Control of sound fields with a circular double-layer array of loudspeakers

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Control of sound fields with a circular double-layer array of loudspeakers

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This investigation is concerned with generating a controlled sound field for listeners inside a circular array of loudspeakers without disturbing people outside the array. Ideally this configuration would have the advantage that reflections from the surroundings would be of no concern. Inspired by the Kirchhoff-Helmholtz integral theorem a double-layer array of loudspeakers is used. Several solution methods are suggested and examined with computer simulations: pure contrast control, pure pressure matching, and a weighted combination. In order to compare the performance of the methods two performance indices are used, i) the ratio of the sound energy in the listening zone to the sound energy in the quiet zone, and ii) a normalised measure of the deviations between the desired and the generated sound field in the listening zone. The best compromise is obtained with the method that combines pure contrast control with a pressure matching technique.

1 INTRODUCTION

For some time it has been technologically possible to control sound fields with loudspeaker arrays. This is reflected in the literature. One group of methods attempts to generate a sound field that approximates a desired sound field;\textsuperscript{1-13} and another group of methods tries to concentrate sound energy in a spatially limited listening zone and at the same time reduce the sound energy in another zone, the quiet zone.\textsuperscript{14-19} The former category of methods generates sound that may disturb people outside the listening zone. The latter category of methods can solve this problem, but methods that deal only with sound energy have no control over wave fronts or propagation directions of waves, and this might well give rise to disturbing perceptual artefacts.

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This paper proposes and examines a method that attempts to generate a sound field that imitates a desired one in a listening zone and at the same time reduces the sound energy in another region. In other words, the objective is to generate a controlled sound field in the listening zone without disturbing others in the quiet zone. In the present case the listening zone is defined as a region inside of a circular array of loudspeakers, and the quiet zone is outside the array. In order to achieve this goal, a double-layer array of loudspeakers is used. Such an array can be realised in practice with pairs of loudspeakers mounted back-to-back.

The work is related to multi-zone reproduction techniques that reproduce different sound fields in several regions. Poletti has proposed a multi-zone reproduction technique that can reproduce different sound fields in several regions the two-dimensional (2D) case. Wu and Abhayapala have studied generation of a 2D zone of quiet and a listening zone inside an array both in free space and in a reverberant space. Jacobsen et al. extended this method to the 2.5D case using a circular array of monopoles in 3D space, and compared this method with acoustic contrast control in computer simulations as well as experiments. Jacobsen et al. extended this method to the 2.5D case using a circular array of monopoles in 3D space, and compared this method with acoustic contrast control in computer simulations as well as experiments. (‘Two and a half dimensions’ means real 3D sources in 3D space but optimised for a 2D sound field on a surface.) Other related studies have aimed to reduce sound outside of arrays primarily for reducing the effect of reflections from the room. For example, Poletti and Abhayapala proposed a method for controlling an interior and an exterior sound field in 2D.

This study considers a 2.5D case in which the quiet zone is outside a circular double layer array of loudspeakers.

2 OUTLINE OF THEORY

2.1 Statement of the Problem

Figure 1 illustrates the circular double-layer array of loudspeakers, the listening zone and the quiet zone. The loudspeakers are located on two circles of radii $r_s+$ and $r_s$. Both the inner and the outer array are composed of $N$ loudspeakers. The loudspeakers of the inner array face inward, and those of the outer array face outward. The listening (or bright) zone $S_b$ is located in the plane inside the loudspeaker array, and the quiet (or dark) zone $S_d$ is in the same plane but outside the array. The listening zone is inside a circle of radius $r_b$, and the quiet zone is a ring-shaped region of which the inner radius is $r_d$ and the width is $\Delta r_d$.

When the listening and the quiet zones are sampled at discrete points, $r_b^{(1)}, r_b^{(2)}, r_b^{(3)}$, and $r_d^{(1)}, r_d^{(2)}, r_d^{(3)}$, the sound pressures in the two zones can be expressed as vectors,

$$
\mathbf{P}_b = \left[ P(r_b^{(1)}) P(r_b^{(2)}) \ldots P(r_b^{(M_d)}) \right]^T, \quad (1)
$$

$$
\mathbf{P}_d = \left[ P(r_d^{(1)}) P(r_d^{(2)}) \ldots P(r_d^{(M_d)}) \right]^T. \quad (2)
$$

In matrix form the sound field generated by all loudspeakers at the given positions in the two zones can be expressed as

$$
\mathbf{P}_b = \mathbf{H}_b \mathbf{q}, \quad (3)
$$

$$
\mathbf{P}_d = \mathbf{H}_d \mathbf{q}, \quad (4)
$$
where $\mathbf{H}_b$ and $\mathbf{H}_d$ represent transfer functions between all source and receiver positions, and $\mathbf{q}$ is a vector with the source strengths (volume velocities). The desired sound field is $\hat{\mathbf{P}}_b$ in the listening zone and $\hat{\mathbf{P}}_d = \mathbf{0}$ in the quiet zone.

### 2.2 Performance Indicators

We need some measures to quantify how well various control strategies work; accordingly, two performance indices are introduced. One is the acoustic contrast, defined as the ratio of the average sound potential energy density in the listening zone to that in the quiet zone,

$$\mu = \frac{M_d \mathbf{P}_b^H \mathbf{P}_b}{M_b \mathbf{P}_d^H \mathbf{P}_d},$$  \hspace{1cm} (5)

where $H$ indicates the Hermitian transpose. The other performance index is the normalised spatial average error between the desired and the reproduced sound field in the listening zone, defined as

$$e_b = \frac{(\hat{\mathbf{P}}_b - \mathbf{P}_b)^H (\hat{\mathbf{P}}_b - \mathbf{P}_b)}{\mathbf{P}_b^H \mathbf{P}_b}.$$  \hspace{1cm} (6)

The purpose of this study can be described as solving the problem of maximising the acoustic contrast expressed by Eqn. (5) with the configuration shown in Fig. 1, and at the same time minimising the spatial error given by Eqn. (6).

### 3 SOLUTION STRATEGIES

Choi and Kim have proposed a method of maximising the acoustic contrast.\textsuperscript{14} In their method, the source strength vector that maximises the contrast is obtained as the eigenvector that corresponds to the maximum eigenvalue of a combination of the spatial correlation matrices, $\mathbf{R}_d^{-1} \mathbf{R}_b$, where $\mathbf{R}_d = \mathbf{P}_d^H \mathbf{P}_d / M_d$ and $\mathbf{R}_b = \mathbf{P}_b^H \mathbf{P}_b / M_b$,

$$\begin{bmatrix} \mathbf{R}_d^{-1} \mathbf{R}_b \end{bmatrix} \mathbf{q}_{ct} = \mu_{max} \mathbf{q}_{ct}.$$  \hspace{1cm} (7)

However, this solution cannot reduce the error between the desired and the reproduced sound field.

On the other hand, a solution that minimises the normalised average error in the listening zone can be obtained simply by matrix inversion,

$$\mathbf{q}_{sub} = \mathbf{H}_b^{-1} \hat{\mathbf{P}}_b,$$  \hspace{1cm} (8)

where $+$ indicates the pseudo-inverse operator. However, this solution does not reduce the energy in the quiet zone.

Yet another solution that matches the reproduced sound field with a desired sound field both in the listening zone and the quiet zone can be obtained by minimising the global error. If the vector $\hat{\mathbf{P}} = \begin{bmatrix} \hat{\mathbf{P}}_b & \mathbf{0} \end{bmatrix}$ represents the desired sound field in both zones, it follows that the source strengths can be
obtained as

\[ q_{mb} = H^T \hat{P}, \quad (9) \]

where \( H = \left[ H_f^T \ H_f^T \right]^T \). This solution increases the acoustic contrast and reduces the error \( e_b \).

A more refined combined solution with greater flexibility can be obtained by introducing the cost function

\[ J = \kappa P_d^H P_d + (1 - \kappa) \left( \left( P_b - \hat{P}_b \right)^H \left( P_b - \hat{P}_b \right) \right), \quad (10) \]

where \( \kappa (0 \leq \kappa < 1) \) is a weighting factor that determines the balance between the potential energy in the quiet zone and the mean square error in the listening zone. Combining with Eqns. (3) and (4) gives

\[ J = q^H \left[ \kappa H_d^H H_d + (1 - \kappa) H_b^H H_b \right] q + \left(1 - \kappa\right) \left[P_b^H P_b - \hat{P}_b^H H_b q - q^H H_b^H \hat{P}_b \right]. \quad (11) \]

It can be shown that this cost function has a global minimum, corresponding to the optimum source strength vector

\[ q_{mb} = \left[ \kappa H_d^H H_d + (1 - \kappa) H_b^H H_b \right]^{-1} (1 - \kappa) H_b^H \hat{P}_b. \quad (12) \]

As the weighting factor \( \kappa \) varies from zero to one the solution varies between minimising the error \( e_b \) and maximising the energy ratio \( \mu \). As \( \kappa \) approaches one the acoustic contrast is maximised; with \( \kappa = 0 \) pure pressure matching (Eq. (8)) is obtained; and with \( \kappa = 0.5 \) the global error is minimised (Eqn. (9)).

4 A SIMULATION STUDY

In order to examine the performance of this system, an example with a circular array with 20 equidistantly spaced loudspeaker pairs is examined in a simple simulation setup. The loudspeakers, the listening zone, and the quiet zone are located in the plane \( z = 0 \). Considering the size of the head of a listener, the radius of the listening zone \( r_b \) is 0.2 m. The radii \( r_{x+} \) and \( r_{x-} \) are 0.9 m and 1 m, respectively, the radius \( r_d \) is 2 m, and the width of the quiet zone \( \Delta r_d \) is 1 m. Free-field conditions are assumed, and the desired sound field in the listening zone is a plane wave with an amplitude of unity propagating in the negative \( x \)-direction.

The spacing between adjacent loudspeakers in the inner array is 28.3 cm, and the spacing between adjacent loudspeakers in the outer array is 31.4 cm. Thus, the Nyquist frequency \( f_N \) at which the spacing in the inner array is equal to half a wavelength is 606 Hz. The frequency range of interest is between 100 Hz and 1 kHz, which includes frequencies below and above the Nyquist frequency. The listening zone and the quiet zones are sampled at discrete points with a spacing of 5 cm between adjacent points. This is less than one sixth of the wavelength at the maximum frequency, 1 kHz.

The solutions are based on Eqns. (7) and (12) with different values of the parameter \( \kappa \). The solutions involve matrix inversion, which occasionally can lead to excessively large source strengths. This is particularly likely to occur at low frequencies where the transfer functions become dependent on each other. The problem can be reduced with regularization.
Figure 2 shows the sound pressure level (normalised to zero at the centre of the listening zone) and the phase of the reproduced sound field obtained with the pure contrast control at 200, 500, and 800 Hz. As can be seen, the level in the quiet zone is less than -30 dB at 800 Hz, even though this is well above the Nyquist frequency, but there is no control over the phase in the listening zone. On the other hand Fig. 3 shows the sound field obtained with pressure matching in the listening zone \( \kappa = 0 \) at 200, 500, and 800 Hz. At all frequencies, the phase increases in the negative \( x \)-direction in the listening zone, which implies that a plane wave propagates in this direction. On the other hand, the level is not lower than -20 dB in most of the quiet zone.

Figure 4 shows the sound field obtained with the combined solution and \( \kappa = 0.5 \) at 200, 500, and 800 Hz. The level in the quiet zone is lower than -30 dB at these frequencies, and the plane wave is reproduced in the listening region.

### 4.1 Performance in the Array Plane

Figure 5 shows a comparison of the two performance indices as functions of frequency obtained with various solutions: pure contrast control, pure pressure matching in the listening zone \( \kappa = 0 \), and the combined solution with \( \kappa = 0.1, 0.5, 0.9 \). The highest contrast is obtained with the highest value of \( \kappa \), whereas the lowest value of \( \kappa \) gives the lowest contrast at all frequencies. The contrast obtained with the combined solution exceeds 40 dB below the Nyquist frequency (606 Hz), and takes values higher than 20 dB above the Nyquist frequency. On the other hand, the minimum spatial error is obtained with the lowest value of \( \kappa \), whereas the spatial error obtained with the highest value of \( \kappa \) is larger than -5 dB. The spatial errors obtained with the combined solution are lower than -20 dB below the Nyquist frequency, and increase to values from -15 to 0 dB at higher frequencies.

### 4.2 Performance in the Vertical Plane

Even though the region of main interest is in the plane of the loudspeaker array, it is also interesting to examine the performance of the various control techniques in the vertical plane. Figure 6 shows the sound field in the \( x-z \) plane at 200, 500, and 800 Hz obtained with the combined solution. As can be seen, the sound field is significantly attenuated outside of the array in the plane of interest \( (z = 0) \), but this is no longer the case outside of this plane: some sound is radiated upwards and downwards.

### 5 REFLECTIONS AND SCATTERING

The fact that a high energy ratio can be obtained in the plane of the array implies that the proposed technique is relatively immune to reflections from walls. However, unless the array is placed in anechoic surroundings, sound waves propagating upwards and downwards can be reflected from the ceiling and the floor and affect the sound field in the plane of interest. This problem can be reduced if the ceiling and floor are treated with sound absorbing material.

Another, perhaps potentially more serious problem could be caused by scattering due to the head of the listener in the listening zone. To examine this the head of a listener has been modelled as a rigid sphere. The main effect of the scattering and turns out to be an increase of the sound pressure level in the quiet zone. However, the effect is quite modest.
6 CONCLUSIONS

A technique based on circular double-layer array of loudspeakers has been proposed for generating a plane propagating sound wave in a listening zone inside the array and a zone of quiet outside the array. A solution based on a relatively modest number of loudspeakers that combines acoustic contrast control with pressure matching in the listening zone has been shown to perform fairly well. Computer simulations with a circular array with a diameter of about 1 m and twenty loudspeaker pairs mounted back-to-back have demonstrated that this system provides acoustic contrast of more than 40 dB, and a normalised spatial error of about -20 dB at frequencies below the Nyquist frequency (about 600 Hz). Up to the highest frequency examined (1 kHz), well above the Nyquist frequency, acoustic contrast of about 20 dB and normalised spatial errors of less than -10 dB have been obtained.

7 ACKNOWLEDGMENTS

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8 REFERENCES


Fig. 1 – A circular double-layer array of loudspeakers, the listening zone and the quiet zone.
Fig. 2 – Sound field obtained with pure contrast control. Left column, sound pressure level relative to the level at the centre; right column, phase. Results at 200, 500, and 800 Hz from top to bottom.
Fig. 3 – Sound field obtained with pure pressure matching in the listening zone. Left column, sound pressure level relative to the level at the centre; right column, phase. Results at 200, 500, and 800 Hz from top to bottom.
Fig. 4 – Sound field obtained with combined solution ($\kappa = 0.5\)$. Left column, sound pressure level relative to the level at the center; right column, phase. Results at 200, 500, and 800 Hz from top to bottom.
Fig. 5 – Acoustic contrast and normalised spatial error; contrast control (Δ), pressure matching in the listening zone (□), and combined solution with $\kappa = 0.1$ (+), 0.5 (*), and 0.9 (×).

Fig. 6 – Sound field in the x-z plane obtained with combined solution ($\kappa = 0.5$). Results at 200, 500, and 800 Hz from top to bottom.