional to mass flow and to the antisymmetric part of the damping distribution, while the symmetric part has no effect on phase shift. This prediction, based on approximate analysis of a strongly simplified model, could hold also for real systems. As a first test of this, a few laboratory measurements for a commercial Coriolis flowmeter with two U-shaped pipes have been conducted\textsuperscript{20}. Fig. 4(a) sketches the setup, and Fig. 4(b) shows the recorded phase shift versus damping ratio for the pipe with no fluid. Energy dissipation additional to the air and structural pipe damping was imposed in a contactless manner, by fixing neodymium magnet disks asymmetrically near one flowmeter pipe. Varying the distance between the magnet stack and a small copper plate attached to the pipe allows for controlling the level of damping. Vibrations of the pipe are measured by a laser velocity transducer, and the built-in flowmeter electronics and software was used for recording phase shift. The damping ratio for the mode in question was estimated using B&K PULSE modal analysis software, using standard impulse hammer testing and averaged frequency response functions. As appears from Fig. 4(b), the phase shift grows almost linearly with damping (apart from noisy measurements at very low levels), as predicted theoretically for this case of asymmetric damping. Attempts have been made to test also the complementary part of the theoretical prediction, i.e. that phase shift is invariant to symmetric damping, but henceforth with less success, due to practical difficulties in realizing and measuring perfect damping symmetry.

Fig. 4. (a) Experimental setup for determining resonance frequencies, damping ratios, and phase shift for a Coriolis flowmeter. (b) Square symbols: measured phase shift \( \Delta \psi \) versus imposed modal damping ratio \( \xi \) for the drive mode; solid line: linear fit to measurements (\( \Delta \psi = 0.004 \xi - 0.0003 \)).
5. Conclusions

We have demonstrated how effects of small imperfections for vibrating pipes with fluid flow can be analyzed using systematic perturbation analysis. The results come as simple analytical expressions, relating measures of imperfection to vibration parameters of interest, such as the spatial phase shift relevant for Coriolis flowmeters. Analytical results were shown to agree with results of pure numerical analysis. Some particular results were derived, e.g. that the asymmetric part of rotational support damping changes the spatial phase shift along the pipe in the same manner as mass flow, so that neither two factors can be determined by measuring only phase shift. More details on the perturbation and the numerical analysis are presented separately, and work along similar lines with consideration to other kinds of imperfection is in progress.

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Predicting phase shift effects for vibrating pipes conveying pulsating fluid.

ABSTRACT

How do perturbations related to the fluid flow influence the dynamic behaviour of vibrating fluid-conveying pipes? This is relevant to know, e.g., when exploiting flow-affected oscillations of pipes to determine the fluids mass flow or density, as done with Coriolis flowmeters. An oscillating flow, e.g. caused by gear, piston or peristaltic pumps in the pipe system or valve openings and closings [1], can generate severe vibrations in a pipe system. Problems may arise with the function of Coriolis flowmeters, e.g. when the flow pulsation frequency is near one of the resonant frequencies of the meter [2]. The effect of flow pulsations on Coriolis flowmeters has been the subject of various studies, e.g. [2, 3, 4, 5, 6, 7]. The results of these studies, in particular [3], are valuable but lack a systematic perturbation analysis to fully uncover the importance of the involved parameters. Computational models for numerical simulation of real flowmeters are available. However they offer little insight into the essential physical phenomena taking place, and typically require extensive time for exploring parameter dependencies. A simplified model and systematic analysis tools are believed to be useful for providing such insight. Also the results for simplified systems may be used to create hypotheses on the effect of flow pulsations for more complicated, realistic systems, which can be tested against, e.g., experimental results or computational fluid dynamics simulations. These could be helpful in designing future Coriolis flowmeters, e.g. for minimizing the flowmeter’s sensitivity.

The Coriolis flowmeter working principle is based on fluid conveying pipes driven at resonance, where Coriolis forces are developed due to directional changes in the moving fluid induced by the pipe oscillations. A distortion of the pipes driven motion is induced by these forces. This produces a phase shift between transversely vibrating points along the pipe, which under certain ideal circumstances is proportional to fluid mass flow rate. Thus phase shift is the measured quantity, supposed to be linearly related to mass flow and independent of other factors. But in reality phase shifts could be influenced by a variety of imperfections - like flow pulsations, non-ideal boundary conditions, nonlinearity, and uneven distribution of mass, stiff-
ness and damping, and nonlinearity - some of which may then reflect as apparent changes in mass flow, i.e. as erroneous flowmeter readings.

In this work a simple model of a Coriolis flowmeter, a single, straight, supported pipe (Fig. 1), allows a clearer understanding of how the phase shift depends on variations of the amplitude and oscillating frequency of the fluid velocity. Possible future extensions of the model may include, e.g., a bended pipe, clamped pipe ends, non-uniform mass distribution of the pipe material or non-uniform flow profile.

Transverse motions \( u(t) \) of the system in Fig. 1 are described by the non-dimensionalized equation of motion

\[
\ddot{u} + \alpha (2\dot{v}u' + v^2u'' + \dot{v}u') + u''' + cu = -p_a \delta(x - x_p) \cos(\Omega_p t)
\]

where \( v \) is the time-varying flow speed, \( \alpha \) the ratio of fluid mass to total mass, \( c \) the damping coefficient, \( p_a \) the amplitude of the harmonic force at \( x = x_p \) having the normalized frequency \( \Omega_p \), \( \delta \) Dirac’s delta function, and dots and primes denote, respectively, differentiation with respect to time \( t \) and axial coordinate \( x \).

A systematic perturbation analysis involves solving (1) using, e.g., the method of multiple scales. This can be used to derive analytical expressions for how the phase shifts are affected by flow pulsation. The analysis procedure is similar to the one described in [8], which enables a rather general application and adaption to different configurations.

References

Predicting phase shift effects for vibrating fluid-conveying pipes due to Coriolis forces and fluid pulsation.

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Predicting phase shift effects for vibrating fluid-conveying pipes due to Coriolis forces and fluid pulsation

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Abstract

Knowing the influence of fluid flow perturbations on the dynamic behaviour of fluid-conveying pipes is of relevance, e.g., when exploiting flow-induced oscillations of pipes to determine the fluids mass flow or density, as done with Coriolis flow meters (CFM). This could be used in the attempts to improve accuracy, precision, and robustness of CFMs. A simple mathematical model of a fluid-conveying pipe is formulated and the effect of pulsating fluid flow is analysed using a multiple time scaling perturbation analysis. The results are simple analytical predictions for the transverse pipe displacement and approximate axial shift in vibration phase. The analytical predictions are tested against pure numerical solution using representative examples, showing good agreement. Fluid pulsations are predicted not to influence CFM accuracy, since proper signal filtering is seen to allow the determination of the correct mean phase shift. Large amplitude motions, which could influence CFM robustness, do not appear to be induced by the investigated fluid pulsation. Pulsating fluid of the combination resonance type could however influence CFMs robustness, if induced pipe motions go unnoticed and uncontrolled during CFM operation by feedback control. The analytical predictions offer an immediate insight into how fluid pulsation affects phase shift, which is a quantity measured by CFMs to estimate the mass flow, and lead to hypotheses for more complex geometries, i.e. industrial CFMs.

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validity of these hypotheses are suggested to be tested using laboratory experiments, or detailed computational models taking fluid-structure interaction into account.

**Keywords:** Coriolis, mass flowmeter, fluid oscillations, perturbation analysis, instrumentation

1. Introduction

How do perturbations related to the fluid flow influence the dynamic behaviour of fluid-conveying pipes? This is relevant to know, e.g., when exploiting flow-induced oscillations of pipes to determine the fluids mass flow or density, as done with Coriolis flowmeters (CFM). Oscillating fluid flow, e.g. caused by gear, piston or peristaltic pumps in the pipe system or fast valve openings and closings [1], can generate severe vibrations in a pipe system. It is stated in [2] that the operation of the CFM gets disturbed, when the frequency of the flow pulsations is equal to one of the resonant frequencies of the meter. The measuring errors are most severe when the frequency is equal to the Coriolis frequency [2]. In addition it has been shown in [3] that flow pulsation frequencies are likely to disturb CFMs, when equal to the sum and difference of the drive and Coriolis mode frequency of the flowmeter. A model and analysis tools are necessary in order to systematically examine perturbation effects on phase shifts for fluid conveying pipes.

The dynamics and stability of fluid-conveying pipes have been the subject of investigations for more than sixty years [4, 5]. What is of interest in the present study is how the axial shift in vibration phase depends on variations of the amplitude and oscillation frequency of a pulsating fluid velocity. This is of interest for CFM manufacturers, since CFMs measure this phase shift to estimate fluid mass flow.

The study [6] describes a straight pipe filled with an oscillating fluid and without any external harmonic excitation. The results indicate the a pulsating flow in a fluid-conveying pipe can cause parametric resonance.
The dynamics of a pipe conveying fluid with steady velocity and harmonically varying flow velocity and boundary conditions other than simply-supported are investigated in [7]. This paper states that the equation of motion derived in [6] is erroneous since it neglects the axial movements of the pipe due to axial acceleration of the fluid associated with the imposed velocity perturbations.

An investigation of parametric and combination resonances of a continuous flexible cantilevered pipe due to a pulsating flow with clamped pipe ends is presented in [8]. It is concluded that parametric and combination resonances are possible if the flow velocity is harmonically perturbed.

A straight pipe hinged at two immovable supports conveying a fluid at a velocity with a harmonically varying component, but not externally forced excitation is investigated in [9, 10]. The method of multiple scales is used and stability, bifurcation and pipe response is investigated.

In [3] a simple CFM, comprising of a straight pipe, rigidly built-in at its two ends and conveying a pulsating flow, is considered. The study involves a simple, but not systematic, perturbation analysis, i.e. the order of magnitude of neglected terms is not recorded. Effects of pulsating flow on the detector signals are investigated. For pulsating flow the detector signals are shown to contain components involving at least four frequencies, whereas detector signals in the steady flow case contain significant components only at the drive mode and Coriolis mode frequency. Flow pulsation frequencies, which might cause problems, are identified to be the sum and difference of the drive mode and Coriolis mode frequency. In this case an infinite amplitude motion corresponding to the Coriolis mode frequency is excited by the flow pulsation.

The experimental response of CFMs has been investigated in [11]. The tests show that the CFM used gives erroneous mass flow readings for flow pulsations at the Coriolis mode frequency, and at a frequency equal to the difference between the drive and Coriolis mode frequency; this confirms the findings of [3].

A procedure for modelling pulsating flow in CFMs using the ANSYS FE code is presented in [12]. It is concluded that flow measurement errors in the presence
of pulsating flow are due to the signal processing.

The dynamic response of CFMs to flow pulsations is investigated [13] using a straight tube meter, finite element simulations of flow tubes and experiment with commercially available flowmeters. The tested flowmeters show the presence of sensor signal noise at the Coriolis frequency in the measured response.

The results of [2, 3, 12, 13] are valuable, but appear to lack a systematic perturbation analysis to fully uncover the importance of the involved parameters. Purely computational detailed models for numerical simulation of real flowmeters are available. However they offer little insight into the essential physical phenomena taking place, and typically require extensive time for exploring parameter dependencies. The aim of this paper is to provide a simplified model and systematic analysis tools, which are believed to be useful for providing such insight. Also, the results for simplified systems may be used to suggest hypotheses on the effect of flow pulsations for more complicated, realistic systems, which can be tested against, e.g., experimental results or computational fluid dynamics simulations. These could be helpful in designing future CFMs, e.g. for improving measurement and accuracy stability in the presence of pulsating flow.

The governing equation and boundary conditions for a fluid-conveying pipe are presented in section 2. Section 3 presents a multiple scales perturbation analysis for solving the equation of motion. This is used to derive analytical expressions of the transverse pipe displacement and axial shift in vibration phase depending on the flow pulsation. The analysis procedure is similar to the one described in [14], which enables a rather general application and adaption to different configurations. The results are used to suggest hypotheses describing the importance of flow pulsations on the dynamic behaviour of fluid conveying pipes. In Section 5, these hypotheses are tested against numerical simulation.

2. Mathematical model

The model system Fig. 1 is a straight, fluid-conveying, simply supported pipe. A nondimensional equation of motion governing transverse pipe motions $u(x, t)$
can be derived from expressions for the kinetic and potential energies employing Hamilton’s principle [15], and is similar to what can be found in [14]:

\[
\ddot{u} + u'''' + c\dot{u} + \alpha (2v\dot{u}' + v^2u'' + \dot{v}u') = -p_a \delta(x - x_p) \cos(\Omega_p t + \phi_0),
\]

with \(v\) being the axial fluid velocity, and \(x \in [0;1]\) the axial coordinate. Equation (2) describes the assumed simply supported boundary conditions. Results based on clamped-clamped conditions would be closer to installations seen in real CFMs, however they are less transparent due to elaborate modes, and are not used here, since they would eventually lead to the same hypotheses as results based on simply supported conditions. The fluid velocity \(v\), which is a function of time \(t\), not of position \(x\), has a harmonic pulsation around the mean velocity \(v_0\), defined by the pulsation amplitude \(q \in [0;1]\) and pulsation frequency \(\omega_f\), i.e.

\[
v(t) = v_0(1 + q \cos(\omega_f t)).
\]

All parameters and variables in (1)-(3) are nondimensionalised:

\[
\begin{align*}
x &= \frac{\tilde{x}}{I}, & u &= \frac{\tilde{u}}{I}, & t &= \tilde{\omega} \tilde{t}, & v &= \frac{\tilde{v}}{\tilde{\omega} I}, & \Omega_p &= \frac{\tilde{\Omega}_p}{\tilde{\omega}}, & \omega_f &= \frac{\tilde{\omega}_f}{\tilde{\omega}}, & \tilde{\omega}^2 &= \frac{EI}{I^4(\rho_p A_p + \rho_f A_f)}, \\
\alpha &= \frac{\rho_f A_f}{\rho_p A_p + \rho_f A_f}, & c &= \frac{\tilde{c}}{\tilde{\omega}(\rho_p A_p + \rho_f A_f)}, & p_a &= \frac{\tilde{p}_a}{I^2 \tilde{\omega}^2 (\rho_p A_p + \rho_f A_f)},
\end{align*}
\]

where tildes denote physical variables or parameters, the coordinate \(x\) and pipe deformation \(u\) are nondimensionalised by the pipe length \(l\), the time \(t\) by the characteristic frequency \(\tilde{\omega}\) and the fluid speed by the characteristic wave speed.

Figure 1: Simply supported pipe conveying pulsating fluid.
\( \dot{\omega} l \), \( \rho A \) is the mass per unit length with subscripts \( f \) and \( p \) referring to fluid and pipe, \( \alpha \) the ratio of fluid mass to total mass, \( EI \) the flexural stiffness per unit length, \( c \) the pipe damping coefficient, \( p_a \) the amplitude of the harmonic force applied external to the pipe at \( x = x_p \) with the normalized frequency \( \Omega_p \), and \( \delta(x) \) is Dirac’s delta function. Differentiation with respect to axial coordinate \( x \) and time \( t \) is denoted \( (\cdot)' \) and \( (\cdot)'' \).

Equations (1)-(4) are developed using the following assumptions: The pipe is considered to be a slender beam made of a linear elastic material with small linearly viscous damping, uniform cross section and mass and stiffness distribution, pipe deformations are assumed to occur only in the transverse direction and rotations of the pipe are assumed small, and shear deformation, longitudinal inertia and rotatory inertia is neglected. The fluid is inviscid, incompressible, with a homogeneous density, filling out the entire inner cross-section area and perfectly coupled to the motion of the pipe. It is assumed that the pipes motion is not significantly affected by the internal fluid pressure and frictional drag.

The first three terms in (1) represent, respectively, the transverse inertia of the pipe and fluid, the flexural stiffness of the pipe, and the damping. The fourth and fifth term represent, respectively, Coriolis and centripetal accelerations, and are due to the fluid flowing at speed \( v \) through pipe segment with instantaneous curvature \( 1/u'' \), causing pipe rotations at angular velocity \( \dot{u}' \). Terms similar to these occur in many other studies regarding fluid-conveying pipes, e.g. [16, 5, 17]. However (1) differs from the corresponding equation in, e.g., [17] by the presence of the forth term \( \dot{v}u' \) representing the effect of time-varying fluid speed.

To indicate the assumed magnitude order of terms, the fluid velocity (3) is written as:

\[
v = \varepsilon v_0 (1 + q \cos(\varepsilon \omega ft)),
\]

where a bookkeeping parameter \( \varepsilon \) is introduced to mark terms that are assumed small compared to other terms, i.e. \( \varepsilon \ll 1 \). This implies that the non-dimensional mean fluid speed \( v_0 \) is small compared to unity, i.e. the (dimensional) mean fluid speed is small compared to the characteristic wave speed \( \dot{\omega} l \);
this is typically fulfilled for real CFMs even at full flow rate conditions. The case of slowly-pulsating fluid, typically caused by displacement pumps, is described by $s = 1$, whereas the case of non-slowly pulsations, e.g. caused by fast-closing valves, is described by $s = 0$. High-frequency pulsations, described by $s = -1$, are mathematically possible. However from a practical point of view, they are uninteresting to investigate, since common displacement pumps and valves employed in CFM applications would most likely not excite pulsations in this frequency range.

Equation (1) models CFMs, which are lightly damped structures driven at resonance, therefore the damping and forcing terms can be assumed to be small, i.e. $c \rightarrow \varepsilon c$ and $p_a \rightarrow \varepsilon p_a$. With (5), this leads to a re-scaling of the non-dimensional equation of motion (1) into:

$$
\ddot{u} + u''' + \varepsilon v_0 \left[ 2 \dot{u}'(1 + q \cos(\varepsilon \omega_f t)) + \varepsilon u''v_0(1 + q \cos(\varepsilon \omega_f t))^2 
- u' \varepsilon q \omega_f \sin(\varepsilon \omega_f t) \right] + \varepsilon c \dot{u} = -\varepsilon p_a \delta(x - x_p) \cos(\Omega_p t + \phi_0),
$$

with boundary conditions still given by (2).

3. Analytical predictions using perturbation analysis

3.1. Solution approach

To investigate the effect of pulsating fluid velocities given by (5), the method of multiple scales is used to solve (6) approximately, following the generally applicable technique established in [14], and the assumptions stated above. The computed solution $u(x, t)$ will be used to set up a simple analytical prediction for the difference in vibration phase $\Delta \psi$ between two symmetric pipe points $x_{1,2} = \frac{1}{2} \pm \Delta x$, $\Delta x \in [0; \frac{1}{2}]$, which, under certain ideal circumstances, is proportional to fluid mass flow rate and therefore a variable employed in Coriolis flowmetering.

Being interested in a perturbation solution in form of a two-timescale expansion, by the method of multiple scales [18] one assumes a uniformly valid
expansion of the form:

$$u(x, t) = u_0(x, T_0, T_1) + \varepsilon u_1(x, T_0, T_1) + O(\varepsilon^2),$$

(7)

where $u_0$ and $u_1$ are functions describing, respectively, the unperturbed motion of the pipe in its fundamental mode and the perturbation correction, $T_0 = t$ is the fast time scale, and $T_1 = \varepsilon t$ the slow time scale. Insert (7) into (6), equate to zero coefficients of like powers of $\varepsilon$, and obtain the zero ($\varepsilon^0$) order problem, i.e. the partial differential equation and boundary conditions for $u_0$:

$$D_i^2 u_0 + u''''_0 = 0,$$

(8)

$$u_0(0, T_0, T_1) = u''_0(0, T_0, T_1) = u_0(1, T_0, T_1) = u''_0(1, T_0, T_1) = 0,$$

(9)

with the partial differential operator $D_i^j$ being defined as $D_i^j = \partial^i / \partial T_j$. Equations (8)-(9) are the same for all considered cases of $s$ in (5). The general solution of (8), may be written as a series expansion in terms of the unperturbed mode shape functions $\varphi_{0j}$:

$$u_0(x, T_0, T_1) = \sum_{j=1}^{\infty} A_{0j}(T_1) \varphi_{0j}(x) e^{i\omega_{0j}T_0} + cc,$$

(10)

where $A_{0j}$ are complex-valued amplitude functions of the slow time scale $T_1$, $i$ the imaginary unit, $cc$ denote complex conjugates of the preceding terms, $\omega_{0j}$ the unperturbed linear natural frequencies, and $\varphi_{0j}$ the corresponding mode shape functions:

$$\omega_{0j} = (j\pi)^2, \quad \varphi_{0j}(x) = \sqrt{2}\sin(j\pi x), \quad j = 1, 2, \ldots$$

(11)

The mode shapes $\varphi_{0j}$ satisfy orthogonality and normalisation conditions, i.e.:

$$\int_0^1 \varphi_{0j} \varphi_{0k} dx = \delta_{jk},$$

$$\int_0^1 \varphi_{0j}^{(m)} \varphi_{0k} dx = \omega_{0j}^2 \delta_{jk}, \quad j, k = 1, 2, \ldots$$

(12)

where $\delta_{jk}$ is Kronecker’s delta. Since CFMs are excited near the frequency $\omega_{01}$, corresponding to the fundamental symmetric mode $\varphi_{01}$, one can keep only the corresponding part of (10), and let $A_{0j} = 0$ for $j = 2, 3, \ldots$, so that:

$$u_0(x, T_0, T_1) = A_{01}(T_1) \varphi_{01}(x) e^{i\omega_{01}T_0} + cc,$$

(13)
which describes the unperturbed motion of the pipe in its fundamental mode. The purpose of the perturbation analysis is then to determine how perturbations, i.e. the $\varepsilon$-terms in (6) with boundary conditions given by (2), influence this fundamental motion.

In the following, solutions of the first ($\varepsilon^1$) order problem for $u_1$ will be sought for two cases of fluid pulsations, "slow" and "non-slow". The aim is transparent analytical expressions, allowing for direct insight into the effect of these fluid pulsations on transverse pipe displacement and axial shift in vibration phase.

3.2. Slow fluid pulsation

In this case we consider pulsations described by (5) with $s = 1$, i.e. $\omega_f << \Omega_p \approx \omega_{01}$ so that the flow speed oscillates much slower than the primary transverse pipe drive. Insert in (6), ignore terms of order $O(\varepsilon^2)$, and obtain:

$$\ddot{u} + u'''' + \varepsilon 2\alpha v_0 (1 + q \cos(\omega_f T_1)) \dot{u} + \varepsilon c \dot{u} = -\varepsilon p_a \delta(x - x_p) \cos(\Omega_p T_0 + \phi_0),$$

(14)

Insert (7) in (14), equate to zero coefficients of like powers of $\varepsilon$, and obtain the first ($\varepsilon^1$) order problem for $u_1$:

$$D_0^2 u_1 + u_1'''' = -2 D_0 D_1 u_0 - 2 \alpha v_0 (1 + q \cos(\omega_f T_1)) D_0 \dot{u}_0' - c D_0 u_0 - p_a \delta(x - x_p) \cos(\Omega_p T_0 + \phi_0),$$

(15)

$$u_1(0) = u_1''(0) = u_1(1) = u_1''(1) = 0.$$  

(16)

Insert (13) in (15), express the nearness of the excitation frequency $\Omega_p$ to the fundamental natural frequency $\omega_{01}$ by introducing:

$$\Omega_p = \omega_{01} + \varepsilon \sigma_1,$$

(17)

stating that $\Omega_p$ differs from $\omega_{01}$ by the small quantity $\varepsilon \sigma_1$, with $\sigma_1$ being a detuning parameter, and obtain:

$$D_0^2 u_1 + u_1'''' = F_1(x, T_1, A_{01}) e^{i \omega_{01} T_0} + cc,$$

(18)
where

$$F_1(x, T_1, A_{01}) = -i \omega_{01} \varphi_{01} [2(D_1 A_{01}) + cA_{01}] - \frac{1}{2} p_a \delta(x - x_p) e^{i(\sigma_1 T_1 + \phi_0)}$$

$$- i \alpha v_0 \omega_{01} A_{01} \varphi'_{01} \left(2 + q e^{i\omega_f T_1} + e^{-i\omega_f T_1}\right),$$

(19)

with boundary conditions given by (16). Equation (18) is a linear differential equation with $T_0$-harmonic inhomogeneous terms. We seek a particular solution for $u_1$ in terms of a Galerkin-expansion in the unperturbed mode shapes $\varphi_{0j}$, i.e.:

$$u_1(x, T_0, T_1) = \sum_{j=1}^{\infty} \varphi_{0j}(x) B_{1j}(T_1) e^{i\omega_{0j} T_0} + cc.$$  

(20)

Insert (20) in (18), multiply by $\varphi_k(x)$, $k = 1, 2, \ldots$, integrate over $x$, apply the relations of orthonormality (12), and obtain an equation for the amplitude functions $B_{1j}$:

$$(\omega_{0j}^2 - \omega_{01}^2) B_{1j} = -i \omega_{01} [2(D_1 A_{01}) + cA_{01}] \int_0^1 \varphi_{01} \varphi_{0j} dx$$

$$- i \alpha v_0 \omega_{01} A_{01} \left[2 + q e^{i\omega_f T_1} + q e^{-i\omega_f T_1}\right] \int_0^1 \varphi'_{01} \varphi'_{0j} dx$$

$$- \frac{1}{2} p_a e^{i(\sigma_1 T_1 + \phi_0)} \int_0^1 \delta(x - x_p) \varphi_{0j} dx.$$  

(21)

Equation (21) can be used to calculate the first order amplitude functions $B_{1j}(T_1)$ for $j = 2, 3, \ldots$ It can be seen from (21), and the definition of $\omega_{0j}$ in (11), that the magnitude of $B_{1j}$ decreases rapidly with increasing mode number $j$. We are looking for a two-mode approximation to $u_1$. With $j = 1$ (21) implies that $B_{11}$ is arbitrary, thus we let $B_{11} = 0$, since vibrations in the fundamental form $\varphi_{01}$ are already taken into account at the $\varepsilon^0$-level through the function $u_0$ given by (13). The amplitude function $B_{12}$ of the second mode, which is small in magnitude, is particularly important for Coriolis flowmeter applications. Using amplitude functions up to the second mode will be sufficient to include all essential effects in the investigated case of a system under resonant excitation of the fundamental mode. We therefore ignore contributions from higher modes than the second, i.e. $B_{1j} = 0$ for $j > 2$. The amplitude function for the second mode is calculated by
inserting \( j = 2 \) in (21):
\[
B_{12} = -\frac{i8\alpha_0\omega_0 A_{01}}{3(\omega_0^2 - \omega_0^{21})} \left( 2 + qe^{i\omega_j T_1} + qe^{-i\omega_j T_1} \right) - \frac{p_0 \varphi_0(x_p)}{2(\omega_0^2 - \omega_0^{21})} e^{i(\sigma_1 T_1 + \phi_0)}.
\]
(22)

For \( j = 1 \) the left hand side of (21) is zero, yielding a solvability condition:
\[
-i\omega_0 (2D_1 A_{01} + cA_{01}) = \frac{1}{2} p_0 \varphi_0(x_p) e^{i(\sigma_1 T_1 + \phi_0)},
\]
(23)

which is a first order ordinary differential equation from which the unknown amplitude function \( A_{01}(T_0) \) can be determined. The solvability condition appears here as a byproduct of the Galerkin discretisation process. The fulfillment of (23) ensures the solution \( u_1 \) to (18) to be free of secular terms, which would violate the assumption underlying (7) that \( |\varepsilon u_1| \ll |u_0| \) at all times \( T_0 > 0 \). Now expressing \( A_{01} \) in polar form:
\[
A_{01}(T_1) = \frac{1}{2} a_{01} e^{i(\sigma_1 T_1 + \phi_0 + \eta_{01})},
\]
(24)

with \( a_{01} = a_{01}(T_1) \) and \( \eta_{01} = \eta_{01}(T_1) \) being real-valued slowly varying functions of time. Insert (24) into (23), separate real and imaginary parts, and obtain the corresponding first order equations for \( a_{01} \) and \( \eta_{01} \), i.e. the modulation equations,
\[
D_1 a_{01} = \frac{p_0 \varphi_0(x_p)}{2\omega_0} \sin(\eta_{01}) - \frac{1}{2} c a_{01},
\]
\[
a_{01}(D_1 \eta_{01}) = \frac{p_0 \varphi_0(x_p)}{2\omega_0} \cos(\eta_{01}) - \sigma_1 a_{01}.
\]
(25)
The two-mode approximate pipe response \( u(x, t) \) can be now calculated using (7), (13), (17), (20), (22), (24), \( \varepsilon = 1 \), \( T_0 = t \), and \( T_1 = \varepsilon t \):
\[
u(x, t) = a_{01} \left[ \varphi_{01} \cos(\Omega_p t + \eta_{01}) + \varepsilon \frac{16 \alpha_0 \omega_0 \varphi_0}{3(\omega_0^2 - \omega_0^{21})} \left[ 1 + q \cos(\omega_0 t) \right] \sin(\Omega_p t + \eta_{01}) \right] - \varepsilon \frac{p_0 \varphi_0(x_p)}{\omega_0^2 - \omega_0^{21}} \cos(\Omega_p t + \phi_0)
\]
(26)

where terms of order \( \varepsilon^2 \) and smaller and mode contributions higher than the second in (20) have been ignored, and the slowly varying amplitude and phase functions \( (a_{01}, \eta_{01}) \) are solutions to (25).

From (26) we can see that the pipe, in case of slow fluid pulsations, mainly vibrates in its fundamental symmetric mode \( \varphi_{01} \); the corresponding term \( u_0 = \)
\[ a_{01} \varphi_{01} \cos(\Omega_p + \phi_0 t + \eta_{01}) = O(\epsilon^0), \text{ i.e. not small.} \]

Recall that terms involving the mean mass flow \( \alpha v_0 \) and external excitation amplitude \( p_a \) are assumed small, i.e. \( O(\epsilon^1) \). It appears from (26), that on top of the motion of the fundamental mode, there will be small additional asymmetric motions of the second mode \( \varphi_{02} \), caused by the mean unidirectional mass flow term involving \( \alpha v_0 \), the pulsating mass flow term involving \( \alpha v_0 q \cos(\omega_f t) \), and the external asymmetric excitation term involving \( p_a \). The part of the pipe response related to the mean and pulsating part of the mass flow appears to be phase-shifted 90° with respect vibrations of the resonantly excited fundamental mode \( \varphi_{01} \). The resulting motion is a traveling wave, so that different points of the pipe axis do not cross the equilibrium line simultaneously.

Equation (26) describes transient as well as stationary vibrations. We are interested in stationary vibrations, when possible effects of disturbances have settled. We employ that stationary solutions of (26) are characterised by having constant amplitude and phase, e.g. \( a_{01}(T_1) = \bar{a}_{01} \) and \( \eta_{01}(T_1) = \bar{\eta}_{01} \), so that \( D_1 \bar{a}_{01} = D_1 \bar{\eta}_{01} = 0 \). Insert this in (25), solve the resulting algebraic equations, and obtain:

\[ \bar{a}_{01} = \frac{p_a \varphi_{01}(x_p)}{2 \omega_{01} \sqrt{\frac{c^2}{4} + \sigma_1^2}}; \quad \bar{\eta}_{01} = \phi_0 - \arctan \left( \frac{c}{2 \sigma_1} \right). \]  

At resonance, i.e. normal CFM operating conditions, \( \sigma_1 = 0 \), and the corresponding resonant amplitude \( \bar{a}_{01} = \bar{a}_{01}^* \) and phase \( \bar{\eta}_{01} = \bar{\eta}_{01}^* \) is:

\[ \bar{a}_{01}^* = \frac{p_a \varphi_{01}(x_p)}{c \omega_{01}}; \quad \bar{\eta}_{01}^* = \phi_0 \pm \frac{\pi}{2}. \]  

Equation (26) implies that, with pulsating flow \( q \neq 0 \), the detector signals will contain frequency components additional to the drive frequency \( \Omega_p \). In CFM applications where one, e.g., could be interested in knowing only the correct short term mean mass flow, i.e. averaged over several periods \( 2\pi/\Omega_p \) of pipe excitation cycles, one would apply proper filtering of the signals to remove data at other frequencies than the drive frequency \( \Omega_p \). The time-harmonic product term in (26) can be written as a sum of two time-harmonic functions at frequencies \( \Omega_p \pm \omega_f \),

\[ a_{01} \varphi_{01} \cos(\Omega_p + \phi_0 t + \eta_{01}) = O(\epsilon^0), \text{ i.e. not small.} \]
respectively. If the filter bandwidth around \( \Omega_p \) is smaller than \( 2\omega_f \) (i.e. essentially \( O(\varepsilon^2) \)), since in this section \( \omega_f = O(\varepsilon) \), these frequencies are in the stopband, and the filtered pipe displacement \( \hat{u}(x,t) \) is obtained simply as the parts of (26) oscillating at frequency \( \Omega_p \), i.e.:

\[
\hat{u}(x,t) = a_{01} \left[ \varphi_{01}(x) \cos(\Omega_p t + \eta_{01}) + \frac{16\alpha v_0 \omega_{01}\varphi_{02}}{3(\omega_{02}^2 - \omega_{01}^2)} \sin(\Omega_p t + \eta_{01}) \right]
\]

\[ - \varepsilon \frac{p_a \varphi_{02}(x_p)}{\omega_{02}^2 - \omega_{01}^2} \cos(\Omega_p t + \phi_0) \quad (29) \]

From (29) the filtered phase shift can be determined as follows: The excitation of CFMs is maintained in sharp resonance with the fundamental mode, i.e. \( \sigma_1 = 0 \) or \( \Omega_p = \omega_{01} \), and usually applied at the middle of the pipe, \( x_p = \frac{1}{2} \). Using (11), (28), (29) and trigonometric identities, one obtains the resonant pipe motion:

\[
\hat{u}(x,t) = \sqrt{2} \frac{p_a c \omega_{01}}{\sqrt{\varphi_{01}(x)}^2 + (k_1 \alpha v_0 \varphi_{02}(x))^2} \cos \left( \Omega_p t + \phi_0 \pm \frac{\pi}{2} + \Psi^* \right), \quad (30)
\]

where:

\[ k_1 = \frac{16\omega_{01}}{3(\omega_{02}^2 - \omega_{01}^2)} \]

(31)

and the phase shift \( \Psi^* = \Psi^*(x_t) \) is given by:

\[ \Psi^* = \arctan \left( -k_1 \alpha v_0 \frac{\varphi_{02}(x)}{\varphi_{01}(x)} \right) \approx -k_1 \alpha v_0 \frac{\varphi_{02}(x)}{\varphi_{01}(x)} \]

(32)

and the approximation in (32) is consistent with the assumed smallness of the r.h.s \( (v = O(\varepsilon)) \). The difference \( \Delta \Psi^* \) in phase between two measurement points, located symmetrically around the midpoint, i.e. \( x_{1,2} = \frac{1}{2} \pm \Delta x \), \( \Delta x \epsilon [0; \frac{1}{2}] \), is:

\[
\Delta \Psi^*(x) = \Psi^* \left( \frac{1}{2} + \Delta x \right) - \Psi^* \left( \frac{1}{2} - \Delta x \right). \quad (33)
\]

Employing (31), (32) and (33), leads to a simple result for the predicted phase shift:

\[
\Delta \Psi^*(x) = s(\Delta x) \alpha v_0, \quad (34)
\]

with the flowmeter sensitivity \( s(\Delta x) \), a factor of proportionality between the phase shift and mass flow, been given by:

\[
s(\Delta x) = \frac{64\omega_{01}}{3(\omega_{02}^2 - \omega_{01}^2)} \sin(\pi \Delta x). \quad (35)
\]
From (35) one can see that the flowmeter sensitivity increases with nearness of the natural frequency of the fundamental mode ($\omega_{01}$) to that of the second mode ($\omega_{02}$), confirming well-known results [19]. Equation (34) implies that the phase shift measured by the CFM, in case of slowly pulsating fluid and after narrow-band filtering of the detector signals, is proportional to mass flow $\alpha v_0$.

If the detector signals are not filtered, or are narrowband-filtered with a bandwidth larger than $2\omega_f = O(\varepsilon)$, the phase shift can be calculated from (26) following the assumption and procedure stated above, yielding:

$$\Delta \Psi^*(x, t) = s(\Delta x) \alpha v(t),$$

with the sensitivity $s(\Delta x)$ still be given by (34). Equation (36) implies that the phase shift will be proportional to the mass flow $\alpha v(t)$, even if the mass flow is not constant, but pulsating slowly, as long as the sensor signals are not too sharply narrowband-filtered. This result may not hold for larger $\omega_f$, cf. the following sections.

3.3. Non-slow fluid pulsations

In this case we consider pulsations described by (5) with $s = 0$, ignore terms of order $O(\varepsilon^2)$, and obtain, instead of (14):

$$\ddot{u} + u^{''''} + \varepsilon \alpha v_0 \left(2(1 + q \cos(\omega_f T_0)) \dot{u}' - q \omega_f \sin(\omega_f T_0) u'\right) + \varepsilon c \dot{u} = -\varepsilon p_a \delta(x - x_p) \cos(\Omega_p T_0 + \phi_0),$$

This leads, after inserting (10), and equating to zero coefficients to like powers of $\varepsilon$, to the first ($\varepsilon^1$) order problem:

$$D_0^2 u_1 + u_1^{''''} = -2D_0 D_1 u_0 - \alpha v_0 \left[2(1 + q \cos(\omega_f T_0)) D_0 u_0' - q \omega_f \sin(\omega_f T_0) u_0'\right]$$

$$- c D_0 u_0 - p_a \delta(x - x_p) \cos(\Omega_p T_0 + \phi_0),$$

with boundary conditions (16). Insert (13) in (38) and obtain:

$$D_0^2 u_1 + u_1^{''''} = -i2\omega_{01} \left[ \varphi_{01} D_1 A_{01} + \alpha v_0 \varphi_{01}' A_{01} + \frac{1}{2} c \varphi_{01} A_{01} \right] e^{i\omega_{01} T_0}$$

$$- i \alpha v_0 q \varphi_{01}' \left[ A_{01}(\omega_{01} - \frac{1}{2} \omega_f)e^{i(\omega_f + \omega_{01}) T_0} + \overline{A_{01}}(\omega_{01} + \frac{1}{2} \omega_f)e^{i(\omega_f - \omega_{01}) T_0} \right]$$

$$- \frac{1}{2} p_a \delta(x - x_p)e^{i(\Omega_p T_0 + \phi_0)} + cc$$

(39)
The oscillating inhomogeneous (r.h.s.-) terms may be resonant whenever their frequency is near a natural frequency $\omega_{0j}$ of the homogeneous (l.h.s.-) part of (39), where $\omega_{0j}, j = 1, 2, \ldots$ is given by (11). For terms depending on the fluid pulsation frequency $\omega_f$, this appears to occur when $\pm \omega_f \pm \omega_{01} \approx \omega_{0j}$. Of particular interest for Coriolis flowmeter applications are the fundamental cases $j = 1$ and $j = 2$, that is: $\omega_f \approx 2\omega_{01}$ (primary parametrical resonance), and $\omega_f \approx \omega_{02} - \omega_{01}$ (first combination resonance), which are both investigated below. Equation (39) predicts, that the (r.h.s.-) terms depending on $\omega_f$ will not be resonant, when the pulsation frequency is $\omega_f \approx \omega_{01}$; this case is therefore covered by the non-resonant case, Sec. 3.3.3. Higher resonances are less likely to be excited by pumps and valves in typical CFM applications, and therefore not investigated.

3.3.1. Primary parametric resonance ($\omega_f \approx 2\omega_{01}$)

Principle parametric resonance indicates that a small parametric excitation can produce a large system response, when the excitation frequency (here of the pulsating fluid $\omega_f$) is near twice a natural frequency of the system (here the fundamental frequency $\omega_{01}$) [18]. A detuning parameter $\sigma_2$ defines this nearness by:

$$\omega_f = 2\omega_{01} + \varepsilon \sigma_2. \quad (40)$$

Insert in (39) and obtain:

$$D_0^2u_1 + u_1''' = F_1(x, T_1, A_{01}) e^{i\omega_{01}T_0} + F_3(x, T_1, A_{01}) e^{i3\omega_{01}T_0} + cc, \quad (41)$$

$$F_1(x, T_1, A_{01}) = -i\omega_{01}\varphi_{01}[2D_1A_{01} + cA_{01}] - \frac{1}{2} p_0 \delta(x - x_p) e^{i(\sigma_1 T_1 + \phi_0)}$$

$$- i\alpha v_0 \varphi_{01}'[2\omega_{01}A_{01} + (\frac{1}{2} \omega_f - \omega_{01}) q A_{01} e^{i\sigma_2 T_1}] \quad (42)$$

$$F_3(x, T_1, A_{01}) = -i\alpha v_0 q A_{01} \varphi_{01}'(\frac{1}{2} \omega_f + \omega_{01}) e^{i\sigma_2 T_1}. \quad (43)$$

A particular solution for (41) in terms of a Galerkin-expansion in the unperturbed mode shapes $\varphi_{0j}$ is:

$$u_1(x, T_0, T_1) = \sum_{j=1}^{\infty} \varphi_{0j} B_{1j} e^{i\omega_{0j}T_0} + \sum_{j=1}^{\infty} \varphi_{0j} C_{1j} e^{i3\omega_{0j}T_0} + cc. \quad (44)$$
Following the procedure of Sec. 3.2, one obtains the following amplitude functions:

\[ B_{11} = C_{11} = 0, \]  
\[ B_{12} = -\frac{i8\alpha v_0 a_{01}}{3(\omega_{02}^2 - \omega_{01}^2)} \left[ \omega_{01} e^{i(\sigma_1 T_1 + \eta_01)} + \frac{\eta}{2} \left( \frac{1}{2} \omega_f - \omega_{01} \right) e^{i(\sigma_2 - \sigma_1) T_1 - \eta_01} \right] \]  
\[ - \frac{p a \varphi_02(x_p)}{2(\omega_{02}^2 - \omega_{01}^2)} e^{i(\sigma_1 T_1 + \phi_0)}, \]  
\[ C_{12} = -\frac{i8\alpha v_0 q a_{01}}{6(\omega_{02}^2 - 9\omega_{01}^2)} \left( \frac{1}{2} \omega_f + \omega_{01} \right) e^{i(\sigma_1 + \sigma_2) T_1 + \eta_01} \]  

(45) \quad \text{(46)} \quad \text{(47)}

The approximate pipe response \( u(x,t) \) can be calculated using (7), (13), (24), (40), (44)-(47), \( T_0 = t \), and \( T_1 = \varepsilon t \):

\[
u(x,t) = a_{01} \left\{ \varphi_{01} + \varepsilon \frac{8}{3} \alpha v_0 q \varphi_{02} \left[ \frac{1}{2} \omega_f - \omega_{01} \right] \left[ \frac{1}{2} \omega_f - \omega_{01} + \omega_{02}^2 - \omega_{01}^2 \right] \sin(\omega_f t) \right\} \cos(\Omega_p t + \eta_01) \\
+ \varepsilon \frac{8}{3} \alpha v_0 \varphi_{02} \left[ \frac{2\omega_{01}}{\omega_{02}^2 - \omega_{01}^2} + \frac{1}{2} \omega_f + \omega_{01} \left[ \frac{1}{2} \omega_f - \omega_{01} \right] \right] q \cos(\omega_f t) \right\} \times \sin(\Omega_p t + \eta_01) \]  
\[ - \varepsilon \frac{p a \varphi_02 \varphi_{02}(x_p)}{\omega_{02}^2 - \omega_{01}^2} \cos(\Omega_p t + \phi_0). \]  

(48)

In the case of principle parametric resonance, the pipe vibrates in its fundamental symmetric mode \( \varphi_{01} \). In addition there are small antisymmetric motions (terms involving \( \varphi_{02} \)) due to mass flow and external excitation. Comparing to (48) to (26), one can see that an additional motion is introduced to the pipe response in case of principle parametric resonance, i.e. the term involving \( \sin(\omega_f t) \), with the amplitude of this term depending on the pulsation, drive and Coriolis frequency. The factor multiplying \( \cos(\omega_f t) \) is also changed compared to the equivalent one in (26) and predicted to depend also on the pulsation frequency. Stationary solutions to (48) have a constant amplitude \( a_{01} = \overline{a}_{01} \) and phase \( \eta_{01} = \overline{\eta}_{01} \); these turn out still to be given by (28). Equation (48), in combination with (28), predicts, that principle parametric resonance in this case will not lead to (in theory) infinite amplitude motions, such as is otherwise usually the case when there are no nonlinearities to limit the response [18]. The reason for this is, that the term with \( \overline{A}_{01} \) in (39), which could excite resonant motions for \( \omega_f \approx 2\omega_{01} \), is proportional
to $\varphi'_{01}$, and thus orthogonal to the mode $\varphi_{01}$ which could be resonantly excited. This property, i.e. that $\int_0^1 \varphi_{01} \varphi'_{01} \, dx = 0$, holds for any smooth function $\varphi_{01}(x)$ satisfying $\varphi_{01}(0) = \varphi_{01}(1)$ and thus also for all symmetric boundary conditions.

Another way to understand the lack of parametric resonance of the fundamental mode $\varphi_{01}$ is to consider the parametric excitation terms in the equation of motion (1): the fifth term in (1) is negligibly small under the relevant conditions, while the fourth and sixth term are proportional to the pipe slope $u'$ or angular velocity $\dot{u}'$. Thus, with dominantly symmetric (wrt. $x = \frac{1}{2}$) vibrations, the small transverse forces induced by the fluid averages out to zero over the pipe length, rather than amplifying motions.

Repeating (cf. Section 3.2) the filtering away of data at other frequencies than the drive frequency $\Omega_f$, in the approximation predicting the pipe motion (48), yields again (29), which turns out also to predict the filtered detector signal in case of principle parametric resonance. From this it follows that phase shift and sensitivity will still be given by, respectively, (34) and (35).

Employing instead the unfiltered pipe response (48), and following Sec. 3.2, one obtains a prediction for the phase shift, ignoring terms of order $O(\varepsilon^2)$ and smaller:

$$
\Delta \Psi^*(x, t) = s(\Delta x) \alpha v_0 \left[ 1 + \frac{\omega_{02}^2 - 5\omega_{01}^2 + 2\omega_{01}\omega_f}{\omega_{02}^2 - 9\omega_{01}^2} q \cos(\omega_f t) \right],
$$

(49)

with the flowmeter sensitivity $s(\Delta x)$ still given by (35). For a non-slow pulsating mass flow, (49) implies that the phase shift will not be proportional to the time varying mass flow $\alpha v(t)$. The flowmeter will output correct mean flow $\alpha v_0$, but incorrect flow oscillation amplitude $\alpha v_0 q$.

3.3.2. Primary combination resonance ($\omega_f \approx \omega_{02} \pm \omega_{01}$)

In this particular case the oscillating inhomogeneous terms of (39) may be resonant when their frequency is near the natural frequency $\omega_{02}$ of the homogeneous part. To capture motions at this frequency, it is necessary to include the corresponding term in (10), i.e. instead of (13) we use:

$$
u_0(x, T_0, T_1) = A_{01}(T_1) \varphi_{01}(x) e^{i\omega_{01}T_0} + A_{02}(T_1) \varphi_{02}(x) e^{i\omega_{02}T_0} + cc.
$$

(50)
Insert into (38) and obtain:

\[ D_0^2 u_1 + u_1''' = -i\omega_01(2\varphi_01 D_1 A_01 + [2\alpha v_0\varphi_0'_{01} + c\varphi_01] A_01) e^{i\omega_01 T_0} \]
\[-i\omega_02(2\varphi_02 D_1 A_02 + [2\alpha v_0\varphi_0'_{02} + c\varphi_02] A_02) e^{i\omega_02 T_0} \]
\[-i\alpha v_0 q \left( \left[ (\omega_{01} + \frac{1}{2}\omega_f) e^{i(\omega_{01} + \omega_f) T_0} + (\omega_{01} - \frac{1}{2}\omega_f) e^{i(\omega_{01} - \omega_f) T_0} \right] \varphi'_{01} A_01 \right. \]
\[+ \left. \left[ (\omega_{02} + \frac{1}{2}\omega_f) e^{i(\omega_{02} + \omega_f) T_0} + (\omega_{02} - \frac{1}{2}\omega_f) e^{i(\omega_{02} - \omega_f) T_0} \right] \varphi'_{02} A_02 \right) \]
\[-\frac{1}{2}p_a \delta(x - x_p) e^{i(\Omega_p T_0 + \phi_0)} + cc. \] (51)

Note that (51) would be the same as (39), if we ignored motions of the second mode \(\varphi_{02}\). To express the nearness of the pulsations frequency \(\omega_f\) to \(\omega_{02} - \omega_{01}\), introduce:

\[ \omega_f = \omega_{02} - \omega_{01} + \varepsilon\sigma_3, \] (52)

with \(\sigma_3\) being a detuning parameter, insert in (51) and obtain:

\[ D_0^2 u_1 + u_1''' = H_1 e^{i\omega_01 T_0} + H_2 e^{i\omega_02 T_0} + H_3 e^{i(2\omega_{02} - \omega_{01}) T_0} + H_4 e^{i(2\omega_{01} - \omega_{02}) T_0} + cc, \] (53)

with

\[ H_1 = -i\omega_01 \left( [2D_1 A_01 + cA_01] \varphi_{01} + 2\alpha v_0\varphi_{01}'\omega_{01} A_01 \right) - i(\omega_{02} - \frac{1}{2}\omega_f) \alpha v_0 q \times \varphi'_{02} A_02 e^{-i\sigma_3 T_1} - \frac{1}{2}p_a \delta(x - x_p) e^{i(\sigma_1 T_1 + \phi_0)}, \] (54)

\[ H_2 = -i\omega_02 \left( [2D_1 A_02 + cA_02] \varphi_{02} + 2\alpha v_0\varphi_{02}'\omega_{02} A_02 \right) - i(\omega_{01} + \frac{1}{2}\omega_f) \alpha v_0 q \times \varphi'_{01} A_01 e^{i\sigma_3 T_1}, \] (55)

\[ H_3 = -i(\omega_{02} + \frac{1}{2}\omega_f) \alpha v_0 q \varphi_{02}' A_02 e^{i\sigma_3 T_1}, \] (56)

\[ H_4 = -i(\omega_{01} - \frac{1}{2}\omega_f) \alpha v_0 q \varphi_{01}' A_01 e^{i\sigma_3 T_1}. \] (57)

with the nearness \(\sigma_1\) of the excitation frequency \(\Omega_p\) to the fundamental natural frequency \(\omega_{01}\) being expressed by (17).
expansion of the unperturbed mode shapes $\varphi_{0j}$ is:

$$u_1(x, T_0, T_1) = \sum_{j=1}^\infty \varphi_{0j}(x) B_{1j}(T_1) e^{i\omega_{01} T_0} + \sum_{j=1}^\infty \varphi_{0j}(x) C_{1j}(T_1) e^{i\omega_{02} T_0} + \sum_{j=1}^\infty \varphi_{0j}(x) D_{1j}(T_1) e^{i(2\omega_{02} - \omega_{01}) T_0} + \sum_{j=1}^\infty \varphi_{0j}(x) E_{1j}(T_1) e^{i(2\omega_{01} - \omega_{02}) T_0} + cc. \tag{58}$$

Following the procedure presented in Sec. 3.2, one obtains the following amplitude functions:

$$B_{11} = C_{12} = D_{12} = E_{11} = 0, \tag{59}$$
$$B_{12} = -\frac{i}{16} \alpha v_0 \omega_{01} A_{01} - \frac{p_0 \varphi_{02}(x_p)}{3(\omega_{02}^2 - \omega_{01}^2)} e^{i(\sigma_1 T_1 + \phi_0)}, \tag{60}$$
$$C_{11} = \frac{i}{16} \alpha v_0 \omega_{02} A_{02}, \tag{61}$$
$$D_{11} = \frac{i}{3} (\omega_{02}^2 + \frac{1}{2} \omega_f) \alpha v_0 q A_{02} e^{i\sigma_3 T_1}, \tag{62}$$
$$E_{12} = -\frac{i}{3} (\omega_{01}^2 - \frac{1}{2} \omega_f) \alpha v_0 q A_{01} e^{-i\sigma_3 T_1}. \tag{63}$$

The amplitude functions $A_{01}$ and $A_{02}$ can be determined from the two solvability conditions, which appear again as a byproduct of the Galerkin discretisation process:

$$i \omega_{01}(2D_1 A_{01} + c A_{01}) = i \frac{\omega_0}{3} (\omega_{02} - \frac{1}{2} \omega_f) \alpha v_0 q A_{02} e^{i\sigma_3 T_1} - \frac{1}{2} p_0 \varphi_{01}(x_p) e^{i(\sigma_1 T_1 + \phi_0)}, \tag{64}$$
$$i \omega_{02}(2D_1 A_{02} + c A_{02}) = -i \frac{\omega_0}{3} (\omega_{01} + \frac{1}{2} \omega_f) \alpha v_0 q A_{01} e^{i\sigma_3 T_1}. \tag{65}$$

Expressing $A_{01}$ and $A_{02}$ in polar form:

$$A_{01}(T_1) = \frac{1}{2} a_{01} e^{i\psi_{01}}, \quad A_{02}(T_1) = \frac{1}{2} a_{02} e^{i\psi_{02}}, \tag{66}$$

with $a_{0j} = a_{0j}(T_1)$ and $\psi_{0j} = \psi_{0j}(T_1)$, for $j = 1, 2$, being real-valued slowly varying functions of time, insert (66) into (64) and (65), separate real and imaginary
parts, and obtain the corresponding first order equations for \(a_{01}\) and \(\psi_{01}\):

\[
\omega_{01} D_1 a_{01} = -\frac{1}{2} p_a \varphi_{01}(x_p) \sin(\sigma_1 T_1 + \phi_0 - \psi_{01}) + \frac{8}{6} (\omega_0 - \frac{1}{2} \omega_f) \alpha v_0 q \times \quad (67)
\]

\[
a_{02} \cos(\psi_{02} - \psi_{01} - \sigma_3 T_1) - \frac{1}{2} \omega_0 c a_{01}
\]

\[
\omega_{01} a_{01} D_1 \psi_{01} = \frac{1}{2} p_a \varphi_{01}(x_p) \cos(\sigma_1 T_1 + \phi_0 - \psi_{01}) + \frac{8}{6} (\omega_0 - \frac{1}{2} \omega_f) \alpha v_0 q \times \quad (68)
\]

\[
a_{02} \sin(\psi_{02} - \psi_{01} - \sigma_3 T_1),
\]

and for \(a_{02}\) and \(\psi_{02}\):

\[
\omega_{02} D_1 a_{02} = -\frac{8}{6} (\omega_0 + \frac{1}{2} \omega_f) \alpha v_0 q a_{01} \cos(\psi_{01} - \psi_{02} + \sigma_3 T_1) - \frac{1}{2} \omega_0 c a_{02} \quad (69)
\]

\[
\omega_{02} a_{02} D_1 \psi_{02} = -\frac{8}{6} (\omega_0 + \frac{1}{2} \omega_f) \alpha v_0 q a_{01} \sin(\psi_{01} - \psi_{02} + \sigma_3 T_1), \quad (70)
\]

To eliminate \(T_1\) define two new dependent variables:

\[
\beta \equiv \psi_{02} - \psi_{01} - \sigma_3 T_1, \quad \gamma \equiv \sigma_1 T_1 + \phi_0 - \psi_{01}, \quad (71)
\]

insert (71) in (67)-(70) and obtain:

\[
D_1 a_{01} = -\kappa_1 \sin \gamma + \kappa_2 a_{02} \cos \beta - \frac{c}{2} a_{01}, \quad (72)
\]

\[
a_{01} D_1 \gamma = -\kappa_1 \cos \gamma - \kappa_2 a_{02} \sin \beta + \sigma_1 a_{01}, \quad (73)
\]

\[
D_1 a_{02} = -\kappa_3 a_{01} \cos \beta - \frac{c}{2} a_{02}, \quad (74)
\]

\[
a_{02} (D_1 \beta - D_1 \gamma) = \kappa_3 a_{01} \sin \beta - (\sigma_1 + \sigma_3) a_{02}. \quad (75)
\]

with

\[
\kappa_1 = \frac{p_a \varphi_{01}(x_p)}{2 \omega_0}, \quad \kappa_2 = \frac{4(\omega_0 - \frac{1}{2} \omega_f) \alpha v_0 q}{3 \omega_0}, \quad \kappa_3 = \frac{4(\omega_0 + \frac{1}{2} \omega_f) \alpha v_0 q}{3 \omega_0}. \quad (76)
\]

Since we are looking for stationary solutions \(a_{01}(T_1) = \bar{a}_{01}, a_{02}(T_1) = \bar{a}_{02}, \beta(T_1) = \bar{\beta}, \) and \(\gamma(T_1) = \bar{\gamma}\) to (72)-(75), we require that:

\[
D_1 \bar{a}_{01} = D_1 \bar{a}_{02} = D_1 \bar{\beta} = D_1 \bar{\gamma} = 0, \quad (77)
\]

yielding stationary values under sharp external pipe excitation \((\sigma_1 = 0)\):

\[
\bar{a}_{01}^2 = \frac{\kappa_1^2}{\frac{1}{4} c^2 + \frac{\kappa_2 + \frac{1}{4} c^2 \kappa_3 + \kappa_2 + \frac{1}{4} c^2 \kappa_3}{c^2 + \sigma_3}}, \quad \bar{a}_{02}^2 = \frac{\kappa_1^2}{\kappa_2 + c^2 \left( \frac{\kappa_2 + \frac{1}{4} c^2 \kappa_3}{4 \kappa_3} + \frac{1}{4} c^2 \sigma_3^2 \right)}, \quad (78)
\]

20
\[
\psi_{01}^* = \phi_0 - \arctan \left( \frac{c}{2\sigma_3} \left[ \frac{\kappa_3 \sigma_0^2}{\kappa_2 \sigma_0^2} - 1 \right] \right), \\
\psi_{02}^* = \phi_0 + \arctan \left( -\frac{2\sigma_3}{c} \right) - \arctan \left( \frac{c}{2\sigma_3} \left[ \frac{\kappa_3 \sigma_0^2}{\kappa_2 \sigma_0^2} - 1 \right] \right) + \sigma_3 T_1.
\] (79)

The two-mode approximated pipe response \(u(x,t)\) can then be calculated using (7), (10), (24), (52), (58)-(63), \(T_0 = t\), and \(T_1 = \varepsilon t\):

\[
u(x,t) = a_{01} \left\{ \begin{array}{l}
\varphi_{01} - \varepsilon \frac{2(\omega_{01} - \frac{1}{2}\omega_f)}{3\omega_{01}(\omega_{02} - \omega_{01})} \sin(\omega_f t) \cos(\omega_{01} t + \psi_{01}) \\
+ \varepsilon \alpha v_0 \varphi_{02} \left[ \frac{16\omega_{01}}{3(\omega_{02}^2 - \omega_{01}^2)} + \frac{(2\omega_{01} - \omega_f)q}{3\omega_{01}(\omega_{02} - \omega_{01})} \cos(\omega_f t) \right] \sin(\omega_{01} t + \psi_{01}) \\
+ a_{02} \left[ \begin{array}{l}
\varphi_{02} + \varepsilon \frac{2(\omega_{02} + \frac{1}{2}\omega_f)}{3\omega_{02}(\omega_{01} - \omega_{02})} \sin(\omega_f t) \cos(\omega_{02} t + \psi_{02}) \\
+ \varepsilon \alpha v_0 \varphi_{01} \left[ \frac{16\omega_{02}}{3(\omega_{02}^2 - \omega_{01}^2)} + \frac{(2\omega_{02} + \omega_f)q}{3\omega_{02}(\omega_{01} - \omega_{02})} \cos(\omega_f t) \right] \sin(\omega_{02} t + \psi_{02}) \\
- \varepsilon p_a \varphi_{02} \varphi_{02}(x_p) \cos(\Omega_p t + \phi_0), \end{array} \right. \end{array} \right. (80)
\]

Stationary solutions to (80) have constant amplitudes \((a_{01} = \overline{a}_{01}, \ a_{02} = \overline{a}_{02})\) and phases \((\psi_{01} = \overline{\psi}_{01}, \ \psi_{02} = \overline{\psi}_{02})\), these are given by (78)-(79) with the approach to sharp combination resonance \((\sigma_3 \rightarrow 0)\), the amplitude \(a_{01}\) decreases while \(a_{02}\) increases, but both remain finite. As appears the pipe is forced to vibrate mainly in its fundamental symmetric mode \(\varphi_{01}\). The fluid pulsation is seen to also induce motions corresponding to the antisymmetric mode \(\varphi_{02}\). The resulting dominating pipe motion is predicted to be a combination of the fundamental symmetric and antisymmetric mode. Terms in (80) involving the mean mass flow \(\alpha v_0\) and external excitation amplitude \(p_a\) are assumed small, as implied by the bookkeeping parameter \(\varepsilon\). These terms will contribute small additional motions of the antisymmetric and symmetric type. The amplitude of the motions due to the unsteady mass flow, i.e. terms involving \(q\), depends on the pulsation, drive and Coriolis frequency, and differ from the ones seen in (26) and (48).

Filtering (80) as in Section 3.2 leads again to (29) for the filtered approximated pipe response \(\hat{u}(x,t)\), with \(\Omega_p = \omega_{01}\) and \(\eta_{01} = \psi_{01}\). The phase shift and sensitivity are then still given by (34) and (35) respectively, so that with proper
filtering at least the mean mass flow \(\alpha v_0\) should be correctly captured, even under combination resonant fluid pulsations.

### 3.3.3. Non-resonant case (\(\omega_f\) away from \(2\omega_{01}\) and \(\omega_{02} - \omega_{01}\))

In this case the pulsation frequency \(\omega_f\) is not causing resonant inhomogeneous (r.h.s.-) terms in (39), which can be written as:

\[
D_0^2 u_1 + u_1''' = J_1(x, T_1, A_{01}) e^{i\omega_{01}T_0} + J_2(x, T_1, A_{01}) e^{i(\omega_{01} + \omega_f)T_0}
+ J_2(x, T_1, A_{01}) e^{i(\omega_{01} - \omega_f)T_0} + cc,
\]

where

\[
J_1(x, T_1, A_{01}) = -i\omega_{01}\varphi_{01}[2(D_1 A_{01}) + 2\alpha v_0\varphi_{01}' A_{01} + cA_{01}]
- \frac{1}{2}p_a \delta(x - x_p)e^{i(x_0 + \phi_0)},
\]

\[
J_2(x, T_1, A_{01}) = -i\alpha v_0 q(\omega_{01} - \frac{1}{2}\omega_f)\varphi_{01}' A_{01}
\]

\[
J_3(x, T_1, A_{01}) = i\alpha v_0 q(\omega_{01} + \frac{1}{2}\omega_f)\varphi_{01}' A_{01}
\]

Equation (81) can be solved following the procedure of Sec. 3.2: The solution to (81) is given in terms of a Galerkin-expansion of the unperturbed mode shapes \(\varphi_{0j}\), i.e. (20), though with additional expansions due to terms involving \(e^{i(\omega_{01} + \omega_f)T_0}\) and \(e^{i(\omega_{01} - \omega_f)T_0}\) in (81). This will yield a solvability condition given by (23) and a two-mode approximate pipe response for \(u(x, t)\):

\[
u(x) = a_{01} \left\{ \left[ \varphi_{01} - \frac{16\alpha v_0 q\omega_{01}\varphi_{02}}{3(\omega_{01}^2 - \omega_{02}^2)} \sin(\omega_f t) \right] \cos(\Omega_p t + \eta_{01}) 
+ \left[ \frac{16\alpha v_0 q\omega_{01}\varphi_{02}}{3(\omega_{02}^2 - \omega_{01}^2)} + \frac{8\alpha v_0 q\omega_{01}\varphi_{02}}{3(\omega_{01}^2 - \omega_f^2)^2} \cos(\omega_f t) \right] \sin(\Omega_p t + \eta_{01}) \right\}
- \varepsilon \frac{p_a \varphi_{02} \varphi_{02}(x_p)}{\omega_{02}^2 - \omega_{01}^2} \cos(\Omega_p t + \phi_0)
\]

Stationary solutions to (85) have a constant amplitude \(a_{01} = \bar{a}_{01}^*\) and phase \(\eta_{01} = \bar{\eta}_{01}^*\), these are given by (28). The filtered approximation \(\tilde{u}(x, t)\) based on (85) is again given by (29), and the phase shift and sensitivity by, respectively, (34) and (35).
4. Hypotheses

The results of the previous sections regarding the simplified model are now used to set up hypotheses regarding the robustness, accuracy and precision of a Coriolis flowmeter. These hypotheses will later be tested by solving the full model with numerical methods and comparing the results.

4.1. Robustness

Robustness can be defined as the ability of the CFM to cope with unpredictable variations in its operating environment with minimal loss of functionality. Could a pulsating fluid induce large pipe vibrations, which could lead to fatigue failure and in the end destroy the flowmeter? To answer this question, the amplitude of the vibrating measuring pipe in a Coriolis flowmeter will be considered, when the pipe is resonantly driven at its first mode by an external force, and the fluid pulsation frequency \( \omega_f \in [0; \sup \{2\omega_{01}, \omega_{02} - \omega_{01}\}] \).

In case of slow fluid pulsations, i.e. \( \omega_f << \omega_{01} \), or non-slow fluid pulsations, that are not combination resonant (i.e. \( \omega_f \) away \( \omega_{02} - \omega_{01} \)), the pipe will vibrate mainly in its first symmetric mode \( \varphi_{01} \), i.e. the drive mode, with amplitude, cf. (28):

\[
\bar{a}_{01} = \frac{2\kappa_1}{c},
\]

with \( \kappa_1 \) given in (76). The pipe response amplitude \( a_{01} \) is predicted to be independent of fluid related quantities, e.g. velocity \( v_0 \), pulsation amplitude \( q \) or frequency \( \omega_f \). For the fluid pulsations it applies for, (86) leads to the hypothesis, that these fluid pulsations not necessarily influence the robustness of a CFM, since they cannot cause vibration amplitudes, which cannot be controlled by adjusting the amplitude of the external force \( p_u \).

In case of primary combination resonance, \( \omega_f \approx \omega_{02} - \omega_{01} \), fluid pulsations are predicted to induce pipe motions in the symmetric and antisymmetric mode \( \varphi_{01} \)
and \( \varphi_{02} \), with dominant stationary amplitudes, cf. (78):

\[
\bar{a}_{01}^* = \frac{\kappa_1}{\sqrt{\frac{1}{4} c^2 + \frac{\kappa_2 \kappa_3 (\frac{1}{2} c^2 + \kappa_2 \kappa_3)}{\kappa_3^2 + \sigma_3^2}}}, 
\]

(87)

\[
\bar{a}_{02}^* = \frac{\kappa_1}{\sqrt{\kappa_2^2 + c^2 \left( \frac{\kappa_2}{2 \kappa_3} + \frac{1}{4} c^2 + \sigma_3^2 \right)}}}, 
\]

(88)

with \( \kappa_1, \kappa_2, \kappa_3 \) given in (76), indicating that stationary amplitudes for \( \omega_f \approx \omega_{02} - \omega_{01} \) depend not only on the pipe damping \( c \) and forcing amplitude \( p_a \) but also the pulsation amplitude \( q v_0 \). For decreasing pipe damping \( c \rightarrow 0 \) and \( \sigma_3 << 1 \) both amplitudes are predicted to attain finite values, \( \kappa_1 |\sigma_3| / \kappa_2 \kappa_3 \) and \( \kappa_1 / \kappa_2 \), respectively. For fluid pulsations close to sharp combination resonance, i.e. \( \sigma_3 \rightarrow 0 \), (87) - (88) predict that the dominating amplitudes will generally decrease for increasing fluid pulsation amplitude \( q v_0 \), though below a certain pulsation amplitude (vanishing as \( c^2 \rightarrow 0 \)), the amplitude of the second mode increases with \( q v_0 \). This contradicts the results in [3], which predicts infinite amplitude motions being excited by combination resonant fluid pulsations. It is also predicted that vibration amplitudes of the antisymmetric mode \( \varphi_{02} \) could outgrow those of \( \varphi_{01} \).

This is illustrated by Fig. 2(a), which shows the dominating amplitudes \( \bar{a}_{01}^* \) and \( \bar{a}_{02}^* \) for increasing pulsation amplitude \( q v_0 \) with parameters as given in the figure caption. In CFMs the forcing amplitude \( p_a \) is feedback-controlled: when detecting a decreasing amplitude \( \bar{a}_{01}^* \) of the symmetric mode for increasing pulsation amplitude \( q v_0 \), the control schemes would increase the forcing amplitude \( p_a \) to maximise \( \bar{a}_{01}^* \). However, (88) predicts that this would also increase the amplitude of the antisymmetric modes \( \bar{a}_{02}^* \). Figure 2(b) depicts the amplitudes \( \tilde{a}_{01,e}^* \) and \( \tilde{a}_{02}^* \) in case of feedback amplitude control, i.e. adjusting \( p_a \) so that \( \tilde{a}_{01,e}^* \) takes a constant value for all values of \( q v_0 \). Certain control schemes in the feedback-algorithm could leave increased vibrations of the antisymmetric modes unnoticed, e.g. if the measured signal is taken simply as the mean (would be zero) of measured motions at symmetrically located measurement coils, or if the feedback signal is based on measured signals narrowband-filtered at the drive frequency \( \omega_{01} \). This
leads to the hypothesis, that fluid pulsations of the sharp combination resonance type could affect the robustness of CFM, by exciting unsupervised antisymmetric modes of vibration; if large these vibrations could cause fatigue failure of the CFM.

4.2. Accuracy

The measurement accuracy is defined as the degree of closeness of measurements of a quantity to its actual value. The analytical prediction for the phase shift (34) yields the following hypothesis: CFMs should be able to meter the correct mean mass flow $\alpha v_0$ in case of pulsating fluid, if the signals measured at the detectors is filtered so that they only contain oscillations at the drive frequency $\Omega_p$. 
4.3. Precision

The precision of a measurement system, also called repeatability, is the degree to which repeated measurements under unchanged flow conditions show the same (not necessarily correct) results. Two factors could influence the flowmeter’s precision: a) sensitivity changes, or b) zero-shifts. The sensitivity is predicted not to be influenced by the presence of a pulsating fluid, since it does not depend on fluid related quantities, cf. (35). A zero-shift, i.e. a phase shift induced even in case of zero fluid flow, is also not seen to be induced by fluid pulsations, cf. (34). Therefore the repeatability of the flowmeter could be excellent even in case of pulsating fluid flow.

4.4. Summary of the hypotheses

In Sec. 5, pure numerical analysis of the original system (1)-(2) will be employed to test the following hypotheses:

**H1** Fluid pulsations, that are not of the combination resonance type, do not induce large uncontrollable vibration amplitudes.

**H2** Fluid pulsations of the sharp combination resonance type induce vibration amplitudes of the antisymmetric mode $\varphi_{02}$ that could be of the same order of magnitude as those of the driven resonant mode $\varphi_{01}$; both modal amplitudes are predicted to decrease in magnitude for increasing fluid pulsation amplitude $q v_0$.

**H3** Mean phase shifts can be measured by proper filtering of the sensor signals.

**H4** Fluid pulsations do not change the flowmeter sensitivity.

5. Numerical testing of analytical predictions

5.1. Numerical solution to equation of motion

The non-dimensional governing equation (1) is discretised using a standard Galerkin approach,

$$u(x, t) = \sum_{j=1}^{N} \varphi_j(x) z_j(t),$$  \hspace{1cm} (89)
where \( N \) is the number of expansion term, \( \varphi_j \) the appropriate comparison functions satisfying the boundary conditions (2), and \( z_j \) the generalized coordinates of the discretised systems. Eigenfunctions \( \varphi_j(x) \) of the corresponding unforced, undamped, no-flow problem with the same boundary conditions as the pipe under consideration are used as comparison functions, i.e.:

\[
\varphi_j(x) = \sqrt{2} \sin(j \pi x),
\]

(90)

and \( z_j \) thus become also modal coordinates. Substitute (89)-(90) in (1), multiply by \( \varphi_k(x) \), and integrate over \( x \) to yield a system of equations governing the time evolution of \( z_j \). In matrix notion this leads to a linear system of ordinary differential equations with harmonic forcing:

\[
M \ddot{z} + C \dot{z} + K z = f_k(t),
\]

(91)

where the components of \( M, C, K \) and \( f \) are given by:

\[
M_{jk} = \int_0^1 \varphi_j \varphi_k dx,
\]

(92)

\[
C_{jk} = 2 \alpha v \int_0^1 \varphi'_j \varphi_k dx + c \int_0^1 \varphi_j \varphi_k dx,
\]

(93)

\[
K_{jk} = \alpha v^2 \int_0^1 \varphi''_j \varphi_k dx + \alpha \dot{v} \int_0^1 \varphi'_j \varphi_k dx + \int_0^1 \varphi''''_j \varphi_k dx,
\]

(94)

\[
f_k(t) = -p_a \varphi_k(x_p) \sin(\Omega_p t + \phi_0), \quad j, k = 1, N.
\]

(95)

It should be noted that the fluid velocity is time varying, i.e. \( v = v(t) \), leading to time-varying stiffness and damping matrices. The pipe is assumed to be excited resonantly by \( f_k \) at its fundamental natural frequency. To solve (91), it will be re-written in first-order form:

\[
\dot{y}_1 = y_2,
\]

\[
\dot{y}_2 = -M^{-1}Cy_2 - M^{-1}K y_1 + M^{-1}f,
\]

(96)

with the initial conditions \( y_1(0) = z_0 \) and \( y_2(0) = \dot{z}_0 \), and where \( y_1 = z \) and \( y_2 = \dot{z} \), respectively, is the vector of displacements and velocities. Equations (96) can be re-written as a single first-order vector equation:

\[
\dot{y}(t) = A y(t) + \tilde{f}(t),
\]

(97)
with the initial conditions $y(0) = y_0$, $A$ being the state matrix:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}C & -M^{-1}K \end{bmatrix},$$

(98)

and

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \quad \ddot{f}(t) = \begin{bmatrix} 0 \\ -M^{-1}f \end{bmatrix}, \quad y_0 = \begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix}.$$  

(99)

The $2N$ equations in (97) are solved for $y$ to determine $z$ and thus $z_j$ using MATLAB’s standard solver ODE45 for ordinary differential equations. The natural frequencies are calculated based on the constant part of $C$ and $K$, i.e. with $v \to v_0$. This requires that $\omega_f$ is not high enough to create high frequency smoothening and stiffening effects [20]. But as previously stated in Section 3.3 $\omega_f$ is not high in the considered cases of fluid pulsation. A non-zero flow speed and small damping results in eigenvalues of $A$ in the form of complex conjugated pairs, with the imaginary parts $\text{Im}(\lambda_j)$ defining the $j$'th natural frequency $\omega_j$. The excitation frequency for resonant excitation of the fundamental mode, is thus determined by $\Omega_p = \text{Im}(\lambda_1)$. Knowing $z_j$, and $\varphi_j$ given by (11), the displacement of the pipe is given by (89).

Choosing the number $N$ of modes high enough, the numerical results produces close to "exact" results, which by contrast to the analytically approximated results of Section 3 do not depend on the smallness of certain parameters. For the results to be presented in Section 5 we used $N = 9$, since for higher values changes were insignificant.

5.2. H1 and H2 - Test of predicted vibration amplitudes

To test H1, the analytical prediction for the vibration amplitude $a_{01}$ of the first symmetric mode $\varphi_{01}$ given by (86) is compared to the results of numerical simulations.

Fig. 3(a) shows the vibration amplitude $a_{01}$ as a function of time, when the fluid is slowly pulsating, i.e. $\omega_f \ll \omega_{01}$. The results for the analytical prediction are calculated using (86), with parameters as given in the figure legend. The same
parameters are also used in the numerical simulations, with the initial conditions for the simulation being \( z(0) = 0 \). It can be seen that the numerical vibration amplitude is increasing until it reaches a constant value. This constant value is the same as the analytical prediction, confirming (86) and thus the hypothesis. For Fig. 3(b) the simulations were repeated with a non-zero initial condition, so that the first mode is enforced to be initially strongly vibrating. It can be seen that the vibration amplitude decreases, converging to a constant value, which is the same as the one predicted by the analytical model (86). The convergence towards the stationary amplitude appears to be exponential in both cases, while the convergence to the same value is a simple consequence of the linearity of the system model: there can be at most one equilibrium state, and stationary behaviour is independent of initial conditions.

When the fluid is non-slowly pulsating in parametric resonance with the first pipe mode, i.e. \( \omega_f \approx 2\omega_{01} \), the analytical vibration amplitude \( a_{01} \) is again given by (86). Like for the case with \( \omega_f << \omega_{01} \), the numerical simulations and analytical predictions give the same stationary results, for both sets of initial conditions, yielding Fig. (3). Generally, for elastic systems, even small parametric excitations are usually producing a large system response (infinite, in the linear setting), when

![Figure 3: Vibration amplitude \( a_{01} \) as function of time for slow fluid pulsation, \( \omega_f = 0.001 << \omega_{01} \), from numerical simulation (S) and stationary analytical prediction (A). Initial conditions: (a) \( z(0) = 0 \); (b) \( z_1(0) = 0.3, z_j(0) = 0, \) for \( j = 2, ..., 9 \). Other parameters: \( \alpha = 0.3, v_0 = 0.1, q = 0.001, p_a = 0.001, x_p = \frac{1}{2}, c = 0.01 \).](image)
the excitation frequency is close to twice a natural frequency of the system. This appears not to be the case here, as predicted theoretically and explained in Section 3.3.1, by the nature of the specific fluid-induced parametric excitation. Even by the cases, where the first mode is initially excited and the fluid mean velocity \( v_0 \) is increased, the hypothesis is supported, that the stationary vibrations will remain small by adjusting the amplitude of the external force, and therefore do not influence the robustness of the flowmeter.

For the case of combination resonance, \( \omega_f \approx \omega_{02} - \omega_{01} \), analytical predictions for the dominating vibration amplitudes \( a_{01} \) and \( a_{02} \) of, respectively, the symmetric and antisymmetric second mode \( \varphi_{01} \) and \( \varphi_{02} \) given by (87) and (88) are calculated and compared to the results of numerical simulations. Fig. 4 shows the dominating vibration amplitudes as function of time, for small pulsation amplitude \( qv_0 = 0.0008 \), and with initial conditions for the numerical simulation being zero (Fig. (a)), and the modes \( \varphi_{01} \) and \( \varphi_{02} \) initially activated (Fig. (b)). The vibration amplitude ratio \( \frac{a_{02}}{a_{01}} \approx 0.035 \) is the same for both cases, and as appears the vibration amplitudes converge to stationary values predicted analytically. To test if \( \pi_{02}^* \) could reach the same order of magnitude as \( \pi_{01}^* \), the pulsation amplitude is increased. Figure 5(a) depicts the results for \( qv_0 = 0.003 \), with other parameters

![Figure 4: Vibration amplitudes \( a_{01} \) and \( a_{02} \) as function of time for \( \omega_f = 29.6116 \approx \omega_{02} - \omega_{01} \), from numerical simulation (S) and analytical prediction (A). Parameters: (a) \( z = 0 \), and (b) \( z_1(0) = 0.2, z_2(0) = 0.02, z_j(0) = 0 \) for \( j = 3, ..., 9 \). Parameters: \( v_0 = 0.4, q = 0.002 \), others as for Fig. 3.](image)
as given in the figure caption. There is good agreement between numerical and stationary analytical results, and the amplitude ratio being \( \frac{a_{02}}{a_{01}} = 0.19 \) implying that the amplitudes are not of the same order of magnitude. The pulsation amplitude is increased further to \( qv_0 = 0.015 \), and it appears from Fig. 5(b), showing the results, that the analytical predictions (87) and (88) have no longer the same accuracy as the numerical solution. However, as predicted for increasing \( qv_0 \), \( a_{01}^* \) and \( \overline{\alpha}_{01}^* \) have decreased (cf. Fig. 5(a)). The ratio between the dominating amplitudes is increased to \( \frac{a_{02}}{a_{01}} = 0.72 \), implying both amplitudes to be of the same order of magnitude. The time necessary for the solution to reach a stationary solution has increased considerably compared to Fig. 5(a).

5.3. H3 and H4 - Test of phase shift prediction and sensitivity

H3 will be tested employing data from numerical simulations solving (1) as described in Sec. 5.1. CFM pipes normally vibrate in a resonantly driven symmetric mode \( \varphi_{01} \), with a small overlay of vibrations of an antisymmetric mode \( \varphi_{02} \), excited by fluid flow. Since the vibration of the antisymmetric mode is 90° out-of-phase wrt. the symmetric drive mode, the combined pipe motion will be a travelling wave, so that pipe axis points to not cross the equilibrium line \( u = 0 \) at the same time, leading to a time shift \( \Delta t_0 \) in zero-crossing between different axis points. The techniques used to determine the phase shift between the
detector signals vary in commercially available meters from manufacturer to manufacturer and are confidential, and therefore not available for comparison with the predictions based on perturbation methods. The CFM literature [3] gives an expression for the relationship between the time shift $\Delta t_0$ in zero crossing and the corresponding phase shift $\Delta \Psi$ [3, 11, 21]:

$$\Delta \Psi = \Omega_p \Delta t_0,$$

(100)

assuming the signal to be properly filtered to extract oscillations at the drive frequency $\Omega_p$. The phase shift $\Delta \Psi$ is, after a calibration determining the calibration constant $C$, assumed to be a measured of the mass flow $\alpha v$, i.e.

$$\Delta \Psi = C\alpha v.$$  

(101)

The numerical results are narrowband-filtered using a 4th order Butterworth filter at the drive frequency $\Omega_p$ only, the time shifts $\Delta t_0$ are calculated from the filtered data, and the phase shifts are calculated. Figure 6 shows the phase shift as a function of mean mass flow $\alpha v_0$, comparing the results in case of non-pulsating fluid flow ($q = 0$) based on analytical approximation (solid line) and numerical simulations (symbol marker ◦). The analytical phase shifts have been calculated using (34), the numerical ones with (100), all with parameters as given in figure caption. Phase shifts for pulsating fluids are also shown in Fig. 6 (symbol markers ◦, △, □), these are calculated using filtered data from numerical simulations. An insert shows a zoom into the results. It appears from Fig. 6 that the analytical approximation agrees rather accurately with the numerical simulation, implying that the simple expression gives about the same accuracy as the numerical solution. This applies for slowly and non-slowly pulsating fluids, the results do therefore support H3. Small discrepancies are seen only for large mass flow, reflecting the decrease in accuracy of the analytical approximation as parameters assumed small (here $\alpha v_0$) increase. They also support H4, as a possible change in sensitivity would show as a change in line slope. This appears not to be the case.
Figure 6: Phase shift $\Delta \Psi^*$ as a function of mean mass flow $\alpha v_0$. No fluid pulsation ($q = 0$): Numerical simulation (○) and perturbation analysis (34) (solid line). Fluid pulsations: $\omega_f = 0.001 << \omega_{01}$ (◇), $\omega_f = 19.654 \approx 2\omega_{01}$ (□), $\omega_f = 29.614 \approx \omega_{02} - \omega_{01}$ (⊿). Parameters: $\Delta x = \frac{1}{4}$, others as for Fig. 3.

Equation (100) does not allow a straightforward calculation of the phase shift in case of non-slow pulsating fluid, since “time shifts” in this case are not well defined; the elastic waves at different frequencies propagate at different speeds, each with its own time shift,. It can therefore not be used to test, e.g. (49), which is based on displacement data containing also components at other frequencies than the drive frequency $\Omega_p$. Fig. 7 is a representative example of the pipe displacement in case of fluid pulsations at points located in the antinodes of the antisymmetric mode $\varphi_{02}$, representing common detector locations employed in CFMs. The analytical results are calculated using (26) with parameters given in the figure legend, which are also used in the numerical simulation. Good agreement between the analytical and numerical results is show. From Fig. (b) in particular it can be seen that the two sensor signals are not simply time shifted,
Figure 7: (a)-(c) Displacement of two symmetrically located points along a fluid-conveying simply supported pipe, from numerical simulation (symbol markers) and analytical prediction (lines), with (b)-(c) showing representative zooms. Points located in the antinodes of the antisymmetric mode $\varphi_{02}$, i.e. $x_1 = \frac{1}{4}$ (○, solid line), and $x_2 = \frac{3}{4}$ (□, dashed line). $\omega_f \approx 2\omega_{01} = 2\pi^2$, other parameters as for Fig. 3.

i.e. the time shift between the signals at any two time instances is non-constant. This supports that (100) cannot be employed directly to test, e.g., (49) quantitatively. A proper filtering of the data, as described in Sec. 3.2, is necessary to correctly capture the mean phase shift.

6. Conclusion

It has been demonstrated how the effect of small fluid pulsations on vibrating fluid-conveying pipes can be analysed approximately using a systematic perturbation analysis. The results were derived for a simple model of single straight, simply supported pipe. The model ignores substantial features of industrial CFMs, which typically have two curved pipes, clamped rather than simple supports, and are mounted with sensors and actuators. However, the main physical properties
are assumed to be unaffected by this complexity. The results of analysing this model are simple analytical expressions relating measures of the fluid pulsations to vibrations parameters of interest, here the axial phase shift of transverse pipe vibrations, which is of particular interest for Coriolis flowmeter producers, who employ this phase shift to measure the mass flow. The analytical predictions offer immediate insight into how fluid pulsation affects phase shift. They are approximately valid under realistic assumption on the smallness of certain parameters.

Predictions based on the simple model are used for creating the following hypothesis: Proper filtering of the detector signals allows CFMs to measure the mean phase shift correctly, even in case of pulsating fluid flow. However, this filtering could leave unwanted vibrations in certain modes undetected and therefore uncontrolled, in particular those that are of the combination resonance type could influence the robustness of CFMs.

Representative examples were tested against pure numerical solutions, and good agreement was demonstrated. For the specific case of combination resonance further investigations are recommended to point out sources for an implied energy removal from the (assumed) dominating modes. The simple analytical expressions presented would be even more valuable for application, if their predictions were validated employing real CFMs. An experimental test of the predictions using real CFMs is therefore highly recommendable. The works [11, 13] offer suggestions for how an experimental setup for testing the hypotheses could be set up. Alternatively detailed computational models taking two-way fluid-structure interaction into account could be used [22].

The particular combination of simple modelling and perturbation analysis has been employed also to study other kinds of imperfections relevant for Coriolis flowmeters, e.g., imperfect pipes supports or excitation, and nonlinearity [14, 23]. This approach may appear unnecessary elaborate for specific cases, where more heuristic or ad hoc approximations could provide similar results with fewer calculations. Its strength is seen to lie in the generality of which various kinds of imperfections can be handled, e.g. weak nonlinearity can readily be included.
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Effect of asymmetric actuator and detector position on Coriolis flowmeter and measured phase shift.

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Effect of asymmetric actuator and detector position on Coriolis flowmeter and measured phase shift

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Abstract
Coriolis flowmeters (CFM) are forced to vibrate by a periodic excitation usually applied midpipe through an electromagnetic actuator. Hands-on experience with industrial CFMs it appears, that the electromagnetic actuator has to be located as symmetric as possible. For CFM design and trouble-shooting it is of relevance to know how and if imperfections, related to the excitation location, influence the dynamic behavior of the vibrating fluid-conveying pipes employed in CFMs. A simple model of an imperfectly excited, simply supported, straight, single pipe CFM is investigated using a multiple time scaling perturbation analysis. The result is a simple analytical expression for the approximated phaseshift, which offers a direct insight into how the location of the actuator influences the phase shift. It appears, that asymmetrical forcing combined with fluctuating pipe damping could be a factor contributing to lack of zero shift stability observed with some industrial CFMs. Tests of the approximated solution against results obtained by pure numerical analysis using Galerkin expansion show very good agreement. The effect of asymmetric detector positions is also investigated. Any asymmetry in the detectors position, e.g. due to manufacturing variations or improper handling of the CFM, induces a phase shift that leads to changes of the meter’s sensitivity, and could therefore result into erroneous measurements of the mass flow. This phase shift depends on the mass flow and does not contribute to a lacking zero-point stability. The validity of the hypotheses, which are assumed to be basically similar for more complicated geometries, e.g. bended and/or dual pipe CFMs, with or without multiple actuators, is suggested to be tested using laboratory experiments with purpose built non-ideal CFMs.

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1. Introduction

Coriolis flowmeters (CFM) exploit flow-induced phase differences between two points on vibrating pipes. Under certain conditions, this phase shift can be assumed to be proportional to mass flow, a property measured by CFMs. This phase shift could be influenced by a number of factors other than the mass flow violating the assumed proportionality, e.g. structural non-uniformities or imperfections such as non-ideal supports or resonant flow pulsations. Some of these can be compensated for during the initial calibration of the CFM, others could fluctuate in time or space and therefore contribute to erroneous flowmeter readings. CFMs are usually driven by an electromagnetic actuator in a feedback loop to ensure the correct resonant excitation [1]. During the assembly of a real CFM, the actuator is usually individually mounted on each flowmeter pipe. The same applies for the detectors used to measure the displacement of the vibrating pipes, from which the phase shift is then calculated using signal processing. Hands-on experience with real CFMs leads to the hypothesis, that it is crucial that the actuator is mounted symmetric w.r.t. the pipe geometry, e.g. midpipe in a straight pipe configuration. No knowledge seems to exist on the effect of asymmetrically mounted detectors. This leads to the question: Do asymmetric excitations or asymmetrically mounted detectors actually influence the phase shift of a vibrating fluid-conveying pipe, and if yes, how?

Computational models for numerical simulation of real flowmeters are available but offer little insight into the essential physical phenomena taking place, and typically require extensive time for exploring parameter dependencies. A simplified model and systematic analysis tools are believed to be useful in providing such insight. The results for simplified systems may also be used to create hypotheses for complicated, realistic systems, which can be tested against, e.g. experimental results or numerical models. These hypotheses can be useful in designing future Coriolis flowmeters.

The dynamics and stability of fluid-conveying pipes have been the subject of investigation for more than sixty years. A recent overview lists a number of imperfections influencing phase shift...
A perturbation-like approach has been used to derive an analytical expression for fluid-conveying pipes [3]. Imperfections related to non-ideal boundary conditions have been investigated [4–6]. Flow pulsations have also been investigated [7,8]. Studies related to non-ideal boundary conditions have been found [9,10]. Investigations regarding the electromagnetic actuator and detectors either focus on how they affect the phase shift as added masses [11–13], or investigate measurement resonance-control-systems and algorithms controlling the feedback loop to ensure the correct resonant excitation of the fluid-conveying pipe [14–18]. No studies are available investigating the effect of deviations from ideal actuator and detector positions on the phase shift of vibrating, fluid-conveying pipes. A generally applicable technique [5,6], adaptable to different imperfection configurations, will be employed to develop a mathematical model and derive simple analytical expressions, which will enable the prediction of the effect of asymmetric actuator and detector positions on the detector signals and the axial shift in vibration phase. This could help to understand some of the phenomena seen with real Coriolis flowmeters, e.g. an apparently drifting zero shift, or unexplainable measurement errors. The techniques used in commercially available meters to determine the difference in phase between detector signals are not only strictly confidential but vary from manufacturer to manufacturer. This paper will not provide suggestions, how CFMs generally could compensate for possible additional phase shifts, due to effects, which were not present under the calibration of the flowmeter.

This paper is structured as follows: The governing equation and boundary conditions are presented in Section 2. In Section 3, a systematic perturbation analysis will be carried out, solving the governing equation using the method of multiple scales and a two-mode approximation; the latter being sufficient for Coriolis flowmeters operating in the fundamental symmetric mode. An analytical expression for the phase shift will be derived. These results will be used to create hypotheses for real CFMs. A numerical method using a standard Galerkin expansion for testing the analytical approximations is presented in Section 4. Section 5 illustrates the effect of the driving force position on the phase shift measured by a CFM. The effect of asymmetrically mounted detectors is dealt with in Section 6. Conclusions are given in Section 7.

2. Mathematical model

The model system Fig. 1 is an imperfectly excited, simply supported, pipe CFM. The analysis will be done under the assumption, that effects are basically similar for more complicated geometries, e.g. bended and/or dual pipe configurations, with attached detectors and single or multiple actuators. Using simply supported or clamped–clamped boundary conditions will in the end lead to the same hypotheses. Results based on clamped–clamped conditions, which are closer to installations seen in real CFMs, are less transparent due to elaborate mode shapes. The equation of motion governing transverse pipe motions \( u(x, t) \) is derived from expression for the kinetic and potential energies employing Hamilton’s principle [19,20]:

\[
\ddot{u} + u''' + c \dot{u} + \alpha (2 \dot{u} + \dot{u}') = -sp_0 \delta (x - \xi_p) (\Omega_p t), \tag{1}
\]

where \( c = c/p_0 A_p \) the pipe’s linear viscous damping per unit pipe length, \( p_0 \) the amplitude of the harmonic force applied external to the pipe at \( x = \xi_p \) with the normalized frequency \( \Omega_p \) and \( \delta(x) \) Dirac’s delta function. Differentiation with respect to space \( x \) and time \( t \) is denoted, respectively, \( (\cdot)' \) and \( (\cdot)'' \) in (1).

The assumptions for (1) and (2) are as follows: The pipe is considered to be a slender beam made of a linear elastic material with small damping, uniform cross section and mass and stiffness distribution, pipe deformations are assumed to occur only in the transverse direction and rotations of the pipe are assumed small, and shear deformation, longitudinal inertia and rotary inertia is neglected. The external forcing amplitude \( p_0 \) is small, since the pipe is driven at resonance. The fluid is inviscid, incompressible, with a homogeneous density, filling out the entire inner cross-sectional area and perfectly coupled to the motion of the pipe. It is assumed that the motion of the pipes is not significantly affected by internal fluid pressure and frictional drag. The coordinate \( \tilde{x} \) and pipe deformation \( \tilde{u} \) are nondimensionalized by the pipe length \( l \), the time \( t \) by the characteristic oscillation frequency \( \omega \), cf. [21], and the flow speed by the characteristic wave speed \( \tilde{c} \), \( \rho A_0 \) the mass per unit length with subscript \( f/p \) referring to fluid and pipe, \( \alpha \) the ratio of fluid mass to total mass, \( El \) the flexural stiffness per unit length, \( c \) the pipe’s linear viscous damping per unit pipe length, \( p_0 \) the amplitude of the harmonic force applied external to the pipe at \( x = \xi_p \) normalised frequency \( \Omega_p \) and \( \delta(x) \) Dirac’s delta function. Differentiation with respect to space \( x \) and time \( t \) is denoted, respectively, \( (\cdot)' \) and \( (\cdot)'' \) in (1).

The assumptions in (1), that are small compared to the other terms, have been multiplied by \( \varepsilon \). The parameter \( \varepsilon \) serves as a book-keeping parameter throughout the analysis by quantifying the assumed order of magnitude of the terms. The assumed order of magnitude of the different terms is physically realistic for Coriolis flowmeters. The book-keeping parameter \( \varepsilon \) is utilised in a way that it indicates that all terms due to fluid flow in (1) are small compared to the other terms, it does not indicate the order of magnitude of the two fluid-related terms \( 2\alpha \nu \dot{u}' \) and \( \alpha \nu^2 \ddot{u}' \) compared to each other.

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The first three terms in (1) represent, respectively, the transverse inertia of the pipe and fluid, the flexural stiffness of the pipe, and the effect of damping. The fourth and fifth term, representing, respectively, Coriolis and centripetal accelerations, are due to the fluid flowing at speed \( \nu \) through pipe segment with instantaneous curvature \( 1/\nu' \), causing pipe rotations at angular velocity \( \dot{\nu} \). Terms similar to these occur in many other studies regarding fluid-conveying pipes, e.g. [3,22,23].
3. Analytical predictions using perturbation analysis

3.1. Method

For CFM manufacturers it is of interest to be able to predict the phase shift depending on parameters, such as flow speed and various imperfections, e.g. non-midpipe excitation, pulsating flow, support conditions, or pipe stiffness distribution, since this knowledge could, e.g., lead to more precise and accurate CFMs or help to optimize the flowmeter’s sensitivity. A simplified model and systematic analysis tools are believed to be useful for providing such insight. Computational models for numerical simulation of real CFMs may also provide predictions for the phase shift for every possible operational condition. However these models offer little insight into the essential physical phenomena taking place, and typically require extensive time for exploring parameter dependencies. Therefore an approximate solution $u(x, t)$ is sought to the linear partial differential equation (1) with the boundary conditions (2) and under the assumptions stated above. The solution can be employed to predict the phase shift between transversely vibrating points along the pipe, which, under certain ideal circumstances, is proportional to fluid mass flow rate and therefore a parameter employed in Coriolis flowmetering. The generally applicable technique presented in [5] used the method of multiple scales for investigating the effect of non-ideal support conditions on the phase shift of vibrating fluid-conveying pipes. In this study this technique is employed to investigate imperfections related to the locations of pipe excitation and detectors and their effect on the phase shift.

3.2. General approximate solution

Being interested in a perturbation solution in form of a two-timescale expansion, by the method of multiple scales one then assumes a uniformly valid expansion of the form:

$$u(x, t) = u_0(x, T_0, T_1) + \varepsilon u_1(x, T_0, T_1) + O(\varepsilon^2),$$

(4)

where $u_0$ and $u_1$ are functions describing, respectively, the unperturbed motion of the pipe in its fundamental mode and the perturbation correction, $T_0 = t$ the slow timescale and $T_1 = \varepsilon t$ the fast timescale. The expansion is carried out to order $O(\varepsilon^0)$, therefore terms of order $O(\varepsilon)$ and smaller are omitted. The perturbation analysis will determine, how small perturbations, i.e. $\varepsilon$-terms in (1), will change the fundamental motion $u_0$. Insert (4) into (1)–(2), equate to zero coefficients of like power of $\varepsilon$, and obtain the zero ($\varepsilon^0$) order problem for $u_0$:

$$D_0^2 u_0 + u_0'' = 0,$$

(5)

$$u_0(1) = u_0(0) = u_0'(1) = u_0'(0) = 0,$$

(6)

and the first ($\varepsilon^1$) order problem for $u_1$:

$$D_0^2 u_1 + u_1'' = -2D_0 D_x u_0 - cD_0 u_0 - \alpha(2D_0 u_0 v + u_0'' v^2) - p_0 \delta(x - x_p) \cos(\Omega_0 T_0),$$

(7)

$$u_1(1) = u_1'(0) = u_1'(1) = u_1'(0) = 0,$$

where the operator $D_0^j = \partial^j / \partial T_0^j$ denotes the partial differentiation of order $j$ with respect to time $T_0$. Eq. (5) is a linear partial differential equation, whose general solution can be expressed as a series expansion in terms of mode shape functions $\phi_0$:

$$u_0(x, T_0, T_1) = \sum_{j=0}^{\infty} A_0(T_1) \phi_0(x) e^{i0 T_0} + cc,$$

(9)

with $A_0$ being complex-valued amplitude functions of slow time, i.e. the imaginary unit, and $cc$ denoting complex conjugates of the preceding terms. The linear natural frequencies $\omega_0$ and mode shapes $\phi_0$ for the unperturbed ideal system are [24]:

$$\omega_0 = (\pi n)^2, \quad \phi_0(x) = \sqrt{2} \sin(j \pi x), \quad j = 1, 2, \ldots, \quad (10)$$

with the latter satisfying the ordinary differential equation with boundary conditions for the free system. Since the system under consideration is excited near the fundamental frequency $\omega_0$, one lets $A_0 = 0$ for $j = 2, 3, \ldots, \in (9)$, so that the unperturbed motion of the pipe is:

$$u_0(x, T_0, T_1) = A_0(T_1) \phi_0(x) e^{i0 T_0} + cc.$$  

(11)

Insert (11) into (7)–(8) and obtain the equation for the perturbation correction $u_1$:

$$D_0^2 u_1 + u_1'' = F(x, A_0, T_1) e^{i0 T_0} + cc,$$

(12)

with

$$F(x, A_0, T_1) = -i2\omega_0 \omega_0 D_x A_0 - i\alpha \omega_0 A_0 -$$

$$- \sigma (i2\omega_0 \phi_0'' + v^2 \phi_0') A_0 + \frac{1}{2} p_0 \delta(x - x_p) e^{i\Omega_0 T_1}.$$  

(13)

Near-resonant terms have been converted to resonant terms by introducing a detuning parameter $\sigma$ measuring the nearness of the excitation frequency $\Omega_0$ to the unperturbed, fundamental natural frequency $\omega_0$:

$$\Omega_0 = \omega_0 + \varepsilon \sigma.$$  

(14)

Eq. (12) is a linear partial differential equation with $T_0$-harmonic inhomogeneous terms. A particular solution $u_1$ in terms of a Galerkin expansion of the lowest unperturbed mode shapes $\phi_0$ is:

$$u_1(x, T_0, T_1) = \sum_{j=1}^{\infty} \phi_0(x) B_j(T_1) e^{i0 T_0} + cc.$$  

(15)

To determine the first-order amplitude functions $B_j$ follow the standard Galerkin procedure; insert (15) into (12)–(13), multiply with $\phi_0(x)$, $k = 1, 2, \ldots$, integrate over $x$, employ relations of orthogonality:

$$\int_0^1 \phi_0^m \phi_0^k dx = \alpha_0^2 \delta_{mk},$$  

(16)

$$\int_0^1 \phi_0^m \phi_0^k dx = \delta_{jk},$$  

(17)

and obtain:

$$(\omega_0^2 - \alpha_0^2) B_j =$$

$$= -i2\omega_0 (2D_x A_0 + i\alpha A_0) \int_0^1 \phi_0^m \phi_0^k dx$$

$$- \alpha A_0 \left( i2\omega_0 v \int_0^1 \phi_0^m \phi_0^k dx + v^2 \int_0^1 \phi_0'' \phi_0^m dx \right)$$

$$+ \frac{1}{2} p_0 \delta(x - x_p) \int_0^1 \phi_0^0(x) dx.$$  

(18)

It can be seen from (18), that the magnitude of the functions $B_j$ decreases rapidly with increasing mode number $j$. Since we are only interested in a two-mode approximation for $u_1$, we can let $B_0 = 0$ for $j > 2$. We also let $B_1 = 0$, since vibrations in the fundamental form $\phi_0$ are already taken into account at the $\varepsilon^0$-level through the function $u_0$. This leads to the solvability condition for $A_0$:

$$i2\omega_0 (D_x A_0) - p_0^2 \alpha^2 A_0 + i\alpha \omega_0 A_0 = \frac{1}{2} p_0 \phi_0^0 (x_p) e^{i\Omega_0 T_1}.$$  

(19)

The fulfillment of (19) ensures, that the solution $u_1$ for (12) is free of secular terms, i.e. those proportional to $e^{-i\Omega_0 T_1}$. Secular terms would lead to a violation of the assumption $|\varepsilon u_1| \ll |u_0|$ for $T_0 > 0$. Eq. (19) is a first-order ordinary differential equation for $A_0$. The complex-valued function $A_0(T_1)$ can be written as:
In Coriolis flowmetering, the resonance frequency control system with the slowly varying amplitude and phase functions (driven fundamental symmetric mode order $O$) is important for CFM applications, it is obtained by inserting (21) into (18):

$$B_{12} = -18\frac{\text{pa}_{10}\text{pa}_{21}}{3(\text{oa}_{02} - \text{oa}_{01})^2} e^{i(\eta_1 + \eta_0)} + \frac{p\text{pa}_2(x_p)}{2(\text{oa}_{01} - \text{oa}_{02})} e^{i\eta_1}.$$  

For calculating the two-mode approximate pipe response $u(x, t)$ insert (11), (14), (15), (20) and (23) in (4):

$$u(x, t) = a_0\text{pa}_1 \cos(\Omega_2 t + \eta_0) + \text{pa}_2(x_p) \left( \frac{16\text{pa}_1\text{pa}_2(\text{oa}_{01} - \text{oa}_{02})}{3(\text{oa}_{02} - \text{oa}_{01})^2} \right) \sin(\Omega_2 t) + \frac{\text{pa}_2(x_p)}{\text{oa}_{02} - \text{oa}_{01}} \cos(\Omega_2 t).$$  

with the slowly varying amplitude and phase functions ($a_0$, $\eta_0$) being solutions of (21)–(22).

From (24), we can see how the pipe basically vibrates in its driven fundamental symmetric mode $\text{pa}_1$, this term is in (24) of order $O(\varepsilon^3)$. On top of this fundamental motion are small additional antisymmetric motions of the second mode $\text{pa}_2$ and order $O(\varepsilon^4)$. The two-mode expansion is based on the assumption that the CFM is driven in its fundamental symmetric mode. An analysis of CFMs working at higher modes would require more mode expansions to be included in the analysis. The antisymmetric motions of the second mode $\text{pa}_2$ are excited by asymmetric causes, i.e. the non-zero unidirectional mass flow $\alpha v$ and in case $\text{pa}_2(x_p) \neq 0$ the external asymmetrical forcing $p\text{pa}_2(x_p)$. The antisymmetric motions vibrate $90^\circ$ out of phase with respect to the fundamental motion. As a result, the combined pipe motion $u$ is a traveling wave, meaning that points on the pipe axis do not cross the equilibrium line simultaneously. The CFM working principle exploits this phenomenon. After a suitable flowmeter calibration, the time shift in zero-crossing between two symmetric points on the pipe axis is assumed to be proportional to the mass flow. So by measuring the time shift, or corresponding phase shift, one can actually estimate the mass flow.

### 3.3 Phase shift under non-midpipe sharply resonant excitation

Eq. (24) describes stationary as well as transient solutions to (1)–(2). Since we are interested in steady-state solutions, these are characterised by having a constant amplitude $a_0(\text{pa}_1) = \bar{a}_0$ and phase $\eta_0(\text{pa}_1) = \bar{\eta}_0$ in (21)–(22), which leads to:

$$\bar{a}_0 = \frac{p\text{pa}_1(x_p)}{2\text{pa}_0(1 + \sigma + \frac{\pi^2\alpha v^2}{\text{oa}_{01}^2})},$$

$$\bar{\eta}_0 = \arctan\left(\frac{\text{co}_{01}}{2\sigma\text{oa}_{01} + \pi\alpha v}\right).$$

In Coriolis flowmetering, the resonance frequency control system is commonly based on a phase feedback loop ensuring a phase shift of $\frac{\pi}{2}$ between the pipe deflection and excitation [16]. The resonant detuning parameter $\sigma^*$, enabling this, is:

$$\sigma^* = -\frac{\pi^2\alpha v^2}{2\text{oa}_{01}}.$$  

The excitation frequency $\Omega_2$, is given by (14) and the pipe is forced to vibrate at the fundamental resonance frequency of the pipe under fluid flow $\omega_{1*}^*$, i.e. $\Omega_2 = \omega_{1*}^*$, which is given by:

$$\omega_{1*}^* = \omega_1 - \frac{\pi^2\alpha v^2}{2\text{oa}_{01}}.$$  

This expression for the resonance frequency differs from previous findings presented in [25]. The reason for this lies in the equation of motion (1) and the way how the book-keeping parameters $\varepsilon$ has been utilised. The corresponding resonant amplitude $\bar{a}_0^*$ and phase $\bar{\eta}_0^*$ with respect to the external force are:

$$\bar{a}_0^* = \frac{p\text{pa}_1(x_p)}{\text{co}_{01}}, \quad \bar{\eta}_0^* = \frac{\pi}{2},$$

Insert (29) into (24), apply trigonometric identities and obtain

$$u(x, t) = p = \sqrt{A(x)^2 + B(x)^2} \sin(\omega_{1*}^* t - \Psi(x)),$$

with

$$A(x) = \frac{\text{pa}_2(x_p)}{\text{oa}_{02} - \text{oa}_{01}} \left( \frac{16\text{pa}_1\text{pa}_2(\text{oa}_{01} - \text{oa}_{02})}{3(\text{oa}_{02} - \text{oa}_{01})^2} \right) + \text{pa}_2(x_p),$$

$$B(x) = -\frac{\text{pa}_2(x_p)}{\text{oa}_{02} - \text{oa}_{01}}.$$

and

$$\Psi(x) = -\arctan\left(\frac{A(x)}{B(x)}\right).$$

Since the ratio $A(x)/B(x)$ is of order $O(\varepsilon)$ and therefore small, we can assume that:

$$\Psi(x) \approx -\frac{A(x)}{B(x)}.$$  

The difference in phase $\Delta \Psi$ between two symmetric measurement points $x_{1,2}$, each offset a distance $\Delta x_d$ w.r.t. the midpipe position, i.e.

$$x_{1,2} = \frac{1}{2} \pm \Delta x_d,$$

with $\Delta x_d \in [0, \frac{1}{2}]$, is given by:

$$\Delta \Psi = \Psi\left(\frac{1}{2} - \Delta x_d\right) - \Psi\left(\frac{1}{2} + \Delta x_d\right).$$  

Insert (34) into (36), employ (10), (31) and (32), and obtain:

$$\Delta \Psi = -\frac{4\text{pa}_1}{\text{oa}_{02}^2 - \text{oa}_{01}^2} \left[ \frac{16}{3} \alpha v + 2\cos(\pi x_p) \right] \sin(\pi \Delta x_d).$$

When producing Coriolis flowmeters, one tries to keep the deviation from the ideal actuator position as small as possible. This can be expressed by the following definition of actuator position $x_p$:

$$x_p = \frac{1}{2} \pm \varepsilon \Delta x_p,$$

with $x_p = \frac{3}{2}$ being the ideal actuator location, $\Delta x_p$ the deviation from the ideal location, and $\varepsilon$ indicating that the deviation from the ideal location is small. Insert (38) into (37), Taylor expand, and keep terms with the two highest orders of magnitude. Since the term related to the mass flow $\alpha v$ is of order $O(\varepsilon^3)$ and therefore the highest order in (37), terms of order $O(\varepsilon^4)$ are also to be kept. This yields into:

$$\Delta \Psi = -\frac{4\text{pa}_1}{\text{oa}_{02}^2 - \text{oa}_{01}^2} \left[ \frac{16}{3} \alpha v + 2\pi \cos(\pi x_p) \right] \sin(\pi \Delta x_d).$$  

For real CFMs (39) implies, that the effect of a small error in driver location cannot be ignored, in particular for cases with very small mass flows, i.e. when $\alpha v = O(\varepsilon^2)$. 

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Eq. (39) leads to the following hypotheses: If the actuator is placed at its ideal position, i.e. \( \Delta x_p = 0 \), the second term in (39) will be zero. However, if the force is not exactly applied at midpipe, i.e. \( \Delta x_p \neq 0 \), there will, according to (39), be a contribution to the phase shift related to this asymmetrical excitation. The second term in (39) will be denoted “zero shift”, since it is also be present in case of zero mass flow, i.e. \( \alpha v = 0 \). One can compensate for this error during the initial meter calibration. However, this zero shift appears to depend on the pipe’s viscous damping \( c \). The pipe’s damping may fluctuate in time, e.g. due to temperature changes, wear or vibration level, and therefore show up as changes in the measured mass flow. Asymmetrical forcing combined with fluctuating pipe damping could therefore be a factor contributing to lack of zero-point stability observed with industrial CFM’s.

4. Method for numerical validation

To validate (39), the equation of motion (1) is discretized directly using a standard Galerkin approach [26]. The unknown solution \( u(x, t) \) is expanded into:

\[
\begin{equation}
    u(x, t) = \sum_{j=1}^{N} q_j(t) \phi_j(x),
\end{equation}
\]

where \( q_j(t) \) is a new set of variables, \( \phi_j(x) \) suitable expansion functions satisfying all boundary conditions (2), and \( j = 1, \ldots, N, \). Insert (40) into (1), multiply by \( \phi_k(x) \), integrate over the pipe length, and obtain:

\[
\begin{equation}
    \mathbf{M} \ddot{q} + \mathbf{C} \dot{q} + \mathbf{K} q = f \cos(\Omega_p t),
\end{equation}
\]

with \( \mathbf{q}(t) \) being the modal coordinates, and where the components \( (j, k) \) of the modal mass, stiffness, and damping matrix, and modal force vector are given by:

\[
\begin{align}
    M_{jk} &= \int_{0}^{1} \phi_j \phi_k dx, \\
    C_{jk} &= 2 \alpha v \int_{0}^{1} \phi_j' \phi_k' dx + c \int_{0}^{1} \phi_j \phi_k dx, \\
    K_{jk} &= \alpha v^2 \int_{0}^{1} \phi_j'' \phi_k dx + \int_{0}^{1} \phi_j''' \phi_k dx, \\
    f_k &= -\rho_0 \phi_k(x_0).
\end{align}
\]

Eq. (41) is a system of ordinary differential equations, governing the time evolution of the modal coordinates \( q_j(t) \).

The pipe is assumed to be resonantly excited in its fundamental mode, with the corresponding resonance frequency changing with fluid flow, as indicated by (28). To determine the resonance frequencies, let \( f = 0 \) in (41), insert a time harmonic solution \( q_j(t) = \phi_j e^{i\omega t} \), and solve the resulting algebraic eigenvalues problem numerically for \( \lambda_j \). A non-zero flow speed and small damping results in eigenvalues made of complex conjugated pairs, with the real part Re(\( \lambda_j \)) the damping ratio \( \zeta_j = \text{Re}(\lambda_j)/\omega_0\) of mode \( j \). The excitation frequency for resonant excitation of the fundamental mode is therefore \( \Omega_p = \omega_0 = \text{Im}(\lambda_1) \).

To obtain the stationary response under resonant excitation of the fundamental mode insert in (41) a harmonic solution form:

\[
\begin{equation}
    \dot{q}_j(t) = a_j \sin(\Omega_p t) + b_j \cos(\Omega_p t),
\end{equation}
\]

which is a system of linear algebraic equations, from which \( a_j \) and \( b_j \) can be determined. Insert (46) into (40) and obtain:

\[
\begin{equation}
    u(x, t) = A(x) \sin(\Omega_p t + \Psi^*).
\end{equation}
\]

The difference in phase \( \Delta \Psi^* \) between two measurement points, located symmetrically around midpipe and given by (35), can than be obtained by inserting (50) into (36), which yields into:

\[
\begin{equation}
    \Delta \Psi^* = \arctan \left( \frac{\sum_{j=1}^{N} b_j \phi_j(\frac{1}{2} + \Delta x_d)}{\sum_{j=1}^{N} a_j \phi_j(\frac{1}{2} + \Delta x_d)} \right) = \arctan \left( \frac{\sum_{j=1}^{N} b_j \phi_j(\frac{1}{2} - \Delta x_d)}{\sum_{j=1}^{N} a_j \phi_j(\frac{1}{2} - \Delta x_d)} \right).
\end{equation}
\]

Eq. (51) gives for \( N \to \infty \) the “exact” result for the comparison with the approximated phase shift (39).

5. Numerical example: effect of imperfect pipe excitation on phase shift

The theoretical approximation (39) is now tested against the solution of the full numerical model employing (51) for different levels of excitation asymmetry \( \Delta x_p \) and different values of the pipe’s damping ratio \( \zeta \). Since the pipe is driven at resonance the amplitude of the excitation force can be chosen small, i.e. \( p_0 = 0.001 \).

Fig. 2 illustrates the variation of the phase shift \( \Delta \Psi^* \) with mass flow \( \alpha v \), for five levels of excitation asymmetry \( \Delta x_p = \{-0.1, -0.05, 0, 0.05, 0.1\} \) and with the pipe’s damping ratio \( \zeta \) being \( (a) 0.05 \%, (b) 0.2 \% \) and \( (c) 0.5 \% \), with \( (a) \) being a realistic value for CFM pipes. The middle line corresponds in all three cases to the case of midpipe excitation, i.e. \( \Delta x_p = 0 \). The analytical approximation (39) (symbol markers) agrees well with the numerical solution (lines). This means that the simple expression has about the same accuracy as the numerical solution, but it provides much more insight into which imperfection affects the phase shift, and how. The general observation made from (39) is also illustrated by Fig. 2(a)–(c), i.e. an asymmetric excitation will cause a phase shift even at zero fluid flow. It also appears from Fig. 2(a)–(c), that the magnitude of the zero shift induced by the asymmetric excitation strongly depends on the magnitude of the pipe damping, i.e. the smaller the pipe damping, the less pronounced the induced zero shift will be.

6. Effect of asymmetrical detector position on phase shift

The mathematical model and results presented in Sections 2 and 3 can also be used to investigate the effect of asymmetrical detector positions. The detectors of real CFMs are usually located near the antinodes of the flowmeter pipes’ second mode shape.
Taylor expand for small \( \Delta x_1 \) and one obtains:

\[
\Phi = \Psi \left( \frac{1}{2} + \Delta x_2 + \varepsilon \Delta \delta_1 \right) - \Psi \left( \frac{1}{2} + \Delta x_d + \varepsilon \Delta \delta_2 \right) - \tau \left( \frac{1}{2} + \Delta x_d + \varepsilon \Delta \delta_2 \right).
\]  

(53)

Insert (34) into (53), employ (10), (31) and (32) and obtain:

\[
\Phi = \frac{32 \rho_0 (\xi \omega_0)}{3(\omega_0^2 - \omega_0^2)} \left[ \cos \left( \pi \left( \frac{1}{2} + \Delta x_d + \varepsilon \Delta \delta_2 \right) \right) \right] - \cos \left( \pi \left( \frac{1}{2} + \Delta x_d + \varepsilon \Delta \delta_2 \right) \right). 
\]  

(54)

Taylor expand for small \( \varepsilon \) the cosine-terms in (54):

\[
\Phi = \frac{32 \rho_0 (\xi \omega_0)}{3(\omega_0^2 - \omega_0^2)} \left[ 2 \sin (\pi \Delta x_d) \right] 
\]  

and

\[
+ \pi (\Delta x_2 - \Delta x_1) \cos (\pi \Delta x_2) \right]. 
\]  

(55)

Here the first term represents the phase shift caused by the mass flow measured between symmetrically located detectors and is of order \( O(\varepsilon^2) \). The order of magnitude of the second term is \( O(\varepsilon^3) \). The effect of asymmetrically located detectors is largest for detector positions near midpipe, i.e. \( \Delta x_d \to 0 \), and vanishes monotonously as detectors are placed near the supports, i.e. \( \Delta x_d \to \frac{1}{2} \). We can see from (55) that an asymmetry of the detector positions will give rise to an additional phase shift. This phase shift will only be present in case of non-zero mass flow \( \alpha \), which means that asymmetrically located detectors will not induce a zero shift. Asymmetrical detector positions could however change the CFM sensitivity. It is possible that the position of the detectors changes slightly asymmetrically during the lifetime of the CFM. In this case it appears from (55) that the measured mass flows will be erroneous, if the sensitivity, determined by the CFM calibration, does not take possible future geometrical changes into account.

To test and illustrate the analytical prediction (55), the solution of the full numerical model (51) is employed. Fig. 3(a) and (b) illustrate the variation of the phase shift \( \Phi \) for different levels of detector position asymmetry defined by (52). To obtain Fig. 3(a) the symmetric offset from the midpipe position is chosen to be \( \Delta x_d = \frac{1}{2} \), corresponding to the position of the antinodes of the pipes second mode shape, so that \( x_1 = 1/4 + \Delta x_1 \) and \( x_2 = 3/4 + \Delta x_2 \). In addition, \( \Delta x_1 = -0.1, -0.05, 0 \) and \( \Delta x_2 = 0 \). From top to bottom the symbols and lines show different levels of force location asymmetry \( \Delta x_d = (-0.1, -0.05, 0, 0.05, 0.1) \). Other parameters: \( \Delta x_d = \frac{1}{2}, \alpha = 0.3, \rho_0 = 0.001 \).

Fig. 2. Effect of asymmetrical external forcing \((x_d = \frac{1}{2} + \Delta x_d)\) on phase shift \( \Phi \) for varying mass flow \( \alpha \), obtained by analytical approximation (39) (symbol marker), and by numerical solution (51) (line) for different ratios \( \xi \) (a) 0.05%, (b) 0.2%, (c) 0.5%. From top to bottom the symbols and lines show different levels of force location asymmetry \( \Delta x_d = (-0.1, -0.05, 0, 0.05, 0.1) \). Other parameters: \( \Delta x_d = \frac{1}{2}, \alpha = 0.3, \rho_0 = 0.001 \).

Fig. 3. Effect of asymmetrical detector position on phase shift \( \Phi \) for varying mass flow \( \alpha \), obtained by analytical approximation (55) (symbol marker) and numerical solution (51) (line) (a) From top to bottom: \( x_1 = 1/4 + \Delta x_1 \) with \( \Delta x_1 = [-0.1, -0.05, 0] \) and \( x_2 = 3/4 \). (b) From top to bottom: \( x_1 = 1/4 + \Delta x_1 \) and \( x_2 = 3/4 + \Delta x_2 \) with \( \Delta x_1 = [-0.025, 0, 0.025] \) and \( \Delta x_2 = [0.025, 0, -0.025] \). Employed parameters: \( \Delta x_d = \frac{1}{2}, \alpha = 0.3, \rho_0 = 0.001 \).
7. Conclusion

Industrial CFMs exploit flow-induced phase shifts of vibrating fluid-conveying pipes to measure mass flow. It has been demonstrated how an asymmetrically applied driving force and asymmetrically positioned detectors influence this phase shift. The analytical model, from which the results are obtained using a systematic perturbation analysis and a two-mode expansion, the literature does not seem to provide a simple and accurate expression for the phase shift in case of asymmetrically applied driving force and asymmetrically positioned detectors. The analytical predictions presented in this paper offer this and therefore also an immediate insight into how asymmetries influence the phase shift. It appears that asymmetric forcing combined with fluctuating pipe damping could be a factor contributing to the lack of zero-point stability observed with industrial CFMs. In addition it seems that asymmetrically positioned detectors give rise to phase shifts which result into changes of the CFM’s sensitivity and possibly erroneous measurements of the mass flow. Representative examples were tested against pure numerical solutions and good agreement was demonstrated. The employed analytical model ignores substantial features of industrial CFMs, assuming that the main physical properties are assumed to be unaffected by this complexity, e.g. the presence of two curved pipes, actually mounted single or multiple actuators and detectors, and that the results are either directly transferable, or can be used for creating hypotheses. The presented hypotheses should therefore be tested, e.g., experimentally or with details computational models taking two-way fluid-structure interaction into account. The presented simple expressions would be even more valuable for application, if their predictions were to be tested with real and later modified, or initially imperfect purpose-built CFMs. The simple expressions could possibly replace computational simulations, which are highly time consuming and offer little insight into the physical phenomena taking place.

Simple modelling and systematic perturbation analysis has been employed to investigate other imperfections relevant for CFMs, e.g. pipe supports, structural imperfections and pulsating fluid flows, and are partly reported in [5,6,8,27].

Acknowledgements

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References

Dynamics of fluid-conveying pipes: Effect of velocity profiles.

DYNAMICS OF FLUID-CONVEYING PIPES: EFFECTS OF VELOCITY PROFILES

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Summary Varying velocity profiles and internal fluid loads on fluid-conveying pipes are investigated. Different geometric layouts of the fluid domain and inflow velocity profiles are considered. It is found that the variation of the velocity profiles along the bended pipe is considerable. A determination of the resulting fluid loads on the pipe walls is of interest e.g. for evaluating the dynamical behaviour of lightly damped structures like Coriolis flow meters.

INTRODUCTION

How do internal flow conditions, such as flow velocity profiles, affect the dynamic behaviour of fluid-conveying pipes? This is relevant to know, e.g., for exploiting flow-induced oscillations of pipes to determine the fluids mass flow or density, as done with Coriolis flow meters. Velocity profile effects for straight pipes have been investigated [1, 2]. It is stated in [1] that inconvenient velocity profiles result in disturbed flow conditions within the flow meter. Bended measuring tubes are not considered in [1, 2]. It is relevant to investigate bended pipes, as they influence the fluid vibrational field and result in variable velocity distributions of the measured flow along the pipe length [1]. The aim is to understand velocity profile effects for flow meters having bended pipe. This can be used to gain knowledge of how internal flow conditions influence the dynamics of fluid-conveying pipes, e.g., the amplitude and frequency of flow-induced pipe oscillations, pipe mode shapes, and stability of equilibriums.

COMPUTATIONAL MODEL AND ANALYSIS

The pipes fluid volume is determined by the internal pipe diameter $D$, the lengths of the straight pipe elements $L_i$, and the radii $r_i$ and associated angles $\theta_i$ of the bended sections, $i = 1, 2, ..., $ see Figure 1. The spatial distribution and time evolution of the fluid flow through the pipe is governed by the continuity and momentum equations. The flow is assumed to be incompressible, Newtonian, and isothermal. The standard $k - \varepsilon$ model of turbulence leads to the governing equations [3, 4]:

$$\nabla \cdot \mathbf{u} = 0$$  \hspace{2cm} (1)

$$\rho_f \left( \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) \right) = -\nabla p + \nabla \cdot (\mu_{eff} \nabla \mathbf{u} + \mu_{eff} (\nabla \mathbf{u})^T) + \mathbf{B}$$  \hspace{2cm} (2)

which are solved numerically using ANSYS CFX with the finite-volume method [4]. Here $\rho_f$ is the fluid density, $\mathbf{u}$ the velocity vector, $p$ the pressure, $\mu_{eff} = \mu + \mu_t$ the effective viscosity accounting for turbulence, $\mu$ the dynamic viscosity and $\mathbf{B}$ the body forces. The turbulence viscosity $\mu_t$ is linked to the kinetic energy $k$ and the dissipation $\varepsilon$, which are determined from two additional transport equations [4, 5]. From the evolution of the fluid flow in a straight pipe at rest with a constant inlet flow velocity $V$, one obtains the fully developed inlet velocity profile at $x = 0$. For the whole fluid domain, no-slip conditions are applied to the wall, which furthermore is assumed to be smooth. At the outlet of the domain, an ambient pressure is prescribed. The domain is split into finite volume cells equally distributed in the axial direction. Due to high velocity gradients near the pipe walls, inflation layers are applied to these areas.

Figure 1: Model system of a bended fluid-conveying pipe.
**NUMERICAL RESULTS**

Figure 2 exemplifies typical results for different pipe geometries\(^1\). Figure (a) and (b) show the velocity profile at the inlet and outlet of a bended pipe. It appears that bended pipes lead to lower velocities and asymmetric velocity profiles. Furthermore, it appears that the magnitude of the asymmetry in the velocity profile depends on the chosen pipe geometry, in particular the bends in the pipe. This could be of relevance for the dynamic behaviour of fluid-conveying bended pipes used to measure the mass flow, as done with Coriolis flow-meters. Figure (c) shows the pressure on a bended pipe wall. The fluid loads on the wall are seen to be non-uniform. The load on a specific cross-section of the wall is seen to depend on the axial location of this section. If large enough, variations in fluid load and velocity profile may cause a vibrating pipe to behave quite differently than predicted by using the greatly simplifying assumptions of uniform flow.

![Figure 2: Example results for (a) Inlet velocity profile (b) Outlet velocity profile (c) Pressure on pipe wall.](image)

**CONCLUSIONS**

Pipe bends can lead to considerable variation in the fluid load and velocity profile. For applications such as Coriolis flow meters, it is of interest to know how deviations from uniform flow affect the error in estimating the mass flow from measured pipe deformation quantities. To uncover this, one can compare simulated results from a full fluid-structure interaction model of a vibrating bended pipe, to results from a classical simplified model assuming uniform flow and no structure-to-fluid action. This paper describes the first stage of this research, which is to adequately model the fluid-to-structure action for a pipe at rest.

**References**


\(^1\)Pipe geometry: \(D = 3.5\, \text{mm}, L_1 = L_5 = 30\, \text{mm}, L_2 = L_4 = 40\, \text{mm}, L_3 = 20\, \text{mm}, r_1 = r_4 = 6\, \text{mm}, r_2 = r_3 = 4\, \text{mm}, \theta_1 = \theta_2 = \theta_3 = \theta_4 = 45^\circ\). Ambient pressure at outlet: 1.013 bar.
Dynamics of fluid-conveying pipes: Numerical investigation of velocity profile effects.

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Dynamics of fluid-conveying pipes: Numerical investigation of velocity profile effects

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Abstract

Vibrating fluid-conveying pipes are investigated using a commercial finite element and finite volume code, with focus on axial shifts in vibrations phase caused by fluid flow with imperfect velocity profiles. This is of relevance for the general understanding of elastic wave propagation and of particular interest for troubleshooting and design of phase-shift measuring devices such as Coriolis flowmeters (CFMs). The results of the numerical simulations yield predictions for how imperfect fluid velocity profiles can affect phase shift. This leads to the hypothesis that velocity profile effects could influence CFM accuracy and precision, since the flowmeter sensitivity appears to depend on the velocity profile of the conveyed fluid. The validity of this hypothesis could be tested using simplified analytical models providing direct insight into the effects at play or laboratory experiments with commercial CFMs.

Keywords: mass flow meter, Coriolis, fluid structure interaction, ANSYS, CFX, velocity profile effects, wave propagation

1. Introduction

How do internal flow conditions, such as imperfect velocity profiles, affect the dynamic behaviour of fluid-conveying pipes? This is relevant to know when exploiting fluid-structure interactions between vibrating pipes and fluid flow, e.g.,
to measure mass flow. When a fluid-conveying pipe is forced to vibrate, a vibrational fluid velocity field is induced which interacts with the undisturbed fluid velocity field, causing Coriolis-like forces on the fluid particles. These forces are transferred as fluid loads on the pipe wall giving rise to a distortion of the forced pipe motion, and the combined pipe motion is then a travelling wave. In Coriolis flowmetering one exploits, that the resulting time shift in zero crossing - or the corresponding axial shift in vibration phase - between transverse vibrations of any two pipe points under certain ideal circumstances can be assumed proportional to mass flow, and thus be considered a measure of mass flow through the pipe.

According to the known scientific literature, CFMs are said to be affected by velocity profile effects, which denote variations of the CFM sensitivity due to non-ideal velocity profiles of the measured fluid flow [1]. A CFM manufacturer on the other hand claim that CFMs are largely immune to these effects [2].

A review of the literature on transient phenomena in liquid-filled pipe systems is given in [3]. It is emphasized that fluid-structure-interaction must be taken into account for correctly modelling liquid and pipe vibrations.

An early study of CFMs [4] employs finite element modelling (FEM) and a simple flow model, i.e. uniform flow with constant velocity. The employed numerical model does not enable the investigation of velocity profile effects; nevertheless it is concluded, that CFMs are sensitive to these effects.

The focus of [5] is on numerical efforts to model the flow field in rotating pipes, without actually modelling pipe walls. The effects of Coriolis and centrifugal forces are examined. The results imply that fluid velocity flow profiles influence pipe motion.

The effect of flow conditions on a CFM having a straight and slender measuring pipe is investigated in [6], where computational fluid dynamics (CFD) analyses are carried out to study the flow in measuring pipes using different inlet conditions, i.e. fully developed, asymmetric triangular and swirl flows. Varying Reynolds numbers, frequencies of vibration and different measuring pipe dimensions were investigated. The results imply that the sensitivity might decrease
in case of flow with small Reynolds numbers. The numerical model employed in [6] does actually not allow the determination of the pipes mode shape due to fluid forces. To overcome this problem, it is suggested to couple a solid dynamics analysis of the pipe to the CFD analysis.

A numerical “finite element and finite volume” model (hereafter FE/FV model) of a CFM is presented in [7, 8]. The considered model represents the simplest CFM design, i.e. a straight and slender measuring pipe with clamped ends vibrating in its first symmetric lateral mode. In [7] the FV code COMET 2.1 is coupled to the FE code Abaqus 6.3, whereas the FV code Star-CD v3.26 and the FE code Abaqus 6.5 are used in [8]. All employed codes are commercial. The considered coupled fluid-structure problem is in both studies solved by adopting a partitioned analysis approach. The numerical model in [7] is validated by comparison with solutions of a Euler beam and one-dimensional flow model, and good agreement is shown. Results in [7] are given in terms of fundamental natural frequencies of vibrating pipes as well as axial shifts in vibration phase, but velocity profile effects are not investigated. The model and simulation procedure presented in [8] are used for estimating the velocity profile effect on CFMs. From calculated time differences between symmetrically positioned pipe points it can be seen that the calculated sensitivities decrease when the flow switches from turbulent to laminar.

In this paper the dynamic behaviour of fluid-conveying pipes will be investigated employing a coupled FE/FV model established using the commercial FE code ANSYS and the FV code CFX. The considered pipe configurations have both straight and/or bended pipe segments. This yields new knowledge on how phase shifts are affected by fluid flow with imperfect velocity profiles. This is of relevance for the general understanding of elastic wave propagation and of particular interest for trouble-shooting and design of phase-shift measuring devices such as CFMs. In Sec. 2 the FE/FV model of vibrating fluid-conveying pipes and the method of analysis are presented. In Sec. 3 results obtained with the coupled numerical FE/FV model are presented. This yields hypothesis on how imper-
fect velocity profiles could influence CFM accuracy and precision. Concluding remarks are given in Sec. 4.

2. Model formulations and method of analysis

2.1. Model definition

The simplified CFM pipes considered in this paper are schematically presented in Figs. 1 and 2. The structural domain of the model in Fig. 1 consists of a straight pipe defined by the pipe length $L$, internal pipe radius $r_i$ and wall thickness $h$. The effect of actually mounted detectors and actuators is neglected. The fluid domain of the model corresponds to a cylinder inside the pipe with length $L$ and radius $r_i$. The investigated bended pipe configurations are similar to the straight one, however they are made of straight and bended pipe elements as shown in Fig. 2.

2.2. Governing equations for the fluid domain

The equations describing the fluid are the continuity and Navier-Stokes equations, assuming conservation of mass and momentum, respectively. The fluid is

![Figure 1: (a) Definition of velocity vector $\mathbf{v} = \{v_r, v_\theta, v_z\}$. (b) Model of single, straight, fluid-conveying, clamped-clamped pipe.](image)

![Figure 2: Model of single, bended pipe, fluid-conveying clamped-clamped pipe. Velocity vector $\mathbf{v}$ given by Fig. 1(a).](image)
assumed to be isothermal, Newtonian and incompressible. This yields the following governing equations, assuming also turbulent flow [13]:

\[ \nabla \cdot \mathbf{v} = 0, \]  

(1)

\[ \rho_f \left( \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) \right) = -\nabla p + \nabla \cdot \mu_{\text{eff}} \left( \nabla \mathbf{v} + (\nabla \mathbf{v})^T \right) + \mathbf{B}, \]  

(2)

with \( \mathbf{v} = (v_x, v_y, v_z) \) being the flow velocity vector, \( p \) is the pressure, \( \mu_{\text{eff}} = \mu + \mu_t \) is the effective viscosity accounting for turbulence, \( \mu \) is the dynamic viscosity (being fluid temperature dependent), \( \mu_t \) is the turbulent viscosity (depending on the employed turbulence model), and \( \mathbf{B} \) denotes the body forces. The choice of the turbulence model is not a crucial factor [12], therefore the standard \( k - \varepsilon \) model of turbulence is employed; details on how this particular turbulence model is incorporated in CFX can be found in [13].

2.3. Governing equations for the structural domain

The equation of motion for the pipe is [15]:

\[ \mathbf{M} \ddot{\mathbf{u}}_n + \mathbf{C} \dot{\mathbf{u}}_n + \mathbf{K} \mathbf{u}_n = \mathbf{P}_n \]  

(3)

with \( \mathbf{M} \), \( \mathbf{C} \) and \( \mathbf{K} \) being the mass, damping and stiffness matrix, respectively, \( \ddot{\mathbf{u}}_n \), \( \dot{\mathbf{u}}_n \) and \( \mathbf{u}_n \) vectors of nodal acceleration, velocity and displacement amplitudes at time step number \( n \), respectively, and \( \mathbf{P}_n \) the time-dependent forcing function.

2.4. Initial and boundary conditions of the fluid domain

The flow in real straight-pipe CFM is usually fully-developed, having a boundary layer and a certain velocity profile. However, when designing or studying CFMs, one typically assumes the ideal case of plug flow, i.e. a uniform flow with the fluid velocity being assumed to be constant across any cross-section of the pipe perpendicular to the pipe axis. For the investigation of vibrating fluid-conveying pipes this is a reasonable assumption when studying the effects of imperfections relative to the the primary effect of mass flow, e.g. the effect of non-ideal support conditions or pipe imperfections [9, 10]. Since the focus of
this paper is on gaining knowledge of the possible effect of imperfect fluid velo-
city profiles, the plug-flow assumption is not applicable. Instead the flow field is
assumed to be three-dimensional and either fully-developed or disturbed velocity
profiles are applied at the inlet of the fluid domain, with the first serving as mean
of comparison when evaluating the effect of the later. The maximum fluid velo-
city investigated is \( v_{\text{max}} \approx 10 \text{ m/s} \); the flow can therefore be assumed subsonic.
Medium turbulence is assumed by defining the turbulence intensity at the inlet
to be 5 % in all simulations.

The fully developed velocity profiles are estimated by the power-law equa-
tion [16]:

\[
v(r) = v_c \left(1 - \frac{r}{r_i}\right)^{1/m},
\]

with \( v(r) \) being the radial velocity in the pipe cross-section (cf. Fig. 1), \( v_c \) the
centerline velocity corresponding to the maximum velocity encountered in the
cross-section, \( r = \sqrt{x^2 + y^2} \) the pipe radius, \( x \) and \( y \) directions in the coordinate
frame (cf. Figs. 1 and 2). The power-law exponent \( m \) in (4) is estimated using
\( m = -1.7 + 1.8 \log(Re) \) [16], with the Reynolds number being \( Re = 2v_c r_i / \nu \).
Figure 3 exemplifies the fully developed profiles described by (4).

The employed disturbed velocity profiles, which are exemplified in Fig. 4, are
obtained from steady state CFD analyses of fluid domains in bended pipes carried
out prior to the FSI analyses, see App. A for details.

The outlet of the fluid domain an average static pressure of 1 bar is applied,
corresponding approximately to ambient atmospheric pressure at sea level.

The pipe wall is assumed to be hydraulically smooth and a no-slip condition
is applied, i.e. the fluid has zero relative velocity at the solid domain. For the
transient analyses it is also defined as a Fluid Solid Interface boundary. The later
is a pre-defined boundary type available in ANSYS SIMULATION and CFX,
Release 11.0 and enables the automatic load transfer between the fluid analysis
and the structural analysis of the pipe when the simulation is executed. Two-
way fluid-structure-interaction (FSI) is employed: The pipe vibrations yield mesh
Figure 3: Example of fully developed fluid velocity profile employed as inlet conditions of fluid domain with maximum velocity $v_{\text{max}} = 1 \text{ m/s}$.

Figure 4: Examples of disturbed fluid velocity profiles employed as inlet conditions of fluid domain. (a) Maximum velocity $v_{\text{max}} = 1.15 \text{ m/s}$, (b) $v_{\text{max}} = 5.75 \text{ m/s}$, (c) $v_{\text{max}} = 9.93 \text{ m/s}$.

displacements, which are transferred to the fluid domain hereby changing its configuration and therefore the fluid flow; the fluid flow in the pipe creates forces which are transferred back as pipe loads on the inner pipe wall.

Mesh motion is enabled at the pipe wall allowing the fluid domain to follow the motion of the vibrating pipe. The mesh at the inlet and outlet is defined as stationary.

In addition to the above stated, the transient analyses of the fluid domain are initialised using the results from steady-state CFD analyses of the same domain with the same boundary conditions but the pipe wall being at rest, see App. B for details. The fluid flow is therefore already developed when the pipe is forced to vibrate.

2.5. Boundary conditions and loading of the solid domain

CFM pipes normally vibrate in a resonantly driven symmetric mode. The numerically investigated pipe configurations are therefore resonantly forced to vi-
brate in their fundamental symmetric mode by the excitation force $\mathbf{F}_n$, with $\mathbf{F}_n(t) = (F_x(t), F_y(t), F_z(t))$ being harmonic in time and applied at midpipe. All components other than the x-component are assumed zero, so that the pipe vibrates in the $y$–$z$ plane:

$$
\mathbf{F}_n(t_n) = (F_a \sin(2\pi f_d t_n), 0, 0),
$$

(5)

where $F_a$ is the forcing amplitude, and $t_n$ is the $n$th time step. Since the pipe is driven at resonance, the forcing amplitudes can be chosen small. The natural frequency $f_d$, also denoted drive frequency by CFM manufacturers, yielding vibrations of the fundamental symmetric mode is determined by modal analyses of the fluid-filled pipe in ANSYS prior to the coupled FSI simulation, see App. C for details.

The pipe ends at $z = 0$ and $z = L$ are in all analyses assumed to be clamped, as it is typically the case with real CFM pipes, i.e.:

$$
\mathbf{u}_n(0) = \mathbf{u}_n'(0) = \mathbf{u}_n(L) = \mathbf{u}_n'(L) = 0.
$$

(6)

The inner pipe wall is, like the wall of the fluid domain, defined as a Fluid Solid Interface boundary.

2.6. Method of analysis

Figure 5 shows the work flow chart for a FSI analysis using the ANSYS Workbench framework employing DesignModeler, SIMULATION, ANSYS, CFX Mesh and CFX.

The pipe to be investigated is created in DesignModeler, with the structural and fluid domain representing the pipe wall and the fluid inside the pipe, respectively.

SIMULATION and CFX Mesh are used to generate the mesh for the solid and fluid domain, respectively.

A modal analysis of the solid domain is performed in ANSYS to determine the investigated pipe’s natural frequency corresponding to the fundamental symmetric mode. Details regarding this analysis can be found in App. C. The determined natural frequency is used to calculate the excitation force using (5), which
will then be applied as an external force in the subsequent the dynamic response analysis.

A dynamic response analysis for the solid domain is defined in SIMULATION. Linear elastic theory can be used, since deformations of the measuring pipe are assumed to be small. The time step for dynamic analysis of structural domain and the CFD analysis is decided. The solver file for the ANSYS solver is written.

A steady state analysis of the fluid domain with the pipe being at rest is carried out in CFX, establishing the initial conditions for the transient fluid analysis, see also App. B.

At last the transient analysis of the fluid domain is defined employing the initial values from the prior steady state analysis. The number of stagger-iterations necessary for each coupling time step to obtain equilibrium conditions at the
fluid-structure-boundary is decided as well as convergence criteria for the solution of the fluid and solid domain. The solver file for the CFX solver is written.

The coupled analysis is started from within CFX. The CFX and the ANSYS solver (both release 11.0) are then used to compute the response of the fluid and structural domain, respectively, by solving (1)-(3) with the specifications given in the solver files. The shared geometry model under the framework of ANSYS Workbench eliminates data transfer errors. The solvers are coupled using a staggered coupling procedure: The result of the transient simulation for the fluid domain at the previous time step $t_{n-1}$ is used by the fluid solver as initial condition at time step $t_n$ with the purpose of determining the load on the pipe wall induced by the fluid. The load is then transferred to the solid domain and used at time step $t_n$ as boundary condition together with the state of the pipe obtained in the preceding time step $t_{n-1}$. The motion of the pipe wall is then calculated, transferred to the fluid domain and the fluid mesh is updated. The fluid solver proceeds then to time step $t_{n+1}$, and determines the load on the pipe walls using the updated mesh and fluid data of the preceding time step $t_n$. The loads are automatically transferred across the meshes using the Fluid Solid Interface boundary; should dissimilarities of the meshes occur a globally conservative interpolation method is used to interpolate the loads.

2.7. Discretisation of computational domain

The typically discretisation of the solid domain is shown in Fig. 6(a), while the discretisation of the fluid domain is shown in Fig. 6(b). For the discretised fluid domain a high density of elements is here used near the pipe wall to capture the significant variations of velocity gradients in this region [17]. The solid and fluid domains are discretised so that the two meshes coincide at the inner pipe wall. The discretisation shown in Fig. 6(c), which shows an equidistant mesh in the z-longitudinal direction applies to the wall of the fluid domains, as well as the inner and outer pipe wall of the solid domain. The mesh densities shown in Fig. 6 are instructive, the actually employed number of elements depends on
the actual geometry investigated. For modelling of the solid domain an element has been used that allows three-dimensional modeling of solid structures. The employed element type is defined by eight nodes having three degrees of freedom at each node: translations in the nodal $x$, $y$ and $z$ directions. For the fluid domain an element was used that could model both the fluid medium and the interface in fluid/structure interaction problems. The employed element has eight corner nodes with four degrees of freedom per node: translations in the nodal $x$, $y$ and $z$ directions and pressure.

The mesh qualities have been tested before proceeding to the coupled analyses. In dynamics, the mesh should be adequate to represent the highest mode of interest. The quality of the mesh for the solid domain is tested by static analysis using loads that produce an approximation of the fundamental symmetric mode. The mesh density for the fluid domain is increased until adequate solutions are obtained for the steady state simulations described in App. B.

2.8. Phase shift estimation

The coupled numerical analyses of the fluid and the pipe yield results, which can be used to predict how vibrating, fluid-conveying pipe responds to fluid flow with
varying velocity profiles. The investigated pipes vibrate, like CFM pipes, in a resonantly driven symmetric mode, with a small overlay of vibrations of an antisymmetric mode excited by fluid flow. Since the vibration of the antisymmetric mode is $90^\circ$ out-of-phase with respect to the symmetric drive mode, the combined pipe motion will be a travelling wave. This leads to a time shift $\Delta t_0$ in zero-crossing between different axis points. The CFM literature [8, 22, 19, 23] gives an expression for the relationship between this time shift and the corresponding axial shift in vibration phase $\Delta \Psi$:

$$\Delta \Psi = 2\pi f_d \Delta t_0,$$

with $f_d$ being the drive frequency. Time-varying displacement data of two symmetrically located nodes is employed to calculate the time shifts in zero-crossing, which yield, after insertion into (7), the phase shifts due to the varying flow conditions. The nodal displacement data is taken from two nodes located in the antinodes of the pipe’s first antisymmetric mode, which is the mode excited by fluid flow; this corresponds to a location typically used in real CFMs designs. This will yield hypotheses on how velocity profile variations could affect CFM accuracy and precision.

3. Simulation results

3.1. Investigated pipe geometries

The analysed straight pipe with geometry as shown in Fig. 1 is characterised by the pipe length $L = 400$ mm, the internal pipe radius $r_i = 8$ mm and the wall thickness $h = 2$ mm. The investigated bended pipe configuration with geometry as presented in Fig. 2 is characterised by five straight pipe segments with the lengths $L_1 = L_5 = 25$ mm and $L_2 = L_3 = L_4 = 1$ mm and four bended pipe segments with $r_1 = r_2 = r_3 = r_4 = 100$ mm and $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 90^\circ$. The pipe is assumed to be made of steel with Young’s modulus $E = 200$ GPa, Poison ratio $\nu = 0.3$, and density $\rho = 7850 \frac{\text{kg}}{\text{m}^3}$. The fluid is in all simulations water with temperature $T = 15^\circ$ C, density $\rho_f = 999 \frac{\text{kg}}{\text{m}^3}$, dynamic viscosity $\mu =$
1.14 \times 10^{-3} \, \text{kg/m}^3\text{s} \text{ and kinematic viscosity } \nu = 1.14 \times 10^{-6} \, \text{m}^2\text{s}. \text{ For all conducted coupled simulations, each oscillation cycle has been divided into 60 equal time steps, and the simulations were run until transient pipe vibrations had died out.}

### 3.2. Phase shift predictions

Figure 7 shows the phase shift $\Delta \Psi$ as function of mass flow $\dot{m}$ calculated from nodal displacement data obtained by numerical investigations of a straight pipe conveying fluid flows with either fully-developed (□) or disturbed (○) velocity profiles. The mass flows are calculated using [16]:

$$\dot{m} = \rho f \pi r_i^2 v_{\text{max}}, \quad (8)$$

with $v_{\text{max}}$ being the maximum fluid velocity encountered at the inlet of the fluid domain.

It readily appears from Fig. 7, that fluid flow with disturbed velocity profiles induces smaller phase shifts than fluid flow with fully-developed profiles. For some parts of the considered mass flow range, the relationship between phase shift and

![Figure 7: Phase shift $\Delta \Psi$ as function of mass flow $\dot{m}$. Flow through straight pipe with fully-developed (□) and disturbed (○) velocity profiles. Data from simulations (symbol markers) connected by trendlines. Pipe geometry as specified in Sec. 3.1.](image-url)
mass flow seems to be linear. The factor of proportionality (hereafter denoted sensitivity) is for both cases not the same throughout the entire mass flow range. The different slopes of the trendlines for the two cases indicate, that the sensitivity depends on the velocity profile of the fluid flowing through the pipe. The effect appears to be most significant for mass flows smaller than 1 kg/s. Previous studies [6, 8] predict that CFM sensitivity changes might occur when the flow transitions from laminar to turbulent. Figure 7 shows results for turbulent fluid flow, since the involved Reynolds numbers are in the range from $1.4 \times 10^4$ to $14 \times 10^4$. This implies, in addition to the findings in [6, 8], that sensitivity changes might also occur while the flow is turbulent and not just during the transition from laminar to turbulent flow.

The results depicted in Fig. 7 yield the hypothesis that the sensitivity of CFMs could depend on the velocity profile of the fluid flowing through the meter. This may influence CFM accuracy and precision, since the flow conditions under which the meter is calibrated to determine the sensitivity will typically differ from the conditions encountered in service. In practical applications employing straight pipe CFMs, this can yield erroneous flowmeter readings.

Figure 8 shows the phase shift $\Delta \Psi$ as function of mass flow $\dot{m}$, calculated from nodal displacement data obtained from the numerical investigation of a bended pipe conveying fluid flows with either fully-developed ($\Box$) or disturbed ($\circ$) velocity profiles. This case is of particular relevance for CFM manufacturers, since bended pipes configurations are frequently employed in CFM designs. Disturbances will be induced while the fluid is flowing through the bended pipe. This results in the fluid flow not remaining fully-developed, even though a fully developed flow, like the one shown in Fig. 3, is applied at the inlet of the fluid domain.

It appears from Fig. 8, that there again is a linear relationship between phase shift and mass flow for some parts of the measurement range. Similar to the case of the straight pipe dealt with in the previous section, the sensitivity does not seem to be the same throughout the entire mass flow range. Generally it can be concluded that different fluid velocity profiles for similar mass flow induce
Figure 8: Phase shift $\Delta \Psi$ as function of mass flow $\dot{m}$. Flow through bended pipe with fully-developed ($\Box$) and disturbed ($\circ$) velocity profiles applied at inlet of fluid domain. Data from simulations (symbol markers) connected by trendlines. Pipe geometry as specified in Sec. 3.1.

different phase shifts. But for the investigated bended pipe there is no clear trend on how disturbed velocity profiles applied at the inlet of the fluid domain influence the phase shift: For mass flows of 0.8 kg/s and smaller, the phase shift was apparently reduced, whereas it was increased for the mass flows larger than 1 kg/s. To the author’s knowledge there are no other studies available where the effect of velocity profiles on bended pipe configurations is investigated.

The results shown in Fig. 8 lead to the hypothesis, that the sensitivity of CFMs utilising vibrations of bended pipes could depend on fluid’s velocity profile. For practical applications employing these CFMs to measure mass flow, this could yield erroneous flowmeter readings, since CFM accuracy and precision could be affected.

4. Conclusion

In this paper it has been demonstrated how numerical simulations can be used to study the effect of fluid velocity profiles on the dynamic behaviour of simple fluid-conveying pipes, employing two-way fluid-structure interaction in
the commercial finite element (FE) code ANSYS and finite volume (FV) code CFX. The considered geometries have been straight and bended pipe configurations. The results are presented in terms of the axial shift in vibration phase between two symmetrically located pipe points. For CFMs, which employ phase shifts as a mean for measuring mass flow, this yields the hypothesis: Velocity profile effects could influence CFM accuracy and precision, since the flowmeter sensitivity appears to depend on the fluid velocity profile. In real applications this could yield erroneous mass flow measurements. The obtained results predict, that sensitivity changes might occur while the flow is turbulent and not just during the transition from laminar to turbulent flow, as predicted in [6, 8].

The hypothesis is based entirely on numerical simulations, which are limited to the particular parameters and conditions investigated. Since the magnitude of velocity profile effects on commercial CFMs might depend on constructional and operational parameters, the hypothesis should be tested experimentally employing different CFMs.

Detailed computational models of real CFM can be straightforwardly set up in commercially available FE and FV codes following the procedures described in this paper. This allows for simulation of any thinkable system specification and operating condition. The presented numerical simulations of simple configurations are already highly computational demanding and limited to the particular parameters and conditions simulated. It is reasonable to assume that simulations of real CFM designs with more complex geometries will be even more resource demanding. The computational times involved may prohibit the investigation of parameter-dependencies in any depth. In future investigations regarding the effect of velocity profiles one should therefore consider to employ also analytical pipe models and approximate analysis as described [9, 10], which might provide direct insight into the parameters at play and help increase the benefit of numerical simulations.
Acknowledgments

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References


A. Determination of disturbed velocity profiles

The disturbed velocity profiles, applied at the inlet of the vibrating straight pipes, are determined by steady state CFD analyses of the fluid domain in a bended pipe configuration with pipe radius $r_i = 8 \text{ mm}$. The considered fluid domain is shown in Fig. 9. During all simulations, the velocity $v_c$ at the inlet is assumed to be constant as illustrated in Fig. 9. The flow develops while flowing through the straight part of the domain, yielding the second velocity profile right before the bended segment. It

![Diagram of fluid flow through straight and bended segments](image)

Figure 9: Velocity profiles encountered in fluid flow through straight and bended segments. Employed boundary conditions: constant fluid velocity at inlet, smooth wall and no-slip condition, and 1 bar of average static pressure at outlet.
can be seen that the velocity profile appears fully-developed. Once the fluid is flowing through the bended segment, the velocity profile gets disturbed yielding the velocity profile shown at the outlet of the fluid domain. Employing different inlet velocities in the range from 1 to 10 m/s yields different disturbed velocity profiles at the outlet, examples are shown in Fig. 4. These outlet profiles are then used as inlet conditions for the fluid domain during 1) the steady state CFD simulations determining the initial state of the fluid prior to the transient simulations (see also App. B), and 2) the coupled transient simulations of the fluid and the pipe.

B. Initialisation of transient analysis

Steady state analyses of the fluid domain are run prior to the transient fluid analyses simulating fluid flow while the pipe is at rest. This yields initial values, e.g. for the fluid velocity and pressure. In the steady state and the subsequent transient analyses of the fluid domain the same boundary conditions are applied at the inlet, outlet and pipe wall. Figures 10 and 11 exemplify the results of the steady state analyses of straight pipes at rest, showing the inlet velocity profiles (a) and the pipe wall pressure (b). Figure 10(a) shows a fully developed velocity profile applied at the inlet, Fig. 11(a) shows a disturbed velocity profile applied at the inlet. Figures 12 and 13 exemplify the results of the steady state analyses of bended pipes at rest, velocity profiles at the inlet, midpipe and outlet of the fluid domain are shown in Figs. (a)-(c), whereas the pipe wall pressure can be seen in Fig. (d). Figure 12(a) shows a fully developed velocity

![Figure 10](image1.png)

(a) (b)

Figure 10: Initial conditions for fluid domain during transient simulation, obtained by steady state analysis of pipe at rest. (a) Fully developed velocity profile applied at inlet with velocity $v_{\text{max}} = 4.99$ m/s. (b) Fluid induced pressure on pipe wall.
Figure 11: Initial conditions for fluid domain during transient simulation, obtained by steady state analysis of pipe at rest. (a) Disturbed inlet velocity profile with $v_{\text{max}} = 5.75$ m/s. (b) Fluid induced pressure on pipe wall.

Figure 12: Initial conditions for fluid domain during transient simulation, obtained by steady state analysis of bended pipe at rest. (a) Fully developed velocity profile at inlet with $v_{\text{max}} = 4.89$ m/s. (b) Disturbed velocity profile midpipe of fluid domain with $v_{\text{max}} = 4.45$ m/s. (c) Disturbed velocity profile at outlet of fluid domain with $v_{\text{max}} = 4.76$ m/s. (d) Fluid induced pressure on pipe wall.

profile applied at the inlet, Fig. 13(a) shows a disturbed velocity profile applied at the inlet.
Figure 13: Initial conditions for fluid domain during transient simulation, obtained by steady state analysis of bended pipe at rest. (a) Disturbed velocity profile at inlet with $v_{\text{max}} = 4.62$ m/s. (b) Disturbed velocity profile midpipe of fluid domain with $v_{\text{max}} = 4.53$ m/s. (c) Disturbed velocity profile at outlet of fluid domain with $v_{\text{max}} = 4.76$ m/s. (d) Fluid induced pressure on pipe wall.

C. Modal analyses of fluid-filled pipes

In Coriolis flowmetering the pipe is resonantly forced to vibrate, typically by resonant excitation of the fundamental symmetric mode. The natural frequencies of the considered pipe geometries for this particular mode are determined by modal analyses in ANSYS prior to the FSI simulations. The geometries of the fluid-filled pipes are modelled in ANSYS, with the finite element models consisting of uniaxial elements with six degrees of freedom at two nodes and element input data including the pipe outer diameter, wall thickness and internal fluid density. The pipe material is defined, and the pipe ends are clamped. The number of elements of the numerical model is increased until the results, i.e. the natural frequencies of the pipe, converges. The results of the modal analyses, i.e. the natural frequency corresponding to the fundamental symmetric mode, the later being exemplified in Fig. 14, are inserted into (5) to determine the time-varying force to be applied midpipe. The natural frequencies for the straight and the bended pipe configuration corresponding to the fundamental
symmetric mode are determined to be 643 Hz and 162 Hz, respectively.

The literature offers an analytical prediction for the natural frequencies of straight fluid-filled pipes [18]. For the straight pipe configuration considered in this paper, these predictions are compared to the results of numerical simulations (see Fig. 15) and good agreement is seen between the analytical predictions and the numerical results for the two lowest modes supporting the accuracy of the numerical simulations.

Figure 14: Fundamental symmetric mode shape of straight, fluid-filled, clamped-clamped pipe ($L = 0.4$ m, $r_i = 8$ mm, $h = 2$ mm, material constants as given in Sec. 2.1) determined by modal analysis in ANSYS and corresponding to natural frequency 643 Hz.

![Figure 14: Fundamental symmetric mode shape of straight, fluid-filled, clamped-clamped pipe](image)

Figure 15: Natural frequencies $f$ of straight, fluid-filled pipe ($L = 0.4$ m, $r_i = 8$ mm, $h = 2$ mm, material constants as given in Sec. 2.1) by analytical prediction (symbol marker $\times$) [18] and numerical simulation ANSYS (symbol marker $\square$).

![Figure 15: Natural frequencies $f$ of straight, fluid-filled pipe](image)
Assessment of the applicability of the weight vector theory for Coriolis flowmeter.

ASSESSMENT OF THE APPLICABILITY OF THE WEIGHT VECTOR THEORY FOR CORIOLIS FLOWMETERS

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Abstract – The weight vector theory for Coriolis flowmeters has been the subject of research presented by Hemp and co-workers in various articles. The underlying theory may not be easily understood. This paper explains the application of the weight vector theory for Coriolis flowmeters. The theory is applied to simple theoretical meter configurations consisting of a single straight pipe. The application of the weight vector approach is of relevance when investigating velocity profile effects, e.g., in Coriolis flowmeters. Promising results have been found in recent literature showing the vulnerability of straight pipe Coriolis flowmeter configurations to velocity profile effects. The application of the weight vector theory is shown to be either limited to the investigation of few parameters or employs unrealistic boundary conditions and lacks comparative studies, making a more comprehensive study desirable. The usefulness of the weight vector theory to predict velocity profile effects for bended tube is not apparent from today’s state-of-the-art literature, but of great interest for flowmeter manufacturers since bended tubes designs are frequently used in today’s Coriolis flowmeters.

Keywords: weight vector, velocity profile effects, Coriolis flow meter

1. INTRODUCTION

The concept of the weight vector has been developed by J.A. Shercliff [1] and M.K. Bevir [2]. For electromagnetic flowmeters, this approach is extensively applied and experimentally validated. The weight vector theory for Coriolis flowmeters has been developed with the purpose of predicting velocity distribution effects [3].

The extensive work done by J. Hemp has evidently had an impact on the development of the weight-vector approach for electromagnetic and Coriolis flowmeters. He initially worked with the theory for electromagnetic flowmeters, see for example [4,5], and has in the recent years put his focus on the application of the same theory for Coriolis flowmeters.

Generally speaking, the weight vector theory for Coriolis flowmeters provides means to express the phase difference between sensing signals as a function of the steady flow field in the tube and a weight-vector field, which depends on vibrational flows in the appropriately vibrating tube without the steady flow.

The basic weight vector theory for Coriolis flowmeters has been described in [6]. A technique is presented for developing an analytical expression for the weight vector. A first application of the same technique for Coriolis flowmeters is presented in [3] and [7]. The former shows the derivation of the weight vector theory for Coriolis flowmeters, whereas the latter presents the calculation of the Coriolis flowmeter sensitivity if the effect of fluid viscosity is to be taken into account. More recent studies are published in [8], [9] and [10]. A review of the state-of-the-art findings and open questions regarding velocity profile effects in Coriolis mass flowmeters is presented in [11]. The presented study related to the weight vector theory considers straight tube configurations and employs results from [3,6].

The weight vector theory for Coriolis flowmeter may not be understood from the related literature. To remedy this and encourage further application and testing, the theory is briefly revised, the necessary equations are determined and their application is illustrated for single straight tube configurations. The aim of this paper is to discuss the applicability of the weight vector theory, e.g. to predict velocity profile effects of Coriolis flowmeters, and point out its vulnerability.

2. WEIGHT VECTOR THEORY FOR CORIOLIS FLOWMETERS

The measuring tube is assumed to be a straight circular cylindrical shell with its geometry being defined by its length $L$, wall thickness $h$ and inner tube radius $R_i$, Fig. 1. According to the weight vector theory for Coriolis flowmeters [10], the flow induced phase difference $\Delta \phi$ between sensor signals can be determined assuming linearity between the measured signals and the flow field.
\[
\Delta \phi = \int_{-\infty}^{\infty} \int_{0}^{2\pi R_i} \mathbf{V}_0 \cdot \mathbf{W}_\phi \, dr \, d\theta \, dx
\]  

(1)

where \( \mathbf{V}_0 \) is the steady fluid velocity vector in absence of pipe vibrations and \( \mathbf{W}_\phi \) the weight vector for the phase difference. The weight vector depends on certain vibrational flow fields in the absence of steady flow. The integral is taken over the entire volume of the fluid. The weight vector for the phase difference \( \mathbf{W}_\phi \) is defined as [10]

\[
\mathbf{W}_\phi = \text{Im} \left( \frac{\mathbf{W}}{(1)(x_s, \theta_s)} \right)
\]

(2)

where \( \mathbf{W} \) represents the weight vector and \( v_{ij}^{(1)}(x_s, \theta) = i \omega u_{ij}^{(1)}(x_s, \theta) \) the radial tube velocity of the working mode at the sensing point \((x_s, \theta)\) with \( \omega \) being the angular operation frequency and \( u_{ij}^{(1)}(x_s, \theta) \) the radial displacement of the working mode. The weight vector \( \mathbf{W} \) can be defined using [3,6,10]

\[
\mathbf{W} = -\rho \left[ \left( \mathbf{v}^{(2)} \cdot \nabla \right) \mathbf{v}^{(1)} - \left( \mathbf{v}^{(1)} \cdot \nabla \right) \mathbf{v}^{(2)} \right]
\]

(3)

where \( \rho \) is the density of the fluid and \( \mathbf{v}^{(1)} \) and \( \mathbf{v}^{(2)} \) are fluid vibrational velocity fields. The field \( \mathbf{v}^{(1)} \) is a result of the tube vibration without steady flow in the symmetric working mode, i.e. driven by a central force. The field \( \mathbf{v}^{(2)} \) results from the antisymmetric tube vibration, without the presence of a flowing fluid, driven by equal and opposite unit forces applied at the sensing points \((x_s, \theta)\) and \((x_s, \theta)\).

3. APPLICATION EXAMPLES

To exemplify the application of the theory presented in section 2, two examples from the state-of-the-art literature will be summarized. Major results will be presented and a reflection will be given on the value of these examples with respect to a future application on other Coriolis flowmeter designs.

3.1. Determination of weight vector and phase difference for straight Coriolis flowmeter with non-supported ends

Hemp [8] investigated a straight Coriolis flowmeter, consisting of a single tube defined by the tube length \( L \) and the inner tube radius \( R_i \), with non-supported free ends infinitely close to but unattached to adjacent piping. Neglecting viscosity and compressibility of the fluid, the equations for \( \mathbf{v}^{(1)} \) and \( \mathbf{v}^{(2)} \) are

\[
\begin{align*}
&i\omega \rho \mathbf{v} = -\nabla p \\
&\nabla \cdot \mathbf{v} = 0
\end{align*}
\]

(4)

(5)

with \( p \) being the pressure on the fluid. Equation (4) and (5) correspond to the momentum and continuity equation derived by employing mass conservation and Newton’s second law on a fluid element.

Equations (4) and (5) have the approximate locally rigid tube solutions

\[
v_r = V \cos \theta; \quad v_\theta = -V \sin \theta; \quad v_x = \Omega r \cos \theta;
\]

\[
p = -i\omega \rho V r \cos \theta
\]

(6)

where \( V \) is the local linear velocity of the tube and \( \Omega \) the local angular velocity of the tube. Inserting (6) into (3) results in

\[
\mathbf{W} = \rho \left( \Omega^{(2)} V^{(2)} - \Omega^{(1)} V^{(1)} \right) k
\]

(7)

indicating that the weight vector away from the tube ends is independent of \( r \) and \( \theta \).

Assume a fully developed flow velocity profile, i.e. the phase shift \( \Delta \phi \) can be determined using an axisymmetric velocity profile \( \mathbf{v} = v(r) \mathbf{k} \). Equation (1) turns out to be [8]

\[
\Delta \phi = \int_{0}^{R_i} 2\pi r v(r) W_\phi(r) \, dr
\]

(8)

where the axisymmetric weight function for phase shift \( W_\phi(r) \) is defined using (2)

\[
W_\phi(r) = \text{Im} \left( \frac{W(r)}{(v_s)p} \right)
\]

(9)

with the axisymmetric weight function \( W(r) \) being

\[
W(r) = W_0(r) + W'(r)
\]

(10)

and

\[
W'(r) = \rho \int_{0}^{L} \left( \Omega^{(2)} V^{(2)} - \Omega^{(1)} V^{(1)} \right) d\xi
\]

(11)

(12)

\begin{align*}
W(r) &= -0.941 + 0.816 \left( \frac{r}{\xi} \right)^2 - 0.214 \left( \frac{r}{\xi} \right)^4 + 0.551 \left( \frac{r}{\xi} \right)^6
\end{align*}

(13)

As a final result [8] states, that the phase difference \( \Delta \phi \) neglecting end effects can be determined using

\[
\Delta \phi = M \frac{L^3}{EI} \cdot F\left( \frac{\zeta}{L} \right)
\]

(13)

where \( M \) is the mass flow rate, \( EI \) the tube rigidity, \( \zeta \) the sensor position and \( F(\zeta/L) \) the non-dimensional sensor position function, cf. [8].

The study presented in [8] illustrates, in a straightforward way, the application of the weight vector theory to determine an expression for the phase difference between sensor signals \( \Delta \phi \) of a straight tube Coriolis flowmeter.

The chosen unrealistic boundary conditions, i.e. unsupported pipe ends unattached to adjacent piping, are a major handicap of the study. A more realistic investigation would incorporate clamped or at least hinged pipe ends, since these are better representations of actual Coriolis
flowmeter designs. The reason for choosing the investigated boundary condition is not stated in [8]. The cause for this could for example be, that alternative boundary conditions, e.g. simply-supported pipe ends, would just complicate the results without changing the conclusions, or that the theory presented in [8] simply does not hold for other boundary conditions.

In addition, the developed formulas have not been illustrated by numerical calculations. The results from [8] have neither been compared to nor confirmed by analytical or numerical results obtained with alternative solution procedures. On the basis of the information given in [8], it cannot be concluded whether the presented formulas are beneficial or not.

### 3.2. Weight vector study of velocity profile effects in straight tube Coriolis flowmeter

A straight Coriolis flowmeter with clamped ends is considered in [10]. Fig. 1. The measuring tube has the following dimensions inner tube radius \( R_i = 10 \text{ mm} \), wall thickness \( h = 0.5 \text{ mm} \) and length \( L = 200 \text{ to } 600 \text{ mm} \) and material properties density \( \rho = 4510 \text{ kg/m}^3 \), Young’s modulus \( E = 102.7 \text{ GN/m}^2 \) and Poisson’s ratio \( \nu = 0.34 \). The results are presented for the working mode, with the distance between the sensing points being \( s = L/2 \) and in terms of variations of the flowmeter’s mass flowrate sensitivity [10]

\[
K = \frac{\Delta \phi}{q_m} \tag{14}
\]

corresponding to the ratio between the phase difference \( \Delta \phi \), determined using (1), and the mass flowrate \( q_m \)

\[
q_m = \pi \rho V_0 R_i^2 \tag{15}
\]

where \( V_0 \) is the mean flow velocity. In [10] the velocity profile effect is presented as variations in the ratio between the mass flowrate sensitivities for chosen velocity profiles \( K \) and for the flat, plug-flow profile \( K_0 \)

\[
K = \frac{\int_0^\infty r V_{0x}(r) \overline{W}_z(r) \, dr}{\int_0^\infty r \overline{W}_z(r) \, dr} \tag{16}
\]

where \( \overline{W}(r) \) is the axisymmetric weight function, see [10].

Equation (16) is illustrated by Fig. 2, which shows variations of the mass flowrate sensitivities with aspect ratio \( L/R_i \) for laminar and turbulent flow, respectively \( K_{lam} \) and \( K_{turb} \), relative to the sensitivity for the flat velocity profile \( K_0 \), assuming a circumferential mode where the tube cross-section is not deformed during the tube vibration. Fig. 2 can be seen that the ratio between the turbulent flow and flat velocity profile sensitivity is almost 1 for long tubes. This means that there is no difference between assuming a simple plug flow rather than a more realistic turbulent flow to determine the flowmeter sensitivity for long tubes using weight vector theory. This does not apply for short tubes as it can be seen in Fig 2. Similar conclusions are drawn when laminar flow is assumed. For short tubes, the sensitivity predicted using a plug flow assumption is larger than the sensitivity predicted using the laminar flow assumption. This also applies for long tubes, however the difference not as pronounced as for short tubes.

Table 1 shows mass flowrate sensitivities calculated for the two lowest circumferential modes, i.e. assuming the tube cross-section to be, respectively, non-deformed and deformed during the tube vibration, and for different tube aspect ratios using a direct and the weight vector solution procedure. This corresponds to a quantitative test of the theory for a few parameters. The direct solution procedure is described in [10]. It can be seen in Table 2 that the results, with a few exemptions, are in agreement. This illustrates the applicability of the weight vector theory as it is presented in [10].

The applicability of the weight vector theory to determine the sensitivity of a straight Coriolis flowmeter has been shown in [10]. A quantitative test employing a few parameters shows agreement between the results obtained by the weight vector theory and a direct solution procedure. Indications are found that the simple plug flow assumption can be used to estimate the sensitivity in case of high aspect ratios, e.g. long tubes. Furthermore it is shown that short tubes are more vulnerable to velocity profile effects than long tubes. The weight vector theory predicts a higher sensitivity using a plug flow assumption is larger than the sensitivity predicted using the laminar flow assumption. This also applies for long tubes, however the difference not as pronounced as for short tubes.

### Table 1. Comparison of the mass flowrate sensitivities from the direct and the weight vector solution procedure, adapted from [10].

<table>
<thead>
<tr>
<th>Aspect ratio ( L/R_i )</th>
<th>Mass flowrate sensitivity ( K_6 ) (rad/(kg/s))</th>
<th>Non deformed tube cross-section</th>
<th>Deformed tube cross-section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Direct solution</td>
<td>Weight vector</td>
<td>Direct solution</td>
</tr>
<tr>
<td>20</td>
<td>4.349(10^{-3})</td>
<td>4.349(10^{-3})</td>
<td>2.996(10^{-2})</td>
</tr>
<tr>
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<td>1.705(10^{-1})</td>
</tr>
<tr>
<td>60</td>
<td>1.062(10^{-2})</td>
<td>1.062(10^{-2})</td>
<td>4.259(10^{-1})</td>
</tr>
</tbody>
</table>
tivity when shifting from laminar to turbulent flow, which agrees with practical experiences from a Coriolis flowmeter manufacturer. Compared to [3,8], [10] offers indications that the weight vector theory in fact can be used to evaluate velocity profile effects for Coriolis flowmeters. It is apparent from the results presented in [10], that velocity profile effects cannot generally be neglected for Coriolis flowmeters, especially when a short tube design is employed. This is valuable information, since velocity profile effects often are ignored when designing Coriolis flowmeters. The work [10] leaves open questions, since it does not investigate the influence of other parameters, e.g. alternative boundary conditions or curved measuring tube shapes, on velocity profile effects.

4. DISCUSSION AND CONCLUSION

The early literature regarding the weight vector theory for Coriolis flowmeters does not provide sufficient information to enable a straightforward application of the theory on Coriolis flowmeters. The usefulness of the weight vector theory to evaluate velocity profile effects for Coriolis flowmeters is not shown.

In the recent literature, promising results have been published showing that especially short tube designs are vulnerable to velocity profile effects, so that velocity profile effects cannot generally be neglected for Coriolis flowmeters. However the quantitative test, to compare the results from weight vector theory calculations to results obtained by direct solution procedures, is limited to a few parameters, so a more comprehensive study is necessary.

The major lack of the state-of-the-art weight vector theory for Coriolis flowmeters is that it is limited to straight pipes with more or less realistic boundary conditions.

Even though it would be of great interest for manufacturers of Coriolis flowmeters with bended tube designs, the usefulness of the weight vector theory to predict velocity profile effects of bended tube configurations is not apparent from the today's literature. A first step in this direction is made by J. Hemp, who has determined the weight vector for the three straight sections of rigid u-tube flowmeter, however, without determining the weight vector in the curved corners of the meter, cf. [3,6], which would lead to a more complicated expression for the weight vector. The three obtained constant expressions for the weight vectors are parallel to the tube axis and pointing in the flow direction in each straight section. This indicates that the influence of bended tubes cannot be neglected when determining the weight vector for bended tube designs. If it could be shown, that the weight vector theory for Coriolis flowmeters is also useful to study velocity profile effects in bended tube configurations, this would provide a powerful tool, e.g. for flowmeter manufacturers. Its major advantage will be that it can validate and possibly replace the time-consuming and computational demanding simulations which are used today, i.e. numerical fluid-structure-interaction simulations.

The steady flow assumption indicates that the presented theory is only valid when the flow in the flowmeter is laminar. However, in real Coriolis flow meter applications the flow is usually turbulent. J. Hemp argues in [3], that the weight vector theory probably still is valid, since filtering of the sensor signals should remove the effect of turbulence related velocity fluctuations. An experimental validation of this statement using real Coriolis flowmeters is not apparent from the literature. Others have used mathematical expressions for describing turbulent velocity profiles, cf. [10], and used these as input in the presented theory. This approach gives promising results. However this approach does not replace the necessity of further investigations to confirm the applicability of the weight vector theory in case of turbulent flow. Numerical methods and/or experiments with real Coriolis flowmeters can, in this context, be tools to employ.

Computational fluid dynamics can be used to clarify whether fluid properties, e.g. viscosity and compressibility, and pipe characteristics which influence the fluid flowing in the vibrating pipe, e.g. internal pipe wall roughness, can be neglected or how they might change the weight vector theory for Coriolis flowmeters. Under certain assumptions, e.g. neglecting the effect of viscosity, J. Hemp argues based on ultrasonic flowmeter theory, that the expression for determining the weight vector is the same for compressible and incompressible flow [3]. This indicates that neglecting the effect of viscosity has an influence on the applicability weight vector theory for Coriolis flowmeters. This indication should be checked to test it and its extend, e.g. by using numerical methods, since it has not been studied further according to the known literature.

To sum up, it has been shown that the weight vector theory is applicable for predicting velocity profile effects in Coriolis flowmeters. The theory is however vulnerable, since comprehensive studies are missing and realistic tube designs and boundary conditions have not been investigated. This leads to the still open question: Can the weight vector theory for Coriolis flowmeters be easily applied to real Coriolis flowmeter designs, or not? The theory seems to hold a significant potential but also does not seem straightforward in practical application involving real Coriolis flowmeters. The required fluid vibrational velocity fields may, e.g., not be readily set up, and may even involve computational demanding and time-consuming numerical simulations.

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Experimental investigation of zero phase shift effects for Coriolis flowmeters due to pipe imperfections.

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Experimental investigation of zero phase shift effects for Coriolis flowmeters due to pipe imperfections

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Abstract

Theoretical investigations of a single, straight, vibrating, fluid-conveying pipe have resulted in simple analytical expressions for the approximate prediction of the spatial shift in vibration phase. The expressions have lead to hypotheses for real Coriolis flowmeters (CFMs). To test these, the flexural vibrations of two bended, parallel, non fluid-conveying pipes are studied experimentally, employing an industrial CFM. Special attention has been on the phase shift in case of zero mass flow, i.e. the zero shift, caused by various imperfections to the ”perfect” CFM, i.e. non-uniform pipe damping and mass, and on ambient temperature changes. Experimental observations confirm the hypothesis that asymmetry in the axial distribution of damping will induce zero shifts similar to the phase shifts due to fluid flow. Axially symmetrically distributed damping was observed to influence phase shift at an order of magnitude smaller than the primary effect of mass flow, while small added mass and ambient temperature changes induced zero shifts two orders of magnitude smaller than the phase shifts due to mass flow. The order of magnitude of the induced zero shifts indicates that non-uniform damping, added mass as well as temperature changes could be causes contributing to a time-varying measured zero shift, as observed with some commercial CFMs. The conducted experimental tests of the theoretically based hypotheses have shown that simple mathematical models and approximate analysis allow general con-

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Inclusions, that may provide direct insight, and help increasing the benefit of time consuming numerical simulations and laboratory experiments.

Keywords: Coriolis, mass flowmeter, experimental, fluid conveying pipe, zero shift drift, imperfection, sensitivity

1. Introduction

Coriolis flowmeters (CFM) exploit an assumed linearity between measured vibration phase shift and mass flow. Deviations from the "perfect" CFM could lead to a deviation from this linear relationship. Simple analytical models and their solutions offer a direct insight into which imperfections influence phase shift, and therefore also the mass flow measurement. Hypotheses can be created for more complicated designs, i.e. real CFM, using these simple models. Experimental investigations are important for testing these hypotheses. Full computational models involving fluid-structure-interaction are an alternative to experiments, but they typically require extensive time and computational resources to fully explore parameter dependencies and do not offer hands-on experience and understanding of the real flowmeter.

The experiments presented in this paper are inspired by the analytical works [1, 2, 3]. They investigate effects of symmetrical and asymmetrical damping, mass and stiffness distribution on the phase shift experienced by a single, vibrating, fluid-conveying pipe and lead to hypotheses for more complex systems, i.e. CFMs. The early research paper [4] presents the investigation of phase shift effects in CFMs employing simplified mathematical models and experiments with real CFMs. Asymmetries in damping were identified as sources for phase shifts. CFMs with straight pipes have previously been investigated using a lumped-parameter model to explain processes causing density and mass flow reading errors [5]. It is stated that these errors, among other things, could be caused by spatially asymmetric damping of the flowmeter’s measuring pipe. Our experiments will be used to test not only the hypotheses by [1, 2, 3] but also the statement in [5] that error measurements of CFM seem to be due to the damping not being symmetric along
the measuring pipe. The signal pick-ups and actuator are in many CFMs installed
directly on the measuring pipe, and can therefore be seen as added masses. This
has initiated theoretical investigations on how size and placement of theses affect
phase shift [6, 7, 8].

Experimental investigations of CFMs found in the scientific literature deter-
mine, generally speaking, how the CFM is metering the ”true” mass flow under
certain flow or external conditions, e.g. pulsating flow [9, 10], two-phase flow
[11, 12], batch flow measurement [13, 14, 15], or external vibrations [16]. Investi-
gations deal then either with the sensor part of the flow meter, i.e. the vibrating
pipes, or the transmitter technology, i.e. the electronics and drive control, and
signal processing algorithms. The experimental work published in the scientific
journals generally focuses on monitoring the deviation from the true mass flow
due to a specific reason, using e.g. timers and weight scales, and on developing
error compensating algorithms, but not on explaining the reason for the error
itself. The available literature documents very few experimental investigations
dealing with factors influencing the phase shift in case of zero mass flow, see e.g.
[3, 4]. We will employ a simple experimental setup, which does not require an
actual fluid flow to explore parameters influencing the phase shift. This is im-
portant for understanding a possible drift of the zero shift during the flowmeter
lifetime, as experienced with some CFMs.

Section 2 presents the general CFM theory as well as a summary of the math-
ematical model, analytical predictions and hypotheses, which motivated the ex-
perimental work. Section 3 describes the experimental setup and test procedure,
and Section 4 the results. In Sec. 5 the results are discussed in light of the
hypotheses stated, and concluded on.

2. Mathematical model and analytical prediction

2.1. General Coriolis flowmeter theory

The phase shift measured by CFMs is [17]:

\[ \Delta \Psi = \tilde{\omega}_d \Delta \tilde{t}, \]  

(1)
with $\omega_d$ being the angular frequency of the pipe corresponding to the drive mode, and $\Delta t$ the time shift in zero-crossing between two pipe axis points, assuming the signal to be properly filtered to extract oscillations at $\omega_d$. A tilde denotes here and in the following a physical variable or parameter. After suitable calibration, this phase shift is assumed to be a measure of the mass flow through the meter.

The calibration factor $\tilde{C}$ for a CFM can be determined using [18]:

$$\tilde{C} = \frac{\Delta \tilde{t}}{\tilde{m}},$$

(2)

with $\tilde{m}$ representing the mass flow. Equation (1) and (2) yield then the following expression for the phase shift $\Delta \Psi$:

$$\Delta \Psi = \tilde{C} \omega_d \tilde{m},$$

(3)

assuming a linear relationship between mass flow and phase shift. This expression does not take CFM imperfections into account, which could disturb the assumed linearity and therefore lead to erroneous flowmeter readings, i.e. over- or under-estimations of the true massflow through the CFM.

2.2. Extended mathematical model of straight, vibrating, fluid-conveying, structurally imperfect pipe

Figure 1 shows a single, straight, fluid-conveying, simply supported and imperfect pipe. Imperfections considered in this particular model include non-uniform mass, stiffness and damping distribution, uniform but not proportional damping, weak nonlinearities, and two types of boundary conditions: simply supported and clamped-clamped (the latter not shown in Fig. 1). Transverse motions $u(x,t)$ of the system in Fig. 1 are described by the non-dimensionalized equation of motion [1], and partly [2, 3]:

$$\ddot{u} + u'''' + \varepsilon \left( \alpha_p(x) \dot{u} + [EI(x)u'']'' + \alpha_f [v^2 u'' + 2v \dot{u}'] - \mu^2 \left[ \eta + \frac{1}{2} \int_0^1 (u')^2 dx \right] u'' \\
+ \gamma u^2 + L_k[u] + L_c[\dot{u}] + \beta f(\dot{u}) \right) = \varepsilon p \delta(x - x_p) \cos(\Omega t),$$

(4)
Figure 1: Model of transversely vibrating, simple supported, fluid-conveying, pipe with structural non-uniformities and excited by force at midpipe.

with ε being a bookkeeping parameter of no physical meaning marking terms that are assumed small compared to unity, and \( L_k \) and \( L_c \) linear spatial differential operators describing arbitrarily distributed damping and stiffness:

\[
L_k = k_u(x) + \frac{d}{dx}[k_\theta(x) \frac{d}{dx}], \quad L_c = c_u(x) + \frac{d}{dx}[c_\theta(x) \frac{d}{dx}].
\] (5)

This equation of motion is an extension of what can be found in [6, 19]; it can be derived using Newton’s second law or Hamilton’s principle, see e.g. [19, 20, 21] for details. Prime denotes differentiation w.r.t. the axial coordinate \( x \in [0;1] \).

All parameters, variables and functions in (4) are nondimensionalised:

\[
x = \frac{\tilde{x}}{l}, \quad u = \frac{\tilde{u}}{l}, \quad t = \tilde{\omega}_0 \tilde{t}, \quad \tilde{\omega}_0^2 = \frac{EI_0}{l^4 A_0}, \quad v = \frac{\tilde{v}}{\tilde{\omega}_0 l}, \quad \rho A_0 = \rho_p A_p + \rho_f A_f \\
\alpha_f = \frac{\rho_f A_f}{\rho A_0}, \quad \alpha_p(x) = \frac{\rho_p A_p(x)}{\rho A_0}, \quad \mu^2 = \frac{A_p l^2}{I_p}, \quad \gamma = \frac{\tilde{\gamma} l}{\rho A_0 \tilde{\omega}_0^2}, \quad EI(x) = \frac{EI_p(x)}{EI_{p0}}, \\
c_u(x) = \frac{\tilde{c}_u(x)}{\tilde{\omega}_0 \rho A_0}, \quad c_\theta(x) = \frac{\tilde{c}_\theta(x)}{\tilde{\omega}_0^2 \rho A_0}, \quad k_u(x) = \frac{\tilde{k}_u(x)}{\tilde{\omega}_0^2 \rho A_0}, \quad k_\theta(x) = \frac{\tilde{k}_\theta(x)}{\tilde{\omega}_0^2 \rho A_0}, \\
\Omega = \frac{\tilde{\Omega}}{\tilde{\omega}_0}, \quad p = \frac{\tilde{p}}{l^2 \tilde{\omega}_0^2 \rho A_0}, \quad f(\tilde{u}) = \frac{\tilde{f}(\tilde{u})}{l \tilde{\omega}_0^2 \rho A_0}
\] (6)

Here \( u \) is the transverse pipe deflection, \( x \) the axial coordinate, \( t \) the time, \( l \) the pipe length, \( v \) the flow speed, \( \alpha_f \) the ratio of fluid mass to total mass, \( \alpha_p \) ratio of pipe mass to total mass, \( \rho_p A_p(x) \) the axially varying pipe mass per unit length being the sum of a dominating mean value (indicated by the subscript zero) and small non-uniformities given by functions of the axial coordinates, \( \rho_f A_f \) fluid mass per unit length, \( \mu \) the slenderness ratio, \( \gamma \) the coefficient of asymmetric stiffness w.r.t. \( u = 0, \eta \) a measure of the axial deformation, \( EI(x) \) the axially varying
flexural stiffness, \( k_{u,\theta} \) and \( c_{u,\theta} \) functions describing the axial distribution of, respectively, stiffness (additional to that of the pipe itself) and viscous damping per unit length, with subscript \( u/\theta \) indicating transverse / rotational distributions that may be non-uniform and discontinuous, \( \beta \) the coefficient of generalized damping, \( f(\dot{u}) \) a velocity dependent possibly nonlinear generalized damping function, and \( p_a \) the amplitude of a time-harmonic excitation force at \( x = x_p \) having frequency \( \Omega \).

Equation (4) is valid under the following assumptions: Only transverse deflections in the plane of the excitation force are important. Deflection slopes during the vibrations are small, i.e. \((u')^2 \ll 1\), and the midplane stretching is small and uniform. The pipe is considered slender with near-uniformly distributed flexural mass and stiffness and small variations of cross section. In addition is also assumed that the coefficients of damping \( c_{u,\theta} \), non-uniform stiffness \( k_{u,\theta} \), asymmetric stiffness \( \gamma \), uniform generalised damping \( \beta f(\dot{u}) \) and external forcing amplitude \( p_a \) are small. The fluid is assumed to be incompressible, flowing with a constant velocity \( v \) and a flat velocity profile towards \( x = 1 \). These assumptions hold approximately for Coriolis flowmeter applications under standard operating conditions.

To obtain an analytical prediction for axial shifts in vibration phase the procedure suggested in [2] is used. Since the purpose of this paper is to present experimental observations testing the theoretical predictions derived in [1], we here only summarise the solution process for solving the equation of motion: The solution for the transverse pipe vibrations is approximated by a Galerkin expansion using the first two linear mode shapes, separating the problem into a spatial and time depending problem, which are solved separately. The mode shapes are determined from the solution of the unperturbed eigenvalue problem, using appropriate boundary conditions. The time dependent modal amplitude functions are found using a systematic perturbation approach. The results can then be calculated analytically from the full forced response. The full solution of the two-mode approximation can be found in [1], and for a closely related case in [2]. In
case of midpipe excitation \((x_p = \frac{1}{2})\), the difference in phase, denoted phase shift \(\Delta \Psi\), between the transverse motion of two pipe points \(x_{1,2}\) positioned symmetrically around the middle of the flowmeter pipe, i.e. \(x_{1,2} = \frac{1}{2} \mp \Delta x, \Delta x \in [0; \frac{1}{2}[,\) is given by the following simple prediction [1]:

\[
\Delta \Psi(x_1) = \frac{2\varepsilon_1 \omega_2(x_1)}{\varphi_1(x_1)(\omega_2^2 - \omega_1^2)} \left(2\alpha_f v \int_0^1 \varphi_2 \varphi_1' dx + \int_0^1 \varphi_2 L_c[\varphi_1] dx \right) + O(\varepsilon^2),
\]

with \(\varphi_j = \varphi_j(x_j)\) and \(\omega_j, j = 1, 2,\) being, respectively, the mode shapes (normalised to \(\int_0^1 \varphi_j^2 dx = 1\)) and natural frequencies for the fluid filled pipe with actual boundary conditions (simply supported or clamped). For simple supports \((\omega_j = (j\pi)^2, \varphi_j = \sqrt{2}\sin(j\pi x))\) this is identical to Equation (42)-(43) in [2]. Equation (7) holds for the simple model pipe; it cannot be used to predict quantitatively the phase shift for real CFMs of complex geometry. But it can be used to pose the following testable hypotheses for what to expect for real CFMs:

**H1** The linear meter sensitivity, i.e. the factor of proportionality between phase shift \(\Delta \Psi\) and mass flow \(\alpha_f v\) employed by CFM manufacturers increases with a smaller difference between \(\omega_2\) and \(\omega_1\); this is a well-known result [22]. For a real CFM \(\omega_1\) and \(\omega_2\) are to be understood as, respectively, the drive frequency (corresponding to the driven mode \(\varphi_1\)) and the so-called Coriolis frequency (corresponding to the mode \(\varphi_2\) excited by the flow).

**H2** Fluid flow will induce a phase shift, which increases in proportion to mass flow \(\alpha_f v\).

**H3** Non-uniformly distributed damping may cause phase shifts, which are inseparable from the phase shift caused by mass flow; this occurs if the integral with \(L_c\) does not vanish. For common forms of \(L_c\) the integral vanishes for damping distributions, which are symmetrical w.r.t. the midpipe, i.e. \(c(x) = c(1 - x)\). Thus asymmetric damping distributions cause phase shifts in proportion to their asymmetry. The effect can be of the same order of magnitude as the effect of mass flow and be present even at zero fluid flow, and therefore be mistaken as a change in mass flow.
H4 Uniformly distributed generalized damping \( f(\dot{u}) \) (not necessarily linear) does not affect measured phase shift, to the order of accuracy used in the approximation. If there is an effect, it must be at least one order of magnitude smaller than the primary effect of the mass flow, and non-uniform damping, if present.

H5 Imperfections expressed by generally non-uniform perturbations of the pipes mass and stiffness do not to affect the measured phase shift, at least to the order of approximation employed. For practical applications a possible effect could be overshadowed by the effects of other imperfections.

H6 Temperature may change pipe natural frequencies (by a change in fluid density) as well a damping properties, and thus affect phase shift.

The approximate analytical prediction (7), and similar ones for other imperfections, have already been tested against results obtained by pure numerical analysis using Galerkin expansion, showing good agreement [1, 2, 3]; thus the mathematical approximations for the simple model are adequate, under the assumptions made. The question is then to which extend these predictions hold also for real CFMs. We asses that by testing H3, H5, and H6 using laboratory experiments employing real CFMs of complex geometry, we can answer this. Hypothesis H1 is already well supported and cannot be tested using the test rig. Since there is no actual fluid flow through the CFM, H2 cannot be tested. The test rig does not enable investigating hypothesis H4. From previous research work, e.g. [23], other effects of temperature, than the one stated in H6, are already well-known, e.g. that temperature changes affect the elastic properties of CFMs, cause pipe expansions and increase the axial forces on the CFM pipe.

3. Experimental setup and procedures

3.1. Setup

Laboratory measurements for a commercial CFM with two U-shaped pipes have been conducted to test H3, H5 and H6. The same excitation and measure-
ment equipment has been used during all experiments. To allow for the experiments, the CFM was stripped for its protecting casing, and also not mounted in a piping system the recommended way; this implies more sensitivity to external disturbances than for a similar CFM under normal operating conditions, and thus larger variability in measured phase shifts. Vibrations of one of the pipes were measured by a laser velocity transducer (B&K 3544). The damping ratio for the mode of interest was estimated using B&K PULSE modal analysis software and hardware (B&K front-end 3560C), in combination with standard impulse hammer (B&K 8203) testing, and averaged frequency response functions.

The built-in CFM software and electronics were used for obtaining the measured phase shifts. To estimate background noise the phase shift was recorded during 19 h 22 min with the CFM not being affected by fluid flow, external damping, added masses or temperature changes. The phase shift was drifting with a 4% deviation from its mean value in the beginning and end of the measurement, it was constant for 17 hours of the experiment. The standard deviation of the measured data is $\sigma(\Delta\Psi) = 3.2 \times 10^{-6} \text{ rad}$.

3.2. Procedure

To establish and investigate the effect of an asymmetrical damping distribution, the principle of eddy-current damping was employed to vary the energy dissipation asymmetrically in a contactless manner. This was done by installing neodymium magnet discs asymmetrically near one of the flowmeter’s pipes and attaching a small copper plate to one of the flowmeters’ pipes (Fig. 2(a)). The level of damping was controlled by varying the number of magnets and the distance between the magnets and the copper plate. The increase in pipe damping was recorded by measuring the damping ratio of the driven mode using B&K PULSE model analysis software with standard impact hammer testing. The results were checked against simple ring down tests with estimation of logarithmic decrement, and good agreement was obtained.

The neodymium magnets enable an easy control of the damping and allow a
Figure 2: Experimental setup for determining resonance frequencies, damping ratios and phase shift for an industrial Coriolis flowmeter in case of (a) asymmetrical damping, (b) symmetrical damping and (c) external added masses.

High resolution of the investigated damping range. However, due to difficulties in realising the same amount of damping at different pipe locations, they cannot conveniently be employed to test the effect of symmetrical damping. This was instead done by taping strips of damping gel tape (Geltec GT-1) in a symmetric manner to the flowmeter pipes close to the driver and sensor coil, as indicated in Fig. 2(b). In this case the level of damping was controlled by varying the amount and position of tape attached to the pipes. It is not possible to measure the degree of symmetric damping, however significant effort was put in the setup to mount the strips symmetrically. The gel tape was also used to investigate the effect of asymmetric damping by attaching it either close to sensor coil 1 or 2, however the resolution of the damping range was not as fine as for the case employing neodymium magnets.

For investigating the effect of added mass on the phase shift in the analytical model [1] mass was, for convenience of interpreting results, redistributed from one
side of the flowmeter pipe to the other. This could be realized by removing pipe material from the real CFM. However, since it would be difficult to re-establish the symmetric design afterwards, the effect of asymmetrical and symmetrical additional external mass was realized instead by gluing small brass weights to the flowmeter pipe using beeswax (Fig. 2(c)). Up to 9% of the flowmeter pipes’ mass was added in this way.

4. Results

Hypothesis H1 is already well supported and could not be tested using the setup described in Sec. 3, since it does not allow an actual fluid flow through the tested flowmeter. Still we need to evaluate whether the phase shifts due to the various pipe imperfections could be of the same order of magnitude as those due to mass flow, and thus significant for applications. For this purpose experimental data provided by Siemens A/S is evaluated in Sec. 4.1, testing at the same time hypothesis H2. The effect of asymmetrical damping (also H3) is investigated in Sec. 4.2, whereas the effect of symmetrically distributed damping (H3) is tested in Sec. 4.3. The focus of Sec. 4.4 is on the effect of imperfect pipe mass (H5), while in Sec. 4.5, we investigate how ambient temperature changes influence the phase shift measured by the employed CFM (H6).

4.1. H2 - Mass flow

Fig. 3 shows the phase shift $\Delta \Psi$ caused by the mass flow of water through the CFM, as well as the order of magnitude of the zero-shift stability requirement $\Delta \Psi_0$ for the employed dual-pipe CFM. The phase shift due to mass flow is obtained by inserting the metered mass flow in (3), knowing the CFMs calibration factor $\tilde{C}$ and its drive frequency $\tilde{\omega}_d$. A linear relationship between mass flow and phase shift can be identified in Fig. 3(a), i.e. supporting H2.

Fig. 3(a) will be used as a reference, when evaluating whether the phase shift induced by a given imperfection is significant. Two key performance indicators are taken from Fig. 3(a): The phase shift $\Delta \Psi_{\text{max}} = 11.2 \times 10^{-3}$ rad corresponding to
Figure 3: Phase shift $\Delta \Psi$ as function of measured mass flow based on experimental data (provided by Siemens A/S). (a) Entire measurement range, phase shift $\Delta \Psi_{\text{max}}$ due to full rate mass flow, and order of magnitude of zero-shift stability requirement $\Delta \Psi_0$ of employed industrial dual-pipe CFM. (b) Representative zoom into measurement range. Different marker symbols indicate different runs.

the full rate mass flow measured, and the zero-shift stability requirement $\Delta \Psi_0 = 4.7 \times 10^{-7}$ rad, which defines how much the metered phase shift is allowed to fluctuate in case of zero mass flow.

4.2. H3 - Asymmetric damping

4.2.1. Effect of eddy-current damping

Fig. 4 shows the measured phase shift $\Delta \Psi$ as function of the flowmeter pipes’ damping ratio $\zeta$. Damping ratios are measured for the drive mode; this applies to all damping measurements presented. Employing up to eight magnets, and varying the distance to the flowmeter pipe in small steps, enables the investigation of the damping range with high resolution. As appears asymmetrically applied damping increased the pipe damping from the base value 0.064% up to $\zeta = 0.164\%$, inducing phase shifts up to $\Delta \Psi = 1.02 \times 10^{-3}$ rad. We also identify a linear relationship between applied damping and induced phase shift. Comparing this to Fig. 3(a), we see that the phase shift due to the maximum level of applied asymmetric damping is about 1/10 of the full rate phase shift $\Delta \Psi_{\text{max}}$; it could be mistaken as a change in mass flow during a flow measurement. The experimental results depicted in Fig. 4 thus supports H3. In a practical application the
phase shifts due to asymmetric damping would be inseparable from those caused by mass flow, unless either the mass flow or damping asymmetry is known, or known to be constant in time. The phase shifts induced by asymmetric damping, even the ones caused by small damping of, e.g., $\zeta = 0.07\%$, are much bigger than the zero-shift requirement $\Delta \Psi_0$ of the tested flowmeter. A possible zero shift could be removed by a proper initial meter calibration. However, damping could change subsequent to the meter calibration, e.g. due to wear, lubrication, temperature changes, or air bubbles in the fluid. So, a fluctuating damping distribution could be a factor contributing to the lack of zero-shift stability occasionally observed with industrial CFM.

4.2.2. Effect of gel tape damping

Fig. 5 shows the effect of asymmetrical damping on measured phase shift $\Delta \Psi$, with the damping being induced by gel tape. The largest effect of gel tape on pipe damping was observed when placing it at the pipe antinodes, i.e. close to sensor coil 1 or 2, cf. Fig. 2(b). This indicates the primary effect of gel tape, in this case, is to increase air damping, rather than to absorb surface strains near the pipe nodes, as initially intended. The phase shifts caused by asymmetric
damping induced by attached damping tape are of the same order of magnitude as those due to mass flow (cf. Fig. 3). The phase shifts increase roughly in proportion to the asymmetrically induced damping. These results also support H3. Comparing Figs. 4 and 5, it appears that gel tape increased pipe damping 2-3 times more than the magnets, giving higher phase shifts; however magnets allowed for a higher resolution of the damping range.

Fig. 5(b) shows that increasing negative phase shifts are induced when the damping is applied near sensor coil 2. This change in the sign of the phase shift is actually also predicted by (7), as shown in the appendix. One consequence of this is, that if part of the pipe damping changes slowly in time, then this may increase as well as decrease the measured phase shift (and thus meter readout), depending on which half of the pipe length is affected by the damping change.

4.3. H3 - Symmetric damping

Fig. 6 shows the measured phase shift $\Delta \Psi$ as a function of the pipe damping ratio $\zeta$. The pipes’ damping was increased by attaching gel tape as symmetrically as possible w.r.t. the axial coordinates. In contrast to the experiments resulting
in Fig. 5, the damping tape was here attached close to the driver coil. The mean phase shift $\Delta \Psi = -9.43 \times 10^{-5}$ rad and standard deviation $\sigma(\Delta \Psi) = 5.24 \times 10^{-4}$ rad are shown by the dashed and dotted lines. The phase shifts induced by increasing the pipes damping symmetrically are approximately one order of magnitude smaller than the phase shifts due to asymmetric damping (cf. Fig. 5), thus supporting H3. The results in Fig. 6 also imply that symmetrically increased damping does not give rise to linearly increasing phase shifts, as with asymmetrically increased damping. Comparing Fig. 5 and 6, it appears that asymmetrically applied damping leads to larger changes in the measured phase shift than symmetrically applied damping, even though the damping ratio was increased about twice as much for the symmetric case.

4.4. H5 - Imperfect pipe mass uniformity

Fig. 7 shows the measured phase shift $\Delta \Psi$ as a function of added mass. To investigate the effect of imperfect pipe mass uniformity, up to 9 grams of added mass have been applied in an asymmetrical way, resulting in the measured phase shifts shown in Fig. 7(a)-(e), whereas up to 12 grams (about 5% of total pipe mass) have been attached as symmetrically as possible to obtain Fig. 7(f). We
can see that any amount of added mass, applied either symmetrically or asymmetrically, induces a phase shift, and that the induced phase shifts tend to increase in magnitude with added mass. The phase shifts induced by small added mass are at least one to two orders of magnitude smaller than the phase shift due to mass flow, cf. Fig. 3. This supports H5. The experimental results suggest that added mass should not lead to significant errors in the flow measurement. However, the phase shifts induced by added mass are larger than the CFM’s zero-shift requirement $ΔΨ_0$. This indicates that a developing non-uniformity in pipe mass could explain a possible drift of the zero-shift during the CFM’s lifetime, caused by, e.g., corrosion, cavitation, or particles attached to the inner CFM pipe walls of an improperly cleaned CFM.

H5 states, that a possible effect of imperfect pipe mass uniformity could be overshadowed by the effects of other imperfections. The result shown in Fig. 8 support this hypothesis. Fig. 8(a) shows the measured phase shift $ΔΨ$ as a
function of asymmetrically added mass. In contrast to the previous experiments, up to 20 grams have been added to one flowmeter pipe, which corresponds to approximately 9% of the pipes’ total mass. We can see that the measured phase shift appears to be little affected by adding small amounts of mass up to 3 grams. However, adding more mass than that significantly increases the measured phase shift, up to the same order of magnitude as those due to mass flow. Mass changes of that order of magnitude are less likely to be seen in real CFM applications, however if they occur, they could be mistaken for a change in mass flow.

The size of the surface of the 4 to 20 g weights led to the question whether they could have acted also as air dampers? Fig. 8(b) shows the measured phase shift $\Delta \Psi$ as a function of the pipes’ damping ratio $\zeta$, with the damping being measured after each weight has been added. As appears adding mass also increased the pipes damping considerably, from 0.06% for the case of flowmeter pipes without added imperfections to 0.3% with 20 grams of mass added to one flowmeter pipe. This increase in pipe damping, induced asymmetrically, could explain the significant phase shifts induced by large added mass. The effect of added mass, which is predicted to be small as long as the mass is small, could in this case be overshadowed by the effect of asymmetrical damping, offering a possible explanation for the order of magnitude of the phase shifts shown in Fig. 8(a).

To illustrate measurement variability, Fig. 9 shows the effect of adding mass
Figure 9: Measured phase shift $\Delta \Psi$ (symbol markers) as function of (a) asymmetrically or (b) symmetrically added mass. Error bars show mean value $\bar{\Delta \Psi}$ and standard deviation $\sigma(\Delta \Psi)$.

asymmetrically (a), or symmetrically (b), using a setup like the one for Fig. 7(a) and (f). Measurements of phase shift were repeated five times for each added mass resulting in a variability in measured phase shift as shown by error bars in the figure. As appears, a change in phase shift with added mass is present, considering the variability, for small masses applied asymmetrically, however the change is two orders of magnitude smaller than those due to mass flow $\Delta \Psi_{\text{max}}$.

4.5. H6 - Temperature

Our experiments indicated that even small temperature changes might induce phase changes. To investigate this further, a heat gun was used to heat up the flowmeter pipes (Fig. 10). In real applications this could be caused by sudden temperature changes of either the fluid in the CFM or the ambient surroundings. Neither gel tape nor magnets and copper plate were influencing or attached to the flowmeter’s pipe during these investigations. The temperature of the room, in which the experiments were conducted, was 20-21°C.

Fig. 11(a) shows the phase shift caused by heat impact from air with tem-
temperature of 58 °C measured by a thermometer at the heat gun outlet. The grey lines show the sampled data logged by the CFM, and the black lines the running average; here we only consider the latter, low pass filtered data. The heat gun was turned on after $t_{s,h}=290$ s, and turned off again after 570 s at $t_e=860$ s. The phase shift is seen to increase after the heater gun is turned on and fluctuates around a higher value, approximately $1.2 \times 10^{-5}$ rad, corresponding to about 0.1 % of the full rate phase shift $\Delta \Psi_{\text{max}}$ (cf. Sec. 4.1). The phase shift decreases to its original value after the heat gun is turned off, though only after the unexplained period.

Fig. 11(b) shows the phase shift caused by heat impact with changing temperature. A different heat gun had to be employed to enable this temperature variation. The heat gun was turned on at $t_{s,h}=60$ s blowing air with 29 °C onto the flowmeter’s pipes. The air temperature was reduced after 480 s to 22 °C, i.e. at $t_{s,c}=540$ s. After another 480 s, at $t_{s,e}=1020$ s, the heat gun was turned off. It can be seen that the phase shift increases to $7 \times 10^{-6}$ rad, when the heat gun is turned on, and decreases to $-7 \times 10^{-6}$ rad, when the air temperature is reduced. The scattering or high-frequency variations of the logged data around the running average is increased when blowing with a heat gun on the flowmeter’s pipes, probably reflecting noise and air flow induced pipe vibrations. Once the heat gun is turned off, the scattering of the data is decreased and the CFM meters phase
Figure 11: Phase shift measured by CFM exposed to various temperature variations of ambient environment as explained in the text. Grey lines: Data logged by CFM. Black line: Running average CFM data (Window size: 50 samples). Events: $t_{s,h}$ heat gun started emitting hot air, $t_{s,c}$ heat gun started emitting cold air, $t_e$ heat gun turned off.

shifts of the same magnitude as before the heat impact.

The phase shift induced by sudden temperature changes are in both cases larger than the requirement for zero-shift stability requirement. It appears therefore that temperature changes could explain a drifting zero-shift during the CFMs lifetime. For each of the two cases the measured phase shift returned to its original value, once the heat gun was turned off. This strongly indicates that the phase shift change was indeed caused by the change in temperature. Here we can only speculate on the actual link between temperature and phase shift: Due to the positioning of the heat gun, the two sensor coils experience different temperatures and amounts of air flow. The sensor coils can be seen as small dampers, i.e. pistons inside a cylinder. Changing the air temperature with the heat gun, changes the viscosity of the air. Hence, asymmetrically distributed damping could have been induced when blowing air on the flowmeter’s pipes, resulting in phase shift.
changes according to H3, which is now well supported.

5. Conclusion

This paper presents an experimental investigation of the effect of pipe-imperfections on a Coriolis flowmeter. The purpose is to test theoretically based hypotheses derived from analytical expressions derived for greatly simplified models. These expressions are given so that one can observe the influence of particular pipe-imperfections on the axial shift in vibration phase, which is exploited in Coriolis flowmetering.

We have experimentally tested hypotheses regarding the influence of asymmetrical and symmetrical pipe damping, added mass and increasing ambient temperature on phase shift measured by CFMs.

With respect to axially non-uniform damping, experiments showed that axially asymmetrically distributed damping induces phase shifts, even in the absence of mass flow. This supports the theoretically based hypothesis. The observed zero shifts can be of the same order of magnitude as the ones due to actual fluid flow, and could be mistaken for changes in mass flow.

Axially symmetrically distributed damping was demonstrated experimentally to induce zero shifts, even though the damping is applied as symmetrically as possible. However, these zero shifts are one order of magnitude smaller than the phase shifts due to actual fluid flow, supporting the theoretically based hypothesis regarding this matter.

Imperfectly distributed pipe mass was observed to induce zero shifts, however due to their order of magnitude these are ignorable for typical applications measuring actual mass flow. Therefore it appears that there is agreement between the theoretically based hypothesis and what we see during the experiments.

Changing ambient temperature was observed to induce zero shifts, which are two orders of magnitude smaller than the phase shifts due to actual mass flow. The link between temperature and phase shift under zero flow conditions was hypothesised to be an asymmetrical change in the air damping provided by the
vibrating measurement coils. Without referring to specific evidence [23] points at temperature changes as a possible source for changes in zero shift.

Some CFMs have occasionally shown a drifting zero shift. Imperfections related to damping, added mass, and temperature changes could, based on the order of magnitude of the experimentally observed zero shifts, contribute to time-varying zero shifts in case the imperfections themselves vary in time.

The hypotheses tested were derived for a simple model of single straight pipe with uniform plug flow. This model ignores substantial features of real CFMs, e.g. having two curved pipes, attached sensors and driver coils, external disturbances, and non-uniform flow. It was assumed that the main physical properties were unaffected by this apparent complexity. The hypotheses created using the simplified models were then tested with laboratory experiments. The hypotheses passed this test with acceptable accuracy. Our experimental tests have shown that simplified models and approximate analysis are useful, even for complex systems such as a real CFM, in providing direct insight into which imperfections affect phase shift, and in which manner. Only hypotheses on various factors’ zero shift effect was tested. Testing how the same factors affect CFM sensitivity would require an experimental setup with controllable mass flow, and possibly also a higher order analysis of the analytical model.

This paper shows how some pipe imperfections could lead to incorrect mass flow readings, providing data and knowledge not previously shown in the available literature. Others sources, for which hypotheses exist, are still to be experimentally investigated. Non-uniform stiffness distribution is of particular interest, since the analytical approximation predicts no effect on measured phase shift, whereas [23] mentions incorrect mass flow readings due to stiffness changes.

Acknowledgments

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References


Appendix Model prediction of the change in phase shift caused by a symmetrical change in damper position

Let a linear viscous damper be positioned at $x = x_c \in ]0;1[$, with transverse and rotational damping coefficients $c_{0u}$ and $c_{0\theta}$, respectively, so that the damping operator in (7) is:

$$L_c = c_{0u} \delta(x - x_c) + \frac{d}{dx} \left( c_{0\theta} \delta(x - x_c) \frac{d}{dx} \right),$$  \hspace{1cm} (A.1)

where $\delta(x)$ is Dirac’s delta function. Then let $I_c = I_c(x_c)$ denote the second integral in (7), which is responsible for the change in phase shift due to damping. Inserting (A.1) one finds, using integration by parts and the integral properties of $\delta(x)$, that

$$I_c(x_c) = \int \varphi_2 L_c[\varphi_1] dx = c_{0u} \varphi_1(x_c) \varphi_2(x_c) - c_{0\theta} \varphi_1'(x_c) \varphi_2'(x_c).$$  \hspace{1cm} (A.2)

If the damper position is shifted from $x_c$ to $1 - x_c$, i.e. symmetrically w.r.t. the midpipe, one finds, using (A.2) and the symmetry properties of $\varphi_j(x)$ (i.e. $\varphi_j(1 - x) = (-1)^{j-1} \varphi_j(x)$ and $\varphi'_j(1 - x) = (-1)^j \varphi'_j(x)$ for $j = 1, 2, ...$ and clamped-clamped or simply supported boundary conditions), that:

$$I_c(1 - x_c) = -I_c(x_c).$$  \hspace{1cm} (A.3)
Thus, a symmetrical change in damper location is predicted to give a phase shift due to damping, which is equal in magnitude, but opposite in sign.
Review of sources inducing zero-shift stability problems.

Technical report
Review of sources inducing zero-shift stability problems

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ABSTRACT

This document evaluates some possible sources for Coriolis flowmeter (CFM) zero-shift problems. Basis of the evaluation are results of a brainstorming session at Siemens Flow Instruments. These imperfections of the non-ideal CFM are then compared to hypotheses generated from predictions for the phase shift, which are obtained from a simple analytical CFM model. The analytical model offers direct insight into which imperfections affect phase shift, and how they do it. This leads to indications of the relevance of the imperfections, and therefore also to suggestions for necessity of further investigation, i.e. experimental and/or numerical.

Keywords: Coriolis flow meter, phase shift, imperfections, non-ideal supports, material properties, flow pulsation, asymmetrical driving force, asymmetrical pickup location

1. REVIEW AND COMPARISON OF RESULTS

In Coriolis flowmetering one employs an assumed linearity between a fluid-induced phase shift and the metered mass flow. In a perfect world, a Coriolis flowmeter (CFM) will measure zero phase shift in case of zero fluid flow, and therefore zero mass flow. This might however not always be the case. Initially one can correct this error by introducing a calibration factor, which is determined individually during the calibration of each CFM. Hands-on experience with CFMs indicates, that the measured zero-shift, i.e. the phase shift in case of zero flow, can change during the lift time of a CFM; from zero, to which it has been calibrated, to an apparently random positive or negative value. The measured phase shift can continue to jump from the new value back to zero or another random value. Knowing the causes inducing the lack of zero-shift stability as it is experienced by some Coriolis flowmeters can help to understand and produce more accurate CFMs.

A simplified model of a single, straight, fluid-conveying pipe, which has been solved analytically employing simplifying approximations and a perturbation method, is used to predict the phase shift for real CFMs. Our model does not consider two coupled pipes, which are also employed in real CFM designs, and the effect of a dual pipe configuration is not investigated by us nor apparent from the state-of-the-art literature in this field. A simplified model of a CFM is shown in Fig. 1, which illustrates the model of a vibrating, fluid-conveying pipe being simply supported at its ends. The pipe is driven at resonance by an external force applied mid-pipe, as it is done in real flowmeters. This simple model offers a direct insight into the parameters, which might influence the phase shift, and is used to establish hypotheses for real Coriolis flowmeters. Details of the calculations are omitted here and can be found elsewhere.\textsuperscript{1–4} The presentation is limited to results and established hypotheses.

Some possible imperfections causing in zero-point problems are given in the original mindmap by SFI.\textsuperscript{5} This map is based on SFI’s knowledge and experience with CFMs. Imperfections giving zero-point problems, which cannot be supported or investigated by the work carried out at DTU-FAM, are not taken into further consideration, i.e. electrical and electromechanical imperfections, therefore omitted. The result hereof is a reduced mindmap, which can be seen in Fig. 2. Only imperfections related to boundary conditions and mechanics are considered. Dashed circles indicate that the above mention imperfections have been investigated by the group at DTU-FAM; a continuous circle indicates, that future investigations by the group at DTU-FAM will confirm or dismiss the importance of the above mention imperfections.

The work by J.J. Thomsen and J. Dahl\textsuperscript{2} investigates imperfections related to the boundary conditions of a CFM. In an ideal system one typically assumes simply supported (Fig. 1), clamped and hinged supports. These are however not realistic, since it is more likely to have, e.g., finitely small distributed transverse and rotational stiffness and damping, or finitely small support flexibility, i.e. finitely large support stiffness (Fig. 1). The phase shift $\Delta\Psi$ is always determined between two symmetric measurement points $(x_{1,2} = \frac{1}{2} \pm \Delta x)$ represent-
The phase shift:

\[ \Delta \Psi (\frac{1}{2}) = \frac{32\sqrt{2}}{45\pi^2} \alpha v. \]  

(3)

This implies that: a) Small rotational support stiffness has no effect on measured phase shift, to the order of accuracy used in the approximation; b) if there is an effect, it must be at least two orders of magnitude smaller than the primary effect of mass flow \( \alpha v \). For practical purposes, i.e. explaining a lack of zero-point stability, this effect can be considered negligible, or overshadowed by effects of other imperfections.

The presence of rotational stiffness at hinged supports gives the phase shift:

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The presence of transverse damping at flexible supports does not, even if unsymmetrical, affect the phase shift, again, at least not to the order of approximation employed.

The work by N. Fuglede\textsuperscript{3, 6} investigates imperfections related to damping and structural non-idealities, i.e. non-uniform mass and stiffness distribution. The phase shift in case of symmetrically placed detectors and sharply resonant excitation at the pipe centre is determined to be:

\[ \Delta \Psi (\frac{1}{2}) = \frac{2\sqrt{2}}{15\pi^2} \alpha v + \int_0^1 \varphi_2 L_c[\varphi_1] dx, \]  

(4)

with \( L_c[\varphi_1] \) being distributed linear damping, \( a \) the vibration amplitude, and \( \varphi_1 \) and \( \varphi_2 \) to mode shapes due to simple supported boundary conditions. The expression implies that: a) phase shift is proportional to mass flow; b) sensitivity depends on frequency distribution (\( \omega_0^2 - \omega_1^2 \)); c) distributed linear damping may give a zero shift depending on damping distribution; d) non-idealities of flowmeter pipe related to mass, stiffness and non-linearity has negligible influence on phase shift. Generally speaking, any asymmetry in the damping distribution will lead to a phase shift even in case of zero mass flow. Small variations of mass and stiffness properties along the flowmeter pipe, e.g. due to an unbalanced pipe or uneven corrosion, should not cause zero shift stability problems. This hypothesis is very relevant for real Coriolis flowmeters, since SFI has pointed out these very variations as a possible source for zero shift stability problems (Fig. 2).

The work by S. Enz\textsuperscript{4} investigates imperfections related to the fluid flowing in the vibrating pipe. The
brainstorming session lead to assumption, that energy might be lost due to media related coupling, when the oscillation frequency of the fluid \( \omega_f \) is equal to the first natural frequency of the pipe \( \omega_0 \). In the analytical investigations, it is assumed that the fluid velocity has an unsteady component so that \( v(t) = v_0 [1 + q \cos(\omega_f t)] \).

In case of pulsating fluid the detectors signals are predicted to contain frequency components additional to the drive frequency \( \omega_0 \).

For the case of slowly pulsating fluids \( \omega_f \ll \omega_0 \), which could be caused by pumps employed in CFM applications, this yields a phase shift with a constant and unsteady component:

\[
\Delta \Psi \left( \frac{1}{4} t \right) = \frac{32 \sqrt{2}}{45 \pi^2} \alpha v_0 \left[ 1 + q \cos(\omega_f t) \right].
\] (5)

CFMs are predicted to measure the unsteady mass flow \( \alpha v(t) \) correctly, if, and only if, the fluid is pulsating slowly.

In CFMs one would typically employ narrowband filtering to remove data at frequencies other than the drive frequency. For non-slow fluid pulsations (the relevant cases to investigated turned out to be \( \omega_f \approx 2\omega_0 \), \( \omega_f \approx \omega_0 - \omega_0 \), and \( \omega_f \) away from \( 2\omega_0 \) and \( \omega_0 - \omega_0 \)\), which could be caused by fast-closing valves, this yields the "filtered" phase shift:

\[
\Delta \hat{\Psi} \left( \frac{1}{4} t \right) = \frac{32 \sqrt{2}}{45 \pi^2} \alpha v_0.
\] (6)

CFMs are predicted to measure the mean phase shift \( \alpha v_0 \) if the fluid is pulsating non-slowly and the detector signals are narrowband filtered.

The case of \( \omega_f \approx \omega_0 \) is predicted to induce additional frequency components to the detector signal, these can be removed by narrowband filtering, so that the CFM would measure the correct mean mass flow \( \alpha v_0 \), with phase shifts predicted by (6).

High frequency fluid pulsations are less likely to cause problems for CFMs. Pumps and valves commonly applied in real CFM applications operate within a frequency range, which will not lead to fluid pulsations in the high frequency range.

Equations (5) and (6) imply that the presence of a pulsating fluid: a) induces a phase shift, but b) should not cause zero shift stability problems.

S. Enz has also investigated imperfections related to the position of the actuator and two detectors metering the pipe displacement from which the phase shift is determined.

Usually one assumes that the pipe is excited midpipe. However, small manufacturing variations might occur, so that the pipe is not exactly driven at midpipe, i.e. \( x_p = \frac{1}{2} \pm \Delta x_p \), with \( x_p = \frac{1}{2} \) being the ideal actuator location and \( \Delta x_p \) a small deviation from the ideal location. The phase shift in case of asymmetrically placed driver and sharply resonant excitation is determined to be:

\[
\Delta \Psi \left( \frac{1}{4} t \right) = \frac{32 \sqrt{2}}{45 \pi^2} \alpha v_0 \left[ 1 + \frac{6}{16} c \pi \Delta x_p \right].
\] (7)

The expression implies that: a) an asymmetric excitation will cause a phase shift even at zero fluid flow; b) the magnitude of the phase shift depends on the pipe damping; c) the zero shift can possibly be avoided by careful manufacturing of the CFM.

Manufacturing variations might also result into the two detectors not being exactly located in the antinodes of flowmeters Coriolis mode, i.e. \( \Delta x = \frac{1}{4} \). For the two detector positions this can be described by \( x_1 = \Delta x + \Delta x_1 \) and \( x_2 = \Delta x + \Delta x_2 \), with \( \Delta x_1, \Delta x_2 \in [0, \pm \frac{1}{4}] \) being a small deviation from the ideal (symmetric) detector positioning. The phase shift in case of asymmetrically located detectors is determined to be:

\[
\Delta \Psi \left( \frac{1}{4} t \right) = \frac{32 \sqrt{2}}{45 \pi^2} \alpha v_0 \left[ 1 + \frac{7}{16} (\Delta x_1 + \Delta x_2) \right].
\] (8)

Equation (8) implies that asymmetrically located detectors: a) have an effect on the measured phase shift, b) should not cause zero shift stability problems, c) can be compensated for if present before the flowmeter calibration, and d) will lead to measurement errors, if they are caused by, e.g., improper handling of the flowmeter after the meter calibration.

2. FUTURE PERSPECTIVES

Initial experimental investigations have been performed by N. Fuglede to test this experimentally by applying external asymmetric damping to a given CFM by SFI. The promising results are limited to a few parameters and lack a more systematic investigation. This work has been continued by Stefan Neumeyer during the autumn semester 2009. Stefan will carry out systematic experiments to investigate the following (in prioritised order): 1) asymmetric damping, 2) asymmetric mass distribution, 3) symmetric damping, 4) temperature changes of surroundings and of flowmeter pipes, 5) change of boundary conditions. SFI states that temperature changes could induce material damping changes\(^5\) and Niels Fuglede’s hands-on-experience indicate the same. It should therefore be tested more systematically, whether temperature changes induce a phase shift. The results of the experimental investigation will be presented as a journal paper\(^8\) as well as a technical report for Siemens A/S, Flow Instruments\(^9\).

The numerical work by S. Enz\(^10\) will investigate, whether media related coupling can cause zero-point problems. Media related coupling is here related to motions in the flowing fluid induced by the vibrating pipe. Question to be answered: Can pipe motion induce motions in the fluid, which could excite the pipe, e.g., if frequency of fluid motions is equal to natural frequencies of the pipe. The numerical work includes also effects of fluid velocity profiles.

3. CONTRIBUTIONS

In 2008-2009 five people at DTU-FAM (including two PhD students and two project students) have worked
with Coriolis flowmeter related problems. Thomsen and Dahl demonstrated how to mathematically model a simplified CFM with various imperfections, and how to analyse their effects on phase shift using perturbation analysis. This approach was subsequently used in the other analytical studies, i.e. imperfections related to damping and structural non-idealities (Niels Fuglede) and imperfections related to the fluid and excitation of a CFM (Stephanie Enz). Niels Fuglede also initiated experimental investigations, which have been continued by Stefan Neymeyer and Stephanie Enz.

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