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## GENETIC ALGORITHM FOR MULTI-OBJECTIVE OPTIMAL DESIGN OF SANDWICH COMPOSITE LAMINATES WITH MINIMUM COST AND MAXIMUM FREQUENCY

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### ABSTRACT

This paper deals with optimal design of sandwich composite laminates consisting of high-stiffness and expensive surface and low-stiffness and inexpensive core layers. The objective is to determine ply angles and number of core layers in such a way that natural frequency is maximized with minimal material cost. A Genetic Algorithm (GA) procedure is used for simultaneous cost minimization and frequency maximization. The proposed model is applied to a graphite-epoxy/glass-epoxy laminate and results are obtained for various aspect ratios and number of layers.

**Keywords:** Optimal Design, Genetic Algorithm, Composite Laminates, Cost Reduction.

### INTRODUCTION

In recent years, composite materials have been extensively used because of their high strength-to-weight ratio and their potential for specific design by selecting the fiber materials and orientations. Composite laminates are usually employed in aerospace, defence, marine and automotive industries. Most composite structures have components that may be modelled as rectangular plates. To reduce the cost and enhance the mechanical properties of these structures, sandwich design is usually used. Such designs employ the high-stiffness and expensive materials in the surface layers and the low-stiffness and inexpensive materials in the core layers. This idea combines the advantages of two materials. The present study, aims at optimal design of symmetric laminates subjected to free vibrations.

Various integer programming techniques are employed in Haftka and Walsh [1], Nagendra et al. [2], Le Riche and Haftka [3], Gürdal et al. [4], and Kogiso et al. [5], to determine the optimal stacking sequences of laminates under buckling loads. In the design of laminates, maximum frequency problems are of practical importance. Adali et al. [6] used an integer programming approach with boolean variables for frequency maximization of composite laminates undergoing free vibrations. Boyang et al. [7], applied Genetic Algorithm (GA) to find the optimal stacking sequence of a composite laminate for maximum buckling load.

Weight minimization of laminated composite panels subjected to strength and buckling constraints was investigated by Gantovnik et al. [8]. Lin and Lee [9] applied a GA procedure with local improvement for optimum stacking sequence of a composite plate.

Research work on composites optimal design is extensive. However, most papers focused on a single objective and overlooked some important design issues such as material cost.

In this work, a multi-objective optimal design is considered to design a sandwich composite laminate by using a genetic algorithm (GA). The design objective is to achieve the maximum frequency with minimum cost. The paper has been organized as follows: in the next section definition of the basic problem is given. Optimization process of this basic problem is illustrated in section three. Some numerical results are provided and discussed in section four to show the efficiency of the proposed solution technique in optimal design.

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## PROBLEM DESCRIPTION

Consider a simply supported hybrid laminated plate of length  $a$ , width  $b$  and thickness  $h$  in the  $x$ ,  $y$  and  $z$  directions, respectively. The laminate consists of an even number of orthotropic layers made of different materials. The surface layers are made of composite with high stiffness fiber reinforcements and the core layers of a composite with low stiffness reinforcements. Each layer has a constant thickness  $t$  so that  $h = N \times t$  where  $N$  is the total number of layers. Note that the total thickness of the laminate is kept constant as the number of layers is changed in order to compare the performance of equal thickness designs.

The hybrid laminates are made of  $N_i$  inner plies and  $N_o$  outer plies such that  $N = N_i + N_o$ . The equation governing the free vibrations of these laminates is given by:

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (4)$$

In equation (4),  $w$  denotes the deflection in the  $z$  direction,  $\rho$  is the mass density, and  $h$  is the total thickness of laminate.

The bending stiffness  $D_{ij}$  are computed from

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \quad (5)$$

Where  $z_k$  is the distance from the middle plane of the laminate to the top of the  $k$ th layer and  $\bar{Q}_{ij}$  is the plane stress reduced stiffness component of the  $k$ th layer which can be computed as a function of fiber orientations and material properties.

The mass density of a hybrid laminate is computed as a thickness weighted average given by:

$$\rho = h^{-1} \int_{-h/2}^{h/2} \rho^{(k)} dz \quad (6)$$

Where  $\rho^{(k)}$  denotes the mass density of the material in the  $k$ th layer.

The boundary conditions for the simply supported plate are given by:

$$\begin{aligned} w = 0, M_x = 0 & \quad \text{at } x = 0, a \\ w = 0, M_y = 0 & \quad \text{at } y = 0, b \end{aligned} \quad (7)$$

Where  $M_x$  and  $M_y$  represent the bending moments about  $x$  and  $y$  axes, respectively. The influence of bending-twisting coupling stiffness  $D_{16}$  and  $D_{26}$  are assumed insignificant and hence will be omitted in the analysis. The error induced by this assumption is negligible if the following non-dimensional ratios:

$$\gamma = D_{16} (D_{11}^3 D_{22})^{-1/4}, \quad \delta = D_{26} (D_{11} D_{22}^3)^{-1/4} \quad (8)$$

Subject to constraints

$$\gamma \leq 0.2, \quad \delta \leq 0.2 \quad (9)$$

A detail discussion of this condition and its implications is given in Nemeth [1], where it is shown that for buckling problems the constraints (9) are effective in reducing bending-twisting coupling to a negligible level. Due to similarity of expressions for buckling load and frequencies, the same constraints are used to reduce the error introduced by neglecting  $D_{16}$  and  $D_{26}$ .

The solution of the eigenvalue problem (4) subject to the boundary conditions (7) is obtained by taking the deflection  $w$  for the vibration mode  $(m, n)$  as

$$w(x, y, t) = W(x, y) e^{i\Omega t} \quad (10)$$

$$W(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (11)$$

By substituting equation (11) into (4), we compute the eigen-frequency  $\omega_{mn}$  as

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h} \left\{ D_{11} \left( \frac{m}{a} \right)^4 + 2(D_{12} + 2D_{66}) \left( \frac{m}{a} \right)^2 \left( \frac{n}{b} \right)^2 + D_{22} \left( \frac{n}{b} \right)^4 \right\} \quad (12)$$

Where the various frequencies  $\omega_{mn}$  correspond to different mode shapes (different values of  $m$  and  $n$  in equation (12)). The fundamental frequency is obtained when  $m$  and  $n$  are both one.

## OPTIMIZATION PROCESS

Optimal design of a multi-objective composite laminate involves selection of a sequence of ply angles and number of low-stiffness and less expensive layers for maximization of natural frequency and minimization of structure cost. Therefore, a solution (chromosome) in GA algorithm can be represented by a sequence of ply angles and number of layers for each material. The ply angles of laminate and number of layers are coded as binary numbers. Angles can vary between -90 to 90 degrees with increments of 15 degrees. Therefore, there are 13 possible fiber orientations for each layer. Since the composite laminate is symmetric, we can consider half of the sequence as initial sequence. A typical solution is shown in Figure 1.

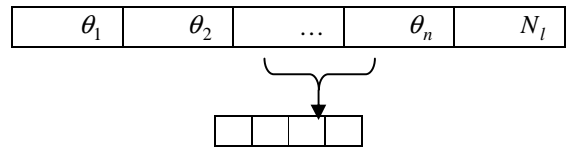


Figure 1: Definition of an individual

Where,  $N_l$  is the number of layers for each material.

In the next step, a fitness function will be defined for the multi-objective optimization problem. The purpose is to maximize the fitness function. This fitness function is the normalized summation of two objectives; 1) material cost and 2) frequency. Constraints are the equations (9) that should be satisfied. These constraints are included as a penalty in the fitness function. A penalty factor will be added to the objective

function if a constraint violates. The general form of objective function is as follows:

$$F = \frac{1}{(k_1 f_1^2 + k_2 f_2^2 + c_1 g_1^2 + c_2 g_2^2)} \quad (13)$$

$$f_1 = \left( \frac{\omega_{\max} - \omega}{\omega_{\max}} \right), \quad g_1 = (\delta - 0.2) \quad (14)$$

$$f_2 = \left( \frac{\cos t}{\cos t_{\max}} \right), \quad g_2 = (\gamma - 0.2)$$

Where  $\omega_{\max}$  and  $\cos t_{\max}$  represent the frequency and material cost when all layers are made of graphite-epoxy.

Also the material cost function is defined as:

$$\text{cost} = ab \frac{h}{N_t} g(\alpha_o \rho_o N_o + \beta_i \rho_i N_i) \quad (15)$$

Where  $h$  is the total thickness of laminate,  $N_t$  is total number of layers,  $\rho_o$  is the density of high-stiff layer material,  $N_o$  is the number of high-stiffness layers,  $\alpha_o$  is the material cost factor of high-stiffness layer,  $\rho_i$  is density of low-stiffness layer material,  $N_i$  is number of low-stiffness layers,  $\beta_i$  is the material cost factor of low-stiffness layer,  $a$  is the length of the plates and  $b$  is the width of the plate.

Coefficients  $k_1$  and  $k_2$  in equation (13) represent the relative importance of frequency and material cost in the objective functions. For instance,  $k_1 < k_2$  means that a lower material cost is more important than a higher frequency for designer. In this work,  $k_1$  and  $k_2$  are the same and coefficients  $c_1$  and  $c_2$  should be chosen so that any violation in the constraints imposes a considerable penalty in the fitness function. Therefore, there is no possibility of accepting a solution if the constraints are not satisfied.

## NUMERICAL RESULTS

A multilayer hybrid laminate consisting of glass-epoxy in inner layers and graphite epoxy in outer layers with geometrical dimensions of  $b=0.25m$ ,  $h=0.002m$  is considered. The properties of materials are as follows:

Graphite/Epoxy (T300/5280) :

$$E_1 = 181Gpa, E_2 = 10.3Gpa, G_{12} = 7.17Gpa$$

$$\nu_{12} = 0.28, \quad \rho = 1600kg/m^3$$

Glass/Epoxy(Scotchply1002):

$$E_1 = 38.6Gpa, E_2 = 8.27Gpa, G_{12} = 4.14Gpa$$

$$\nu_{12} = 0.26, \quad \rho = 1800kg/m^3$$

The idea of locating expensive material in the outer layers and inexpensive material in the inner layers can reduce the material costs while satisfies the design specifications. The material cost per unit weight of graphite-epoxy is eight times more expensive than that of glass epoxy and in the material cost function the cost factors  $\alpha_o$  and  $\alpha_i$  are eight and one, respectively. However the stiffness-to-weight ratio of graphite epoxy is about four times higher than that of glass epoxy.

We prefer to have a laminate with higher frequency and lower cost. It is known that the more graphite layers, the higher frequency and material cost. Therefore it is necessary to find the best trade off between frequency and material cost.

A computer code has been developed in order to find the best sequence of ply orientations and number of glass layers for various numbers of layers and aspect ratios.

To include the effects of number of layers and aspect ratio on the performance of the laminate, the total thickness of the laminate is kept constant and number of layers and aspect ratio are changed. In every case, the computer code was run to get the optimum angle sequence and layer numbers of each material.

Table 1 contains the best sequence of ply angles and number of glass layers for  $N=8$  and different aspect ratios varying from 0.2 to 2.

This table shows that the best sequence of ply angles changes from 0 to 90 if the aspect ratio varies from 0.2 to 2.

Table 1: Optimal sequences of angle- ply laminates for maximum frequency and minimum costs versus the aspect ratio for N=8

$a/b$	$b$	$h$	$N$	$\theta_{best}$	$\omega_{\max}$ (rad / s)	$\text{cost}_{\min}$	$n_{glass}$	Cost reduction (%)	Frequency reduction (%)
0.2	0.25	0.002	8	$[0/0/0/0]_s$	19093	1.1375	6	64.5	20
0.4	0.25	0.002	8	$[0/0/0/0]_s$	4844.3	2.275	6	64.5	21.4
0.6	0.25	0.002	8	$[0/30/-30/-30]_s$	2252.3	3.4125	6	64.5	19.6
0.8	0.25	0.002	8	$[0/45/-45/-45]_s$	1339.5	4.55	6	64.5	25.5
1	0.25	0.002	8	$[-45/45/45/45]_s$	1047.3	5.6875	6	64.5	25.9
1.2	0.25	0.002	8	$[90/-45/45/45]_s$	855.64	6.825	6	64.5	27.7
1.4	0.25	0.002	8	$[90/60/-60/-60]_s$	820.4	7.9625	6	64.5	23.9
1.6	0.25	0.002	8	$[90/-75/60/60]_s$	800.15	9.1	6	64.5	21.2
1.8	0.25	0.002	8	$[90/90/90/90]_s$	790.07	10.238	6	64.5	21.2
2	0.25	0.002	8	$[90/90/90/90]_s$	784.04	11.3750	6	64.5	21.3

The last two columns of the table present the percent of material cost and frequency reduction with respect to the situation which all of layers are made of graphite-epoxy.

As illustrated by Table 1, it is clear that a small decrease in the natural frequency leads to a considerable decrease in material costs. The table also shows that use of glass-epoxy layer in the composite laminate made of graphite-epoxy can decrease material costs considerably while there is a small reduction in the fundamental frequency.

Table 1 shows that for an average decrease of 22.7% in the fundamental frequency, there is a 64.5% reduction in material costs.

Similar results for different number of layers are demonstrated in tables 2 and 3. It is seen in table 2 that when  $N=16$  there is an average decrease of 19.5% in frequency while there is a 64.5% average reduction in material costs. Table 3 shows that when  $N=28$ , a 26.7% average decrease in frequency corresponds to a 71.2% average decrease in material costs.

As seen from tables 1, 2 and 3 the best sequence of ply angles are about 0 for an aspect ratio between 0.2 and 0.4,  $\pm 30$  for an aspect ratio of 0.6,  $\pm 45$  for aspect ratios between 0.8 and 1.2,  $\pm 60$  for an aspect ratio between 1.4 and 1.6 and 90 for aspect ratios between 1.8 and 2.

Figure 2 illustrates variation of fitness with respect to number of generations during optimization process by GA. Horizontal axis represents the number of generations and the vertical axis corresponds to the fitness function values. As it implies, with increasing in generation number, the average fitness values come closer to their maximum values. In other word, as the generation progresses, number of more fit individuals in the population increases and design variables approach to their optimal (or near optimal) quantities.

Moreover, it can be seen that when  $N=8$  and solution space is smaller, average fitness values reach to maximum fitness values faster and after a few generations.

The computational results, show that GA is a powerful tool in optimal design of composite laminates and it is able to

search a wide space of solutions in a reasonable time to find the optimal solution.

## CONCLUSIONS

In this paper, a multi-objective optimal design of composite laminates for minimum cost and maximum frequency is investigated. Hybrid laminates consisting of low-stiffness and inexpensive inner layers and high stiffness and expensive outer layers were considered. A genetic algorithm (GA) has been developed in order to obtain an optimal design for various numbers of layers and several aspect ratios. Maximum frequency and minimum cost of design have been achieved using GA for a graphite-epoxy/glass-epoxy laminate. Several aspect ratios and number of layers have been examined in the optimization process. Computational results show that the use of GA in optimal design of composite laminates can decrease material costs considerably while there is a small variation in the fundamental frequency. This approach is useful in reducing the total production cost of composite laminates. The results also show that GA algorithm is able to solve this problem in a reasonable computational time. Future works may include optimal design of composite laminates with different boundary conditions and material properties.

Table 2: Optimal sequences of angle- ply laminates for maximum frequency and minimum costs versus the aspect ratio for  $N=16$

$a/b$	$b$	$h$	$N$	$\theta_{best}$	$\omega_{max}$ (rad/s)	$cost_{min}$	$n_{glass}$	Cost reduction (%)	Frequency reduction (%)
0.2	0.25	0.002	16	$[0/0/0/0/0/0/0/0]_s$	19093	1.1375	12	64.5	20
0.4	0.25	0.002	16	$[0/0/-15/15/15/0/-15/0]_s$	4862.6	2.275	12	64.5	21.2
0.6	0.25	0.002	16	$[30/-30/30/-30/-30/-30/30/-30]_s$	2247.7	3.4125	12	64.5	19.8
0.8	0.25	0.002	16	$[-45/30/45/45/-45/45/45/-45]_s$	1449.4	4.55	12	64.5	19.3
1	0.25	0.002	16	$[-45/45/45/45/45/45/45/45]_s$	1154.5	5.6875	12	64.5	18.3
1.2	0.25	0.002	16	$[45/-45/-60/-45/-60/-60/-45/-45]_s$	992.2	6.8250	12	64.5	16.2
1.4	0.25	0.002	16	$[-60/60/60/60/60/60/60/60]_s$	883.2	7.9625	12	64.5	18.1
1.6	0.25	0.002	16	$[-60/60/60/60/60/60/75/75]_s$	816.16	9.1	12	64.5	19.7
1.8	0.25	0.002	16	$[90/90/90/90/90/90/90/90]_s$	790.07	10.238	12	64.5	21.2
2	0.25	0.002	16	$[90/90/90/90/90/90/90/90]_s$	784.04	11.3750	12	64.5	21.3

Table 3: Optimal sequences of angle- ply laminates for maximum frequency and minimum costs versus the aspect ratio for  $N=28$

$a/b$	$b$	$h$	$N$	$\theta_{best}$	$\omega_{max}$ (rad / s)	$cost_{min}$	$n_{glass}$	Cost reduction (%)	Frequency reduction (%)
0.2	0.25	0.002	28	$[0/0/0/0/0/0/0/0/0/0/0/0/0/15/15]_s$	16518	0.8429	24	73.7	30.8
0.4	0.25	0.002	28	$[0/15/-15/0/15/0/0/-15/0/0/0/15/0/0]_s$	4286.7	1.6857	24	73.7	30.5
0.6	0.25	0.002	28	$[30/-30/-30/-30/-30/45/30/-30/30/-30/30/-45/-30/-30]_s$	1960.5	2.5286	24	73.7	30
0.8	0.25	0.002	28	$[-45/45/-45/45/-45/45/45/45/-45/-45/45/30/-45/-45]_s$	1277.2	3.3714	24	73.7	28.9
1	0.25	0.002	28	$[45/45/-45/-45/-45/-45/45/-45/45/-45/-45/-45/45/45]_s$	1007.6	4.2143	24	73.7	28.7
1.2	0.25	0.002	28	$[-45/45/45/45/-45/45/-60/-45/45/-45/-45/45/-45/-45]_s$	879.3	5.0571	24	73.7	25.7
1.4	0.25	0.002	28	$[60/-60/-60/60/60/-60/-45/-60/-60/-60/60/-45/-45/60]_s$	849.7	7.275	22	67.5	21.2
1.6	0.25	0.002	28	$[60/-60/60/60/75/-60/60/75/60/-60/-60/-75/60/-75]_s$	785.14	8.3143	22	67.5	22.8
1.8	0.25	0.002	28	$[90/90/90/90/90/90/90/90/90/90/90/90/90/90]_s$	760.08	9.3536	22	67.5	24.2
2	0.25	0.002	28	$[90/90/90/90/90/90/90/90/90/90/90/90/90/90]_s$	754.01	10.393	22	67.5	24.3

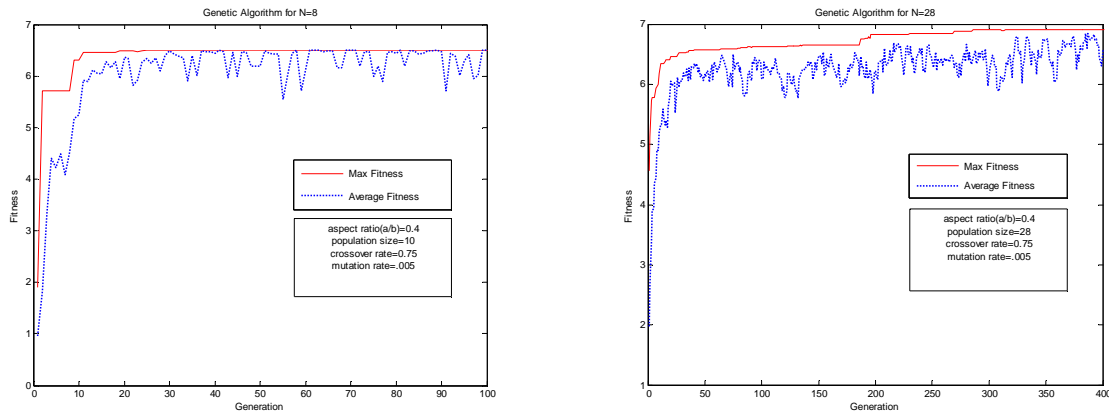


Figure 2: Variation of fitness with number of generations for  $N=8$  and  $N=28$

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