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Abstract

Two wedge problems including slope diffraction are solved: One in which the incident field is a non-uniform plane wave, and one in which it is an inhomogeneous plane wave. The two solutions lead to the same GTD slope diffraction coefficient. This coefficient reveals the existence of a coupling effect between a transverse magnetic (or transverse electric) incident plane wave and the transverse electric (or transverse magnetic) slope-diffracted field. The coupling effect is not described by the existing GTD slope diffraction coefficient.

1 Introduction

Slope diffraction is a scattering mechanism contributing to edge diffraction. It occurs when the amplitude of the incident field possesses a non-zero first-order derivative, i.e., a slope, with respect to the distance normal to the plane of incidence. According to the existing GTD solution [1] an incident transverse magnetic (TM) field gives rise to only a TM slope-diffracted field. Likewise, an incident transverse electric (TE) field gives rise to only a TE slope-diffracted field. Here, 'transverse' is with respect to the plane of incidence or the plane of diffraction.

In this work, a new GTD slope diffraction coefficient is derived by solving two different wedge problems. In contrast to the existing coefficient the new coefficient reveals the existence of a coupling effect between the TM incident plane wave and the TE slope-diffracted field and vice versa.

In the first problem the incident field is a non-uniform plane wave given as the superposition of three uniform plane waves. The transverse field components of the non-uniform plane wave are zero at the diffraction point but possess a linear amplitude variation in the direction normal to the plane of incidence and thus a slope. The GTD solution for the wedge illuminated by the non-uniform plane wave is found by superposition of the GTD solutions for each of the three uniform plane waves [2] [3].

In the second problem the incident field is an inhomogeneous plane wave obtained by an analytical continuation of the expression for a homogeneous (uniform) plane wave. The inhomogeneous plane wave possesses an exponential amplitude variation in the direction normal to the plane of incidence and thus a

slope. The exact solution for the half-plane illuminated by the inhomogeneous plane wave is obtained from an analytical continuation [4]. From a series expansion of the exact field, the contribution to the diffracted field proportional to the slope of the incident field is extracted.

The coupling effect was apparently first reported in [2] for the half-plane case, see also [3]. In a recent paper [5] the existence of the coupling effect is also indicated. The scattering configuration and associated coordinate systems are shown in Fig. 1. A perfectly conducting wedge residing in free space is illuminated by a plane wave. θ^i and ϕ^i are the polar and azimuthal angles of incidence, $\hat{k} = -\hat{x}\sin\theta^i\cos\phi^i - \hat{y}\sin\theta^i\sin\phi^i - \hat{z}\cos\theta^i$ the propagation unit vector, k the wave number, and ζ the intrinsic impedance of free space. The time factor $\exp(j\omega t)$ is assumed and suppressed.
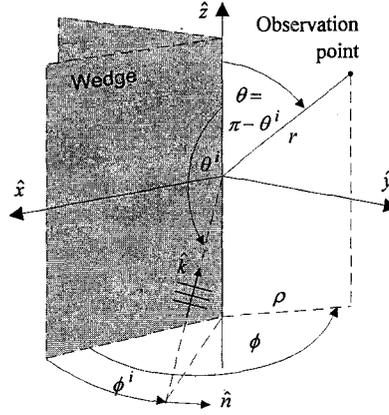

Fig. 1 *Wedge illuminated by plane wave.*

2   Non-uniform plane wave illumination

Two cases exist: quasi-TM illumination and quasi-TE illumination. The quasi-TM non-uniform plane wave is expressed in terms of three uniform plane waves:

$$\vec{E}^{QTMi} = \frac{1}{jk}\lim_{\epsilon\to 0}\frac{\vec{E}_{+\epsilon} - \vec{E}_{-\epsilon}}{2\epsilon} - \hat{\phi}^i\frac{E_0\cos\theta^i}{jk}\exp(-jk\hat{k}\cdot\vec{r}) \tag{1}$$

with E_0 being a constant and

$$\vec{E}_{\pm\epsilon} = \hat{\theta}^i_{\pm\epsilon}E_0\exp(-jk\hat{k}_{\pm\epsilon}\cdot\vec{r}) \tag{2}$$

The unit vectors $\hat{k}_{\pm\epsilon}$ are defined as $\hat{k}$ with the azimuthal angle of incidence ϕ^i replaced by $\phi^i\pm\epsilon$. $\hat{\theta}^i_{\pm\epsilon}$ are the polar unit vectors at $(\theta,\phi^i\pm\epsilon)$. It is emphasized that the limiting sum of the two TM uniform plane waves in (1) will include a TE uniform plane wave which is non-zero at the edge. Since we want to obtain an incident field being zero at the edge, this TE uniform plane wave is subtracted as the second term in (1). The non-uniform plane wave has the form

$$\vec{E}^{QTMi} = \hat{\theta}^i E_0\sin\theta^i\rho\sin(\phi - \phi^i)\exp(-jk\hat{k}\cdot\vec{r}) \tag{3}$$

According to Maxwell's equations the magnetic field is

$$\vec{H}^{QTMi} = -\left[\hat{\phi}^i\rho\sin(\phi - \phi^i) + \hat{k}\frac{1}{jk}\right]\frac{E_0}{\zeta}\sin\theta^i\exp(-jk\hat{k}\cdot\vec{r}). \tag{4}$$

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The GTD field for the non-uniform plane wave illumination is found as the superposition of the GTD fields for the three uniform plane waves. The result is presented in equation (5) below. The case of the quasi-TE non-uniform plane wave is found similarly [2]: the addition of the limiting sum of two TE uniform plane waves and a TM uniform plane wave.

3   Inhomogeneous plane wave illumination

The expression for the incident inhomogeneous plane wave is obtained by a complex continuation with respect to the azimuthal angle of incidence ϕ^i in the expression for the homogeneous plane wave, $\vec{E}_0 \exp(-jk\hat{k} \cdot \vec{r})$. That is, the real angle ϕ^i is replaced by the complex angle $\phi^i_c = \phi^i + j\phi^i_{im}$. It should be emphasized that the complex continuation must be applied to the entire expression for the homogeneous plane wave, including the polarisation vector, in order for the inhomogeneous plane wave to satisfy Maxwell's equations. The plane wave relation becomes $\zeta\vec{H} = \hat{k}_c \times \vec{E}$, where $\hat{k}_c$ is the complex continuation of $\hat{k}$. The real and imaginary parts of $\hat{k}_c$ are perpendicular and parallel to the equi-phase planes, respectively. Consequently, there is a field component perpendicular to the equi-phase planes.

In [6] the complex continuation procedure has been applied to solve the wedge problem for normal incidence ($\theta^i = \frac{\pi}{2}$). In [4] the same procedure has been applied to solve the half-plane problem for oblique incidence.

In this paper the GTD slope diffraction coefficient is derived from the latter solution in three steps. Consider the case of the incident inhomogeneous plane wave obtained from the TM homogeneous plane wave. First, a series expansion in ϕ^i_{im} of the incident field is derived. The $O(1)$ term is a TM homogeneous plane wave $\vec{E}_1$, and the $O(\phi^i_{im})$ term is the sum of a TE homogeneous plane wave $\vec{E}_2$ and the quasi-TM non-uniform plane wave $\vec{E}^{QTMi}$ (3). Second, a non-uniform asymptotic expansion (in k) of the exact diffracted field is derived and only the dominant terms retained. Third, a series expansion in ϕ^i_{im} is derived for the diffracted field. The $O(1)$ term is a TM field which equals the GTD result for $\vec{E}_1$. The $O(\phi^i_{im})$ term contains only a TM field in spite of the incident TE field $\vec{E}_2$. Consequently, the slope-diffracted field due to $\vec{E}^{QTMi}$ must predict the $O(\phi^i_{im})$ TM field *and* cancel the TE diffracted field due to $\vec{E}_2$. This is achieved by the diffraction coefficient in (5).

So far, the work for the inhomogeneous plane wave has been restricted to the half-plane. However, the extension to the wedge is straight forward.

4   The new slope diffraction coefficient

The two procedures described in the preceding sections lead to the same GTD slope diffraction coefficient. The coefficient is not valid close to the optical boundaries; for the non-uniform plane wave because GTD has been used to

obtain the diffracted field, for the inhomogeneous plane wave because a non-uniform asymptotic expansion has been used.

The slope diffracted field becomes

$$
\left\{ \begin{array}{c} E_\theta^d \\ E_\phi^d \end{array} \right\} = \frac{1}{jk\sin\theta^i} \left\{ \begin{array}{cc} \dfrac{\partial D_s}{\partial \phi^i} & \cos\theta^i D_s \\ \cos\theta^i D_h & -\dfrac{\partial D_h}{\partial \phi^i} \end{array} \right\} \left\{ \begin{array}{c} \dfrac{\partial(\vec{E}^i \cdot \hat{\theta}^i)}{\partial n} \\ \dfrac{\partial(\vec{E}^i \cdot \hat{\phi}^i)}{\partial n} \end{array} \right\} \frac{\exp(-jkr)}{\sqrt{r}} \quad (5)
$$

where $D_{s,h}$ are the GTD soft and hard diffraction coefficients for uniform plane wave illumination and $n = \rho\sin(\phi - \phi^i)$ is the distance from the plane of incidence.

The main diagonal terms in the diffraction matrix are the same as in the existing GTD slope diffraction coefficient. The remaining terms in the diffraction matrix reveal that a coupling exists between the TE incident field and the TM slope-diffracted field and vice versa. These coupling terms are of the same order in the wave number k as the main diagonal terms. They vanish in the case of normal incidence $(\theta^i = \frac{\pi}{2})$.

Furthermore, it is interesting to observe from sections 2 and 3 that a plane wave possessing a slope has a field component perpendicular to the equi-phase planes. It has been shown [4] that the slope-diffracted field may be expressed as the diffraction of this perpendicular component.

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