Fatigue Crack Initiation and Growth in Ship Structures

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DEPARTMENT OF
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Preface

This thesis is submitted as a partial fulfilment of the requirements for the Danish Ph.D. degree. The work has been performed at the Department of Naval Architecture and Offshore Engineering of the Technical University of Denmark during the period of September 1994 to January 1998. Professor Preben Terndrup Pedersen and Associate Professor Peter Friis Hansen have supervised the study.

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During my study Professor Petershagen gave me the opportunity to visit the Institut für Schiffbau der Universität Hamburg, Germany — and Professor Sumi and Associate Professor Kawamura arranged for me to visit the Department of Naval Architecture and Ocean Engineering, Yokohama National University, Japan. Two very inspiring and fruitful periods - I thank the staff of the departments for their help and kindness during my visits.

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Executive Summary

Fatigue crack initiation and fatigue crack growth are important damage modes in ship structures. An important example is the large number of longitudinal to web frame connections which make up an essential part of the production costs of a very large crude oil carrier (VLCC). The slot structures are complicated welded details exposed to dynamic loads during the service of the vessel and if not adequately designed significant fatigue cracking may occur and cause major costs of repair. This was the case of the early cracking of the second-generation VLCCs. The objective of the present work has been to present new fatigue analysis tools and to develop a new fatigue-resistant slot design which is suitable for application of welding robots.

In conventional slot structures cracks can often be found at the toe and the heel of the bracket used to transfer load from the pressure on the side shell into the web frame. In the proposed new design the bracket is removed in order to avoid these cracks and to ease the application of welding robots. The result is that the load to be carried by the weld seam between the web frame and the longitudinal is increased and the cutout becomes a hard-spot area. Therefore a finite element based shape optimisation is employed to reduce the stresses in this region and a new shape optimised cutout is formed.

A probabilistic fatigue damage model is established with the purpose of comparing the relative fatigue strength of the new shape optimised slot and a conventional design. The model is based on the “quasi-stationary narrow-band” theory which allows for inclusion of the non-linear effect of the intermittent submergence of the side shell close to the load line of the vessel. On the basis of an assumed sailing route fatigue lives of the two types of structures are estimated with the effects of different heading angles and manoeuvring in bad weather taken into account. The toe of the bracket is predicted to be the weakest point of the conventional structure with an estimated fatigue life of 5 years while 28 years are predicted for the weakest point of the shape-optimised design i.e. the fatigue strength of the new structure is increased by a factor of 6. At the same time manual fitting of the bracket is avoided which can ease the application of welding robots.

Cracks may initiate at imperfections caused by welding and thermal cutting of edges. Conventional fatigue damage approaches are normally based on the assumption that if subsequent crack propagation occurs it is in a self-similar manner. However branched crack propagation can be found in structures containing residual stresses or in details exposed to...
baxial loading, where curved crack propagation may have a significant influence on the crack growth rate. A two-dimensional multipurpose crack propagation procedure is established in order to investigate mixed mode crack propagation in welded structures subjected to high-cycle loads. The method is based on a step-by-step finite element procedure with continuous remeshing of the domain surrounding the propagating crack. Special emphasis is put on the effect of residual stresses and "crack closure". The latter is of major importance to cracks propagating in compressive residual stress fields. The simulation procedure is validated by comparison with experimental and numerical data found in the literature covering both the influence of holes, biaxial loading and residual stresses. A good correlation is obtained.

The crack simulation procedure is applied to the conventional and the shape-optimised slot structures. Residual stresses are estimated on the basis of relatively simple inherent strain distributions. Cracks 1 mm in length are initiated at stress concentrations in the bend of the cutouts and close to the weld between the longitudinal and the web frame. From the simulations it is observed that compressive residual stresses close the faces of the crack initiated close to the weld which reduces the risk of crack propagation. This is an important observation because imperfections from welds can act as crack initiators.

The crack simulation procedure is restricted to constant amplitude loading and a representative cyclic wave is determined for the simulations by the established probabilistic model. However, the stress intensity ranges obtained for the configurations with initial cracks in the bends result in prediction of crack arrest at initial stages when the loads from the equivalent wave are applied. This makes it difficult to conclude that one of the designs is superior to the other with respect to stable crack propagation. Nevertheless, crack simulations performed with threshold values equal to zero indicate that the initial stress intensity range for the optimised cutout is larger than for the conventional cutout, but it drops to zero after a small distance of crack propagation. The stress intensity range for the conventional structure starts at a lower range, but it increases continuously as the crack propagates. The same trend is to be expected at a stress level above the threshold limit. Thus, the optimised slot design seems to be superior to the conventional design.
Synopsis


I traditionelle forbindelser til gennemføringen af longitudinaler i webrammer kan revner ofte findes ved tåen og hælen af det stag som bruges til at færdig funde fra skibssiden ind i webrammen. I det nye design fjernes staget for at undgå disse revner samt for at lette implementeringen af svecserobotter. Derved forøges den kraft, som kan overføres af svecsesømmen mellem longitudinalen og webrammen og ukappet bliver et "hard-spot" område. En finite element baseret formoptimeringsprocedere bruges derfor til at reducere spændingerne omkring udkragingen og et nyt optimeret udkap opnås herved.

En probabilistisk udmattelsesmodel etableres med det formål at sammenligne udmattelsesstyrken af det nye design med styrken af en traditionel samingstype. Udmattelsesmodellen er baseret på en "kvasi-stationær smalbåndsteori" hvilket tillader inkludering af den ikke lineære effekt fra det uregelmæssige vandtryk der virker på skibssiden i området omkring vandlinien. Effekter af forskellige mødevinkler samt manøvrering i dårligt vej er ligeledes inkluderet. Levetiderne af de to samingstyper bestemmes på basis af en antaget sejlroute. Med en beregnet levetid på 5 år forudsiges tåen af staget at have den korteste levetid i det konventionelle design mens det svageste punkt i det formoptimerede udkap forudsiges en levetid på 28 år. Det nye design er altså seks gange stærkere med hensyn til udmattelse og det tillader let anvendelse af svecserobotter.

Revner kan initieres ved defekter som stammer fra svecning eller termisk skæring af kanter. Revnevækstmodeller er normalt baseret på en antagelse om lineær udbredelse af revnen. Men krum revneudbredelse kan forekomme i konstruktionen som indeholder residu Spændinger eller i detaljer som er udsat for bi-axiel last. Den krumme revneudbredelse
can have a stor betydning for revnevækstshastigheden. En to-dimensional revnevækst-
model opbygges derfor til undersøgelse af revneudbredelse i sværeste konstruktioner, som er
udsat for mangegangspåvirkninger. Metoden baseres på en “skridt-for-skridt” finite element
procedure med nyt net i området omkring revnen for hvert skridt. Der vil blive lagt vægt på
effekten af residualspændinger og revnelukning hvor den sidste del er af speciel betydning.

Revnesimuleringsproceduren bruges i et sammenligningsstudie af det nye design og en
traditionel forbindelse. Residualspændinger bestemmes på basis af relativt enkelte udtryk
for tojningsfordelinger hidrørende fra opbygning af konstruktionerne. Revner med en længde
på 1 mm initieres på steder med høj spændingskonzentration, hvor det er i rundingen af
udkappene og ved sveisningen mellem udkappet og longitudinalen. Simuleringerne viser at
kompressive residualspændinger lukker revner, som initieres i udkappet tæt ved sveisningen. Det er en vigtig observation, da revner kan dannes fra defekter i sveisningen.

Revnesimuleringsproceduren er begrænset til dynamiske laste med konstant amplitude og
en repræsentativ cyklisk last findes derfor ved anvendelse af den probabilistiske model. Den
variation i spændingsintensiteten opnås for de to konfigurationer med initielle revner i
rundingen af udkappene resulterer i revnestop på de initielle stadier. Revnesimuleringer
udført med grænseværdien for revnestop sat lig med null indikerer, at det initielle spændings-
intensitetsudsving vil være størst for den optimerede detalje men efter en kort revneudbred-
delse vil udsvingen i spændingsintensitet falde til null for denne detalje. For det traditionelle
design starter spændingsintensitetsudsvinget på et lavere niveau, men det stiger efterhånden
som revnen vokser. Den samme trend kan forventes for et spændingsintensitetsudsving over
grænseværdien for revnestop. Den formoptimerede gennemføring synes derfor at have størst
styrke mod udbredelse af lange revner.
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Chapter 1

Introduction

1.1 Overview and Background

A large number of complicated welded plate joints can be found in an ocean-going ship. During the service time of the vessel these details are exposed to time-varying loads caused by the irregular seaway, the propulsion system, and changes in the loading conditions. Hard spots, such as connections between longitudinal stiffeners and web frames where local rises in stress intensity can be found, may therefore be prone to fatigue cracking. Thus, new production-friendly and more fatigue-resistant designs are always of interest.

The classic brittle fracture of the early welded Liberty ships during World War II triggered a considerable amount of work to prevent fatigue failures. In the intervening half century significant insight has been gained in fatigue and fracture mechanisms found in various types of structures, including ships. Physical models for the description of many common fatigue cracking mechanisms have been developed and experimental work has been carried out to verify and support these models. However, in spite of the efforts made in the field of this subject, cracking problems frequently occur in ship structures today.

The causes are manifold, but a major reason is that the construction of an ocean-going ship which completely resists fatigue cracking is, if possible at present, economically unfeasible. Therefore, fatigue cracking of the structural members is to be expected at some level during the lifetime of a ship. The acceptable level is determined by the safety demands set by national and international societies, the skills of the naval architects, and the cost of repair of cracks.

Sometimes the acceptable level of fatigue cracking is exceeded when significant structural changes are introduced in the ship designs, e.g., when high-tensile steel on a large scale was adopted in the building of the so-called second-generation of very large crude oil carriers (VLCCs). The higher structural strength of the high-tensile steel is not reflected in the
same improvement of the fatigue strength of the material. Therefore, the reductions of the scantlings based on structural strength resulted in increased frequency and severity of fatigue cracking in many new tankers (significant fatigue damages were found in the second-generation VLCCs after few years of trade). A large fraction of the cracks observed in the tankers was in the connections between the transverse web frames and the longitudinal stiffeners in the vertical position between the laden and the ballast waterlines. The non-linear load from intermittent submergence of the side shell makes the details located in this zone especially prone to fatigue cracking.

The large number of longitudinal to web frame connections constitutes an essential part of the production costs of a crude oil carrier. Especially when double-hull structures are built, where the number of these connections is increased significantly compared to the conventional design as it can be seen from Figure 1.1. In order to achieve competitive costs, some shipyards have made an effort to improve the productivity of ship fabrication by the introduction of robots. Hence, new designs which are more suitable for the use of welding robots are of interest. However, before such new designs are applied to locations which are known to be critical with respect to fatigue, such as longitudinal to web frame connections, it is important to carry out fatigue assessment already at the design stage to avoid cases like the early cracking of the second-generation VLCCs.

Design tools for the prediction of fatigue lives of details located at critical points of ships are therefore important. Models for assessment of the residual strength of details containing an initial crack are also of interest, especially for welded and flame-cut details which often contain small imperfections inherent from the manufacture.
The present study deals with models for both types of fatigue assessments—a probabilistic approach for fatigue life estimations of uncracked details and a two-dimensional crack simulation procedure for the prediction of stable growth of initiated cracks. These models will be applied to a conventional and a new slot design for the longitudinal to web frame connection where the new structure is suitable for the use of welding robots.

1.2 Objectives and Scope of the Work

A new design of the slot structure for the connection between the longitudinal stiffeners and the web frames forms the starting point of the present work. The objective is to present two new design tools for fatigue life assessment and to investigate a production-friendly slot design which allows for the application of welding robots and at the same time maintains or even increases the fatigue strength of the structure. A slot structure found in an existing VLCC forms the basis for the work. The first part of the study deals with

- structural optimisation of a new design for the longitudinal to web frame connection by use of a static shape-optimisation procedure;
- establishing a probabilistic fatigue damage model for the estimation of fatigue lives. The model is to include the non-linear effects arising from intermittent submergence of the side shell located close to the mean sea waterline;
- and a comparison of fatigue lives estimated for two designs.

Conventional fatigue damage approaches are normally based on the assumption that the crack propagates in a self-similar manner. However, branched crack propagation can be observed in structures containing residual stresses or in details exposed to biaxial loading. Curved crack propagation can lead to both acceleration and retardation of the crack growth and in some cases even result in arrest of the crack propagation. The second part of the study focuses on the establishment of a multipurpose numerical procedure for the prediction of mixed mode crack propagation in welded plate structures. The model is to include a qualitative estimation of the effects of residual stresses arising from the manufacture of the details. The solution is based on a step-by-step finite element procedure with the basic components given as:

- Model for the prediction of the crack propagation direction for a given stress intensity state at the crack tip.
- Crack arrest criterion.
- A robust mesh generator for continuous remeshing of the arbitrarily shaped two-dimensional domain which surrounds the propagating crack.
Chapter 1. Introduction

- A relatively simple procedure for a qualitative estimation of residual stress fields arising from the welding of thin plates. The method is based on the inherent strain approach.

- Empirical estimation of mixed mode crack closure in the presence of residual stresses.

The procedure is restricted to high-cycle fatigue which implies that the principles of linear elastic fracture mechanics can be employed for the description of the stress state at the crack tip and the crack growth mechanisms. The influence of residual stresses will be concentrated on effects from static residual stress fields only affected by the propagating crack. Effects of shakedown and overloads will not be considered in the present work.

1.3 Structure of the Thesis

The contents of the thesis are presented in nine chapters composed as follows. Chapter 2 deals with two different approaches for relating a cyclic stress state to the fatigue strength of the material. First the common S-N approach is outlined which is followed by a possible application of the cyclic strain approach to high-cycle fatigue with the latter discussion based on the work of Andersen [3].

The fatigue damage model is a significant part of a fatigue damage approach however other important elements have to be addressed for reliable fatigue life estimation of a critical detail. The components of a probabilistic fatigue damage approach are outlined in Chapter 3. The probabilistic model is first applied to fatigue assessment of a conventional slot design. Shape optimisation is applied to a new type of slot structure and the fatigue life of the obtained design is estimated by the probabilistic approach leading to a comparison between the predicted fatigue lives for the two structures. This part of the thesis can also be found in Andersen et al. [4].

The second part of the thesis focuses on mixed crack propagation controlled by the stress intensity state at the crack tip. Chapter 4 outlines the theory needed for the development of a crack simulation procedure while the aspects of numerical implementation are discussed in Chapter 5 together with examples of application.

Crack closure takes place when the two surfaces of the crack faces come into contact. The phenomenon can be found in most types of fatigue crack propagation but it becomes of major importance when cracks are located in compressive residual stress fields. Chapter 6 is therefore devoted to crack closure.

Chapter 7 deals with the inclusion of the effects of residual stresses which arise from welding and flame-cutting. First relatively simple procedures for the estimation of residual stresses caused by welding and flame-cutting of thin plates are outlined. This is followed by a description of the implementation of residual stresses in the crack simulation procedure.
that is input of residual strain and stress fields and the determination of stress intensity factors in the presence of residual stresses.

In Chapter 8 the established crack propagation procedure is applied to the conventional and the optimised slot designs described in Chapter 3.

Finally in Chapter 9 conclusions and recommendations for further work are presented.
Chapter 1. Introduction

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Chapter 2

Fatigue Damage Models

2.1 Introduction

Dynamic stress variations experienced during the service period of a ship can initiate fatigue cracks in details which are inadequately designed, constructed or maintained. Subsequent crack propagation may cause failure of primary structural members leading to catastrophic consequences such as massive oil pollution or loss of the whole ship [46, 72]. Fortunately these cases are rare and usually related to bad maintenance of the vessel. However, even if the fatigue cracking does not lead to complete failure, the cost of inspections and repairs and the consequences of, e.g., minor oil pollution due to leakage can be high [70]. Thus, fatigue cracking should if possible be avoided or kept at an acceptable level.

Fatigue initiation is a localised phenomenon which strongly depends on the structural geometry and stress concentrations. In welded structures such as ships, cracks are known to initiate at stress concentrations caused by flaws from welding procedures and at cutouts and plate joints where abrupt geometrical transitions cause a local rise in stress intensity. Usually, the fatigue strength of a critical detail can be improved by changing the design of the structure. Therefore, in cyclically loaded structures, the designer should evaluate the fatigue strength of details where high stress concentration can be found. Both simple and relatively complex approaches are available for such evaluations. They normally consist of

- a load history description
- the response of the structure to the external load
- and a model for combining the structural response and the material fatigue strength.
- If the structure is exposed to a stochastic load, a fatigue accumulation hypothesis is also required.
All components have to be dealt with in a balanced way for reliable fatigue life estimation.

A probabilistic approach for the estimation of the fatigue life will be presented in Chapter 3 while this chapter focuses on fatigue damage models which relate the cyclic stress state at the critical point to the fatigue strength of the material. Before a discussion of these models, a definition of commonly used stress measures is given.

2.2 Definition of Stresses

Most approaches employed for fatigue life assessment are strongly related to the stress state at the crack tip. A definition of three stress measures often applied for fatigue analysis are therefore introduced. They are: 1) Nominal stress, 2) hot-spot stress, and 3) notch stress. These stress measures are applied in fatigue analysis where the type of stress used for the estimations depends on the problem to be solved and the desired level of accuracy.

Nominal Stress

The nominal stress $\sigma_{\text{nom}}$ is defined as the stress which can be obtained straightforwardly from the sectional forces the moments and the dimensions of the component by simple formulas such as Naviers and Grasshoffs equations. Increase in stresses which arise from discontinuities in the structural geometry or local notches like welds are to be excluded in the determination of the nominal stress. A drawback of this approach is that the definition of the nominal stress can be rather difficult for complicated structural details. The hot-spot approach as well as the notch stress approach can be applied to overcome this difficulty.

Hot-Spot Stresses

The basic idea in the hot-spot approach is to define the local stress so that a notch effect from geometrical discontinuities is considered while the increase in stress arising from the actual geometry of a weld is disregarded. This method has been widely applied to fatigue prediction of critical joints in offshore structures where the modelling of welds can be rather troublesome. The hot-spot stress $\sigma_k$ is calculated as the nominal stress times a hot-spot (structural) stress concentration factor $K_t$.

Notch Stress

The notch stress $\sigma_k$ is the stress which can be found locally at a notch, e.g., at the edge of a cutout or at the toe of a weld. The difference between the hot-spot stress and the notch
stress is that the actual geometry of the notch (e.g., a weld toe) is taken into account in the latter approach. The rise in stress from a weld can be included by a stress concentration factor $K_w$. The three types of stress measures are illustrated in Figure 2.1.

### 2.2.1 Stress Concentration Factors

Stress concentration factors (SCFs) for the different notch effects may be calculated from the theory of elasticity by various methods. Analytical calculations are mostly suited for relatively simple configurations. The finite element method offers a more versatile method which can be applied to both two- and three-dimensional problems. When the finite element method is used for determination of the SCFs of details in welded structures the choice is between modelling the weld or not. The latter approach results in the simplest models but the stress may become singular at plate joints. In these cases extrapolation procedures have to be employed for determination of the structural SCFs as described in Section 3.2.3. Alternatively stress concentration factors for typical details in ships can be found in fatigue guide books (see e.g., [2Γ34]).

All stress risers have to be taken into account in evaluations of the notch stress. An overview of notch factors important to ship structures is given in DNV [34] together with a library of stress concentration factors.

### 2.3 Fatigue Damage Models

There are several models for relating the cyclic stress/strain state at the notch or crack tip to the fatigue strength of a material. Two procedures are presented here. First the often used S-N approach and then a possible application of the cyclic strain approach to high-cycle fatigue. The basic assumption in both procedures is that the fatigue performance of an unnotched small-scale laboratory specimen describes the fatigue endurance of a “real” structure when it is exposed to the same stress history.
2.3.1 The S-N Approach

The S-N approach is based on fatigue tests which describe the relation between the cyclic stress range and the corresponding number of load cycles for failure. The procedure assumes that it is only necessary to consider the range in the principal stresses when the fatigue endurance is estimated. Thus, the influence of the mean stress level is neglected.

The S-N fatigue test can be presented in a Log-Log diagram called a S-N curve. The basic design curve is given as

\[ \log N = \log a - m \log \Delta \sigma \]  

(2.1)

where \( N \) is the number of cycles for failure at the stress range \( \Delta \sigma \) and \( m \) is the negative slope of the S-N curve. The desired level of safety can be adjusted by the material-related parameter \( \log a \). DNV [34] recommends that the design curve is based on the mean test curve minus two times the standard deviation which results in a 97.6% probability of survival.

Damage Accumulation

The S-N design curve given by Eq. (2.1) relates the critical number of load cycles \( N \) to a specific stress range. Thus, a damage accumulation model is needed if the S-N approach is to be applied to structures subjected to fluctuating loads. For practical purposes, the linear Palmgren-Miner rule can be employed. It states that the total damage experienced by the structure may be expressed by the accumulated damage from each load cycle at different stress levels. The fluctuating stresses can be divided into \( j \) levels of constant stress and the damage is calculated as

\[ D = \sum_{i=1}^{j} \frac{n_i}{N_{c,i}} \]  

(2.2)

where \( n_i \) is the number of stress cycles at load level \( i \) and \( N_{c,i} \) is the corresponding critical number of load cycles. Failure is assumed to occur when the damage index \( D \) is equal to 1.0. The Palmgren-Miner approach assumes that the cumulative damage \( D \) has the same form regardless of the applied stress and that \( D \) is independent of the sequence of applied stresses. This is in general not the case. However, the Palmgren-Miner rule is still one of the most accurate models for cumulative damage calculations.
2.3.2 Cyclic Strain Approach Applied to High-Cycle Fatigue

The outlined S-N approach is the most commonly used model for description of the relation between the applied load and the fatigue strength and it is adopted by most classification societies. The S-N approach normally yields good results but 1) it does not take into account local yielding and 2) it neglects the influence of the mean stress level. The effect of the external load-induced mean stress on the fatigue life is not very pronounced if the residual stresses at the notch are close to the yield stress level as they often are in welded connections. But if for scallops and cutouts with a lower residual stress level the mean stress may play an important role. Thus for estimations of the fatigue life of scallops and cutouts subjected to high cyclic stresses it has been proposed to use the cyclic strain approach which accounts for local yielding and the mean stress level.

The cyclic strain approach has normally been used in connection with low-cycle fatigue. An application of the method to high-cycle fatigue problems was analysed in a case study by Andersen [3]. Cracks reported in the longitudinal of a 300 m long bulk carrier after 21 years of trade were used for the investigation (see Figure 2.2). For this configuration fatigue lives were estimated by the cyclic strain approach and subsequently compared with fatigue lives predicted by the S-N approach. Some of the observations and conclusions made in the study will be reviewed here starting with an outline of the cyclic strain approach.

![Figure 2.2: Location of reported crack.](image)

2.3.3 Outline of the Cyclic Strain Approach

Cyclically loaded elastic-plastic materials often undergo changes which result in a dynamic yield curve different from the static yield curve. The cyclic strain approach is based on the dynamic strain/stress relationship taking into account local yielding at the notch. Thus the dynamic yield curve has to be determined for the detail of interest either (1) experimentally (2) by non-linear finite element analysis or (3) by approximation formulas such as the Zeeger or the Neuber rule. The latter method is simple and gives good results compared to the two other methods [41]. Especially the simple Neuber rule can be useful:

\[ \epsilon_a = \frac{\sigma_a}{E} \left( \frac{\sigma_{a,e}}{\sigma_a} \right)^2 \]  

(2.3)
where $E$ is the modulus of elasticity, $\sigma_{n,e}$ is the elastic notch stress amplitude, $\sigma_a$ and $\epsilon_a$ are the local elastic-plastic stress and strain amplitudes, related by a material law. A commonly employed material law for cyclically loaded structures is [42]

$$\epsilon_a = g(\sigma_a) = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{1/n'}$$

(2.4)

where $K'$ and $n'$ are material constants representing the plastic part of the strain amplitude $\epsilon_a$. The local elastic-plastic stress and strain relation can be obtained from Eq. (2.3) and Eq. (2.4) as a function of the elastic notch stress amplitude $\sigma_{n,e}$. Figure 2.3 illustrates this relation for a circular hole in a plate of high-tensile steel. If the structure is exposed to very high loads and general yielding is present in the total net section, the Neuber rule can be modified by an additional factor to yield good results [41].

The damage from each load cycle can be found from the well-known Wöhler curves, but when the cyclic strain approach is applied, the stress has to be replaced by a special damage parameter. The damage parameter $P_{SWT}$ proposed by Smith, Watson and Topper [119] can be employed, taking the effect of the mean stress into account:

$$P_{SWT} = \sqrt{\sigma_{\text{max}} \epsilon_a E} = \sqrt{(\sigma_a + \sigma_m) \epsilon_a E}$$

(2.5)

with $\sigma_{\text{max}}$ being the maximum stress in each load cycle and $\epsilon_a$ the local elastic-plastic strain amplitude. Both $\sigma_{\text{max}}$ and $\epsilon_a$ may be obtained by solving Eq. (2.3) and Eq. (2.4). For negative mean stress $\sigma_{\text{max}}$ should be replaced by $\sigma_a$ in Eq. (2.5).

The Palmgren-Miner rule given by Eq. (2.2) is applied in the case of variable amplitude loading.
2.3. Fatigue Damage Models

Figure 2.4: Sketch of the three different interpretations of the $P_{SWT}$ damage curves.

Observations Made in the Study

The data available for damage curves based on the Smith-Watson and Topper formulation was reviewed during the study. It was observed that the data in the high-cycle region is relatively limited for flame-cut edges in steel HF 36. This left some uncertainty how to establish the damage curves in the high-cycle zone. Heuler [56] has suggested three different interpretations of the available data:

- “Original” by use of all available data. In the case study this interpretation was applied in the form of a bilinear fit of the data.
- “Elementary” the damage curve is characterised by one exponential distribution neglecting the data in the high-cycle area.
- “Reduced fatigue strength” like the “elementary” interpretation but with a cutoff level at half of the fatigue strength (the high-cycle point where the $P_{SWT}$ level is equal to half of the “original” $P_{SWT}$ value).

All three interpretations were applied in the investigations together with a deterministic estimation of the load history of the vessel leading to the very different damage indices $D$.
presented in Figure 2.4. From this figure it is seen that the curves based on the two last assumptions are the most conservative.

It is also important to note that the influence of the mean stress level was found to be quite significant. Two loading conditions were considered in the case study a fully loaded and a ballast condition. Only the ballast condition with a relatively high mean stress level was found to contribute to the damage index while the contribution from the fully loaded condition with a zero mean stress level was negligible.

Comparing the S-N and the Cyclic Strain Approach

The S-N approach was applied in the case study using S-N curves recommended by the DNV [34]. The holes were located in a corrosive environment. Nevertheless, the calculations were carried out for both a corrosive and a non-corrosive environment for comparative reasons since fatigue data for the cyclic strain approach was only available for non-corrosive conditions. Table 2.1 presents the estimated damage indices obtained by the different methods. The results obtained by the S-N approach show good agreement with the two conservative indices estimated by the cyclic strain approach despite the fact that the damage estimated by the cyclic strain approach is from the ballast condition only while the resulting damage index from the S-N approach consists of two equal damage contributions from the two loading conditions. The damage index obtained by the “original” cyclic strain approach is at least a factor of 10 smaller than the rest of the indices which indicates that one of the conservative methods (or an alternative method) should be used when the cyclic strain approach is applied for design purposes. The effect of the corrosive environment is seen to reduce the fatigue life by a factor of 2.5 when the S-N approach is applied. A sensitivity study showed that a 10 % higher stress level causes an increase of the damage index by 125 % when calculated by the cyclic strain approach while the same increase would change the damage index predicted by the S-N approach by approximately 66 %. Hence, the cyclic strain approach is most sensitive to inaccuracies in the determination of the stress level.

<table>
<thead>
<tr>
<th>Damage approach</th>
<th>Damage index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclic strain(\text{original})</td>
<td>0.11</td>
</tr>
<tr>
<td>Cyclic strain(\text{reduced fatigue strength})</td>
<td>1.66</td>
</tr>
<tr>
<td>Cyclic strain(\text{elementary})</td>
<td>2.13</td>
</tr>
<tr>
<td>S-N(\text{air})</td>
<td>1.17</td>
</tr>
<tr>
<td>S-N(\text{corrosive environment})</td>
<td>2.82</td>
</tr>
</tbody>
</table>
2.3.4 Summary of Fatigue Damage Models

The cyclic strain approach takes into account the effect of local yielding and the mean stress level which can be important to the fatigue life predictions of scallops and cutouts. The method has been successfully applied to low-cycle fatigue but the application of the method to high-cycle fatigue is for some materials encumbered with a level of uncertainty due to a relatively limited number of data points available in the high-cycle region of the $P_{SWT}$-damage curves. Depending on the interpretation of this data the procedure has been shown to predict fatigue lives similar to or a factor of 10 less than fatigue lives estimated by the S-N approach [3]. The cyclic strain approach could offer an interesting alternative to the S-N approach if more data becomes available in the high-cycle area. However, the S-N approach will be applied in the following.
Chapter 2. Fatigue Damage Models

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Chapter 3

Probabilistic Fatigue Damage Analysis of Slot Designs

3.1 Introduction

The connections between the longitudinal stiffeners and the transverse web frames have for long time been regarded as weak points in the fatigue strength of a traditional tanker design with the fatigue problems becoming especially pronounced in the second-generation VLCCs which are built with extensive use of high-tensile steel. The connections are complicated welded structures with a large number of potential crack initiation sites. In a study of registered crack in first- and second-generation VLCCs Yoneya et al. [151] found that most of the fatigue cracks are initiated at the toe and the heel of the flat bar stiffeners as illustrated in Figure 3.1. An obvious way to avoid these cracks is to remove the flat bar stiffener. However, an elimination is not without cost. The load to be transferred by the weld seam between the web frame and the longitudinal is raised which results in an increased

Figure 3.1: Detail of typical cracks in side structure.
stress level at the cutout. To overcome this problem, Kamoi [66] introduced a specially shaped cutout (apple shape) suitable for stress relaxation in order to keep the stress level in the slot structure at an acceptable level.

An objective of this study was to design a slot structure which allows for an increased application of automatic welding procedures and at the same time reduces the number of fatigue cracks. To avoid manual fitting of the flat bar stiffener, a design was sought where the stiffener was removed. But the elimination of the bracket has some side effects: the buckling strength of the web frame is weakened and the cutout becomes a hard-spot area. A rational way of reducing the stress level is to shape-optimize the cutout. The finite element based optimisation program ODESSY [100] was employed for that purpose and a new shape-optimised slot design was obtained. The buckling strength of the web frame was secured by mounting a vertical stiffener which did not introduce new hard-spot areas.

A non-linear probabilistic fatigue damage model was established in order to compare the relative fatigue strength of the shape-optimised slot structure and the original detail. The procedure for fatigue life estimation consists of a stochastic generation of the wave climate to be experienced by the vessel during a design life. The ship hull is modelled in the 3-D line program I-ship (developed by the Technical University of Denmark) and the response of the ship due to the simulated environmental loads is obtained by a strip program. When the dynamic loads and the ship response are known, it is possible to calculate the global hull girder loads by use of finite-element models.

A good description of the dynamic stress variations in the vicinity of the slot structure is important to a reliable fatigue life estimation and a local finite-element model with a relatively fine mesh was established for that purpose. A hybrid formulation was employed where the local stress response was calculated on the basis of the loads obtained from the global analysis.

The non-linear damage prediction was based on a quasi-stationary narrow-band model [43] where the short-term response for each simulated sea state was derived by adding the stress components from all global load cases with their correct phase. The S-N approach was applied to the fatigue damage calculations with the stress range determined as the difference between the maximum and the minimum stresses for a load cycle of $2\pi$. The mean damage rate $E[\nu(\Delta\sigma)^m]$ was calculated from the weighted short-term peak distributions over all the sea states and heading directions. Finally, the fatigue life could be estimated on the basis of the simulated stress range $\Delta\sigma$ and the fatigue accumulation model where the linear Palmgren-Miner rule was applied.

Two main issues are presented in this chapter: 1) An outline of a non-linear probabilistic fatigue damage model and 2) development of a new slot design for the longitudinal web frame connection by application of structural shape-optimisation. These two components lead to a comparison of estimated fatigue lives for a conventional and a new shape-optimised design. The fatigue life estimation of the conventional design will be used for the outline of the probabilistic fatigue damage model.
3.2 Probabilistic Fatigue Damage Model

Yoneya et al. [151] registered fatigue cracks in the first- and the second-generation VLCCs. Figure 3.2 shows the distribution of observed cracks in the side longitudinals of the second-generation VLCCs with an average of a little over 10 cracks found in each vessel. The figure shows that in the longitudinal direction most of the cracks were observed in the midship tanks while in the vertical direction the observed cracks were concentrated in the region where the side shell is exposed to non-linear wave pressure going from the mean still water load line to a level of approximately 8 metres below this line. It should also be noted from Figure 3.2 that the starboard side has been more prone to fatigue cracking than the port side. This is probably related to the sailing routes and the loading conditions of the vessels. Similar trends of the location of fatigue cracks were found in a study by Schulte-Strathaus [111] who organised registered cracks from nine survey reports into a database. In this investigation 42% of all cracks was reported to be located in the side shell. With the

Table 3.1: Main particulars for the double hulled VLCC.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall</td>
<td>343.71 m</td>
</tr>
<tr>
<td>Length between perpendicula</td>
<td>327.00 m</td>
</tr>
<tr>
<td>Breadth moulded</td>
<td>56.40 m</td>
</tr>
<tr>
<td>Depth moulded</td>
<td>30.40 m</td>
</tr>
<tr>
<td>Draught laden</td>
<td>19.55 m</td>
</tr>
<tr>
<td>Draught ballast</td>
<td>10.13 m</td>
</tr>
<tr>
<td>Service speed</td>
<td>14.5 kn</td>
</tr>
</tbody>
</table>
just outlined damage statistics in mind a slot structure in a midship tank of an existing 293 000 dwt double-bulled VLCC was chosen as a basis for the present analysis. The main particulars of the vessel are listed in Table 3.1. The detail to be investigated forms part of the T-shaped longitudinal number 52 and the web frame number 174 and it is located two metres below the laden load line as depicted in Figure 3.3.

The fatigue analysis can be split into a number of phases. An overview of the necessary steps in a fatigue life estimation based on direct probabilistic load generation is given in Figure 3.4. The different components presented in the figure will be discussed in the following on the basis of the work of Friis Hansen [43] and the slot structure from the conventional design.

**Direct fatigue life analysis**

![Flow diagram for the fatigue analysis procedure.](Figure 3.4)
3.2. Probabilistic Fatigue Damage Model

3.2.1 Direct Load Modelling

The first step of direct modelling is to establish a load spectrum representative of the considered lifetime of the vessel. For that purpose the computer program PROSHIP [31] can be used. Based on the data available for the different Marsden zones \((H_s, T_z)\) PROSHIP simulates the wave climate for a specified trade. In the present case the route Persian Gulf - Rotterdam was chosen for the analysis. The effect of different heading angles and of manoeuvring in bad weather is handled by PROSHIP. For a detailed description of manoeuvring philosophy and fit of Marsden zone data see Friis Hansen [43]. Loading conditions representative of the entire lifetime of the ship have to be taken into account. For tankers the fully loaded condition and the ballast condition are normally sufficient. In the present case the vessel was assumed to operate in each of these two loading conditions for 50 % of the time. It should be mentioned that the intermediate loading conditions may be expected in the range from 5 % to 20 % of the lifetime of the ship [43].

3.2.2 Load Transfer Function

The previous section describes the establishment of the environment in which the ship is expected to operate. The response of the vessel due to the simulated sea states was in the present formulation determined by a two-dimensional strip program based on the Gerritsma and Beukelman formulation. The output of the program is a two-dimensional prediction of the ship motion and forces due to a regular sinusoidal wave system with given amplitude \(a\) and frequency \(\omega\). The basic assumptions are:

1. A two-dimensional description of ship motion and forces is sufficient.
2. Slamming effects are neglected.
3. The dynamic transients are small and evolve slowly so that the structure responds directly to the local wave sinusoid without significant effect of transients from previous waves.

The two-dimensional description of motion and forces might be rather crude but for the present comparative study it is assumed to cover the major part of the subject.

The effect of slamming was judged to be relatively low in the midship tank hence it is neglected in the fatigue assessment analysis since the stress ranges at higher probability levels are most important to the fatigue life of the structure.

3.2.3 The Local Finite-Element Model

The local finite element model contains a longitudinal spacing in the vertical direction of the side shell and a web frame spacing in the longitudinal direction with half spacing on each side
as depicted in Figure 3.5. Shell elements are used for the modelling with the mesh density concentrated in the vicinity of the slot edge and at the connection between the bracket and the longitudinal. In these regions where a high accuracy is desired, the length of the elements is around 40% of the plate thickness $t$ as recommended by Mikkola [85].

Figure 3.1 shows that the weld at the stiffener toe is a potential crack initiator. This was also observed in the finite element analyses of the slot structure when the side shell was exposed to a unit pressure load. The hot-spot approach is used for the determination of stresses. Hence, the finite element model does not include the weld geometry and the stresses become singular at the interesting spot. An extrapolation method was therefore employed to find the approximate stress level at the weld toe. An extrapolation procedure proposed by DNV [34] was selected for the present analysis where the stress concentration factor is determined by a linear extrapolation to the stiffener toe. Two points located at $t/2$ and $3t/2$ from the intersection line were used for the linear extrapolation as illustrated in Figure 3.6. The calculated SCF due to the structure $K_t$ was equal to 1.49 and the SCF caused by the weld $K_w$ was assumed to be 1.5 as recommended by DNV [34].

The slot construction is a part of the hull structure. Thus, it is exposed to a number of dynamic forces where the main cyclic load contributions are due to wave-induced

- vertical and horizontal hull girder bending
- internal pressure exerted by fluids in the tanks because of vertical acceleration
- dynamic sea pressure loads.
It is important to the above-mentioned stress contributions to take into account that they act with different phases. The effect of wave-induced hull girder torsional stresses is neglected which is usually justifiable for tank ship sections.

A set of boundary conditions for the model was determined by considering the deflection of a side shell with a forced pressure load. The deflection is asymmetric due to the presence of the flat bar stiffener. A larger pilot model covering three web frame spacings verified that the boundary conditions shown in Figure 3.7 give a good approximation to the actual deflection if the web frame and the flat bar stiffener are placed slightly eccentrically in the $x$-direction.

### 3.2.4 FE Model of the Ship and the Local Stress Transfer Function

This section concentrates on the global models which form the boundary conditions (loads) for the local detail just discussed. With the assumptions described in Section 3.2.2 the load contributions are reduced to:

1. Vertical hull girder bending.
2. Deformation of the web frame due to external sea water pressure.
3. Internal pressure exerted by vertical acceleration of ballast water or cargo oil.
4. Sea pressure on the side shell.

The load cases illustrated in Figure 3.8.

The hull girder bending moments are determined by the strip program and the global stress distribution is obtained by means of Navier's theory. These stresses are transformed to edge loads for the boundaries with the label B in Figure 3.7.
Atwo-dimensional beam model is used for calculating the deformation of the web frame due to external sea water pressure arising from a simulated wave height. Afterwardsthe stress distributions are calculated from Grasshoffs and Naviers theories. These stresses are also transformed into edge loads for the boundaries A, C, and D. Similar calculations are made for the internal pressure exerted by simulated vertical acceleration of the fluids in the tanks.

The external sea water pressure is non-linear due to the intermittent submergence of the side shell. Because of this non-linearity the fatigue damage of the combined stress cannot be calculated by a traditional frequency domain analysis. However, the stochastic combination problem of the non-linear stress from the external pressure and stresses caused by the wave-induced moments can be solved by application of a quasi-stationary narrow-band model [43].

The local sea water pressure shows a high correlation with the relative instantaneous wave elevation when the motions of the vessel are accounted for [32]. Given the transfer functions $Z_{\text{beave}}$ and $\Theta_{\text{pitch}}$ (from the strip program) the instantaneous wave elevation above the mean still waterline $Z_{aw}$ at location $x_l$ can be calculated as (by use of a coordinate system with origo as depicted in Figure 3.8)

$$Z_{aw}(x_l, a, \omega) = a \left\{ Z_w(x_l, \omega) - \{Z_{\text{beave}}(\omega) + \Theta_{\text{pitch}}(\omega)x_l\} \right\}$$ (3.1)
3.2. Probabilistic Fatigue Damage Model

Figure 3.8: Load cases for fatigue analysis (shown only for the fully loaded condition).
where $Z_w$ is the incoming wave elevation and small pitch angles are assumed. The local hydrodynamic pressure $P(a, \omega)$ is calculated as

$$P(a, \omega) = \begin{cases} 
0 & \text{for } z_{msw} + z_{aw} < z \\
\rho g \{z_{msw} + z_{aw}(a, \omega)\} & \text{for } z_{msw} < z < z_{msw} + z_{aw} \\
\rho g \{z_{msw} + z_{aw}(a, \omega)e^{k(z_{msw}-z)}\} & \text{for } z < z_{msw} \land z < z_{msw} + z_{aw}
\end{cases} \quad (3.2)$$

where $g$ is the gravity, $\rho$ is the density of the seawater, and $z_{msw}$ is the mean still waterline. The reduction in pressure because of the depth is modelled as $e^{k(z-z_{msw})}$, where $|z - z_{msw}|$ is the distance from the mean waterline to the point considered. When Eq. (3.2) is used, effects of diffracted waves on the hull and waves generated by ship oscillations are neglected, which is a reasonable assumption due to wall-sidedness at the section considered [43].

### 3.2.5 Short- and Long-Term Response

The quasi-stationary model used for prediction of the fatigue damage in the web frame and the side shell assumes that the wave spectrum is narrow-banded, which is justifiable for ship structures [43]. For a narrow-band process, the stress range is twice the amplitude leading to the following Rayleigh distributed stress range within each short term condition:

$$F_{\Delta \sigma}(\sigma) = 1 - \exp \left(-\frac{\sigma^2}{8 m_0}\right) \quad (3.3)$$

where $m_0$ is the spectral moment of order zero. When the S-N approach is applied, fatigue analysis is based on the assumption that it is only necessary to consider the range of cyclic principal stress in the prediction of the fatigue endurance. Principal stresses obtained from each load case are summed taking into account the correct phase of the stress components. For each short-term sea state, the resulting stress for a given amplitude $a$ and frequency $\omega$ is determined by:

$$\sigma(a, \omega, \epsilon) = \sigma_{\text{hull girder bending}}(a, \omega, \epsilon) + \sigma_{\text{local deformation}}(a, \omega, \epsilon) + \sigma_{\text{internal pressure}}(a, \omega, \epsilon) + \sigma_{\text{external pressure}}(a, \omega, \epsilon) \quad (3.4)$$

With the narrow-band theory, it is possible to specify the stress range $\Delta \sigma$ for a given wave amplitude $a$ and frequency $\omega$. It is determined as the difference between maximum and minimum stress within each stress response cycle:

$$\Delta \sigma = \max\{\sigma(a, \omega, \epsilon)\} - \min\{\sigma(a, \omega, \epsilon)\} \quad (3.5)$$
when the phase $\epsilon$ is permitted to run through an entire load cycle $[0, 2\pi]$. 

The mean fatigue damage rate is obtained by unconditioning the short-term peak distribution of Eq. (3.5):

$$
E[D] = \frac{T}{K_{mat}} E[\nu(\Delta \sigma)^m]
$$

$$
= \frac{T}{K_{mat}} \int_{H_s} \int_{T_s} \int_{A} \int_{V} \int_{\Omega} \nu(\Delta \sigma(a, \omega, v, \theta))^m f(a, \omega \mid H_s, T_s)f(v, \theta \mid H_s, T_s))f(h_s, t_s)da\omega dh_s dt_s dv d\theta
$$

(3.6)

where $T$ is the time, $\nu$ the up crossing rate, $K_{mat}$ a material fatigue parameter, $f(a, \omega \mid H_s, T_s)$ is the joint distribution of wave amplitude and frequency within each sea state, $f(v, \theta \mid H_s, T_s)$ accounts for the effect of manoeuvring in severe sea states. It is not possible to perform a closed-form integration of Eq. (3.6). Hence, the integration was evaluated by the Monte Carlo Simulation (MCS) which is a commonly used technique for solving a complex integral. The basic concepts of the method are described in numerous papers and textbooks e.g. in Rubinstein [107].

### 3.2.6 Fatigue Damage Calculation

The mean damage rate is determined at the critical points by the MSC of Eq. (3.6). The simulations are performed with the probabilistic program PROBAN [88] using 10000 simulation points and the outcome is both the mean fatigue damage $E[\nu \Delta \sigma^m]$ and the mean up crossing rate $\nu$. The fatigue life estimation can be based on the linear Palmgren-Miner approach together with design S-N curves. In the present case the S-N curves recommended by DNV [34] were used for the analysis. A combination of Eq. (3.6) and Eq. (2.1) leads to the following expression:

$$
\log N = \log a - \log \left( \frac{1}{\nu} E[\nu \Delta \sigma^m] \right)
$$

(3.7)

from which the equivalent critical number of load cycles $N$ can be determined which leads to a fatigue life prediction of the detail. The material properties $\log a$ and $m$ suggested by DNV [34] are shown in Table 3.2.
Chapter 3. Probabilistic Fatigue Damage Analysis of Slot Designs

Table 3.2: The S-N curve parameters recommended by DNV.
Air or with cathodic protection.

<table>
<thead>
<tr>
<th>S-N Curve</th>
<th>Material</th>
<th>$\log a$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ib</td>
<td>Welded joint</td>
<td>12.76</td>
<td>3.0</td>
</tr>
<tr>
<td>IIIb</td>
<td>Base material</td>
<td>13.00</td>
<td>3.0</td>
</tr>
</tbody>
</table>

3.3 The Fatigue Life of the Conventional Slot Structure

The first step in the fatigue life estimation of a complicated structure is to identify the weak points with respect to fatigue. The different load cases depicted in Figure 3.8 were used to detect the spots with high stress concentrations most likely to be prone to fatigue initiation. Figure 3.9 shows the von Mises stresses at the cutout caused by a unit pressure on the side shell.

![Figure 3.9: Von Mises stress distribution at the cutout of the conventional design.](image)
Stress concentrations are found in the bend and at the connection between the longitudinal and the cutout (the thin lines represent the longitudinal in the shell element modelling). Both the toe and the heel of the bracket are known to be crack initiators. An analysis with equally spaced elements along the line of attachment between the flat bar stiffener and the longitudinal showed that the largest stress concentration occurs at the toe of the present configuration. Mesh refinements were implemented in the region of the bracket toe according the to recommendations of Mikkola [85] and the von Mises stress pattern depicted in Figure 3.10 was obtained for the load case with a unit pressure acting on the side shell.

The fatigue lives of the spots identified as being most likely to experience fatigue initiation were for the conventional design estimated by the established probabilistic model and by use of the material data from Table 3.2. The predicted fatigue life for the toe at the connection between the flat bar stiffener and the longitudinal was 5 years while the estimate was 110 years for the weakest point of the cutout which has the label 1 in Figure 3.11. The calculations confirm that the toe of the flat bar stiffener is a weak point with respect to fatigue which was observed by Yoneya et al. [151].
3.4 Static Shape Optimisation

In the new design, the flat bar stiffener was removed to avoid the stress concentrations at the toe and the heel of the bracket. The load to be carried by the weld seam between the longitudinal and the web frame was thus increased. Stress concentrations appear around the cutout in the web frame as depicted in Figure 3.12. The main concentrations are found at the bends, but unfortunately also at the weld seam connecting the web frame to the longitudinal. By removing the bracket the maximum stress at the cutout is increased by a factor of 1.6. The finite element based optimisation program ODESSY [100] was employed to reduce the general stress level in the web frame and thus to improve the fatigue strength of the construction.

3.4.1 General Considerations

The fatigue damage is caused by loads which act with different phase. But the simulations for the conventional slot design showed that the non-linear local sea water pressure accounts for the majority of expected damage. Thus, the static optimisation of the slot structure could be based on the local sea water pressure alone (load case 4 in Figure 3.8).

For the new design the stress level at the slot edge is of greatest interest. Due to the free edge and to the relatively thin web plate, two of the principal stresses are approximately zero. In this case the von Mises stress given by

\[
\sigma_{ref} = \sqrt{\frac{3}{2} \sigma_{ij} \sigma_{ij} - \frac{1}{2} \sigma_{ii} \sigma_{jj}}
\] (3.8)
is equivalent to the one principal stress different from zero. Hence, the objective function for optimisation is based on the von Mises stress level in the web frame.

The splines describing the shape of the cutout are defined by nodal points. During the optimisation, ODESSY calculates the sensitivity of moving these points in directions defined by design variables. The shape of the cutout is then improved by moving the design points on the basis of the sensitivity analysis. A maximum movement of a nodal point in one step is prescribed to ensure a stable optimisation, which means that the final shape is found by iteration. It should be mentioned that in an ideal shape optimisation every design point would be free to move in all directions and the result would be totally independent of the initial geometry of the cutout. But experience with the applied optimisation program shows that some restrictions have to be made to avoid mesh distortion and to reduce the CPU time to an acceptable level. The final shape of the slot therefore to some degree depends on how the design points and the corresponding variables are defined. Different combinations were tested with the definition given in Figure 3.13 revealing the best results (the slot structure with the lowest stress level which is still suitable for application in the production). The design points depicted as dots are allowed to move in the direction given by the design variables illustrated by the arrows in the figure.

Figure 3.12: Potential crack locations for the conventional design.
3.4.2 Result of the Shape Optimisation

A simplex procedure was used for the optimisation and the final shape of the cutout can be found in Figure 3.14. It is seen that the main stress concentrations are found in the bend of the cutout away from the welding between the web frame and the longitudinal. This is favourable because in the bend there is no weld to initiate cracks. Compared to a traditional rectangularly shaped cutout in a slot design without a flat bar stiffener, the principal stresses from a unit pressure load are reduced by a factor of 1.5 at the location of the weld.

Again, finite element results obtained from the different load cases were analysed to find the potential crack locations in the new design with the detected weak points depicted in Figure 3.15. The lifetime was estimated by the probabilistic model and the predicted fatigue lives of both the new and the conventional design are listed in Table 3.3. It is seen from the table that the fatigue life of the four hard spots in the new slot structure is of approximately

<table>
<thead>
<tr>
<th>Conventional slot design</th>
<th>Estimated lifetime for point 1</th>
<th>110 years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated lifetime for point 2</td>
<td>5 years</td>
</tr>
<tr>
<td>Shape-optimised design</td>
<td>Estimated lifetime for point 3</td>
<td>34 years</td>
</tr>
<tr>
<td></td>
<td>Estimated lifetime for point 4</td>
<td>28 years</td>
</tr>
<tr>
<td></td>
<td>Estimated lifetime for point 5</td>
<td>30 years</td>
</tr>
<tr>
<td></td>
<td>Estimated lifetime for point 6</td>
<td>77 years</td>
</tr>
</tbody>
</table>

the same order of magnitude which is favourable (special attention has been paid to the weld at points 4 and 5). The estimated lifetime for the hard spot at the toe of the flat bar stiffener is about 5 years for the conventional slot structure i.e. six times less than the lifetime for the new design.
3.5 Comments

First of all, too much emphasis should not be placed on the absolute values of the estimated fatigue lives. The calculations are believed to cover the major contributions to fatigue of the slot structures and the predictions are therefore well suited for the comparative study of the two designs. But, as absolute values, the predictions are questionable due to the stochastic nature of the problem and the assumptions made in the study.

The optimisation of the cutout is based on the assumption that the local sea water pressure accounts for most of the fatigue damage in the connections between longitudinals and web frames. Figure 3.16 shows the sum (with the label 1) and the different contributions to the fatigue damage. On the basis of these results, the assumption seems to be verified. It should be noted that the contributions act with different phases and that the damage is non-linear in the stress range.
Figure 3.15: Potential crack locations for new slot.

Figure 3.16 illustrates also how the expected damage is distributed between the ballast and the laden condition. It can be observed that the laden condition accounts for most (approximately 90%) of the predicted damage. The analysis was carried out for a slot structure located in the region where the side shell is exposed to non-linear pressure from the outside sea water in the laden condition. Different patterns are expected for longitudinals located outside this region.

When Kamoi [66] compared his new slot structure with the initial design (slot structure with a flat bar stiffener) he maintained the same stress level at the cutout. Unfortunately, the optimised cutout (from Figure 3.14) does not show the same excellent result. In the present analysis the maximum principal stress was found to be approximately a factor of 1.45 times larger than in the initial slot structure. But the overall stress level is still lower compared to the stress at the toe of the flat bar stiffener.

The points in Figure 3.15 are symmetrically located with respect to the longitudinal. Yet, from Table 3.3 it is seen that the estimated lifetime differs by a factor of 2 for the points in the bends of the cutout. This is caused by the shear forces in the web frame which is respectively increasing/decreasing the principal stresses in the bends. Close to the longitudinal (points 4 and 5) the effect of the shear force is minor due to the geometry of the cutout.

The buckling strength of the web frame is reduced when the transverse flat bar stiffener is removed. A vertical stiffener can be mounted to prevent buckling of the web frame (this is a common way to obtain the required buckling strength). Such a vertical stiffener is ideal for an automatic welding procedure and it does not introduce new weak points in the fatigue strength of the slot structure. Equally important, the fatigue life of the slot is not decreased.
by the influence of the vertical stiffener (a minor improvement can be obtained e.g. for a specific position of the stiffener the estimated lifetime for point 4 was increased from 28 to 30 years).

In the present analysis the flat bar stiffener is without a soft toe but results which are presented in [151] show little influence of a soft toe (the SCF due to pressure is only reduced from 1.7 to 1.6).

3.6 Summary

The heel and the toe of the brackets found in a conventional longitudinal to web frame arrangement are known to be crack initiators. A slot structure found in an existing VLCC formed the basis for a comparative study of a conventional design and a new design where the brackets are eliminated. The exclusion of the stiffeners increases the load to be carried by the weld seams between the longitudinal and the web frame and the cutout becomes a hard-spot area. A finite element based optimisation procedure was applied to reduce the stress level at the cutout and thus increase the fatigue strength of the structure.
Before the shape optimisation the main stress concentrations were at the welds which are known to be crack initiators. The shape optimisation reduced the stress level by a factor of 1.6 in this region and in the new design the main stress concentrations were found in the bends away from the welds.

A probabilistic fatigue damage model was established in order to evaluate the relative fatigue strength of the two structures. The model was founded on the narrow-band theory [43] allowing for the inclusion of the non-linear effect of the intermittent submergence of the side shell found in the region close to the load line.

The probabilistic model was employed in the analysis of the two designs. In the conventional design the toe of the bracket was predicted to be the weakest point with respect to fatigue with an estimated lifetime of 5 years while the predicted lifetimes of hard spots in the shape-optimised cutout were approximately 30 years i.e. the fatigue strength of the new structure is increased by a factor of 6.

From the fatigue life estimations of the slot structures located approximately 2 metres below the laden load line it was observed that the laden condition accounts for 90% of the damage with most of the damage being caused by the external sea water pressure acting on the side shell. Further analysis of the structures can be based on these observations.

Finally it is quite a drastic structural change to remove the flat bar stiffener but the optimised cutout could at a relatively low cost be implemented in the conventional slot design. Thus the probability of cracks at the edge of the cutout would be reduced.
Chapter 4

Two-Dimensional Crack Propagation, Theory

4.1 Introduction

It is good design practice to estimate fatigue lives of critical details in ship structures at the design stage. But the models used for the fatigue life predictions are usually based on one-dimensional crack initiation and propagation laws and they do not cover the occurrence of crack paths which differ significantly from co-planar paths. Such branched crack growth can be observed in structures containing a high level of residual stresses or in details which are biaxially loaded. In some cases the crack will propagate into zones of very high or very low stress intensity and the crack growth will either be accelerated or it will be retarded and if the stress intensity becomes sufficiently low, crack arrest might even occur. Curved crack growth may thus affect the fatigue life of a structure. Hence, the author decided to establish a two-dimensional crack propagation procedure in order to investigate how mixed mode crack growth may affect the fatigue strength of the conventional and the shape-optimised cutouts. This chapter outlines the theory needed for the development of a mixed mode crack propagation procedure in the frames of linear elastic fracture mechanics while the aspects of numerical implementation are discussed in the following Chapter 5.

4.2 Basic Linear Elastic Fracture Mechanics

Fracture mechanics seeks to establish the local stress and strain fields around a crack tip in terms of global parameters such as the loading and the geometry of the structure. Usually fracture mechanical approaches are divided into 1) linear elastic solutions and 2) non-linear methods. Only the first approach is discussed here. The present formulation is also restricted to long cracks subjected to constant amplitude cyclic loading (a distinction between long
and short cracks for normal engineering structures is sometimes assumed to be about 1 mm in length according to Tanaka [130] but it depends on the dimensions of the structure.)

### 4.2.1 Modes of Crack-Tip Deformation

For linear elastic materialsIrwin [62] suggested describing the stresses in the vicinity of the crack by stress intensity factors (SIFs). There are basically three different types of SIFs each describing one of the deformation modes illustrated in Figure 4.1. The superposition of these three modes forms the general case of cracking. The deformation modes can be characterised as follows:

- Mode II is the in-plane tensile mode where the crack surface is symmetrically opened.
- Mode III is the sliding or shear mode which is present when the crack is exposed to skew-symmetric in-plane loading.
- Mode IIII is an anti-plane mode where the crack surface is twisted by forces perpendicular to the crack plane.

In simulations of crack propagation in a plate structureIrwin [62] he found an analytical solution
for the stress distribution in the vicinity of the crack tip:

\[
\sigma_{11} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) - \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left( 2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right) + O(r^{1/2})
\]

\[
\sigma_{22} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + O(r^{1/2})
\]

\[
\sigma_{12} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + O(r^{1/2})
\]

where the stresses are given in the polar coordinate system illustrated in Figure 4.2 and higher order terms are neglected. Eq. (4.1) is therefore only valid in the region close to the crack tip \((r \ll a)\). It is seen that the magnitude of the crack tip stresses is controlled by the stress intensity factors \(K_I, K_{II}\) while the distribution of the stresses is governed by the position relative to the crack tip given by \(r\) and \(\theta\). It is also observed that the distribution of the stresses is singular for \(r = 0\) which is only true for fully brittle materials. That is the main limitation for a linear elastic approach since most materials exhibit some state of plasticity at the crack tip.

Apart from the stresses the displacements are also controlled by the stress intensity factors [54]:

\[
u = \frac{1}{4\mu} \sqrt{\frac{r}{2\pi}} \left[ K_I \left( (2\kappa - 1) \cos \frac{\theta}{2} - \cos \frac{3\theta}{2} \right) - K_{II} \left( (2\kappa + 3) \sin \frac{\theta}{2} + \sin \frac{3\theta}{2} \right) \right]
\]

\[
\sigma_{12} = \frac{1}{4\mu} \sqrt{\frac{r}{2\pi}} \left[ K_I \left( (2\kappa + 1) \sin \frac{\theta}{2} - \sin \frac{3\theta}{2} \right) - K_{II} \left( (2\kappa - 3) \cos \frac{\theta}{2} + \cos \frac{3\theta}{2} \right) \right]
\]
here $\mu$ is the shear modulus of the material and $\kappa$ is given by

$$
\kappa = \begin{cases} 
3 - 4\nu & \text{for plain strain} \\
(3 - \nu)/(1 + \nu) & \text{for plain stress}
\end{cases}
$$

(4.3)

### 4.2.2 Stress Intensity Factors

Eq. (4.1) shows that the stress intensity factors are the fundamental quantities describing the singular elastic stresses. The stress intensity factors are functions of the length and orientation of the crack, the geometry of the body, and the applied load distribution. For the uniaxially loaded case, the mode I stress intensity factors can be calculated from

$$
K = \sigma \sqrt{\pi a F}
$$

(4.4)

where $F$ takes into account the effects of geometry, the crack length, and the applied load. $F$ is equal to 1.0 for a crack located in a uniformly loaded infinite plate perpendicular to the loading direction. Analytical and empirical expressions for stress intensity factors are collected in compendiums such as Tada et al. [128] and Sih [115] for specimens of simple geometries with various crack configurations and load combinations. Unfortunately, structural members used in practical engineering are not generally of simple geometry and they are exposed to complex loading conditions, so specific analysis must be resorted to. In such cases, different methods are available, but for complex mixed mode conditions, the finite element methods seem to offer the best solutions due to their flexibility and ease of computerisation. Methods for determination of SIFs by use of the finite element are described in Chapter 5.

### 4.2.3 Energy Release Rate, $G$

The basic concept of the energy release method was established in 1921 by Griffith. For an ideally brittle material, he stated that crack propagation will occur if the energy supplied at the crack tip is equal to or greater than the energy required for crack growth. This criterion was later reformulated by Irwin [62] by the introduction of the energy release rate, which is a measure of the energy available for an increment of crack extension:

$$
G = -\frac{d\Pi}{dA}
$$

(4.5)

where $\Pi$ is the potential energy and $A$ is the released crack area. Crack initiation occurs when the energy release rate reaches a critical value called the fracture toughness of the
material $G_c$. The fracture toughness is considered a material parameter and theoretically it should be possible to determine this parameter by calculating the energy needed for the nucleation of the atomic bonds in the material. However, materials contain imperfections and crack propagation therefore starts before the theoretical value of the fracture toughness is reached (for steel the experimental value of $G_c$ is about half of the theoretical $G_c$-value). Thus, instead of using the theoretical approach the fracture toughness of a material is usually determined by standardised experiments such as the compact tension (CT) specimen test (described in [231105]).

A very simple relation is often used for linear elastic materials to describe the relation between the fracture toughness and the stress intensity factors in the mixed mode condition:

$$G = \frac{K_I^2}{E' \nu} + \frac{K_{II}^2}{E' \nu^2}$$  \hspace{1cm} (4.6)

where $E'$ is the modulus elasticity $E$ for the plane stress and $E/(1-\nu^2)$ for the plane strain condition. Eq. (4.6) describes a circle in the stress intensity factor plane but investigations have shown that the critical $G$-curve is better described by an ellipse [23]. This indicates that Eq. (4.6) should be modified by a material dependent constant $k$ to obtain a better fit to the experimentally observed data:

$$G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E' k^2}$$  \hspace{1cm} (4.7)

The critical mode I stress intensity factor $K_{IC}$ is sometimes used as a measure for the fracture toughness of the material and it can also be determined by experiments. The relation between $G_c$ and $K_{IC}$ is obtained through Eq. (4.6) with $K_{II}$ equal to zero. From above it can be seen that the fracture toughness of the material is a function of the stress intensity factors but unfortunately it is also a function of the stress state (Eq. (4.6) is a function of $E'$). This means that $G$ has to be related to the present stress state and experiments therefore have to be carried out for both plain stress and plain strain conditions. A way of avoiding that for through thickness cracks is to make sure that plain strain is present in the major part of the crack surface. By analysing the size of the yield zone at the crack tip ($r_{\text{yield}} \approx (K_{IC}/\sigma_{\text{yield}})^2/(6\pi)$ for plain strain) the following expression has been established:

$$B > q \frac{K_{IC}^2}{\sigma_{\text{yield}}^2}$$  \hspace{1cm} (4.8)

where $B$ is the thickness of the specimen and $\sigma_{\text{yield}}$ is the yield stress of the material. The parameter $q$ differs from material to material but an ASTM standard suggests that $q$ equal to 2.5 is to be used which results in a plastic zone with a radius $\approx B/50$. The minimum crack length has to be sufficiently large relative to the plastic zone at the crack tip to ensure “brittle material behaviour”. This is generally the case when the crack length meet the same demand as made on $B$ in Eq. (4.8).
4.3 Direction of Crack Propagation

The crack propagates given that enough energy is supplied at the crack tip. It propagates in different ways depending on the stress state in the vicinity of the crack tip. The simplest case is mode I fatigue where the crack extends straight along a preferred direction due to symmetry. This type of fatigue has been the subject of investigations for several decades and is now quite well established in the frames of linear elastic fracture mechanics. However, structures often contain randomly located cracks which are in a mixed mode stress field by virtue of their orientation with respect to the loading direction. A characteristic of mixed mode (and mode II) fatigue cracks is that they tend to propagate in a non-self-similar manner resulting in changes in the crack propagation direction during the loading period.

Several attempts have been made to establish a criterion for mixed mode fatigue growth. Recent critical reviews of proposed methods are given in [2151G96]. No attempt will be made to repeat their work. There only a few of the commonest approaches are reviewed and discussed. Most of the currently used criteria for prediction of mixed mode fatigue crack propagation are initially developed for monotonic loading and the review starts therefore with the criteria for monotonic loading. The typical configuration for this examination is a plate with a crack which is inclined at the angle $\beta$ with respect to the far field load $\sigma_0$. The nomenclature is given in Figure 4.3.

![Figure 4.3: Typical configuration of the angled crack problem.](image)
4.3. Direction of Crack Propagation

4.3.1 Mixed Mode Crack Propagation under Monotonic Loading

Maximum Tangential/Opening Stress Criterion (Max $\sigma_\theta$ or MTS)

One of the first attempts to predict the crack propagation direction for a plane stationary mixed mode condition was developed by Erdogan and Sih [40]. They investigated the fracture extension in a large plate of brittle material containing an inclined crack. The plate was exposed to a general in plane loading condition. It was proposed that the crack would propagate from the crack tip in a radial direction defined by the plane perpendicular to the direction of greatest tension. Thus the crack would extend in the direction of the largest tangential stress $\sigma_\theta$. In polar coordinates the tangential stress is given by

$$\sigma_\theta = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ K_I \cos \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right]$$  \hspace{1cm} (4.9)

The direction of the maximum tangential stress can be found by setting the derivative of Eq. (4.9) equal to zero. The non-trivial solution is given by

$$K_I \sin(\theta) + K_{II}(3 \cos(\theta) - 1) = 0$$  \hspace{1cm} (4.10)

This equation can be solved

$$\theta_0 = \pm \cos^{-1} \left\{ \frac{3K_{II}^2 + K_I \sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2} \right\}$$  \hspace{1cm} (4.11)

In order to ensure that the opening stress associated with the direction of the crack extension is maximum the sign of $\theta_0$ should be opposite to the sign of $K_{II}$ [113] (the two possibilities are illustrated in Figure 4.4).

![Diagram of crack propagation with positive and negative $K_{II}$](image)

Figure 4.4: Sign of the propagation angle.
The criterion for the crack extension taking place was formulated as

\[ T_c = \sqrt{2\pi} \sigma \theta |_{\theta = 0} = \text{constant} \] (4.12)

where Erdogan and Sih [40] suggested that

\[ T_c = K_{IC} \] (4.13)

From experimental observations Erdogan and Sih concluded that Eq. (4.13) should only be regarded as a practical design tool giving conservative results.

The shortcomings of the maximum tangential stress criterion are: a) the normal stress at the crack tip is singular in all directions when linear elastic fracture mechanics is applied which makes the criterion non-physical; and b) the hypothesis is based on one stress field component while the rest of the components are ignored [5]. The MTS criterion has been widely applied because of its simplicity and supported by many experiments but work which does not support this criterion can also be found (see e.g. [106;129]).

**Strain Energy Density Theory (Min S or SED)**

The strain energy density theory proposed by Sih [116] is an often used method to predict the crack extension angle. The idea is to examine a small element outside a core region defined by \( r_0 \) (see Figure 4.5) and then base estimation of the crack extension on the stress field outside the core region. It is assumed that the energy per unit area (\( dw/dA \)) of the element can be expressed through

\[ \frac{dw}{dA} = \frac{S}{r} \] (4.14)

where the radius \( r \) (see Figure 4.5) is small in comparison with the crack length \( \Gamma \) and \( S \) is the strain energy density factor representing the intensity of the \( 1/r \) energy field [116]. By analysis of a crack extension through the core area the criterion was formulated:

1. Crack initiation occurs in the radial direction \( \theta_0 \Gamma \) along which the local energy density possesses a relative minimum \( S_{min} \):

\[ \frac{\partial S}{\partial \theta} = 0, \quad \left( \frac{\partial^2 S}{\partial \theta^2} > 0 \right) \quad \text{for} \quad \theta = \theta_0 \] (4.15)

2. Unstable crack growth occurs when \( S_{min} \) reaches a critical value \( S_c \Gamma \) that is:

\[ S_{min} = S_c \] (4.16)
For an isotropic and homogeneous material subjected to a plane mixed mode stress field $\Gamma$, the strain density can be found from [117]:

$$S = a_{11} K_1^2 + 2a_{12} K_1 K_{II} + a_{22} K_{II}^2$$  \hspace{1cm} (4.17)$$

in which the coefficients $a_{ij}$ are given by

$$a_{11} = \frac{1}{16\mu}[(1 + \cos \theta)(\kappa - \cos \theta)]$$

$$a_{12} = \frac{1}{16\mu}\sin \theta[2\cos \theta - (\kappa - 1)]$$

$$a_{22} = \frac{1}{16\mu}[(\kappa + 1)(1 - \cos \theta) + (1 + \cos \theta)(3\cos \theta - 1)]$$  \hspace{1cm} (4.18)$$

where $\mu$ is the shear modulus and $\kappa$ is given in Eq. (4.3). Chang [25] investigated the strain energy density theory ($S$-theory) for an elliptic and a slit crack configuration. He found that the $S$-theory is very sensitive to the $r_0/a$ ratio ($r_0/a$ is the distance from the crack tip to the point where the strain density is calculated divided by half the crack length) when applied to an elliptic crack model $\Gamma$ but that it provides predictions which are comparable to other models when it is applied to a slit crack configuration.

Zhengtao and Duo [153] proposed an alternative fracture criterion assuming that the crack would propagate in the direction of the nearest distance from the crack tip to iso-strain-energy-density curves in the vicinity of the crack tip. This also leads to the criterion described by Eq. (4.15) $\Gamma$ but $\Gamma$ as the authors stated, it is on a more physically based background.

**Maximum Energy Release (Max $G$)**

Hussain et al. [58] and Palaniswamy and Knauss [91] independently developed the maximum energy release criterion by analysing the influence of a small virtual extension to an existing crack in the direction $\theta$. The criterion for crack propagation was based on maximising the energy release rate $G$ with respect to the propagation angle $\theta$. The criterion reads:
Chapter 4. Two-Dimensional Crack Propagation, Theory

1. Crack initiation occurs in the radial direction $\theta_0 \Gamma$ which processes the maximum energy release rate $G$:

$$\frac{\partial G}{\partial \theta} = 0, \quad \left( \frac{\partial^2 G}{\partial \theta^2} < 0 \right) \quad \text{for } \theta = \theta_0 \quad (4.19)$$

2. Unstable crack growth occurs when $G$ reaches a critical value $G_c \Gamma$ that is:

$$G = G_c \quad (4.20)$$

As mentioned earlier, the critical value of $G$ can be found experimentally or by setting $K_{II}$ equal to zero in Eq. (4.7) if $K_{IC}$ is known.

Crack Propagation Angles by the Outlined Methods

![Comparison of path-directional criteria](image)

Figure 4.6: Comparison of path-directional criteria.

Ramulu and Kobayashi [99] compared crack extension angles predicted by the three presented methods. They worked on a uniaxially loaded plate with a crack which was inclined with respect to the load direction. The $K_I / K_{II}$-relation was used as the free parameter and the results are shown in Figure 4.6. It can be seen that for a Poisson's ratio equal to 0.3 (~ steel), the methods give very similar results for small values of the incline angle $\theta_0$ (the angle is defined in Figure 4.3).

### 4.3.2 Mixed Mode Crack Propagation under Cyclic Loading

At present, no firmly established cyclic mixed mode criterion exists for the prediction of cyclic crack propagation angles. It is therefore generally accepted that the criteria developed for
monotonic loading are extended also to cover cyclic loading. This is so despite the fact that distinct differences in the characteristics of the crack growth trajectory have been observed for static and cyclic mixed mode crack propagation.

Mageed and Pandey [77] carried out experiments on uniaxially loaded \((\lambda = 0.0\) in Figure 4.3\) aluminium alloy sheets with a centre oblique crack configuration to investigate static and cyclic mixed mode crack initiation and crack paths. The experimentally obtained crack paths were compared to crack paths estimated by two of the criteria discussed in the previous section (and a third criterion called the maximum-normal-strain theory (MTSN) which will not be included in the present discussion). Figure 4.7 shows crack paths for both static and cyclic crack growth where the initial cracks are inclined by two different angles

\[
\beta (the \text{ } \beta \text{ is defined in Figure 4.3}). \text{ In the mode II dominant case } (\beta = 15^\circ) \text{ there is a considerable difference between the cyclic initiation angle and the angles predicted by the theoretical criteria while the static initiation angle is well described by the theoretical approaches. However, differences between the static experiments and the angle predicted by the theories start to occur as the crack propagates. When the mode I stress intensity factor dominates crack paths predicted by the various approaches are in fair agreement with the cyclic experiments. Mageed and Pandey [77] concluded that for } \beta < 30^\circ \text{ the agreement is not very good at the initial stage for cyclic loading. Other investigations have also shown the difficulties in predicting the initial crack angle under dominant mode II loading [96G106].}
\]
Chapter 4. Two-Dimensional Crack Propagation, Theory

It should also be mentioned that mixed mode or mode II crack propagation can cause directional instability at either very small loads or at large loads close to the yield stresses of the material [99]. In the case of significant yielding, the cracks tend to follow planes of maximum shear stress rather than grow in maximum mode I [93, 118]. But in lieu of better approaches and given that the crack usually turns to propagate under mode I conditions [45, 60, 71, 79, 93, 96] the criterion outlined in the previous sections will therefore be used for determination of cyclic initiation and propagation angles.

4.4 Crack Propagation Models for Constant Amplitude Loading

Fatigue may be defined as a process of cycle-by-cycle accumulation of damage in a material undergoing fluctuating stresses and strains [2]. The main goal of crack propagation models is therefore to relate the material damage to the cyclic loads applied. A fatigue process undergoes several stages and from an engineering point of view it is convenient to divide the fatigue life of a structure into three stages [50]:

I Fatigue crack initiation.

II Stable crack propagation.

III Unstable crack propagation.

The first of the three stages covers the crack initiation and the early crack growth state which includes cyclic plastic deformation prior to crack initiation. Initiation of one or more microscopic cracks and coalescence of these micro-cracks to form an initial macro-crack [51]. Notch stress-strain analysis and low-cycle fatigue concepts can be used for the estimation of this initiation period. The second period is characterised by stable crack growth while at the last stage the crack growth accelerates due to interaction between fatigue and fracture mechanisms. The duration of the third period is usually quite short and it can without great loss of accuracy be neglected in the fatigue life estimations. In welded and flame cut structures it is common that flaws and defects exist and the crack initiation period is in the present formulation taken into account by assuming that the cracks have an initial crack length $a_0$. Attention will hereafter be paid to the stage of stable crack growth.

4.5 Mode I Crack Growth

The lifetime of structures with initial cracks is very closely related to the crack propagation period since the initiation time is essentially short. Current experimental and theoretical
linear elastic approaches try to describe the stable and unstable crack growth by a crack propagation rate. This rate is defined as the incremental crack growth $da/dN$ divided by the incremental number of load cycles $dN$ as a function of the stress intensity factor range $AK$ during a load cycle. The SIF range is determined as $K_{\text{Imax}} - K_{\text{Imin}}$. A schematic illustration of a crack growth rate curve is given in Figure 4.8 with characteristics of the curve taken from [103]. The shape of the curve suggests that the crack propagation period is divided into three regions:

- **Region I** bounded by the threshold value where the crack growth rate goes asymptotically to zero as $AK$ approaches $AK_{th}$. Below $AK_{th}$ the crack remains dormant or propagates at crack growth rates which cannot be determined experimentally.

- **Region II** is where stable crack propagation takes place which can be described by linear relationship between $\log da/dN$ and $\log AK$.

- **Region III** crack growth is described by a rapid increase in the crack growth rate going towards “infinity” as the maximum value of the cyclic stress intensity factor $K_{\text{Imax}}$ reaches the fracture toughness of the material $K_{IC}$.

Ritchie [103] reported that the variation of the crack growth curve (for steels) can be characterised in terms of different primary fracture mechanisms (see Figure 4.8). In region II the failure is generally governed by a transgranular ductile mechanism where the influence of microstructures and thickness of the specimen stress is minor. Ritchie [103] reported also
the effect of the mean stress to be minor but significant influence of the mean stress can be found in the case of crack closure (discussed in Chapter 6). In the third region when $K_{\text{max}}$ approaches $K_{\text{IC}}$ the crack growth becomes sensitive to both microstructures and mean stress due to the occurrence of static fracture modes such as cleavage and inter-granular fracture. Similar strong effects of mean stress and microstructures on the crack growth rate can be found in region II where the influence of the environment can also be significant. A detailed discussion of factors affecting the crack growth is given in [103]. Special attention will be given to the threshold region in the next section.

### 4.5.1 Threshold Stress Intensity Range $\Delta K_{\text{th}}$

By integrating the crack growth curve over the critical crack length $a_c$ ($a_c$ is defined as the crack length causing failure of the structures/components) it is possible to obtain the total number of loading cycles resulting in failure for a given structure/component. The crack growth in region I can therefore play an important role since the amount of time spent in this region may be predominant for high-cycle fatigue.

The threshold stress intensity factor range $\Delta K_{\text{th}}$ was initially assumed to be a material constant but numerous studies have shown that it depends on several parameters. A recent review on fatigue thresholds for long-crack fatigue in metallic materials is given by Hadrboletz et al. [53]. They divided the factors affecting the threshold stress intensity range into extrinsic and intrinsic parameters. The extrinsic parameters cover:

- Mean stress/stress ratio effect which again was believed to be influenced by material grain size and the closure of the crack surfaces.
- The effect of overloads (not dealt with in the present formulation) where an overload peak results in a retardation of crack growth due to several mechanisms, e.g., plastic-induced closure and residual stresses.
- Temperature related to changes in material parameters such as Young's modulus $E$ and the yield strength $\sigma_y$. $ET\sigma_y$ and $K_{\text{th}}$ usually decrease with increasing temperature but in special cases an increase in $K_{\text{th}}$ was observed which was explained by closure of crack faces due to corrosion.
- The environment where formation and dissolution of surface layers, closure of crack surfaces due to corrosion and incompressible fluids in the crack are known to be important effects.

The intrinsic parameters include:

- Microstructural features such as grain size, single- and multi-phase microstructures, porosity, solid solution effects, and dislocation arrangements.
The conclusion drawn from the review was that the many intrinsic and extrinsic variables do not permit a quantification of the threshold value derived from basic principles. For a detailed discussion of the parameters listed above see Hadrböletz et al. [53]. The numbers of affecting parameters may however be reduced by the introduction of an effective fatigue threshold stress intensity defined by

\[ \Delta K_{th,\text{eff}} = K_{th,\text{max}} - K_{cl} \]  \hspace{1cm} (4.21)

where \( K_{th,\text{max}} \) is the maximum stress intensity and \( K_{cl} \) is the stress intensity at which crack faces close (investigation of crack closure is the subject of Chapter 6). \( \Delta K_{th,\text{eff}} \) may be obtained from experiments and \( \Delta K_{th,\text{eff}} \) is reported to be in the range 2.4-2.6 MPa m\(^{1/2}\) for steel and in the range 0.9 - 1.9 MPa m\(^{1/2}\) for aluminium alloys [53].

Tanaka and Soya [132] analysed the crack growth behaviour of six grades of steel (\(\sigma_y\) in the range 163 - 888 MPa) under various stress ratios. They found that all the metals could be characterised by the same \( \Delta K_{th,\text{eff}} \) which was outside the range given in [53] namely 3.45 MPa m\(^{1/2}\). An expression for \( \Delta K_{th} \) was also suggested which was found to be a function of \( R \) and \( \Delta K_{th,\text{eff}} \):

\[ \Delta K_{th} = \text{Max}[ (\Delta K_{th,\text{eff}} + K_0)(R_0 - R), \Delta K_{th,\text{eff}} ] \]  \hspace{1cm} (4.22)

where \( K_0 \) and \( R_0 \) are material constants. Their experimental results indicate that \( K_{th} \) is well described for both \( R > 0 \) and \( R < 0 \).

### 4.5.2 Mode I Crack Propagation Laws

The Paris law is a simple but very often used model for description of the crack growth rate in the linear region II. It reads:

\[ \frac{da}{dN} = C\Delta K^m \]  \hspace{1cm} (4.23)

where \( \Delta K \) is the stress intensity factor range and \( c, m \) are material constants. For most materials \( m \) is between 2 and 7 while \( c \) is more material-dependent. A shortcoming of the Paris law is that it neglects the influence of the peak stress and the threshold range.

Attempts have been made to complete the description of the crack growth curve. A popular formula which incorporates the threshold range is:

\[ \frac{da}{dN} = C[\Delta K^m - \Delta K_{th}^m] \]  \hspace{1cm} (4.24)
where the factors affecting the threshold range (discussed in 4.5.1) are taken into account when \( \Delta K_{th} \) is inserted. This equation is valid for region I and II but does not cover unstable crack propagation. The Forman equation is often employed to model the crack growth rate in the regions of stable and unstable crack propagation:

\[
\frac{da}{dN} = \frac{C \Delta K^m}{(1 - R)K_{IC} - \Delta K}
\]  

(4.25)

where the influence of the peak stress is accounted for by the stress ratio coefficient \( R \) (\( \sigma_{\text{min}}/\sigma_{\text{max}} \)). Other modifications have been proposed. A list of nine proposed models is given in [130]. But with the outlined formulas it is possible to describe the whole stress intensity factor range.

### 4.5.3 Crack Closure and Overloads

The closure of crack surfaces during cyclic loading may have significant influence on fatigue crack growth behaviour. Contact between the crack faces may be experienced in many situations as a consequence of e.g. residual compressive stresses, bending loading, crack asperities or roughness and plasticity [18]. A way of dealing with closure is to apply the concept of “effective” stress intensity range \( \Delta K_{\text{eff}} \) which was introduced in Section 4.5.1. Outside the threshold region the effective stress intensity range is determined by replacing \( K_{\text{th, max}} \) with the maximum stress intensity factor \( K_{\text{max}} \) in Eq. (4.21). The details of “crack closure” will be discussed in Chapter 6.

This formulation is restricted to constant amplitude loading and the effect of plasticity induced by overloads (and underloads) and other sequence effects are therefore outside the scope of this work. But for completeness it should be mentioned that an overload may cause retardation of the crack growth. Figure 4.9 shows a schematic representation of a crack curve versus the applied load. The retardation illustrated in the sketch can be explained by the fact that an overload may cause the tensile stress at the crack tip to exceed the yield limit of the material which creates an overload plastic region at the crack tip. When the crack tip is relieved a zone of compressive residual stresses is formed which tends to close the crack surface and thus causes retardation of the crack growth of the subsequent load cycles. The crack growth is resumed when the crack has propagated out of the compressive residual stress region. Relatively simple analytical approaches like the Wheeler model [148] can be used to estimate the retardation effect of overloads. But in some cases these approaches even fail to model the retardation of a single (mode I) overload. Thus more sophisticated approaches are needed for reliable modelling of the sequence effects of variable amplitude loading. Discussions of variable amplitude loading can be found in e.g. Dominguez [35] and Ibso [59].
Mixed Mode Crack Growth

The growth of cracks under mode I and mode II was first systematically studied by Iida and Kobayashi [60]. To obtain a model for plane mixed mode crack propagation, they carried out experiments on a sheet of aluminium alloy exposed to a uniform tensile load. A central crack was inclined at an angle to the loading direction. The results of their experiments showed that even a small $\Delta K_{II}$ contribution would significantly increase the crack growth rate. However, they also observed that the crack tended to grow in the direction of minimum $K_{II}$. An observation which has been supported by numerous investigations, see e.g. [45711995]. Exceptions can be found. Robert and Kibler [104] proved that in special cases crack propagation may occur without reducing the transverse stress intensity component to zero. The latter only takes place under special conditions and some authors therefore suggest basing fatigue life predictions solely on mode I contribution, see e.g. [1211150].

Some models however do take into account the mode II contribution. One way is by introducing an equivalent stress intensity factor $\Delta K_{eq}$ in the Paris equation:

$$\frac{da}{dN} = C(\Delta K_{eq})^m$$  (4.26)
The maximum stress criterion can be used to determine the equivalent mode I stress intensity factor:

\[ K_{eq} = K_I \cos^3 \frac{\theta_0}{2} - 3 K_{II} \cos^2 \frac{\theta_0}{2} \sin \frac{\theta_0}{2} \]  \( (4.27) \)

where \( \theta_0 \) denotes the direction in which the crack is most likely to propagate relative to the crack tip coordinate system \( \Gamma \) and \( \Delta K_{eq} \) is found as the \( K_{eq} \) range during one load cycle.

Tanaka [129] carried out experiments on cyclically loaded \((R = 0.65)\) sheets of pure aluminium with initial cracks inclined to the tensile axis. The original purpose of the study was to establish threshold bounds for cyclic mixed mode fatigue crack growth. As a by-product, the experiments formed the basis for a crack propagation law:

\[ \frac{da}{dN} = C(\Delta K_{eq})^m \]

\[ \Delta K_{eq} = (\Delta K_I^4 + 8 \Delta K_{II}^4)^{1/4} \]  \( (4.28) \)

where \( m \) was found to be 4.4 for the tested aluminium sheets. Eq. \((4.28)\) was developed on the assumption that a) the plastic deformation due to cyclic tension and transverse shear are not interactive \( \Gamma \) and b) the resulting displacement field is the sum of the displacements from the two modes. Application of the law with satisfactory results can be found in [95].

The effect of the mean stress is assumed to be minor in the region of stable crack growth [51]. Nevertheless, Sih and Barthelemy [117] stated that a crack growth expression should \( \Gamma \) in principle \( \Gamma \) contain at least two loading parameters for a proper load definition. Thus, they proposed to extend the strain energy density factor concept to cover the prediction of stable crack propagation:

\[ \frac{\Delta a}{\Delta N} = C(\Delta S_{min})^n \]  \( (4.29) \)

where \( \Delta S_{min} \) is given by

\[ \Delta S_{min} = a_{11}(\theta_0)[K_{I,\text{max}}^2 - K_{I,\text{min}}^2] + 2a_{12}(\theta_0)[K_{I,\text{max}} K_{II,\text{max}} - K_{I,\text{min}} K_{II,\text{min}}] + a_{22}(\theta_0)[K_{II,\text{max}}^2 - K_{II,\text{min}}^2] \]

where \( \theta_0 \) is the angle of crack extension \( \Gamma \) and \( a_{11}, a_{12}, \) and \( a_{22} \) are obtained by setting \( \theta_0 \) equal to \( \theta_0 \) in Eq. \((4.18)\). The equation can be rewritten to include the stress intensity factor range \( \Delta K_j \) and the mean stress intensity factors \( \bar{K}_j \) (for \( j = \text{III} \)):

\[ \Delta S_{min} = 2 a_{11}(\theta_0)\Delta K_I \bar{K}_I + a_{12}(\theta_0)(\Delta K_I \bar{K}_{II} + \Delta K_{II} \bar{K}_I) + a_{22}(\theta_0)\Delta K_{II} \bar{K}_{II} \]

\[ (4.31) \]
Application of the approach can be found in [133] where $C = 0.012 \text{ (cm/cycle)(kN/cm)}^{-1.277}$ and $n = 1.277$ are used as material parameters for aluminium 2024-O. It was found that the estimated number of load cycles corresponded well to the values observed in the experiments. Badalliance [6] also related the crack growth rate to the range in strain energy density $\Delta S_{\min}$ but he used a relatively sophisticated model containing seven empirical constants for the estimation of the crack growth.

Mageed and Pandey [78] compared experimentally obtained fatigue lives with fatigue lives estimated by both the strain energy density criterion and the equivalent stress intensity factor methods given by Eq. (4.27). They found that Eq. (4.27) gave the best fit to the experimental data with the best agreement for mode I dominating cases.

### 4.6.1 Threshold Range for Mixed Crack Propagation

Tanaka [129] was one of the first to investigate threshold limits for cyclic mixed mode loading. Specimens of pure aluminium were used for the analysis. Experimentally obtained threshold values were compared to thresholds predicted by the maximum stress criterion and the strain energy criterion neglecting the effect of the peak stress intensity factors. The two criteria for non-propagation were proposed to be:

$$
\cos \frac{\theta_0}{2} \left[ \Delta K_I \cos^2 \frac{\theta_0}{2} - \frac{3}{2} \Delta K_{II} \sin \theta_0 \right] = \Delta T_c \tag{4.32}
$$

$$
a_{11}(\theta_0) \Delta K_{I}^2 + 2a_{12}(\theta_0) \Delta K_I \Delta K_{II} + a_{22}(\theta_0) \Delta K_{II}^2 = \Delta S_c \tag{4.33}
$$

where $\theta_0$ is the angle of crack propagation predicted by the two criteria. The quantities $\Delta T_c$ and $\Delta S_c$ are the mode I threshold values where $\Delta S_c$ can be found from

$$
\Delta S_c = \frac{1-2\nu}{4\pi \mu} (\Delta K_{I,th})^2 \tag{4.34}
$$

When comparing the limits given by Eq. (4.32) and Eq. (4.33) with experimental results Tanaka [129] found that Eq. (4.32) provides a conservative boundary for mixed mode loading while a non-conservative boundary is predicted on the basis of the strain energy criterion. Tanaka made another fit to the experimental data by assuming that the condition for non-propagation is in a positive quadratic form of $\Delta K_I$ and $\Delta K_{II}$:

$$
A_{11} \Delta K_{I}^2 + 2A_{12} \Delta K_I \Delta K_{II} + A_{22} \Delta K_{II}^2 = 1.0 \tag{4.35}
$$
where the coefficients for pure aluminium were determined as [129]: $A_{11} = 0.0262 \text{ mm}^3/\text{kg}^2 \Gamma$ $2A_{12} = 0.0081 \text{ mm}^3/\text{kg}^2$ and $A_{22} = 0.0381 \text{ mm}^3/\text{kg}^2$.

The fit made by Tanaka [129] is based on a series of experiments with specimens made of pure aluminium exposed to a load with the load ratio $R$ equal to 0.65. Pook [93] used nineteen reported experimental threshold data sets for the generation of a more general failure mechanism map for mixed mode threshold behaviour. Theoretical upper and lower bounds for the mixed mode threshold values were established. The lower boundary for stage II thresholds was expressed as a parabola:

$$\frac{\Delta K_{I}}{\Delta K_{th}} = \left\{ 0.08 \left( \frac{\Delta K_{I}}{\Delta K_{th}} \right)^2 - 0.83 \frac{\Delta K_{I}}{\Delta K_{th}} + 0.75 \right\}^{1/2}$$  (4.36)

which is an appropriate limit for design purposes although it may be unduly conservative [93] in some circumstances.

### 4.6.2 Effects of Biaxial Loading

Many engineering components are exposed to biaxial (multiaxial) static or cyclic loads. Biaxial or non-singular stresses acting parallel or perpendicularly to the crack plane can be observed when a crack is propagating in a welding residual stress field. The effect of biaxial loading on crack propagation has been the subject of many investigations; see e.g. [57Γ1Γ118Γ150]. Smith and Pascoe [118] made a literature review of experimentally observed biaxial loading effect on fatigue crack growth. Unfortunately, no definite conclusion could be drawn from the review. A cyclic tensile principal stress applied perpendicularly to the crack surface was reported to increase to have no influence on and to decrease the rate of crack propagation when compared to uniaxial loading [118]. However, Hoshide and Tanaka [57] carried out experiments for different stress ratios $R$ and different levels of the biaxiality ratio $\eta$ (the ratio between the two non-singular stresses acting parallel or perpendicularly to the crack plane). They eliminated the effect of biaxial loading by showing that the crack growth rate in biaxial stress fields is a unique power function of $\Delta K_{eff}$ (except from cases where significant plasticity is present). Their observation is supported by a later work of Kitagawa et al. [71] who also investigated cracks exposed to various biaxial loading conditions. They found that the crack propagation curve including the threshold region can be characterised solely by $\Delta K_{eff}$. All observations are restricted to small-scale yielding conditions. When significant yielding is present biaxial loading may influence the crack growth.

### 4.7 Limitations of the Linear Elastic Fracture Approach

Linear elastic fracture mechanics does not take into account the crack tip plasticity. Ductile materials like steel always show some order of plasticity which should be taken into consideration. However, in most cases the linear approach (sometimes with modifications) is
Summary of Outlined Theory

The purpose of this review is to establish a basis for predicting the behaviour of long cracks subjected to mixed mode loadings. A characteristic of mixed mode fatigue cracks is that they tend to propagate in a non-self-similar manner. Therefore, under mixed mode loading it is not only the onset and the crack propagation rate which are of interest but also the direction of the crack propagation.

Three of the commonest methods for prediction of the onset of propagation and crack propagation angles have been outlined and discussed. Information needed for determination of the onset of brittle fracture is the fracture toughness of the material given either as $G_c$ or as the critical stress intensity factor $K_{IC}$. The outlined models for estimation of crack extension angles rely purely on the stress intensity factors at the crack tip which can be determined by finite element procedures.

The crack propagation period is usually divided into three stages: a) Crack initiation b) stable crack growth and c) unstable crack growth where only the second stage is explicitly covered by the present formulation. The initiation period is taken into account by assuming that initial cracks exist in the structures of interest. This assumption is justifiable for welded and flame-cut structures where cracks and flaws are normally present at the initial stage due to manufacturing procedures. After the initiation of mixed mode crack growth the crack usually turns to propagate in the mode I dominating direction. Some authors therefore propose to base the fatigue life estimation solely on mode I contributions. This has the advantage that the material properties needed for the fatigue life predictions already exist for a wide range of materials. Another approach is to use an equivalent mixed mode stress intensity factor and then apply fatigue crack growth data from pure mode I tests. This procedure will ensure conservative predictions of fatigue lives [95]. Glinka claims that the maximum effect of the $K_{II}$ contribution is in the order of 10%.
The theory outlined in this chapter forms a solid basis for construction of a program for simulation of mixed mode crack propagation. But some restriction has to be imposed on the formulation due to the complexity of the subject. No attempt has been made to cover the aspects of:

- Fatigue crack growth under variable amplitude loading.
- Out-of-phase mixed mode crack propagation.
- The effect of environment (corrosion, etc.).
- Material changes due to temperature, welding or cutting.
- The growth of short cracks (relative to the dimensions of the structure).
- Non-isotropic materials.

Two other important subjects have not yet been dealt with or they have only been briefly mentioned: The effect of crack closure which will be discussed in Chapter 6 and procedures for treating the effects of residual stresses which will be outlined in Chapter 7.
Chapter 5

Two-Dimensional Crack Propagation, Simulation

5.1 Introduction

Finite element based methods have been successfully applied for solving fracture mechanical problems for several decades. The first applications were for relatively simple purposes such as determination of stress intensity factors for different crack configurations. However, the complexity of the models has increased significantly over the years as the efficiency of the computers has increased. A state-of-the-art review of finite element procedures used in fatigue calculations covering both linear elastic and plastic models is given by Dougherty et al. [36]. The present work will focus on models for two-dimensional crack simulation.

Several procedures have been suggested for the prediction of crack growth in structures which are subjected to complex mixed mode stress conditions. Saouma and Zatz [110] were some of the first to develop a model for automatic tracing of mixed mode crack growth in steel structures. They used a linear elastic quasi-static approach where the crack was extended by a step-by-step scheme. The stress intensity state at each location of the crack tip was used to predict the direction of the next crack increment. The procedure involved remeshing of the domain containing the continuously changing crack geometry and a model for prediction of the crack growth direction. The procedure was applied to the simulation of crack growth in an attachment lug and the obtained results showed a reasonable agreement with experimentally obtained data. The weakness of the approach was the missing ability of the remeshing algorithm to produce a high quality mesh in the domain around the crack tip. Other researchers have worked on mixed mode crack simulation but basically it is the same step-by-step scheme which is applied. Sumi [120121122123124] uses a combination of a finite element and an analytical solution of the stress field at the crack tip to obtain both the mixed mode stress intensity factors and the higher order stress field parameters. The higher order terms allow for the use of curved crack increments based on a first order perturbation
solution together with a local symmetry criterion. This model has with success been applied to the estimation of crack paths and fatigue lives of complicated structures using a super-element technique [121]. Another type of multilevel substructuring technique was developed by Padovan and Guo [52T90]. Displacements determined from a global level model were by a transformation imposed on the boundaries of the moving template containing the crack tip. A high mesh density in the template should then offer an improved description of the stress field around the crack tip. The crack propagation in the global model was handled by the “element death method” where the elements in the wake of the crack are selectively killed. The template technique was tested for several examples including multicrack propagation and showed good correlation.

An objective of the present investigation is to analyse mixed mode crack propagation with special emphasis on the effects of residual stresses and biaxial loading. Different designs of cutouts in VLCCs are to be analysed for possible crack growth behaviours of cracks initiated at points of high stress intensity. This chapter describes the basic components of the two-dimensional crack simulation procedure established for that purpose. An analysis of the crack growth behaviour can reveal if crack arrest is more likely to occur in one design rather than in another. It should be mentioned that effects of residual stresses will first be discussed in Chapter 7.

5.2 Outline of the Simulation Procedure

The present simulation procedure is also based on incremental crack extension assuming that the stress state in the vicinity of the crack tip can be described by a linear elastic
fracture theory which is usually justified for high-cycle fatigue. The input to the procedure is a boundary definition of the initial structure with a crack located at a critical point. The structure can be subjected to both external and internal loads. The resulting stress and the displacement fields are analysed for each crack increment configuration by means of the finite element method. Special crack tip elements are used to describe the singular stress state in the vicinity of the crack tip and the stress intensity factors are computed by either the displacement correlation technique or by use of a $J$-integral procedure. The direction of crack propagation is predicted from the obtained SIFs by one of the theories outlined in Section 4.3 and the crack is extended in the computed direction. The fatigue life can be updated after each extension and local remeshing be carried out in the domain of the crack tip. The procedure is repeated until a stop criterion is reached (i.e. crack arrest, unstable crack propagation, or a maximum number of simulation steps is reached).

The application of the theory outlined in the previous chapter will be briefly discussed in accordance with the flow of the procedure given in Figure 5.1. Special emphasis will be put on the numerical evaluation of stress intensity factors and the remeshing of the domain around the crack tip. The capability of the procedure will be demonstrated by some examples of crack simulations.

### 5.3 Stress Intensity Factors

The stress intensity factors are the governing parameters in a linear elastic approach and a large number of finite element based procedures have been established for the purpose of evaluating the stress intensity factors. In the early studies, a high-density mesh was needed near the crack tip for the singular stress field. Later on, hybrid crack tip elements were introduced which contained the theoretical singularity of the stress field (an early work was presented by Byskov [24]). These elements lacked the capability of the constant strain and the rigid body motion modes [15] and they therefore did not pass the patch test [30] which is a necessary requirement for convergence. Barsoum [14,15] and Henshell and Shaw [55] have independently shown that the quarter-point isoparametric elements allow for a highly accurate computation of the stress field with a relatively coarse mesh around the crack tip. These singular quarter-point elements provide a good tool for both linear and non-linear fracture mechanics calculations.

#### 5.3.1 Singular Isoparametric Quarter-Point Elements

The commonly used singular elements for two-dimensional calculations are: 1) 8-noded quadratic isoparametric elements where the singularity is achieved by placing the mid-side node near the crack tip at the quarter-point or 2) 8-noded triangular elements obtained by collapsing one side of the quadratic elements and placing the mid-side node at the quarter-point (the two types of elements are illustrated in Figure 5.2). The latter method leads
to far the best results which is due to the fact that the square-root singularity is obtained throughout the triangular element whereas for the quadrilateral elements this behaviour has been shown to exist only along the side of the elements [13]. Thus, the triangular collapsed elements were chosen for the present formulation.

5.3.2 The Displacement Correlation Technique

The displacement correlation technique proposed by Shih et al. [114] is a very simple method for calculating mode I stress intensity factors by use of the displacements of the singular quarter-point elements. The expression for the determination of $K_I$ was obtained by combining Eq. (4.2) and the singular part of the displacements along the crack face (Appendix A gives an expression for the displacement along the edge of a quarter-point element). Ingrafea and Mann [61] expanded this approach to a mixed mode condition where the equations read:

$$K_I = \frac{2\mu}{\kappa + 1}\sqrt{\frac{\pi}{2h}} [4u_{2B2} - u_{2IC2} - 4u_{2IB1} + u_{2IC1}]$$

$$K_{II} = \frac{2\mu}{\kappa + 1}\sqrt{\frac{\pi}{2h}} [4u_{1B2} - u_{1IC2} - 4u_{1IB1} + u_{1IC1}]$$

(5.1)

where $\mu$ is the shear modulus of the material and $\kappa$ is given by Eq. (4.3). The quantities $u_1$ and $u_2$ are the sliding and opening displacements of the crack tip element at the points depicted in Figure 5.3 and $h$ is the length of the side of the element which is common to crack face.

5.3.3 Stress Intensity Factors by the $J$-Integral

The rate of change of total potential energy relative to the change in crack length can be calculated by the $J$-integral. It has been shown that this integral is valid from small scale
yield to a state general of yielding and that it is equal to the energy release rate under linear elastic conditions. The critical value of the $J$-integral can be considered as a fracture mechanical material property. The stress intensity factors, the energy release rate and the $J$-integral are in the two-dimensional case related by the following non-linear equations [38]:

\[
J_1 = G_1 = \frac{K_I^2 + K_{II}^2}{E'}
\]
\[
J_2 = G_2 = \frac{-2K_I K_{II}}{E'}
\]

(5.2)

where $J$ and $G$ are components in the global coordinate system. The parameter $E'$ is equal to Young's modulus $E$ for a plane stress condition and to $E/(1-\nu^2)$ for a plane strain condition where $\nu$ is Poisson's ratio.

5.3.4 Procedures for $J$ Computations

The $J$-integral can be computed by two different approaches 1) as a contour integral going from the lower to the upper crack surface for 2) by a virtual crack extension method. Both techniques are summarised in this section where the latter method will be described in the form of an equivalent domain-integral.

Contour Integral Method

The path-independent $J$-integral originally proposed by Rice and Rosengren [102] is a closed contour integral of the strain energy density and the work done by traction around the crack tip. The integral was derived on the assumptions that: a) The body consists of a homogeneous material b) the body is subjected to a two dimensional stress field c) the
crack surface is tension-free and the body forces are zero \([101]\). Generally the contour integral assumes the form:

\[
J_k = \lim_{\epsilon \to 0} \oint_{\Gamma_1} \left[ W n_k - \sigma_{ij} \frac{\partial u_i}{\partial x_k} n_j \right] d\Gamma \quad k = 1, 2
\]  

(5.3)

where \(W\) denotes the strain energy density of the material and \(n\) is a normal vector pointing out into the surrounding domain as defined in Figure 5.4. The body is divided into elements when the finite element method is applied and the contour integral is simply calculated as the sum of the contributions from each element intersected by path \(\Gamma\). The path across the element is free in principle but Bakker [9] strongly suggests that paths are restricted to running through the element integration points to avoid extra interpolation for the determination of stresses and strains within the element. It is also recommended to use a path running parallel to the local element coordinates as illustrated in Figure 5.5 and then apply normal Gaussian integration procedures.

**Numerical Procedure for Contour Integral Method**

Choosing an integration path with the local element coordinate \(\xi\) corresponding to either \(\eta_1\) or \(\eta_2\) as suggested by Bakker the contour integral maps into a summation through Gaussian points of the elements intersected by the contour:

\[
J_{e,x} = \sum_e \sum_{n=1}^{N_e} H^e I^e(\xi) (5.4)
\]

where \(e\) applies to the elements intersected by the contour \(\Gamma\). \(N_e\) is the order of integration for the element and \(H^e\) is the weight factor at the location \(\xi\). The integrand of Eq. (5.4) is in the local element coordinate system given as

\[
I^e(\xi) = (W n_k - \sigma_{ij} u_i n_j) \frac{ds}{d\xi} \quad k = 1, 2
\]  

(5.5)
where $s$ is the path length in the global coordinates. The normal vector components times the derivative of the path length with respect to the local element coordinate can be found from

$$
n_1 \frac{ds}{d\xi} = \frac{dx_2}{d\xi} = \sum_{p=1}^{P} \frac{\partial N_i^p}{\partial \xi} x_2^p
$$

$$
n_2 \frac{ds}{d\xi} = -\frac{dx_1}{d\xi} = -\sum_{p=1}^{P} \frac{\partial N_i^p}{\partial \xi} x_1^p
$$

where $P$ is the number of element nodal points and $N_i^p(\eta_1, \eta_2)$ are the nodal point shape functions at points $(\eta_1, \eta_2)$.

**Equivalent Domain Integral Method (EDI)**

The equivalent domain integral method (EDI) is an alternative way to obtain the $J$-integral. The contour integral is replaced by an integral over a finite-size domain. The EDI approach has the advantage that the effect of body forces can very easily be included.

The standard $J$-contour integral given by Eq. (5.3) is rewritten by the introduction of a weight function $q(x_1, x_2)$ [87]:

$$
J_k = \int_{\Gamma} \left[ W n_k - \sigma_{ij} \frac{\partial n_j}{\partial x_k} \right] q d\Gamma 
$$

The weight function assumes unit value at the inner contour $C_1$ and zero at the outer contour $C_2$ (the contours are depicted in Figure 5.6). The introduction of the $q$-weight function can
be interpreted as a virtual crack extension technique by shrinking the inner contour $C_1$ to the crack tip. The material at the tip is thus given a virtual displacement of one while the surrounding material at the $C_2$ contour is fixed. The weight function may assume an arbitrary value between the two contours describing the area $A_I$. A practical choice would be a linear interpolation between the values at the boundary contours.

The line integral given by Eq. (5.7) can for a traction-free crack surface be transformed into a domain integral by means of the divergence theorem. Thus the equivalent domain integral is derived. It reads:

$$J_k = - \int_{A} \left\{ W \frac{\partial q}{\partial x_k} - \sigma_{ij} \frac{\partial u_i}{\partial x} \frac{\partial q}{\partial x_j} \right\} dA - \int_{A} \left\{ \frac{\partial W}{\partial x_k} - \frac{\partial}{\partial x_j} \left[ \sigma_{ij} \frac{\partial u_i}{\partial x_k} \right] \right\} q dA \quad k = 1, 2 \quad (5.8)$$

The second term of Eq. (5.8) vanishes for structures of homogeneous materials in the absence of body forces. A major advantage is that the integral formulation from Eq. (5.8) is relatively insensitive to the mesh configuration unlike other methods e.g. Irwin's crack closure integral [28]. Problems might however occur in the evaluation of $J_2$ when the non-singular terms near the crack tip become dominant. A way of avoiding this difficulty is described in the following section.

**Improving the EDI Method**

The domain integral given by Eq. (5.8) is valid for evaluation of $J_k$ when the singular stresses dominate the stress field but for general mixed mode problems the stress distribution near the crack tip sometimes contains both singular and non-singular terms. The area integral is derived on the assumption that there is no contribution to $J_k$ from the line integral along the crack surface ($\Gamma^+_s + \Gamma^-_s$ depicted in Figure 5.4). This holds for the calculation of $J_1$ but in the calculation of the $J_2$ there are terms which become non-zero if non-singular stresses are present [98]. Therefore the line integral along the crack surface must be included if general and accurate results are to be obtained for the $J_2$-component. This is unfortunate since all the parameters needed for the numerical integration are known in the Gauss points which are not located along the crack surfaces.

One way of avoiding the line integrals is to apply the decomposition method [28][98][112]. By this approach the displacements and stresses are decomposed into symmetric (mode I) and antisymmetric (mode II) fields with respect to the crack tip. Raju and Shivakumar [98] have shown that the contribution from the line integrals along crack faces vanish in the decomposed stress fields.

Consider two points $P(x_1, x_2)$ and $P'(x_1, -x_2)$ in the vicinity of the crack tip which are symmetrically located with respect to the crack line (see Figure 5.7). Displacements and
Figure 5.7: Definition of points for symmetric and antisymmetric decomposition.

stresses at these points can according to Sha and Yang [112] be used for an analytical separation into the two following fracture modes:

\[
\{u\} = \{u^I\} + \{u^{II}\} = \frac{1}{2} \left\{ \begin{array}{c} u_1 + u'_1 \\ u_2 - u'_2 \end{array} \right\} + \frac{1}{2} \left\{ \begin{array}{c} u_1 - u'_1 \\ u_2 + u'_2 \end{array} \right\}
\]  
(5.9)

and

\[
\{\sigma\} = \{\sigma^I\} + \{\sigma^{II}\} = \frac{1}{2} \left\{ \begin{array}{c} \sigma_{11} + \sigma'_{11} \\ \sigma_{22} + \sigma'_{22} \\ \sigma_{12} - \sigma'_{12} \end{array} \right\} + \frac{1}{2} \left\{ \begin{array}{c} \sigma_{11} - \sigma'_{11} \\ \sigma_{22} - \sigma'_{22} \\ \sigma_{12} + \sigma'_{12} \end{array} \right\}
\]  
(5.10)

A similar decomposition can be made for the strain field. By decomposing the computed stress, strain and deformation fields into mode I and II components, the J-integral may be evaluated as [28]:

\[
J^I_1 = J_1 = \int_A \left\{ \sigma_{ij} \frac{\partial u_i^I}{\partial x_j} - W^I \frac{\partial q}{\partial x_1} \right\} dA + \int_A \left\{ \frac{\partial}{\partial x_j} \left[ \sigma_{ij} \frac{\partial u_i^I}{\partial x_1} - \frac{\partial W^I}{\partial x_1} \right] \right\} q dA
\]  
(5.11)

\[
J^{II}_1 = J_2 = \int_A \left\{ \sigma_{ij} \frac{\partial u_i^{II}}{\partial x_j} - W^{II} \frac{\partial q}{\partial x_1} \right\} dA + \int_A \left\{ \frac{\partial}{\partial x_j} \left[ \sigma_{ij} \frac{\partial u_i^{II}}{\partial x_1} - \frac{\partial W^{II}}{\partial x_1} \right] \right\} q dA
\]

Note that it is the \(J^I_k\) component \((k = 1\) in Eq. (5.8)) which is used for the calculation. The \(J^I_2\) components will be identically zero because of the symmetric conditions, thus there is no contribution from the line integral along the crack surface. Obviously, the decomposition method involves one more step to calculate the new stress, strain and deformation fields, but higher accuracy is obtained and the individual modes of the J-integral are directly available.
Numerical Procedure for Equivalent Domain Integral

When the structure is of homogeneous material and the body forces are absent, the finite element implementation of Eq. (5.11) becomes very similar to the implementation of the contour integral. The only real difference is the introduction of the weight function $q$. With the isoparametric finite element formulation, the distribution of $q$ within the elements can be determined by a standard interpolation scheme by use of the shape functions $N$:

$$q = \sum_{i=1}^{8} N_i Q_i$$  \hspace{1cm} (5.12)

where $Q_i$ are values of the weight function at the nodal points. The spatial derivatives of $q$ can be found by use of the normal procedures for isoparametric elements. When the spatial derivatives of the weight function are known, the equivalent domain integral can be calculated as a summation from the discretised form of Eq. (5.11):

$$J_{EDI}^\alpha = \sum_{\text{elements in } A_I} \sum_{\text{points } P} \left[ \left( \sigma_{ij} \frac{\partial u_i^2}{\partial x_j} - W^{\alpha} \delta_{ij} \right) \frac{\partial q}{\partial x_j} \det \left( \frac{\partial x_m}{\partial \eta_n} \right) \right] w_p \quad \alpha = I, II$$  \hspace{1cm} (5.13)

The terms within $[\cdot]_p$ are evaluated at the Gaussian points $P$, where $w_p$ is the Gaussian weight factor for each point. The present formulation is for a structure of homogeneous material in which no body forces are present.

5.3.5 Evaluation of Numerical Methods for Determination of SIFs

Stress intensity factors obtained by the presented numerical techniques were compared to some benchmark examples. Four cases are given in Appendix B covering pure mode I and mixed mode conditions. A good agreement was obtained for the determination of both the mode I and mode II stress intensity factors. This section focuses on a closed-form example consisting of a biaxially loaded large plate with an inclined crack at the centre. If finite-size effects are neglected, then an analytical expression for stress intensity at the crack tip can be found for the configuration depicted in Figure 5.8. It reads:

$$K_I = \left( \sigma_{22} \sin^2 \beta + \sigma_{11} \cos^2 \beta \right) \sqrt{\pi a}$$

$$K_{II} = \left( \sigma_{22} - \sigma_{11} \right) \sin \beta \cos \beta \sqrt{\pi a}$$  \hspace{1cm} (5.14)

Numerical calculations were performed for uniaxially loaded plates ($\sigma_{11} = 0$) with cracks having different inclination angles. The length $h$ and the width $w$ were taken to be 20 times
the half crack length $a$ (trying to minimise the finite-size effect). This configuration was used to test the determination of stress intensity factors by the displacement correlation technique and the $J$-integral in the formulation of the decomposition method. Figure 5.9 shows mode I and mode II stress intensity factors normalised by $K_I$ at $\beta = 0$ as a function of inclination angle. From the figure it can be seen that the two methods provide accurate results for the mode I factors and that the simple displacement correlation technique also produces quite accurate results for mode II stress intensity factors. Unfortunately, quite the same trend is not observed for the $J$-integral techniques where there is an error in the determination of the $K_{II}$ factor (in the worst case for $\beta = 30^\circ$ the EDI method shows a 7% deviation).

Crack propagation angles $\theta_0$ based on SIFs calculated for $\beta = 30^\circ$ were determined by the MTS criterion (see Eq. (4.11)) to evaluate how the inaccuracies in the numerical determination of $K_{II}$ influence the crack path simulation. The theoretical angle of crack propagation for $\beta$ equal to $30^\circ$ is $43.22^\circ$. The crack propagation angles obtained by the displacement correlation technique and the $J$-EDI method are $42.84^\circ$ and $45.08^\circ$, respectively. The maximum deviation from the theoretical angle is about 4% for the EDI method while the displacement correlation technique shows excellent agreement with a deviation of less than 1% from the theoretical angle. On the basis of these results and the examples presented in Appendix B it is concluded that both DCT and EDI techniques provide mixed mode stress intensity factors with an accuracy suitable for crack path simulation.

### 5.4 Crack Arrest Criterion

When crack arrest is discussed a distinction must be made between static and cyclic crack growth. For the static mixed mode case, crack arrest can be determined from one of the crack extension criteria outlined in Section 4.3.1.
Crack arrest for cyclic crack growth is related to the threshold value $\Delta K_{th}\Gamma$ which is defined as the limit below which the crack remains dormant or propagates at a non-detectable crack growth rate. The crack arrest criterion for cyclic loading is therefore taken to be when the stress intensity at the crack tip falls below the mixed mode fatigue threshold limit.

### 5.5 Extension of the Crack

The crack is extended if crack arrest has not been detected. Three criteria are available for the determination of the crack propagation direction: 1) Maximum tangential stress\(\Gamma\) minimum strain density\(\Gamma\) and 3) maximum energy release rate\(\Gamma\) all discussed in Section 4.3. Given the angle of propagation\(\Gamma\) the new location of the crack tip is solely determined by the size of the crack increment\(\Gamma\) which is a user input in the present formulation. The choice of an appropriate crack increment length is governed by two contradictory requirements\(\Gamma\) 1) accuracy and 2) simulation time. In situations where relatively high ratios of $K_{II}/K_{I}$ are expected or in regions with high stress intensity factor gradients\(\Gamma\) it is proposed to reduce the increment size to improve the accuracy.

### 5.6 Updating of Fatigue Life Estimation

The estimation of fatigue life can be updated for each crack extension. The crack growth equation provides a relation between the crack increment $\Delta a$ and the increment in the number of load cycles $\Delta N$. The number of load cycles equivalent to the crack increment can for cyclically loaded structures be determined by a numerical integration of the governing crack growth equation. If the contribution from the unstable crack growth is neglected\(\Gamma\) then modification of Eq. (4.24) can be used for the integration\(\Gamma\) taking into account the effect of the threshold region:

$$N_{\text{Increment}} = \int_{a_r}^{a_r+\Delta a} \frac{d\alpha}{C[\Delta K_{eq}^m - \Delta K_{eq,th}^m]}$$ (5.15)

where $C$ and $m$ are material parameters\(\Gamma\) $K_{eq}$ is the equivalent mode I stress intensity factor (discussed in Section 4.6) and $K_{eq,th}$ is the corresponding threshold value. The crack length is equal to $a_r$ before it is extended by the increment $\Delta a$. 


5.7 Remeshing of the Crack Domain by the Advancing-Front Method

5.7.1 Introduction

When a step-by-step procedure is applied to automatic tracing of non-collinear fatigue crack propagation, the continuously changing crack path requires local remeshing in the zone covering the crack in each step of a simulation. To avoid user interference, the meshing procedure must be automatic and very robust. It has to meet the normal requirements such as precise modelling of boundaries, applicability to general topologies and it has to create element shapes suitable for finite element analysis. Often a transition from small elements in the vicinity of the crack tip to larger elements at the boundaries is necessary in the case of crack path simulation. Hence, the mesh procedure also has to perform well under such conditions.

In the present formulation, the crack propagation is assumed to take place in a two-dimensional plane. Many approaches are available for plane mesh generation, covering both triangular and quadrilateral element generation. When the mesh generation algorithm is to be chosen, the following items must be considered:

1. The robustness of the method
2. The cost of implementation
3. The speed of the generation
4. The accuracy of the chosen element type relative to the degrees of freedom required for the modelling.

The zone in which the crack is likely to propagate is often relatively small compared to the rest of the structure. This is illustrated by the grey active zone relative to the white passive zone in Figure 5.10. Consequently, only a small part of the structure has to be remeshed in each step of the crack propagation. It is therefore the robustness and the cost of implementation which are of major concern. Among the various techniques for generating unstructured meshes in domains of arbitrary geometries, the advancing front method has proved to be very successful. This technique is therefore chosen for the continuous remeshing of the active zone around the crack tip. In the next section an outline of the method is given and in Section 5.7.9 the application to simulation of crack propagation is described.
5.7.2 Meshing Strategy for the Advancing-Front Method

The mesh is constructed from the boundaries when the advancing-front mesh generation procedure is applied. It can be implemented by two different approaches: (1) all the interior nodes are generated before the construction of the triangular elements \[69\Gamma73\] or (2) the interior nodes and triangular elements are constructed simultaneously \[48\Gamma64\]. The latter approach seems to be the most attractive method since it allows for some degree of control of the shape of each generated element \[33\]. This approach is employed in the present formulation and the flow of the implementation is sketched in Figure 5.11. The description of the triangulation procedure will follow this sketch.

5.7.3 Initialising the Front

The first step of the triangulation procedure is to generate the initial front face list from one or more closed loops of straight-line segments. Basically the front face list contains the discretised boundary of the domain to be meshed. Each face represents a potential side for a new element. The face list is constructed from an input file containing information about the boundary segments, the distribution of the nodes along the segments, and the number of nodes on each segment. The segments are defined by a starting and an ending point (the control points are depicted in Figure 5.12) specified counter-clockwise for the external boundary edges and clockwise for the inner loops which are also depicted in Figure 5.12.
This definition of the boundaries allows for element generation in domains containing both internal and edge cracks.

### 5.7.4 Departure Zone

The departure zone is the selected front face segment from which the new element is generated. Different criteria can be used in the determination of the departure zone but it is still not clear how to define an optimal strategy which covers all geometrical cases [48]. The smallest face segment of the active front was successfully chosen as the departure zone in the present formulation.

### 5.7.5 Make New Element

Creating a new element is the heart of the mesh generation. A sketch of this central part is given in Figure 5.13. After the departure zone has been selected, the main goal is to find the best location for the new third node.
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Potential Third Node Location

A new element is created by use of the departure zone as a basis (line AB in Figure 5.14). The first step is to determine a potential third node location (PTNL). The third node (point C in Figure 5.14) may be constructed so that the triangle ABC is equilateral which ensures elements of a constant size which are well shaped for finite element calculations. Unfortunately this method may cause trouble when a front with small elements is going to approach a front with relatively large elements since the created equilateral elements are of constant size (see Figure 5.15). A way to avoid this problem is to create non-equilateral elements allowing them to grow in size. Problems might still occur if the elements have to grow from a small size to a very large size over a short distance. The mesh density can be forced to grow rapidly in size by applying a control space to govern the size of the elements. At the initial stage the control space defines a desired size of the elements (and thus the initial element height) in all of the domain which is to be meshed (for details see [48]). In the case of rapid element growth however elements of bad shapes will unavoidably be generated with large differences between the length of the three sides of the element. Rapid element growth is almost always a consequence of poor initial boundary discretisation so proper initial boundary definitions are the best way to ensure a successful mesh generation.

An Existing Node in the Vicinity of a PTNL

The second step is to decide if an already existing node can be used for the generation of the new element. If the PTNL is too close to an existing node then it is of importance that the new element “snap” to the existing node. This is ensured by testing if an existing node close to a potential third node location is within a distance defined by a control circle (the control circle is illustrated in Figure 5.14). A node found inside the control circle becomes the new PTNL. It is the size of the control circle and the initial location of the third node which control the growth in element size since a large control circle rather often causes snapping to an existing node.
Checking if the PTNL is Qualified for Element Generation

The third step is to analyse the chosen location of the PTNL (point C in Figure 5.16). This analysis consists of three checks:

1. Is the potential third node location inside an existing element?
2. Is the potential third node location too close to an existing front face?
3. Do the two faces AC and BC cross or overlap one of the existing faces?

The three checks listed above are illustrated in Figure 5.16. The order of the checks is important since the first check is faster to carry out than the third check and the location of the node is not qualified if just one of the checks fails. The first and the third checks should guarantee that a proper element is generated at the present stage while the second check is included to ensure that the remaining zone can be dealt with without encountering difficulties at a later stage that ill-shaped elements are sought to be avoided. If one of the checks fails then the “check procedure” suggests an existing node in the vicinity of the present PTNL as the new third node location (position D or E in Figure 5.16). The new third node location is analysed and the procedure is continued until a suitable node position is found and finally the element is generated.

5.7.6 Update Front/Front Empty?

The front is updated after the generation of the new element. Updating the face list means: 1) Delete the face which was used as a basis for generating the new element, 2) add faces of the element just formed which are not common to any existing elements, and 3) in the cases where an existing node has been selected as the third node location delete the faces which are common to the newly generated element and the existing elements. After updating the front it is checked if the front is empty. In that case the method has converged and the meshing of the domain is completed.
5.7.7 Smoothing of the Generated Mesh

The smoothing process follows upon a complete triangulation. All nodes at the external boundaries are fixed but the location of the interior nodes is shifted by a smoothing procedure. Here, a relatively simple smoothing procedure is employed. The new location of the node $P'$ is determined as the average value of the centre of gravity of the $n$ elements sharing the node $P$ (see Figure 5.17):

$$P' = \frac{1}{n} \sum_{j=1}^{n} G_j$$

(5.16)

where $G_j$ is the centre of the $n$ elements. Smoothing is an iterative procedure and it is repeated until there is no significant change in the nodal position between iterations. This usually happens after a few iterations (five iterations are used in the present implementation). Other smoothing procedures are available such as barycentrality and relaxation but no matter what procedure is used, it has to be ensured that the quality of the mesh remains valid after the smoothing.

5.7.8 Improvement of the Method

The advancing-front method technique requires an extensive search of nodes and faces during the generation of new elements. Data structures such as Quadtree could be implemented to speed up the search routines. However, in the present case the remeshing is restricted to a relatively small region and attempts to improve the speed of the generation are therefore not found to be necessary.

5.7.9 Application to Cracked Structures

Poorly shaped elements may be generated at the crack tip if the meshing algorithm is applied directly to a crack propagation problem. Thus, a template is used at the crack tip to
guarantee well shaped elements around the crack tip and to ensure that the mesh geometry is the same in the vicinity of the crack tip for each step of the crack propagation simulation. The outer geometry of the template is fixed so that the elements within the template (the elements in the shaded area in Figure 5.18) are not affected by the smoothing procedure. Inside the template quadrilateral elements and collapsed quadrilateral elements are used for the meshing. The mesh density in the template area is determined by a refinement parameter which denotes the number of subdivisions. Thus it is easy to check the convergence of different parameters (i.e. the stress intensity factors) as a function of an increasing mesh density around the crack tip.

5.7.10 Examples of Mesh Generation

The proposed mesh generation algorithm was implemented in a C-program and the procedure turned out to be very stable, fast, and robust. Three examples are given here to demonstrate some of the capabilities of the mesh generator. The two first examples depicted in Figure 5.19 and Figure 5.20 show that the generation algorithm is capable of handling domains which contain both convex and non-convex holes and that the algorithm performs well in cases of transition from relatively small element sizes to large element sizes. Figure 5.20 also shows that it is possible to concentrate the mesh density in some regions (by the initial boundary definitions).

After the initial test the mesh generation algorithm was successfully implemented in the step-by-step based crack simulation program. Curved crack paths can be found when the crack propagation is influenced by geometrical discontinuities such as holes. Simulations were performed for a crack propagating in a simply supported beam with three holes. The beam was subjected to a single-force load at the centre of the beam and Figure 5.21 shows the mesh for the second step in a simulation. It is seen that the holes are nicely described and that the mesh density is concentrated in the zone around the crack tip. The figure also indicates that the right side of the structure is modelled by a static mesh which does not change during the crack propagation.
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Figure 5.19: Domain containing both convex and non-convex holes.

Figure 5.20: Merging of two widely different fronts.

Figure 5.21: Example of mesh generation.
5.8 Examples of Crack Path Simulation

Crack paths obtained by the simulation procedure are in this section compared with numerically and experimentally obtained results which cover the influence of holes and biaxial loading.

5.8.1 Influence of Holes on the Crack Growth

It has been found both experimentally and theoretically that the presence of holes may have a significant influence on a propagating crack and holes have even been suggested as crack arresters [123]. Bittencourt et al. [20] described some experiments carried out with statically loaded beams containing holes. The beams were made of the polymer PMMA which is a common choice of material for crack path investigations since it is relatively homogeneous and exhibits brittle fracture behaviour at room temperature. Experiments were made for the two different initial crack configurations called type I and II in Figure 5.22. Crack paths were predicted by the simulation procedure using the equivalent domain integral method for determination of the stress intensity factors and the maximum tangential stress criterion for calculation of the crack propagation angle. The experimental and the simulated crack trajectories are compared in Figure 5.23 for both the initial crack configurations (there is only one crack at a time in the simulations and the experiments). A good agreement is found between the simulated and the experimentally determined crack paths for the type II configuration. The same tendency is found for the type I specimens. However, a deviation is observed when the crack tip approaches the hole. The simulation fails to predict the turn of the crack into the hole. The experimentally observed turn of the crack is believed to be related to plastic deformations of the material and it is therefore not predicted by the linear elastic approach.

The type II configuration was used to test the difference in simulated crack trajectories by application of respectively the J-EDI and the DCT method for calculation of stress...
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Figure 5.23: The experimental and the simulated crack trajectories depicted in the same figure for the two configurations.

Figure 5.24: Simulated crack paths by use of the J-EDI and the DCT method for determination of stress intensity factors. No visual difference.

Figure 5.25: Simulated crack paths by use of the three different criteria for determination of crack propagation angle. No visual difference.
Examples of Crack Path Simulation

Figure 5.26: Experimental and simulated crack paths in a non-symmetrical specimen exposed to a lateral force. The initial mesh for the simulation is also depicted.

Intensity factors. Figure 5.24 shows the two predicted crack paths which were found almost to coincide by use of the maximum tangential stress criterion for determination of crack propagation angles. The same configuration was also employed in the test of the three methods outlined in Section 4.3.1 for estimation of the crack propagation angle. Figure 5.25 shows the obtained crack paths where the DCT was used for calculation of the stress intensity factors. From the two figures it is concluded that the choice of procedures for determination of SIFs and crack propagation angle does not affect the simulation significantly.

5.8.2 Non-Symmetric Specimen Exposed to Lateral Force Bending

The experiments described by Bittencourt et al. [20] were based on the brittle polymer PMMA. Experimental measurements of crack propagating in structural steel \((E = 2.1 \cdot 10^{11}\) Pa and \(\nu = 0.3\)) were made by Theilig et al. [138] using a non-symmetric specimen exposed to a bending moment and a lateral force. A sketch of the specimen is given in Figure 5.26 where the initial mesh for the present simulation is also depicted. Notches were attached along the circular region of the specimen defined by the angle \(\alpha\). Experiments were carried out for three different positions of the initial crack and five specimens were investigated for each initial position of the crack. The experimental scatter band was found to be relatively narrow which can also be seen from the figure. The structure with the thickness \(t\) equal to 15 mm was subjected to a lateral force \(F_Q\) of the magnitude 1 kN and a bending moment \(M_b\) equal to 200 Nm. The experimental scatter bands and the simulated crack trajectories are depicted in Figure 5.26 for \(\alpha\) equal to 40°. It is seen that the simulation provides a good description of the experimentally obtained data. Theilig et
al. [137] used the same experimental configuration for verification of a simulation program based on a higher order method called the MVCCI method (for details see [137]). They also obtained a fine representation of the crack trajectories.

5.8.3 Biaxial Loading

From failures observed in practice it is known that cracks are frequently initiated in regions characterised by complicated geometrical shapes which are asymmetrically loaded [137]. These conditions may result in curved crack trajectories and cracks initiated in members of secondary structural importance can thus propagate into primary members which are vital to the structural integrity. To illustrate the problem Sumi et al. [124] carried out numerical crack path simulations with biaxially loaded cruciform joints. The geometry of the specimens is depicted in Figure 5.27 with the biaxial stress range \( r \) defined as the ratio of the maximum bending stress range applied at the end of the joint. Figure 5.27 also shows the crack paths obtained by Sumi et al. [124]. The simulation procedure discussed in this chapter was applied to the same configurations and the predicted crack trajectories are presented in Figure 5.28. There is a good agreement between the crack paths obtained by the two different approaches. Both procedures predict that small biaxial stress ranges \( r \) can result in that a crack initiated in a secondary component propagates into a primary component due to the biaxial loading (which may cause failure of the whole structure).

5.9 Summary

A step-by-step crack simulation procedure has been presented. The three commonest crack extension criteria were employed. The different components of the procedure were discussed with special focus on the numerical evaluation of the stress intensity factors. In the present formulation these can be determined by both the displacement correlation technique and the \( J \)-integral in the equivalent domain integral formulation with decomposition applied. Crack simulation showed that the choice of methods for determination of crack propagation angle and stress intensity factors does not influence significantly the prediction of the crack paths.

A simple and robust procedure for automatically triangular mesh generation in plane domains of arbitrary geometries has been described. Special attention has been paid to the generation of meshes in the domain surrounding a propagating crack. This implies that the procedure is able of to handle cases where a transition is necessary between regions of small elements found in the vicinity of the crack tip and relatively large elements located at the boundaries of the domain. A template was employed at the crack tip to guarantee well shaped elements around the crack tip and to ensure that the mesh geometry is the same in the vicinity of the crack tip for each step of the crack propagation simulation. The procedure has proved to be well suited to support of discrete crack propagation.
5.9. Summary

![Figure 5.27](image1.png)

**Figure 5.27:** Crack paths simulated by Sumi [124] for a cruciform joint under biaxial in-plane bending.

![Figure 5.28](image2.png)

**Figure 5.28:** Crack paths simulated by the present simulation procedure.
The simulation procedure was validated by comparison with experimental and numerical data covering the influence of both holes and biaxial loading. It is therefore concluded that the presented crack simulation scheme is able to handle crack propagation in general two dimensional domains.
Chapter 6

Crack Closure

6.1 Introduction

Contact between the two crack surfaces during a fatigue load cycle is called crack closure. Several conditions may lead to crack closure. The two most important are: I) Plastic-induced closure caused by the plastic deformed material left in the wake of a propagating crack. The phenomenon has been extensively discussed in the literature since it was first described by Elber [39]. He found that the plastic wake can cause closure of the crack surface even during a fully tensile load cycle. A second source of plastic-induced closure is overloads. An overload may create a plastic region in front of the crack tip. A compressive residual stress zone forms during unloading which results in closure at the crack tip. Plastic-induced closure is believed to be the main source of sequence effects which can affect significantly the crack growth under variable amplitude loading (sequence effects are briefly discussed in Section 4.5.3). II) Geometrical closure arising from a combination of specimen geometry and loads on the structure such as external bending and compressive residual stresses. This type of closure is related to the mean stress at the crack tip often described by the stress ratio $R$ ($R = \sigma_{\text{min}}/\sigma_{\text{max}}$ or $R = K_{\text{min}}/K_{\text{max}}$).

Oxide- and roughness-induced closures are two minor important mechanisms which are only interesting when the stress intensity range at the crack tip approaches the threshold level. A corrosive environment may lead to oxidation of newly cleft crack surfaces. Oxide debris on the crack surfaces then cause a wedging effect imposing a rise in the stress intensity level at which the crack closes. Roughness-induced closure is related to low stress intensity levels at which the dimensions of the plastic zone at the crack are smaller than those of the characteristic microstructure such as grain size. It is therefore highly metallurgical and microstructure-related. The mechanisms of the two latter types of closure are illustrated in Figure 6.1.

A direct analysis of plastic-induced closure is not within the framework of linear elastic fracture mechanics and it is therefore beyond the scope of this formulation. The effect of
plasticity will therefore only be indirectly included by the semi-empirical formulas which relate the linear elastic stress state at the crack tip to the crack growth. Attempts have been made to model both oxide-induced closure [27] and roughness-induced closure [10184-141]. However, these models are specified by a high number of uncertainties and assumptions, e.g., the actual surface topography, including the location of oxide debris and crack surfaces are assumed to be infinitely hard. Hence, the effect of oxide- and roughness-induced closure is also only indirectly taken into account by a semi-empirical description of the crack growth.

Crack closure, whatever its origin, affects the stress intensity at the crack tip and thus also the crack growth. The damaging part of the load cycle is reduced by the closure of the surfaces. When linear elastic fracture mechanics is applied, the reduction is usually handled by the introduction of an effective stress intensity range. The aspect of an effective stress intensity range is outlined in the next section, while a finite element approach which can be used for the modelling of geometrical crack closure is discussed in Section 6.3.

### 6.2 Effective Stress Intensity Range

It is normally assumed that mode I cracks are unable to propagate while they remain closed, and the net effect of closure is a reduction of the nominal stress intensity range to a lower value $\Delta K_{\text{eff}}$ called the *effective stress intensity range*. $\Delta K_{\text{eff}}$ is the portion of the fatigue cycle for which the crack is fully open at the crack tip [89]. The effective stress intensity range can be determined as

$$\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{cl}} \quad \text{for} \quad K_{\text{min}} \leq K_{\text{cl}}$$

and

$$\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{min}} \quad \text{for} \quad K_{\text{min}} > K_{\text{cl}}$$

where $K_{\text{max}}$ and $K_{\text{min}}$ are the maximum and the minimum stress intensity levels applied during a fatigue load cycle. The closure stress intensity $K_{\text{cl}}$ is in the formulation of Packiaraj...
et al. [89] defined as the maximum stress intensity at which the crack surfaces are rigidly pressed together and remain closed during unloading. Sometimes the crack closure intensity in Eq. (6.1) is replaced by the opening stress intensity $K_{\text{open}}$, defined as the minimum stress intensity at which the crack tip is fully open and the compressive contact stresses behind the crack tip are overcome during the loading phase [89]. The different stress intensities are related to the load cycle as illustrated in Figure 6.2. The opening and closure stress intensities are normally reported to be of the same order but not necessarily equal. An example can be found in the work of Packiaraj et al. [89] who investigated threshold values for austenitic stainless steel of the type AISI 316 and as a part of this study they determined experimentally the opening and closure stress intensities. The obtained values of $K_{\text{cl}}$ and $K_{\text{open}}$ were observed to be fairly constant over the entire stress range investigated with $2.95 \pm 0.43$ and $4.14 \pm 0.55$ MPa/$\sqrt{m}$ being reported for $K_{\text{cl}}$ and $K_{\text{open}}$ respectively. Also crack length and $R$-ratio dependencies were found to be small. Chen [26] worked on crack closure in 0.13 % carbon steel. He tried to use both $K_{\text{cl}}$ and $K_{\text{open}}$ in calculations of the effective stress intensity range and both approaches were found to give a good correlating with the experimentally obtained data.

### 6.2.1 Estimation of the Crack Opening Ratio $U$

The crack growth rate has been experimentally determined in terms of the effective stress intensity factor range for a large number of materials (see e.g. [132] where $da/dN = \Delta K_{\text{eff}}$ relations are given for six common steel grades). In order to apply these data it can be useful to express $\Delta K_{\text{eff}}$ as a function of global loading parameters such as $R$ or $UT$ where the crack opening ratio $U$ is defined as

$$ U = \frac{K_{\text{max}} - K_{\text{open}}}{K_{\text{max}} - K_{\text{min}}} = \frac{\Delta K_{\text{eff}}}{\Delta K} \quad 0.0 \leq U \leq 1.0 \quad (6.2) $$

![Figure 6.2: Definition of different stress intensity levels.](image)
McClung [82] carried out a critical review of experimental data and theoretical models found in the literature for the description of a relationship between $U$ and basic fatigue parameters $k_T$applied stress $\sigma$and crack length $a$. Some of the observations and conclusions made by McClung will be outlined here. The opening behaviour was divided into the classes: I) near threshold, II) stable behaviour, and III) loss of constraint. In the near-threshold region, the opening level $K_{\text{open}}$ was generally reported to increase with a decreasing stress intensity factor level. Several mechanisms of closure may be operative in this region including plasticity, surface roughness and oxidation. Above the near-threshold region in class II, the opening stress intensity $K_{\text{open}}$ becomes independent of the stress intensity factor level, but it is still affected by the applied stress level. The dominant mechanism is residual plasticity. Class III closure mechanisms are activated when extensive yielding is present around the crack tip or in the remaining ligament. The elastic constraint is thus lost and the opening stress intensity drops to zero. To get an overview, McClung [82] suggests the construction of a “master diagram” which shows the functional relationship between $K_{\text{open}}/K_{\text{max}}$ and $K_{\text{max}}$ or $\Delta K$ for a fixed load-ratio $R$ and an applied load range. A sketch of such a diagram is given in Figure 6.3. The “master diagram” is unfortunately encumbered with some disadvantages such as: 1) It does not demonstrate the interactions of applied stress and crack length with the influence of the stress intensity factors and 2) it does not show the $R$-dependency. It should be noted that there is a loose correspondence between the classes in the closure map and the three regions of fatigue crack growth. However, a one-to-one correspondence is not always observed. The overall conclusion from the literature review is that no single relationship between crack opening and fundamental fatigue parameters holds under all conditions due to the wide range of parameters affecting the crack opening/closure.

Nevertheless, a practical estimation of $U$ was derived by Tanaka and Soya [131] on the assumption of small-scale yielding. The effect of the plastic zone at the crack tip was taken into account by shifting the actual location of the crack tip by half $(r_p)$ of the yielding zone width $\omega$ as illustrated in Figure 6.4. For this configuration, the linear elastic surface
displacement $v$ in the $y$-direction can by found by

$$v = \frac{4K^*}{E'} \sqrt{\frac{r_p - x}{2\pi}}$$  \hspace{1cm} (6.3)$$

where $K^*$ is the stress intensity for the shifted crack position. Eq. (6.3) does not include the reduction of the $v$ displacement arising from the plastic deformation of the crack wake. A residual plastic wake deformation $\delta_{\text{res}}$ was therefore proposed by Tanaka and Soya [131] and the displacement along the surface thus becomes

$$v_f = v - \delta_{\text{res}}$$ \hspace{1cm} (6.4)$$

The crack opening stress intensity can be found by inserting Eq. (6.3) in Eq. (6.4) and then demanding $v_f$ to be zero for $x = 0$:

$$K_{\text{open}} = \frac{\delta_{\text{res}}E'}{4} \sqrt{\frac{2\pi}{r_p}}$$ \hspace{1cm} (6.5)$$

The effect of roughness and oxide-induced closure can also be included in the reduction of the opening displacement and $\delta_{\text{res}}$ then consists of three terms:

$$\delta_{\text{res}} = \delta_{\text{plastic}} + \delta_{\text{oxide}} + \delta_{\text{roughness}}$$ \hspace{1cm} (6.6)$$

where $\delta_{\text{plastic}}$ is known to be proportional to the opening displacement at the maximum load for $R$ equal to $0\Gamma$ while it is less clear how $\delta_{\text{oxide}}$ and $\delta_{\text{roughness}}$ depend on the loading parameters. Tanaka and Soya suggested that $\delta_{\text{plastic}}$ is calculated as a function $f$ of $R$ multiplied by the stress intensity range $\Delta K\Gamma$ and the sum of $\delta_{\text{oxide}}$ and $\delta_{\text{roughness}}$ is to be
taken as the displacement corresponding to the plastic deformation present when \( K \) reaches a constant value \( K_0 \). These assumptions and Eq. (6.5) lead to

\[
\delta_{\text{res}} = \frac{4}{E} \left( f(R) \Delta K + K_0 \right) \sqrt{\frac{f_p}{2\pi K_{\text{open}}}}
\]  

(6.7)

where material properties are included in \( f(R) \) and \( K_0 \). From Eq. (6.7) and Eq. (6.2) it is possible to obtain the following expression for \( U \):

\[
U = \frac{1}{1 - R} - f(R) - \frac{K_0}{\Delta K} = g(R) - \frac{K_0}{\Delta K}
\]  

(6.8)

in which the function \( g(R) \) describes the \( R \)-dependency and it was assumed to take the form:

\[
g(R) = \frac{1}{(R_0 - R)}
\]  

(6.9)

where \( R_0 \) is a constant representing material parameters such as the tensile strength. The resulting expression for \( U \) can be written as

\[
U = \begin{cases} 
  1.0/(R_0 - R) - \frac{K_0}{\Delta K} & \text{for } K_{\min} \leq K_{\text{open}} \\
  1.0 & \text{for } K_{\min} > K_{\text{open}} 
\end{cases}
\]  

(6.10)

from which it is seen that \( U \) is approximated by a function of \( R \) and \( \Delta K \) and two independent material parameters \( K_0 \) and \( R_0 \). In another work Tanaka and Soya [132] presented \( K_0 \) and \( R_0 \) for six different steel grades and these values are listed in Table 6.1.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \sigma_{\text{yield}} ) [MPa]</th>
<th>( K_0 ) MPa( \sqrt{m} )</th>
<th>( R_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel-A</td>
<td>163.0</td>
<td>8.84</td>
<td>0.862</td>
</tr>
<tr>
<td>Steel-B</td>
<td>269.0</td>
<td>9.66</td>
<td>0.886</td>
</tr>
<tr>
<td>Steel-C</td>
<td>396.0</td>
<td>5.54</td>
<td>1.020</td>
</tr>
<tr>
<td>Steel-D</td>
<td>578.0</td>
<td>3.99</td>
<td>1.030</td>
</tr>
<tr>
<td>Steel-E</td>
<td>762.0</td>
<td>6.06</td>
<td>0.915</td>
</tr>
<tr>
<td>Steel-F</td>
<td>888.0</td>
<td>4.15</td>
<td>1.030</td>
</tr>
</tbody>
</table>

Table 6.1: The material parameters \( K_0 \) and \( R \) for six different steel grades (from [132]).

The steel grades A to C and F were tested for stress ratios \( R \) in the range from 0.1 to 0.8 while the steel grades D and E were tested in a larger range which also covered negative
6.2. Effective Stress Intensity Range

$R$-values going from -1.0 to 0.8. Eq. (6.10) gave a good correlation for all the materials tested [132]. By applying the material data from Table 6.1 and Eq. (6.10) Tanaka and Soya obtained a single $dA/dN-K_{eff}$ crack growth curve representing the six different steel grades. The curve based on regression takes the following form:

$$
rac{dA}{dN} = C(\Delta K_{eff}^m - \Delta K_{eff,th}^m)
$$

(6.11)

with $C$ equal to $1.54 \cdot 10^{-11} \Gamma m = 2.75$ and the effective threshold value determined as 3.45 MPa$\sqrt{m}$. Sumi et al. [125] simulated crack growth in the presence of biaxial loading and residual welding stresses. They used a simplified formulation of Eq. (6.10) where $U$ is assumed to be a function of $R$ only:

$$
U = \begin{cases} 
1.0/(1.5 - R) & \text{for } 0.0 < R \leq 0.5 \\
1.0 & \text{for } 0.5 > R 
\end{cases}
$$

(6.12)

The crack was assumed to propagate in mode I condition and the corresponding crack growth data for KA 36 steel was represented by:

$$
\frac{dA}{dN} = 1.8 \cdot 10^{-11} \left[ (U \Delta K_i)^m - \Delta K_{i,tho}^m \right]
$$

(6.13)

where $\Delta K_{i,tho}$ is the threshold stress intensity at $R = 0$ taken to be 2.45 MPa$\sqrt{m}$ and $m$ is equal to 2.932. No experimental observations were presented regarding the opening stress intensity levels. But a fairly good agreement was found in a comparison of the measured and the simulated fatigue lives.

### 6.2.2 Effective Stress Intensity Range under Mixed Mode Conditions

Several approaches have been proposed for the estimation of the effective stress intensity range in mode I condition taking into account crack closure (as the two models outlined in the previous section). Models for the estimation of crack closure in mixed mode conditions are not very well described in the literature. Despite the fact that the closure effect is even more pronounced in mixed mode conditions than in pure mode I [142]. This probably has to do with the complex nature of mixed mode crack closure which in addition to the parameters affecting mode I closure is also influenced by friction between the crack surfaces including wear.

Tong et al. [140] investigated the mean stress influence on mixed mode threshold values. Mixed mode crack configurations were established by use of four-point bend specimens made
of structural steel BS4360 50D and the obtained threshold values were compared with a theoretical threshold boundary estimated by an equivalent stress range based on the maximum tangent stress criterion (see Eq. (4.27)). A fairly good agreement between experimental data and the theoretical boundary was observed for \( R = 0.7 \) but deviations started to appear as the stress ratio decreased and the effect of crack closure became more pronounced.

Crack propagation angles were determined as a part of the study. Here an interesting observation was made by Tong et al. [140]. The crack propagation angles were found to be insensitive to the mean stress level which suggests that the closure affects equally both mode I and mode II components of the stress intensity leading to the following assumption [140]:

\[
\frac{\Delta K_{I_{\text{eff}}}}{\Delta K_I} = \frac{\Delta K_{II_{\text{eff}}}}{\Delta K_{II}}
\]

Better correlation was obtained between the theoretical boundary and the experimental data when the effective stress ranges given by Eq. (6.14) were used. An experimental verification of Eq. (6.14) by direct measurement of mixed opening stress intensities was unfortunately not carried out. Support of the model can be found by the work of Baloch and Brown [12] who examined crack closure of near-threshold fatigue crack growth under mixed mode loading. They found that the effective parts of the mode I and the mode II stress intensity ranges were proportional to a mixed mode opening ratio \( U \):

\[
U = 1 - \frac{d_{ac}}{d} \left( \frac{\Delta K_{II}}{\Delta K_{II} + F\Delta K_{I}} \right)
\]

where \( d_{ac} \) is the amount of crystallographic crack surface, \( d \) is the average grain size of the material and \( A \) and \( F \) are arbitrary constants. These parameters have to be determined by experiments for the given material which makes the model unsuitable for the present study.

It has been argued that a larger part of the mode II range (the whole range [22]) should be regarded as effective depending on the friction between the crack surfaces. In the present investigations both the range described by Eq. (6.14) and the full range will be applied to see how the mode II range affects the crack propagation in the case of crack closure.

### 6.3 Crack Closure by the Finite Element Approach

The closure of two crack surfaces can in a finite element formulation be regarded as a contact problem which is unfortunately non-linear by nature. A common way of dealing with contact problems is to apply an impenetrability condition, i.e. a surface of one body is not allowed to penetrate the surface of another body. Two finite element approaches can be used to satisfy
6.3. Crack Closure by the Finite Element Approach

the impenetrability condition: 1) Introduction of displacement constraints on the surface of the crack. This can be employed as a direct modification of the stiffness equations or as introduction of appropriate contact forces on the crack boundary. Before applying the displacement constraints the points of contact must be identified. The most straightforward method is to specify a priori the node pairs which might come into contact during the load cycle and then check the condition of these nodes [29]. 2) Use of special non-linear gap elements to couple the possible areas of contact. The gap elements change stiffness with the relative position of the nodes which they couple. If the gap is open the stiffness is set to zero while it is set to a large value if the crack is closed. Gap elements have with success been applied to linear elastic analysis of cracks [11][16][9] they do not introduce new unknowns or demand new solution strategies. The following discussion therefore concentrates on the use of gap elements.

6.3.1 Gap Element Formulation

The first step when gap elements are used is to identify the nodes which are to be coupled. This is relatively easily done in the case of a propagating crack. It is just to ensure an equal number of elements on the upper and the lower crack faces and then match the nodes of the two boundaries. In simulations of crack propagation the gap elements are introduced at the initial stage of the calculations and new elements are continuously added as the crack is extended during propagation. The contact state of each element has to be monitored during the load cycles. The condition (stiffness) of the gap elements can be determined by the normal strain $\varepsilon$ in each gap element (see Figure 6.5 for definition of the normal direction):

$$
\begin{align*}
\varepsilon > -1.0 & : \text{no contact, free state} \\
\varepsilon \leq -1.0 & : \text{contact state}
\end{align*}
$$

(6.16)

Contact is obtained for $\varepsilon = -1.0$ while $\varepsilon < -1.0$ indicates “overlapping” between the crack surfaces. The latter is not allowed and the system has to be recalculated with changed gap element stiffnesses at the points where the surfaces are found to “overlap”. Sliding may occur during the contact state if the friction between the crack surfaces is overcome. The simple Coulomb's law is often applied with relatively good results for engineering problems [11]. Sliding takes place if $\tau \geq \mu |\sigma|$, while the crack faces are assumed to stick together when $\tau < \mu |\sigma|$, where $\mu$ is the friction coefficient and $\tau$ and $\sigma$ are the shear and normal stresses in the gap element.

Properties of the Gap Elements

Many different gap element formulations can be found in the literature. Bai and Zhao [8] have developed gap elements for elasto-plastic calculations with large deformation using standard
2-D and 3-D solid element formulations with modified constitutive relations. Ballarini et al. [11] worked on a special element for the dilate frictional behaviour of rock discontinuities. The derived six-noded gap elements were successfully applied to both elastic-plastic and linear elastic problems.

This discussion focuses on simple gap elements based on modified truss elements neglecting the effect of friction (this is very similar to using springs between the crack surface as originally suggested by Newman [86]). If truss elements are to couple surfaces modelled by quadratic elements, it is important that the stiffness of the gap elements between the "midside" nodes is twice the stiffness of the corner nodes due to the shape function associated with the quadratic elements.

The stiffness state of the elements is determined by Eq. (6.16) and in non-contact regions the gap elements should not offer any resistance to the relative motion between the coupled nodes. In contact areas, the stiffness of the gap elements should be sufficiently large to prevent "overlapping" of the crack surfaces. An appropriate choice of gap element stiffnesses will result in modelling of the two states and still avoid numerical trouble (extreme element stiffness results in an ill-conditioned finite element system). Newman [86] used springs with stiffnesses assigned to $10^7$ times the elasticity modulus of the surrounding material $E_S$ for the modelling of crack closure. A sensitivity analysis carried out in another study of plastic induced crack closure by Dougherty et al. [36] revealed that the model was relatively insensitive to the spring stiffness. They applied springs with stiffnesses equal to 1000 times $E_S$. In the formulation by Bai et al. [8] the elasticity modulus $E$ of the gap elements was assumed to be in the range $10^{-6} E_S$ to $10^{-4} E_S$ in the case of the free state. These values can be used as guidelines for choosing the gap element properties for a given system.
Convergence Criterion

The contact problem is non-linear and it therefore has to be solved by an iteration procedure. Convergence is considered to be reached when \( \epsilon \geq -1.0 \) for all the gap elements in the non-contact region. Convergence in the contact area is more complicated. Here a small overlap described by \( \delta \) is accepted:

\[
-1.0 + \delta \leq \epsilon \leq -1.0
\]

where \( \delta \) is taken to be -0.02 as suggested in [8]. There is a large difference in gap element stiffness between the two states and for complicated contact problems an oscillation of gap element states might occur. A maximum number of iterations is therefore employed in the present formulation to avoid a “never-ending” iteration procedure. An elegant but more complex solution to the problem can be found in [29] where the element stiffnesses are continuously adjusted in the two states (for details see [29]).

6.3.2 Numerical Modelling of Geometrical Closure

Compressive residual stresses are known to close crack surfaces. An example of a crack located in a residual stress field is therefore used to test the capabilities of the gap element formulation. A detailed description of the implementation of residual stresses in the numerical procedure will be presented in the subsequent chapter. This section focuses on some of the aspects regarding the application of gap elements.

![Figure 6.6: Crack closure modelled with gap elements.](image)

Beghini and Bertini [16] experimentally determined residual stress distributions for compact tension (CP) specimens cut from slabs with welding-induced stresses. The dimensions of the specimens were manufactured according to the ASTM standards outlined in Figure 6.6. A residual stress distribution obtained for a specimen with the thickness 12 mm and
the parameter $W$ equal to 48 mm is used in the present analysis. A reproduction of the measured stresses acting perpendicularly to the crack direction is depicted in Figure 6.7 for $a$ and $y$ equal to zero. Compressive residual stresses can be observed in the two ends of the distribution while tensile residual stresses are found in the middle of the specimen. Two configurations were analysed: 1) The crack tip located in the zone where the residual stresses change from a compressive to a tensile state and 2) the crack tip placed in the tensile region of the residual stress field with $a$ being equal to 6 and 12 mm respectively.

CP specimens are normally loaded through the holes by two pins by means of a tensile machine which in the numerical analysis was approximated by two concentrated nodal point loads. The loads $P$ (assumed to be tensile) open the crack while the compressive residual stresses try to close the crack faces. An equilibrium for a given length of crack closure has to be obtained by iteration with gap elements employed to prevent “overlapping” of the crack faces.

Figure 6.8: Different stages of crack closure for the CP specimen with $a$ equal to 6 mm.
Figure 6.8 shows three stages of closure for an increasing load level $P$ acting on the “short crack” configuration: 1) Complete closure 2) partial closure of the crack faces with the crack being open in the region of the crack tip and 3) a completely open crack. The lines between the crack faces in the open stages indicate the gap elements.

The relation between the intensity due to the externally applied loads $P$ and the effective stress intensity is depicted in Figure 6.9 for $a$ equal to 6 mm and in Figure 6.10 for a crack length equal to 12 mm. The different slopes of the curves indicate the stages of crack closure. The crack opening stress intensity level can be estimated from the slope of the curves. It is located at the bend after which the effective stress intensity and the externally applied stress intensity become proportional. The crack opening level $K_{open}$ is rather blurred for the short-crack configuration but estimated to be equal to 12 MPam$^{1/2}$. For $a$ located in the tensile zone it is relatively easy to detect an approximate value of $K_{open}$ which is also equal to 12 MPam$^{1/2}$. Beghini and Bertini [16] both experimentally and numerically determined $K_{open}$ for a whole range of crack lengths and they found that the opening stress intensity varies little with the crack length as observed in the present analysis.

Figure 6.9: The externally applied stress intensity versus the effective stress intensity for $a$ equal to 6 mm.

Figure 6.10: The externally applied stress intensity versus the effective stress intensity for $a$ equal to 12 mm.

Some numerical problems occurred during the analysis. Convergence could not be obtained for some of the partly closed configurations. The stiffness of a few gap elements continued to shift between the two states during the iterations. This problem could to some degree be solved by changing the stiffness of the gap elements but the stiffness had to be kept within some limits. E.g., the stiffness of an open gap element should not affect the determination of the stress intensity at the crack tip. So convergence was not obtained for all configurations of partly closed cracks which explains the location of the marked data point in Figure 6.9.
6.3.3 Comments on Element Modelling of Crack Closure

The finite element method can be applied to an idealised closure problem as it is seen from the just described example of geometrical closure. Plastic-induced closure can also be modelled by use of the finite element method (see e.g. [36Γ83Γ81]) and qualitative understanding of closure mechanisms can be obtained by these models. But finite element modelling of all the factors affecting closure is rather difficult. Therefore the empirical relations outlined in Section 6.2.1 will be applied to modelling of crack closure.

6.4 Summary

Crack closure is affected by a large number of factors such as plasticity/oxide on the crack surfaces and the roughness of the crack surfaces. A qualitative understanding of different closure mechanisms can be obtained by the application of finite element models. However establishing a model which can account for all parameters which influence crack closure is rather difficult. Hence in the present formulation crack closure is modelled by empirical formulas relating the effective stress intensity range to the stress intensity at the crack tip.

Empirical models for determination of the effective stress intensity range have been outlined. However there is some confusion as regards the determination of the effective mode II stress intensity range. Two models are applied: 1) The effective range which follows the effective mode I range and 2) the whole mode II range is regarded as effective. The two models are assumed to represent the boundaries of the effective mode II range.
Chapter 7

Effect of Residual Stresses on Crack Propagation

7.1 Introduction

Residual stresses are self-equilibrating stresses existing in materials or structures under uniform temperature conditions. This type of stresses is present in most engineering components as a product of assembling or manufacturing procedures such as flame-cutting, welding and cold forming. These processes cause residual stresses by inducing plastic deformation of the metal through severe temperature gradients, mechanical forces or microstructure changes [50].

One of the major interests in residual stresses is their effect on the fatigue life of structures subjected to cyclic loading. It is well known that compressive stresses retard the fatigue growth and in some cases even cause crack arrest, while the opposite effect can be observed in regions of tensile stresses. The ability of retarding the fatigue growth by compressive stresses is used to extend the fatigue life of components by techniques such as shot-peening, which is high-velocity bombarding of a surface with shots of steel/glass or ceramics inducing compressive residual stresses near the surfaces of the components. Other surface treatments which create compressive residual stress are described in [44].

Establishing a representative initial stress distribution is an important part of the analysis of the effect of residual stresses and the next section is therefore devoted to the determination of residual stress fields. Given a residual stress field, it is possible to analyse the influence of the residual stresses on a propagating crack. The residual stress effect can be accounted for by a modification of the stress intensity factors by use of the superposition method as described in Section 7.3.1. Special attention also has to be paid in evaluations of the crack growth rates for cracks propagating in residual stress fields, which will be discussed in Section 7.5.
Investigation of mixed mode fatigue crack propagation in residual stress fields is a complicated subject with many still unsolved problems. To achieve some practical results the present formulation is therefore restricted by the following assumptions:

- The initial residual stress distribution is only affected by the propagating crack; thus no redistribution of the initial field is introduced due to shakedown or overloads.
- Only residual stresses arising from welding and flame-cutting are dealt with.
- The external loads are restricted to constant amplitude cyclic loading.
- Effects of multipass welding is not taken into account.

### 7.2 Residual Stress Distributions

The first step in analysing the effect of residual stresses is to establish a residual stress field. The magnitude and the distribution of the residual stresses can be found by different experimental measuring techniques. These techniques can be divided into destructive and non-destructive methods. Two attractive non-destructive approaches are:

1. X-ray stress analysis primarily used for the determination of surface stresses since the applicable X-ray wavelengths have only very short penetration depths (typically 5-10 μm in most materials [74]). The method has the advantage that it can be applied to very small surface elements and it can therefore be used for determination of residual stresses in notches and areas close to the crack tip [76].
2. Neutron diffraction method which can be used for determination of internal stresses. Neutrons are scattered by the atomic nuclei which results in a far deeper penetration than X-rays scattered by electron shells. For many structural materials it is thus possible to investigate internal residual stresses in regions up to several centimetres inside the components (3 cm in steel and 30 cm in aluminium alloys [97]).

The more traditional destructive mechanical approaches like the releasing slice and the hole drilling techniques have the obvious disadvantage that they destroy the structures to be analysed. Nevertheless these methods are widely used due to their accuracy and relatively low cost.

The finite element method offers another suitable tool for the evaluation of residual stress fields and it can be applied together with experimental measurement. Beghini and Bertini [17] developed a technique for modelling of plane residual stress fields. Relatively few strain gauge measurements were combined with finite element calculations. The method was tested on several specimens with different residual stress configurations and good descriptions of the stress field were obtained [17].

Ueda and Fukuda [143] worked on a model for three-dimensional determination of the residual stress fields. The distribution of inherent strain (by definition the strain which causes residual stresses) does not change unless new plastic strains are introduced. Based on this observation Ueda and Fukuda suggested establishing two-dimensional strain fields...
from pieces cut out of the welded structure. By use of inherent strain as a parameter they transformed two-dimensional strain measurements into three-dimensional strain fields. The final residual stress distribution was obtained by using the inherent strain field as initial strains in a linear elastic finite element calculation.

Finite element analysis can also be applied to simulation of a complete manufacturing process (such as a welding procedure) using elastic-plastic calculation and information about the temperature distribution in the material during the process [108, 109, 144, 145, 146]. But all the methods listed above are either based on measurements or "time-consuming" elastic-plastic finite element calculations. For structures composed of thin butt- or fillet-welded plates and stiffeners, simplified approaches have proved to predict residual stresses with reasonable accuracy. One of these approaches is a method proposed by Yuan and Ueda [152] who have shown that welding-induced residual stresses in structural members like I- and T-profiles can be estimated by a combination of simple formulas for inherent strain and elastic finite element calculations. The components of this approach are outlined in Section 7.2.2 while an estimation of residual stresses arising from flame-cutting is discussed in Section 7.2.3.

7.2.1 Welding-Induced Residual Stress Fields

Welding generates a highly localised transient heat input which causes the weld area to be heated up sharply relative to the surrounding material. A main source of residual stresses is produced by the following shrinkage of the weld seam, which exhibits resistance to deformation at the cooling stage from the adjacent material. The second important source is related to volume expansion arising from phase changes in the weld. These two sources induce residual stresses in the direction of the weld and these stresses are here called longitudinal stresses while the stresses generated in the direction perpendicular to the weld are called transverse stresses. The present formulation is restricted to thin plates where the temperature distribution is assumed to be uniform through the thickness and where the stress variation through the thickness of the plates is neglected.

One of the simplest cases of welding-induced residual stresses is the joining of two mild steel plates by a central weld. An idealisation of this case is here used to illustrate how residual stresses can be formed by shrinkage. Figure 7.1 shows a sketch of the residual "shrinkage stresses" arising from the joining of the plates. Typical for this kind of weld is the longitudinal tensile zone located in the narrow area close to the weld. The maximum value of the tensile stress zone is normally at or slightly above the yield limit. Lower compressive stresses are found in the adjacent region, dropping off rapidly as the distance to the weld increases. Transverse stresses are produced by the transverse contraction of the weld seam and indirectly by the longitudinal contraction of the seam. Along the weld, tensile stresses of relatively low magnitude (normally \( \sigma_{22} < 0.2\sigma_y \)) are produced in the middle part and compressive stresses in the ends. This transverse stress distribution can be changed
significantly by displacement constraints at the edges of a plate while the longitudinal component is less sensitive to boundary conditions [80]. The symmetry of the distribution in the longitudinal direction applies only if the welding speed is very high compared to the rate of heat conduction of the material.

The second source of residual stress is related to microstructural changes in the material. A phase transformation (for instance a $\gamma \rightarrow \alpha$-transformation) may take place during the cooling period and cause an expansion of the material. If the yield stress is sufficiently high during the time of phase change, compressive residual stresses can be generated in the weld area. The final distribution is the superposition of the two types of stress fields. Longitudinal

stress distributions are sketched in Figure 7.2 for three different types of weldings. Case a) is for normal mild and low-alloy steel where the "shrinkage" stress is the dominant component which results in a tensile zone in the middle with a magnitude close to the yield limit. Compressive stresses due to the transformation process can be found in aluminium
7.2. Residual Stress Distributions

Figure 7.3: The two analytical estimates of the residual stress distribution and a reproduction of the experimental results by Kanazawa et al. (from [135]).

alloys and high-alloy steels. The peak value in the tensile region is thus reduced as sketched in case b). The stress distribution given in case c) is for a high-alloy steel with ferritic weld material where significant phase change has the result that the stresses in the weld centre go into the compressive range [97]. This formulation concentrates on welding residual stresses appearing in mild and low-alloy steels (i.e., case a).

Longitudinal Residual Stress Distribution for a Butt Weld

When the magnitude of the residual stresses arising from a butt weld is considered, it is seen that the longitudinal stress is the main component. Investigations of the transverse distribution of longitudinal stresses have been carried out by several authors. Terada [134] was one of the first to establish a theoretical estimation of the longitudinal residual stresses arising from the joining procedure. He stated the following requirements for the stress field $f$: (a) The stress must result in zero axial force $\Gamma$; (b) $f(\xi)$ has to be an even function because of symmetry $\Gamma$; where $\xi$ is the coordinate perpendicular to the welding line normalised by the half length of the tensile region $\Gamma$; (c) maximum stress $\sigma_0$ is to appear at the weld $\Gamma$; (d) the effect of the weld has to vanish far from the weld ($f(\xi) = 0$ for $\xi = \pm \infty$)$\Gamma$; and (e) $f(\xi)$ is to decrease monotonically from the centre of the weld until it reaches some negative minimum value and then $f(\xi)$ is to increase monotonically until zero is reached again. To meet these requirements $\Gamma$ Terada chose the following function:

$$f_1(\xi) = \sigma_0 e^{-0.5\xi^2} \left(1 - \xi^2\right)$$

(7.1)

Figure 7.3 shows a reproduction of experiments performed by Kanazawa et al. (from [135]) compared to the residual stress distribution described by Eq. (7.1). A good agreement is found in the tensile area but too large stresses are estimated in the compressive region.
Tada and Paris [127] suggested another function for the estimation of the residual stress field due to a butt weld also based on the requirements listed by Terada:

\[
 f_2(\xi) = \frac{\sigma_0 (1 - \xi^2)}{(1 + \xi^4)}
 \]

(7.2)

This solution gives a good description of the tensile region (see Figure 7.3) whereas Eq. (7.2) predicts too low stresses in the compressive region. Despite the problems in the compressive region, both \( f_1 \) and \( f_2 \) give reasonable estimations of the residual stresses measured by Kanazawa and the two analytical distributions will be used in test examples later on.

### 7.2.2 Simplified Prediction of Residual Stresses by Use of Inherent Strain

**Inherent Strain Distributions**

Carrying out experiments or complete thermal elasto-plastic calculations is very expensive and time-consuming for complex structures. Approximation procedures therefore often have to be applied to get some practical results. A simplified procedure for a qualitative estimation of longitudinal residual stresses in thin wall structures is outlined here. The method was formulated by Ueda and Yuan [146] and is based on inherent strains. These strains can be used as an input to a linear elastic determination of the residual stress distribution in a structure. Ueda and Yuan carried out some numerical experiments in order to study inherent strain distributions in butt welds. Figure 7.4 shows a schematic representation of a computed

![Diagram showing inherent strains in a butt welded plate](image)

**Figure 7.4:** Schematic representation of inherent strains in a single butt welded plate (from [146]).
inherent strain distribution along a butt weld. It is seen that the shape of the longitudinal strain distribution \( \epsilon_x^i \) is prismatic in the middle part of the weld where the effects of the free ends do not interfere. Ueda and Yuan found that the shape of the inherent strain for a cross-section located in the middle part of the weld may be approximated by a trapezoid. Their work showed that the inherent strain pattern varies little with the welding conditions and the size of the plate. The observations made formed the basis for the development of simple formulas for the description of the longitudinal inherent strain \( \Gamma \) taking into account welding conditions, geometrical dimensions and material properties. The procedure was validated by thermal elasto-plastic calculations [152].

The formulation focuses on the distribution of longitudinal inherent strain. From the sketch given in Figure 7.4 it is noted that the transverse inherent strain \( \epsilon_y^i \) only exists in the regions affected by the free ends. The same holds for shear strain (not in the sketch) \( \Gamma \) which also only appears in the vicinity of the ends. The inherent strain distribution illustrated in Figure 7.4 is for a given set of boundary conditions \( \Gamma \) and different boundary conditions could change the strain distribution. However, the longitudinal \( \epsilon_x^i \) is the dominant strain component in thin-plated structures and it is believed to cover the major effect of residual stresses. Therefore, disregard of the effect of \( \epsilon_y^i \) and \( \epsilon_{xy}^i \) is accepted in the present formulation. The biggest error from this assumption will usually occur in the ends of the welds.

By further numerical studies, Yuan and Ueda extended their method for determination of inherent strains and residual stresses in butt welds to more complicated details such as T- and I-profiles [152]. It was found that the inherent strains introduced by the weld in the flange side and in the web side of the fillet welds are almost the same provided the widths of the plates are not very different. The simple formulas for prediction of inherent strains in butt welds were modified for estimation of inherent strains in the T- and I-profiles \( \Gamma \) taking into account new temperature distributions and bending deformations. Predicted residual stress distributions were compared with experimentally obtained distributions and a good agreement was found.

Estimation of Longitudinal Residual Stresses in a Butt Weld by Use of Inherent Strain

The longitudinal inherent strain distribution in the transverse section of a butt weld can be approximated by a trapezoid as illustrated in Figure 7.5a. On the basis of the numerical studies, Ueda and Yuan [146] established some formulas for the description of the characteristic values of the approximated distribution \( \Gamma \). The peak value \( \epsilon_x^i \), the width \( b \) and the width of the heat-affected zone (HAZ) \( y_H \). The following formulas were derived for a butt joint in a plate with a width of 2 \( b \) assuming that the heat source is applied instantaneously to the entire weld line \( (y = 0) \) so that the material deforms uniformly perpendicularly to
the weld. The formulas have the following form [152]:

\[
y_H = 0.242 \frac{Q}{c \rho h (T_m - T_0)}
\]

\[
b = \eta b_0
\]

\[
\dot{\varepsilon}_x = \zeta \dot{\varepsilon}_{x_0}
\]

and

\[
b_0 = \frac{\alpha E Q}{\sqrt{2 \pi} c \rho h \sigma_Y} = \frac{0.242 \alpha E Q}{c \rho h \sigma_Y}
\]

\[
\dot{\varepsilon}_{x_0} = \sigma_Y / E
\]

\[
\eta = 1 - 0.27 \alpha E T_{av} / \sigma_Y
\]

\[
\zeta = \eta - 2 = -1 - 0.27 \alpha E T_{av} / \sigma_Y
\]

\[
T_{av} = Q / \rho A
\]

where

- \(Q\) = line heat input (J/m)
- \(T_m\) = mechanical melting point over which yield stresses disappear (°C)
- \(T_0\) = room temperature (°C)
- \(c\) = specific heat (J/kg°C)
- \(\rho\) = density (kg/m³)
- \(h\) = thickness of the plate (m)
- \(\alpha\) = linear thermal expansion coefficient (1/°C)
- \(E\) = Young’s modulus (Pa)
- \(\sigma_{Y_W}\) = yield stress of weld metal and HAZ (Pa)
- \(\sigma_{Y_B}\) = yield stress of base material (Pa)
- \(A\) = area of the transverse cross-section (m²)

The predicted strain field can be used as input to the calculation of the longitudinal residual stress distribution in the welded structure. A simple example of a resulting stress distribution is shown in Figure 7.5b where it is assumed that the residual stress at the weld is equal to the yield stress of the weld metal, \(\sigma_{Y_W}\). The magnitude of \(\dot{\varepsilon}_x\) is found from the force equilibrium condition (\(\int_0^B \sigma_x h dy = 0\)).
7.2. Residual Stress Distributions

In the case where the average temperature rise is larger than 30°C, significant bending deformations may occur as a consequence of the non-uniform heat input in the web of the T- or I-profile and a second modification has to be made. It starts by dividing the longitudinal strain in the flange into two terms: a) Uniform deformation and b) bending deformation. Inherent strains may be influenced by both types of deformations where the effect of the former can be evaluated by the formulas derived for butt welds. Yuan and Ueda [152] found that the moment introduced by the non-uniform temperature distribution in the web is proportional to the average temperature in the whole section:

\[ M \approx \alpha ET_{av} A \Delta T \quad (7.6) \]
where \( \bar{z} \) is the distance between the weld centre and the neutral axis of the transverse section. The corresponding longitudinal strain becomes \( \epsilon' = \frac{M}{Ez} \approx \alpha T_{av} \frac{A^2}{I} \) with \( I \) being the moment of inertia for the cross section. Therefore the bending part can be accounted for by increasing the average temperature in the determination of \( \eta \):

\[
T_{av}^\prime = T_{av} + \Delta T_{av} \approx (1 + \beta)T_{av}
\]

\[
\beta = \bar{z}^2 A / I
\]

so that

\[
\eta = 1 - 0.27 \alpha ET_{AV}(1 + \beta) / \sigma_{YB}
\]  

(7.7)

An estimation of the longitudinal inherent strain in a T-profile can now be found by applying Eq. (7.3) and Eq. (7.4) to the determination of the magnitude of the strain while Eq. (7.7) and Eq. (7.8) are used to find the distribution.

The inherent strain distribution can also be found in an I-profile by means of the procedure described above. If the welding sequence is to be included in the determination of the residual stresses the following procedure is suggested by Yuan and Ueda [152]: a) Inherent strains are imposed on a stress-free T-profile so that \( \sigma' \) is obtained b) the same inherent strain distribution is applied to a stress-free I-profile which gives \( \sigma'' \) and c) the two stress fields are superposed which results in the final longitudinal residual stress distribution. Numerical experiments were carried out for a typical ship section (by use of the method described above) they showed that the stress distributions in the upper and lower parts of the profile are almost the same. This indicates that the two inherent strain fields could be applied simultaneously neglecting the effect of welding sequence and thus reducing the calculation time without any significant errors in the determination of the residual stress field. It should however be noted that the welding sequence in the case of some problems may have a significant influence especially on the welding-induced deflection.

Comments on the Inherent Strain Method

The methods for estimation of longitudinal inherent strain outlined in the previous sections are developed for thin plates. The definition of “thin plates” in this context is plates in which the temperature distribution and other quantities of interest can be regarded as uniform through the thickness during welding. The methods have been validated for plate thicknesses up to 12 mm but no upper limit has been given. Wu and Carlson [149] carried out a study of stress intensity factors for a half elliptical surface crack located in thin and thick plates containing residual stresses. They found that the thickness effect can be disregarded for plates with a thickness below approximately 25 mm.

Welding speeds are usually in the range of 0.001-0.03 m/s which is relatively high compared to the rate of heat conduction [149]. This justifies the assumption about the heat source being applied instantaneously to the entire weld line.
### 7.2.3 Residual Stresses from Flame-Cutting

Gell investigated the redistribution of residual stresses in flame-cut and welded thin plates [47]. He found that the effect of flame-cutting with an oxygen-acetylene flame is similar to that of welding. A tensile zone is created in the vicinity of the edge similar to the tensile zone along a welding line. Thus, if the heat input is known, the size of the yield zone may be estimated in the same way as for the welding procedure.

However, an accurate prediction of the heat input is rather difficult, especially for thin plates. Alternatively, the residual stresses arising from flame-cutting can be based on a “tension” block method where a block of shrinkage force is loaded into the model (by direct input of strains or by thermal loading). Gell [47] outlined and investigated several proposed methods found in the literature for the determination of the width of the tensile block $c$ in the case of flame-cutting. The formulas were based on experimental work relating the shrinkage force to the plate thickness $t$. One of the formulas has the following form:

$$c = 1100 \frac{\sqrt{t}}{\sigma_{\text{Yield}}}$$  \hspace{1cm} (7.9)

where $c$ and $t$ are in mm and the yield stress $\sigma_{\text{Yield}}$ of the plate is in MPa. It should be mentioned that these procedures are only valid for low-carbon steels because for high-carbon steels, compressive stresses can be created in a zone close to the surface (the width of this zone is approximately 0.5 mm to 1.0 mm) due to martensitic hardening [97].

### 7.2.4 Redistribution of Residual Stresses

A redistribution of the initial residual stress field is often found in cyclically loaded structures such as ships and offshore platforms. The redistribution can occur due to an extreme loading of the structure which causes additional plastic stresses to be induced. For the redistribution can be a consequence of “shakedown”. Shakedown is when the residual stresses gradually disappear in cyclically loaded structures [23]. A large number of factors may affect the residual stress distribution including: (a) The mechanical properties of the material, (b) the direction and the amplitude of the cyclic loading, (c) the number of load cycles, (d) the level and the gradient of the residual stresses, and (e) the temperature. Thus, a theoretical analysis of the redistribution of initial residual stresses is a complicated matter and the redistribution is therefore encumbered with a great number of uncertainties. Nevertheless, Eckerd and Ulfvarson [37] carried out one-dimensional fatigue investigations of cutouts in ship structures in which they included redistribution of residual stresses during the lifetime of the ship. An interesting observation was that, after one voyage in ballast condition followed by one voyage in fully loaded condition, the residual stress distribution became stable according to their calculations. Still a lot of uncertainties are to be solved. Therefore, due to the complexity of multiaxial loading, the present formulation is restricted to a static elastic residual stress field only affected by the propagating crack.
Chapter 7. Effect of Residual Stresses on Crack Propagation

7.3 Stress Intensity Factors in Residual Stress Fields

The initial residual stresses are in general redistributed when a crack is introduced in a body containing residual stresses. An example is a surface without external loads where a physical requirement is that the stresses normal to the surface are zero. To satisfy this condition for a crack which propagates into a residual stress field, the initial stress field normally has to be redistributed. Thus, a model for crack path simulation in residual stress fields has to take into account the redistribution of stresses caused by a propagating crack.

7.3.1 Superposition Method

The superposition technique is a common and relatively simple way of dealing with linear elastic residual stresses and their redistribution. This technique states that the elastic stress field due to two or more load systems can be regarded as the sum of the contribution from each stress field. Thus, the residual stress field is simply added to the externally induced stress field. The redistribution of the residual stresses due to a propagating crack is accounted for by decomposing the total stress field into appropriate part solutions which satisfy the boundary conditions. It should be noted that the superposition principle is valid after the redistribution of stresses due to the introduction of a crack [92f139].

Figure 7.7 shows the superposition technique applied to a cracked body which is subjected to the external load \( T \) on the surface \( S_T \), a residual stress field given by \( \sigma_R \) and \( \tau_R \), and a displacement boundary condition \( V \) on the surface \( S_V \). The system is decomposed into three load cases: (1) An un-cracked body containing the residual stress field. (2) A cracked body with the crack faces being subjected to the reverse sign stresses. The reverse sign stresses are obtained at the line of the potential crack in the un-cracked body without application of external loads. (3) A cracked body subjected to external loads only. The stress intensity factors (SIFs) for a crack with closed crack faces are equal to zero. The stress intensity factor due to the residual stress field thus becomes equal to the contribution from the loaded
crack faces (loaded with $-\sigma_R$ and $-\tau_R$). SIFs due to external loads are computed by normal procedures and the resulting stress intensity factor simply becomes: $K_R + K_{Ex}$.

### 7.3.2 Influence Function/Weight Function Method

The theoretical stress intensity factor for a body subjected to residual stresses on its crack faces can be computed by use of the influence function method (also called the weight function method). The stress intensity factor due to an arbitrary stress distribution $\sigma_R$ on the crack surfaces is calculated from an integral:

$$K_R = \int_{-a}^{a} \sigma_R m(x) dx$$  \hspace{1cm} (7.10)

where $a_-$ and $a_+$ define the two crack tips and the weight function $m(x)$ represents the stress intensity factor for a pair of unit splitting forces $P$ acting perpendicularly to the crack surface $\Gamma$ as illustrated in Figure 7.8. The influence function method is an exact method provided that the correct weight functions are used. These functions can be obtained analytically by experiments or by use of the finite element method. Glinka [50] has collected analytical weight functions which are useful for common through thickness crack configurations. The influence function method makes it possible to calculate stress intensity factors for a variety of crack-residual stress distribution configurations but the complexity of the weight functions [50] often requires a numerical evaluation of Eq. (7.10). It should be mentioned that when Eq. (7.10) is used for compressive stress fields negative $K_I$ values may be obtained which are not meaningful unless some additional external tensile loads are applied (negative $K_I$ values indicate non-physical overlapping of the crack surfaces).

### 7.3.3 Stress Intensity Factors from Residual Stress Fields by the FEM

The stress intensity factors due to residual stresses can also be determined by use of the finite element method. Different approaches are available: (1) A residual stress field which
produces self-equilibrating forces and moments can be introduced as initial stresses/strains in the elements influenced by residual stresses. For (2) the superposition method can be applied and the crack surfaces are subjected to the loads arising from the reverse sign stresses. The superposition method with the crack face loads was found to be easiest to implement in the existing part of the crack simulation procedure and was therefore chosen.

**Implementation of Crack Face Loads**

The computation of SIFs by use of the *Displacement Correlation Technique* (DCT) is not affected by the introduction of a residual stress field as crack face loads. But the evaluations of the $J$-domain integral defined by Eq. (5.11) are. Raju and Shivakumar [98] have shown that the transformation of the $J$-contour integral into a domain integral (see Section 5.3.4) also results in line integrals along crack surfaces which are zero in the case of traction-free faces. However, the line integrals may contribute significantly to the $J$-integral when crack face loads are applied. Thus, the calculation of the $J$-integral must include both the domain and the additional line integrals, and the latter have to be evaluated along the crack faces from the outer boundary of the domain to the crack tip. When the decomposition method is used, the line integrals have the following form [98]:

\[
J_{1,\text{line}}^{L} = 2 \int_{C} \sigma_{22} \frac{\partial u_{2}}{\partial x_{1}} q dx_{1} + 2 \int_{C} \sigma_{22} \frac{\partial u_{2}}{\partial x_{1}} d x_{1}
\]

\[
J_{1,\text{line}}^{H} = 2 \int_{F} \sigma_{12} \frac{\partial u_{2}}{\partial x_{1}} q dx_{1} + 2 \int_{C} \sigma_{12} \frac{\partial u_{2}}{\partial x_{1}} d x_{1}
\]

where $\sigma_{22}$ and $\sigma_{12}$ are stresses acting on the crack faces. The boundaries of the integrals are depicted in Figure 7.9.

**Figure 7.9:** Boundaries for the domain integral.
7.4 Input of Residual Strain and Stress Distributions

In the present formulation the residual stress fields can be given as a direct input or they can be calculated by the inherent strain method. The input is related to an edge in the model which makes it easy to specify stress and strain distributions caused by flame-cutting or welding of two edges. The distributions can assume arbitrary shapes (as functions or discrete distributions). An example of a possible input for inherent strain calculations is given in Figure 7.10.

When the inherent strain approach is applied, the crack surfaces are "stitched" together and the residual stresses arising from the inherent strains are determined in an un-cracked body which is the basis for the superposition method.

7.5 Crack Growth Rates

In the presence of residual stresses, the effective part of the stress range \( K_{\text{eff}} \) can differ significantly from the externally exposed stress intensity range \( K_{\text{Ex}} \). In general the difference is not only a function of the residual stress but also of the applied load, since the crack propagation could be affected by crack closure.

The "effective stress ratio" method originally proposed by Glinka [49] is a commonly used method [16\( \Gamma \), 19\( \Gamma \), 67\( \Gamma \), 68\( \Gamma \), 125] to account for the residual stress effect in the mode I crack growth. The effective stress ratio is simply defined as [125]:

\[
R' = \begin{cases} 
\frac{K_{\text{Hmin}} + K_{\text{IR}}}{K_{\text{Hmax}} + K_{\text{IR}}} & \text{for } K_{\text{Hmin}} + K_{\text{IR}} \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]  

(7.13)
The corresponding effective mode I stress range can be determined by

\[
\Delta K_I = \begin{cases} 
    K_{I_{\text{imax}}} - K_{I_{\text{imin}}} & \text{for } K_{I_{\text{imin}}} + K_{I_{\text{IR}}} \geq 0 \\
    K_{I_{\text{imax}}} + K_{I_{\text{IR}}} & \text{for } K_{I_{\text{imin}}} + K_{I_{\text{IR}}} \geq 0 \text{ and } K_{I_{\text{imin}}} + K_{I_{\text{IR}}} \leq 0 \\
    0 & \text{otherwise}
\end{cases} \tag{7.14}
\]

An effective mode II stress ratio could be defined in a way similar to Eq. (7.13) but to the authors knowledge it is not yet clear how to define an effective mixed mode stress ratio. The mode I stress intensity factor is known to dominate the fatigue crack growth rate. Therefore, it was decided to apply the effective stress ratio given by Eq. (7.13) to mixed mode crack propagation. In mode III stress intensity factors can without problems assume negative values and normal procedures can be applied in the determination of the mode II stress intensity range. An effective range could be determined by Eq. (6.14) as described in the previous chapter.

### 7.5.1 Crack Growth in Transition from Compressive to Tensile Residual Stress Fields

Kang et al. investigated mode I crack growth in both tensile [67] and compressive [68] residual stress fields. They found that the effective stress ratio (the \( R \)-method) can successfully be applied to predict fatigue crack growth in tensile residual stress fields even under compressive cyclic loading. The study of cracks growing in compressive residual stress fields showed that the \( R \)-method can be used for estimation of crack growth in regions of almost uniform compressive residual stresses. On the other hand in regions with transition from compressive to tensile residual stresses an error may occur in the application of the \( R \)-method. This is related to partial closure of the crack faces [68].

However, the detailed behaviour of crack growth/closure of crack propagating through transition regions is not yet clear. Therefore, the effects of the transition are neglected in the present study.

### 7.6 Test Examples

The theory outlined in the previous sections was implemented in the crack simulation procedure. Some examples of calculated stress intensity factors and different crack propagation scenarios will be given in the following. The final example is of a three-dimensional plate assembly where the effects of welding-induced residual stresses are included. Both crack paths and estimated fatigue lives are compared with experimentally obtained results.
7.6.1 Crack Located Symmetrically with Respect to the Weld Line

The first example is stress intensity factors for a crack located symmetrically about a butt-weld line ($L = 0$ in Figure 7.11) with the longitudinal stresses due to the weld described by the functions given in Section 7.2.1. Stress intensity factors were numerically determined by means of both the displacement correlation technique and the $J$-integral method. The obtained results are compared with the SIFs calculated by the influence function method. In the case of a small crack located in a wide plate the weight function $m(x)$ derived for a crack in an infinite plate can be used [50]:

$$m(x)_{\pm a} = \frac{1}{\sqrt{\pi a}} \sqrt{\frac{a + \xi}{a - \xi}}$$  \hspace{1cm} (7.15)

where $\pm a$ refers to the two crack tips as illustrated in Figure 7.11. Both $a$ and the parameter $L$ denoting the eccentricity have been normalised by half the width of the tensile zone $L_t$. By insertion of the weight function given by Eq. (7.15) and a residual stress field described by either $f_1$ or $f_2$ into Eq. (7.10) the following expression for the SIFs is obtained:

$$K_{\pm a} = \frac{1}{\sqrt{\pi a}} \int_{-a}^{a} f_i(\xi) \sqrt{\frac{a + \xi}{a - \xi}} d\xi \hspace{1cm} i = 1, 2$$  \hspace{1cm} (7.16)

Figure 7.12 and Figure 7.13 show the stress intensity factors normalised by $\sigma_0 \sqrt{\pi a}$ as a function of the crack length computed for a crack which is located in the two residual stress fields $f_1$ and $f_2$. The crack length is normalised by the length of the tensile zone of the residual stress distribution and it ranges from 0.5 to 1.0. This implies that the crack is always located inside the tensile zone. The two figures show that the results obtained by the DCT are in very good agreement with the SIFs computed by the influence function method. Moreover the $J$-domain integral solution provides a good estimation of the SIFs in most of the displayed crack length intervals but a small divergence (5 %) is observed for cracks of small lengths.

Figure 7.11: Crack in a longitudinal residual stress field.
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Figure 7.12: SIF as a function of the crack length due to the residual stress field described by Terada [134].

Figure 7.13: SIF as a function of the crack length due to the residual stress field described by Tada and Paris [127].

Table 7.1: SIF due to residual stress field described by Eq. (7.1) and Eq. (7.2), $a = 1.0$, $L = 1.0$.

<table>
<thead>
<tr>
<th></th>
<th>$f_1$ by Eq. (7.1) $K_{\alpha_c}/\sigma_0\sqrt{\pi a}$</th>
<th>$f_2$ by Eq. (7.2) $K_{\alpha_c}/\sigma_0\sqrt{\pi a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical (Eq. (7.16))</td>
<td>0.5913</td>
<td>0.6269</td>
</tr>
<tr>
<td>DCT</td>
<td>0.5885</td>
<td>0.6241</td>
</tr>
<tr>
<td>$J$-integral (EDI)</td>
<td>0.5672</td>
<td>0.6028</td>
</tr>
</tbody>
</table>

7.6.2 Crack Located Eccentrically with Respect to the Weld Line

In the first example the crack tip was always located in the region of tensile residual stresses. In this example the crack is given a displacement along the $\xi$ axis and one end of the crack is then located in the compressive zone while the other crack tip is located in the tensile region. The calculations were carried out for $a = 1.0$ and $L = 1.0$ (see Figure 7.11 for the notation). The computed stress intensity factors obtained by the three methods are presented in Table 7.1 for the crack tip which is located in the tensile region. A good agreement is observed between the SIFs calculated by the three methods. But from the table it can again be seen that the displacement correlation technique provides slightly better results than the $J$-domain integral method in a comparison to the results from the influence function method.

7.6.3 Mixed Mode SIFs in Residual Stress Fields

The previous examples are restricted to cracks located in mode I residual stress fields. But in general the crack propagates through mixed mode residual stress fields. No relevant practical
examples have been found in the literature for cracks located in mixed mode residual stress fields. The configuration illustrated in Figure 7.14 is therefore used to verify the numerical implementation of the superposition method for the mixed condition. The theoretical stress intensity factors for a crack located in an infinite biaxially loaded plate can be computed from:

\[ K_I = \left( \sigma_{22} \sin^2 \beta + \sigma_{11} \cos^2 \beta \right) \sqrt{\pi a} \]
\[ K_{II} = (\sigma_{22} - \sigma_{11}) \sin \beta \cos \beta \sqrt{\pi a} \]  

Figure 7.14: Centre crack located in a mixed mode stress field.

In the numerical models, the width and the height of the plate are equal to 25 times the crack length (the crack length is chosen arbitrarily as \( a = 4 \) m) which reduces the boundary effect to a minimum. Numerically obtained values and theoretical SIFs for different combinations of the inclined angle \( \beta \) and the applied stresses \( \sigma_{11} \) and \( \sigma_{22} \) are listed in Table 7.2. From the table, it can be concluded that the numerical procedure provides an accurate prediction of the mixed mode SIFs arising from a crack located in a biaxially loaded stress field.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \sigma_{11} ) [MPa]</th>
<th>( \sigma_{22} ) [MPa]</th>
<th>Theoretical</th>
<th>Numerical J-EDI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( K_I ) [MPa(\sqrt{m} )]</td>
<td>( K_{II} ) [MPa(\sqrt{m} )]</td>
</tr>
<tr>
<td>120</td>
<td>0.0</td>
<td>100.0</td>
<td>265.0</td>
<td>-153.0</td>
</tr>
<tr>
<td>60</td>
<td>0.0</td>
<td>100.0</td>
<td>265.0</td>
<td>153.0</td>
</tr>
<tr>
<td>120</td>
<td>100.0</td>
<td>0.0</td>
<td>88.6</td>
<td>153.0</td>
</tr>
<tr>
<td>60</td>
<td>100.0</td>
<td>0.0</td>
<td>88.6</td>
<td>-153.0</td>
</tr>
</tbody>
</table>
Brittle fracture often initiates from weld defects associated with conditions such as undercut and lack of penetration. When a crack is initiated along a weld line, it is affected by various conditions like a) the applied external stress b) the welding residual stresses and c) the degradation of the heat-affected zone (HAZ). A significant degradation (reduction of fracture toughness) of the material in the HAZ can be found in weldings of high-strength steel. A crack initiated along a weld line in a high-strength steel is therefore most likely to propagate along the weld. The decrease of fracture toughness along the HAZ is less pronounced in structures made of mild steel and it has been experimentally observed that a brittle crack initiated along a weld line turns and propagates into the base material [122].

A simplification of the latter case is here used to illustrate the capabilities of the simulation procedure. An initial crack with a length of 100 mm is introduced in the zone between the HAZ and the base material in a square plate of 500 mm in length. A longitudinal self-equilibrating welding residual stress is applied perpendicularly to the weld line (see Figure 7.15) while the transverse residual stresses arising from the weld are neglected in the present case. The isotropic material properties are 0.3 and 206 GPa for Poisson's and Young's modulus respectively. Simulations were carried out for three levels of external stresses \( \sigma_a \) applied to the two edges perpendicular to the weld. In the simulation, the crack tip is given a small initial kink of 2.0 degrees towards the base material. The obtained crack paths are plotted in Figure 7.15 from which it is seen that in all the cases the crack turns to the base material and that the shape of the crack path depends on the applied external stress.

Sumi and Yamamoto [122] carried out simulations for the same configuration using the method where the stress state in the vicinity of the crack tip is determined by a superposition...
of an analytical and a finite elements solution. Their simulated results are depicted in Figure 7.16. It is seen that the crack paths predicted by the two different methods are in good agreement.

7.6.5 Mixed Mode Crack Propagation in Residual Stress Fields

Experimental results recently presented by Sumi et al. [125] will be used to evaluate the numerical model for prediction of mixed mode fatigue crack propagation in residual stress fields. An outline of the experiments is given here (for details see [125]). Crack growth data including crack path measurements was obtained for crack propagation in welded plate structures consisting of an I-profile with a curve bar welded to the upper flange by a double fillet weld. A sketch of the specimens which were made of ship structural steel of class KA36 and the rest of the experimental set-up is given in Figure 7.17. From the sketch it is seen that an initial crack is introduced in the bend between the horizontal and the vertical part of the bar stiffener. The specimens were exposed to the cyclic load $P$ at the top of the bar stiffener and simply supported at the ends of the I-profile. In [125] crack growth data was presented for different notch configurations and biaxial stress ratios with control of the latter feature by the stiffness of the I-profile relative to the rest of the structure.

The residual stresses arising from the attachment of the bar to the I-profile were experimentally determined by the conventional sectioning technique for a configuration called B3. The detailed residual stress distribution presented in [125] shows a relatively high level of longitudinal compressive residual stresses in the range of -200 MPa to -300 MPa with a peak attainment approximately 30 mm from the edge of the specimen. Normal residual stresses

![Figure 7.17: Sketch of test layout for mixed mode crack propagation in a residual stress field.](image-url)
acting in the vertical direction were found to be negligible for the present experimental set-up [125].

Numerical fatigue crack simulations were carried out for the configuration B3 with the cyclic load $P$ oscillating between 4.9 kN and 88.0 kN (as in the experimental set-up). The initial finite element mesh used in the simulations is depicted in Figure 7.18 with the crack tip location in the high mesh density zone.

As discussed in Section 6.2 it is not yet clear how to determine the effective part of the $\Delta K_{II}$ in the presence of crack closure. Two interpretations were applied in the numerical simulations: 1) $\Delta K_{II}$ taken as the full range ignoring the friction between the two crack surfaces and 2) an effective $\Delta K_{II}$ obtained by Eq. (6.14) where $\Delta K_{II}$ is assumed not to contribute through the period of crack closure. The two interpretations can be regarded as bounds for the $\Delta K_{II}$ range. The crack is assumed to be two-dimensional in the simulations though the real crack geometry is three-dimensional and rather different crack lengths were observed for some of the experiments carried out by Sumi et al. [125]. However the experimentally obtained crack path for the B3 configuration showed almost the same crack growth rates for both sides of the specimen. A reproduction of the measured crack paths on the front and back side is given by the two bold curves in Figure 7.19 with the labels A and B respectively. Simulations with $\Delta K_{II}$ taken as the full range were performed for two different sizes of crack increments ($\Delta a_1 \approx 3 \Delta a_2$) to test the mesh sensitivity of the calculations. These are denoted E and F in Figure 7.19. The curve D represents the reduced $\Delta K_{II}$ while the last curve C is the numerically obtained crack path when the effect of residual stresses is neglected. It is seen from the figure that the best agreement between the experimental paths and the simulated paths is observed when $\Delta K_{II}$ is taken as the full range. The simulation with the reduced $\Delta K_{II}$ is very close to the crack path determined for the configuration without residual stresses. The increment size is seen to have local influence on the simulated crack path. But the global pattern is the same for the predictions E and F.
Crack growth curves for the same experimental set-up are presented in Figure 7.20 with curves A and B being reproductions of measured $a$-$N$ curves from [125]. The crack growth equations given by Eq. (6.13) and Eq. (6.12) were used for the numerical estimations. It is seen that the negligence of residual stresses (curve C) results in very conservative fatigue life estimations. The measured $a$-$N$ curves are for crack lengths longer than 35 mm “surrounded” by the curves based on the two different determinations of $\Delta K_{II}$ effective. The full-range interpretation results in conservative estimations while the opposite is observed for the reduced $\Delta K_{II}$ method. But again the best correlation between experiments and simulations is obtained for the full-range method. Hence it can be concluded that the full-range interpretation gives the best description of the experimental crack growth data for the B3 configuration.

7.7 Summary

Relatively simple procedures for qualitative estimation of longitudinal residual stresses arising from welding and flame-cutting have been outlined. The methods are based on simple formulas for estimation of the inherent strains in the structures. The inherent strain distributions can be used as an input for a linear elastic determination of residual stresses.
Chapter 7. Effect of Residual Stresses on Crack Propagation

The superposition method has been applied to account for residual stresses in the determination of stress intensity factors and the necessary modification of the $J$-integral has been introduced. Test examples showed a good agreement between theoretical SIFs and the results obtained by the numerical procedures applied. The two latter examples showed that the numerical crack simulation model provides good crack path estimations for fatigue crack growth and brittle fracture close to welds. The mesh density was shown to have local influence on the crack paths but the global behaviour was not significantly affected by changes in crack increment size.

Two different determinations of $\Delta K_{II}$ in the presence of compressive residual stresses were used in the fatigue crack simulations. The best results were obtained for the “full-range” philosophy. However, more simulations and experiments are needed for a general conclusion. Other examples of mixed mode crack growth in residual stress fields have not been found in the literature.

On the basis of the presented test examples, it is concluded that the numerical procedure outlined in Chapter 5 combined with the modifications for crack closure and residual stresses provides a reliable estimation of crack propagation under the influence of residual stresses and the model will be applied to the cutouts described in Chapter 3.
Chapter 8

Crack Simulation Procedure Applied to Cutouts

8.1 Introduction

In this chapter the two-dimensional crack simulation procedure will be used in a comparative study of cracks propagating from the cutouts in the conventional and the shape-optimised slot structures described in Chapter 3. Crack propagation starting from the toe of the bracket used in the conventional design will not be included in the present analysis. Cracks 1 mm in length are initiated at points of high stress concentration and a possible crack growth is investigated in an attempt to estimate the relative fatigue strength of the two designs. A qualitative estimation of the residual stress effect which arises from the manufacture of the slot structure is included. The finite element models of the slot structures and the approximations made are discussed in the following section.

8.2 Modelling of the Problem

A good geometrical description of the cutouts is important to the investigations. Hence the cutouts are modelled with a relatively high-density mesh concentrated in the regions around the initial cracks as depicted in Figure 8.1 and Figure 8.2 for configurations with initial cracks in the bends. The longitudinal transfers load from the side shell into the web frame. The actual stress distribution within the longitudinal is of minor importance to cracks propagating in the web frame. Thus the body of the longitudinal is modelled by membrane elements which ensures a proper transfer of sea water pressure into the web frame while the side shell and the flange of the longitudinal are modelled by truss elements in order to reduce the number of nodal points.
Chapter 8. Crack Simulation Procedure Applied to Cutouts

The bracket in the conventional design will cause unsymmetrical bending, which results in out-of-plane deformation of the web frame. The Mode III deformation (see Figure 4.1) can not be accounted for by the simulation procedure and a simplification has to be made. Two brackets, symmetrically located with respect to the web frame and with half of the thickness of the original stiffeners, are used to transfer the load from the longitudinal to the upper part of the web frame so that in-plane deformation of the web is ensured. A 3-D view of the mesh used for the crack tracing in the conventional slot structure is given in Figure 8.3. The truss elements used for the modelling of the side shell and the flange of the longitudinal are omitted in the visualisation. It should be noted that the use of the truss elements neglects the connections along the weld seam between the side shell and the web frame.

8.2.1 Loads and Boundary Conditions

The crack simulation procedure is restricted to constant amplitude loading. Hence, a representative constant cyclic loading has to be determined for the simulations. The results of the probabilistic fatigue life estimations presented in Chapter 3 can be used as a starting point. In the discussion of Figure 3.16 it was noted that the laden condition accounts for approximately 90% of the damage and from the same figure it could be concluded that most of the damage is caused by the external sea water pressure acting on the side shell. From these observations it was decided to base the simulations on the vessel being in laden condition with an appropriate cyclic wave pressure acting on the side shell. The established probabilistic model was used for the determination of an equivalent cyclic wave. The simulation was based on two observations: 1) A non-linear relation exists between the stress range at the crack tip and the damage to be expected (see e.g. Eq. (3.6) or Eq. (4.23)) and 2) the cyclic stress state at the crack tip is strongly related to the sea water pressure on the
These observations lead a determination of an equivalent wave height $h_{eq}$ by the following relation:

$$h_{eq} = \left[ \frac{\sum_{i=1}^{N}(h_i)^m}{N} \right]^{1/m} \quad (8.1)$$

where $h_i$ is a wave height simulated by the probabilistic model and $N$ is the number of simulation points. The damage caused by large wave heights relative to the damage induced by small wave heights is controlled by the parameter $m$. On the basis of the crack growth equations outlined in the Sections 3.2.6 and Section 6.2.1 and the assumptions described in this section, it was decided to use $m$ equal to 3.0. The outcome of the simulation was a wave amplitude of 0.93 m.

An examination of the loads presented in Figure 3.8 reveals that only the three latter load cases are of interest to the simulation of crack propagation in the web frame. The contribution from each of these cases is determined on the basis of the equivalent wave. The internal pressure from the cargo shows some variation because of the vertical acceleration of the ship. But these variations are relatively small (especially for the midship tank) and the cargo pressure is assumed to be constant during the simulations. Thus, the resulting
load case consists of a static contribution from internal cargo pressure and fluctuating loads controlled by the external wave height.

The equivalent wave and the boundary conditions for the conventional design are depicted in Figure 8.4. The web frame is supported at the edge pointing in the direction of the cargo tank and symmetry conditions are applied at the end of the longitudinal (located in the midpoint between two web frames). Similar boundary conditions are applied to the shape-optimised design except for the support at the end of the brackets.

8.3 Crack Propagation without Residual Stresses

From the von Mises stress plots presented in Chapter 3 it was found that in the conventional design cracks are most likely to initiate at the lower part of the bends and at the weld seam between the longitudinal and the web frame. 1 mm cracks were initiated at these locations and the simulation procedure was used. But with the applied cyclic loads given by the equivalent wave crack arrest was predicted for both configurations at the initial stage. For a matter of comparison simulations were carried out with the threshold value equal to zero. Figure 8.5 shows the obtained crack path for the step where the crack initiated in the bend has propagated 0.047 m while the obtained crack path for a crack initiated at the weld seam is depicted in Figure 8.6.

The crack initiated in the bend turns and propagates in a direction which is almost perpendicular to the side shell while the crack starting at the location of the weld propagates towards the intersection between the longitudinal and the web frame. The simulation procedure allows for tracing of cracks propagating in three-dimensional plate structures.
the cracks are assumed to propagate in a two-dimensional plane which in the present case is defined by the undeformed web frame. A possible crack propagation from the web into the longitudinal is therefore not within the scope of the present formulation. Thus the simulation is terminated when the predicted crack path enters the domain of the longitudinal. Another limitation is that the material of the structure is assumed to be isotropic. Hence material changes due to manufacturing procedures are not taken into account.

In the shape-optimised structure a crack is initiated in the bend of the cutout where the main stress concentration is found. Crack arrest was also predicted at the initial stage for this configuration and again a simulation was carried out with the threshold value set to zero. Partial closure was present during the simulation which reduced the effective mode I range. The crack path presented in Figure 8.7 is for a reduced $\Delta K_{II}$ range while the result of a simulation with full $\Delta K_{II}$ range is depicted in Figure 8.8. The crack paths are very similar and both simulations predict full crack closure for the depicted locations of the crack tips ($K_I$ is not above zero during the whole load cycle). But the crack trace with full $\Delta K_{II}$
range stops one iteration after the trace with the reduced mode II range (for the same crack increment sizes).

Figure 8.9: Equivalent stress intensity range $\Delta K_{eq}$ as a function of the crack length $a$.

The equivalent mixed mode stress intensity range (see Eq. (4.28)) is plotted as a function of the increasing crack length for the four simulated crack paths in Figure 8.9. The equivalent stress intensity range is below the threshold values given in Section 6.2.1 for the whole range of simulated paths. Nevertheless, when the possible crack arrest is disregarded, an interesting trend can be observed. At the initial stage of the crack propagation the stress intensity range is relatively large for the optimised structure; however, it drops to zero after a short distance of crack propagation and the crack faces are closed. The equivalent stress intensity range for the conventional design starts at a lower level but it increases continuously during the crack propagation (see Figure 8.9). Thus, closure of crack surfaces is not observed for this configuration.

8.4 Including the Effect of Residual Stresses

This section presents a qualitative estimation of how residual stresses can affect the crack propagation in the two types of structures. Residual stresses inherent from the manufacture of the structures are considered while other sources of residual stresses, such as initial stresses due to the rolling of the plates, are disregarded.

8.4.1 Modelling of Residual Stresses

The longitudinal to web frame connections are complicated welded structures as it can be seen from Figure 8.10 where the bold lines indicate either a weld seam or a thermally cut edge.
The estimation of the residual stresses due to the welding of the slot structures is based on the inherent strain procedure outlined in Section 7.2.2. The welding parameters needed for the calculation are listed in Table 8.1 with the labels corresponding to the numbers in Figure 8.10. Residual stresses originating from the manufacture of the cutout are estimated by the experimentally obtained Eq. (7.9) which is valid for flame-cut edges. Plasma-cut edges with a different residual stress distribution because of the heat input are not considered.

![Diagram of welding and cutting lines in the slot structure](image)

**Figure 8.10: Identification of welding and cutting lines in the slot structure (conventional design).**

<table>
<thead>
<tr>
<th>Table 8.1: Welding parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) + (2) + (3) + (6)</td>
</tr>
<tr>
<td>Welding speed [cm/min] 42.0</td>
</tr>
<tr>
<td>Voltage [V]                27-28</td>
</tr>
<tr>
<td>Current [A]                270-280</td>
</tr>
<tr>
<td>(4)</td>
</tr>
<tr>
<td>Welding speed [cm/min] 17.0</td>
</tr>
<tr>
<td>Voltage [V]                25</td>
</tr>
<tr>
<td>Current [A]                215-220</td>
</tr>
<tr>
<td>(5)</td>
</tr>
<tr>
<td>Welding speed [cm/min] 110</td>
</tr>
<tr>
<td>Voltage [V]                27± 3</td>
</tr>
<tr>
<td>Current [A]                750 ± 50</td>
</tr>
</tbody>
</table>

Figure 8.11 shows the distribution of one of the resulting principal strain components which is used as input for the calculation of the residual stresses in the conventional slot structure. The inherent strains from the different contributions have simply been added.
This causes some errors where two distributions meet (strains from two different sources cannot simply be added [47]). However, a theoretical estimation is rather difficult. Another source of inaccuracy is that the applied inherent strain procedure is not strictly valid at the ends of the welds where the boundary effects manifest themselves. For the present comparative study, these inaccuracies have been accepted. Nevertheless, a more accurate determination of the residual stress distributions could be obtained by special-purpose programs such as SYSTUS+ [126] and the obtained stress distributions could be applied as input for the crack simulation procedure.

The web frames considered are made of mild steel with a yield stress level close to 235 MPa while the yield strength of the weld material is assumed to be 50% higher (as suggested in [146]). In the web frame, the largest inherent strains (negative) can therefore be found at the location of the weld lines which have the labels 1 and 3 in Figure 8.10.

The deformation of the cutout imposed by the inherent strains can be obtained by linear elastic finite element calculations. The flame-cutting causes the edge to contract and tensile stresses at the yield stress level can be found along the edge. Similar contractions occur by the welding of the longitudinal and the bracket. Figure 8.12 shows the deformation pattern for the conventional structure when it is exposed to the inherent strain distribution depicted.
in Figure 8.11. The deformations are relatively symmetric with respect to the longitudinal despite the slightly non-symmetric input of inherent strain caused by the difference in mesh density on the two sides of the longitudinal.

![Deformation of the conventional structure](image)

Figure 8.12: Deformation of the conventional structure when it is exposed to the inherent strain distribution in Figure 8.11. The deformations are multiplied by a factor of 200.0.

Figure 8.13 shows the normal residual stress component in the direction perpendicular to the weld. The contraction of the weld seams transforms the tensile stresses from the flame-cutting into compressive normal stresses at the connection between the web frame and the longitudinal.

Tensile stresses with the main components parallel to the edge are present in the bends. The flame-cutting results in residual stresses at the yield level of the cut edges. When the contraction of the welds is superposed the residual stresses predicted by the linear elastic analysis exceed the yield stress of the material at the bends. This zone is fortunately limited to a narrow band in the middle of the bend. In the present formulation the stresses are cut off at the yield stress level by use of the von Mises stress criterion. For a more accurate determination the redistribution of stresses due to plasticity should be included.

The discussion of residual stresses has focused on the conventional slot structure. Analogous calculations were performed for the shape-optimised structure. The obtained residual stress distribution is presented in the form of von Mises stresses in Figure 8.15. The elimination of the bracket changes the residual stress distribution at the upper part of the cutout otherwise no significant changes can be found between the residual stress distribution found in the conventional and the shape-optimised slot structures.
Figure 8.13: The component of the normal residual stresses in the direction perpendicular to the welds.

Figure 8.14: The resulting von Mises stress distribution for the conventional cutout.
8.4. Including the Effect of Residual Stresses

Figure 8.15: The resulting von Mises stress distribution for the shape-optimised cutout.

8.4.2 Result of Crack Path Simulation

Cracks were initiated at the same location as for the simulations without residual stresses. An interesting observation was made for the crack initiated close to the weld seam. Compressive normal residual stresses forced the crack surfaces together throughout the whole load cycle and caused crack arrest at the initial stage.

The residual stresses due to welding and thermal cutting were assumed to be static and only affected by the propagating crack. Thus, the cyclic stress range was not changed by the introduction of residual stresses and the application of the cyclic loads given by the equivalent wave still resulted in crack arrest at the initial stage for the configurations with initial cracks in the bends. Again, the threshold value was set to zero and “crack stop” was defined as the mode I stress intensity being zero during a whole load cycle. Crack simulations were carried out and the patterns found in the simulations without residual stresses could be recognised. For the conventional slot structure almost no influence was observed from the change in $R$-value. Figure 8.16 shows the obtained path where the crack has been allowed to grow to a length of 0.15 m. The slow increase in stress intensity range observed in the simulations without residual stresses (see Figure 8.9) was also found here and “crack stop” was not observed. The optimised cutout exhibited some influence of the changed $R$-level which resulted in an increased crack length compared to the simulations without residual stresses. But again closure of the faces was detected in the present case for the crack configuration depicted in Figure 8.17.
In an evaluation of the influence of residual stresses, the assumptions made in the qualitative prediction of the stress fields should be kept in mind. Inaccuracies are to be expected at the ends of the welds and where the effects of two manufacturing procedures interfere.

From the simulations carried out for the conventional structure, it was observed that compressive stresses at the location of the weld reduce the risk of crack propagation in this area, which is an important observation because imperfections from the welding are often found in this region.

The application of an “equivalent wave concept” resulted in prediction of crack arrest at the initial stage of the simulations. Figure 8.18 shows the distribution of the simulated wave heights with the equivalent wave height indicated by the dashed line. From the figure it can be seen that waves which are significantly larger than the equivalent wave can be expected during the lifetime of the vessel. The actual wave heights which result in propagation of cracks initiated in the two structures have not been determined. But the stress intensity ranges depicted in Figure 8.9 together with the threshold values outlined in Section 6.2.1 indicate that crack propagation may occur during the service of the vessel. The effect of peak loads can not be captured by the established high-cycle fatigue model. A more sophisticated model which includes sequence effects would have to be applied for an investigation of crack propagation caused by peak loads.

The prediction of crack arrest at the initial stages makes it difficult to predict the relative fatigue strength of the two structures. Nevertheless, crack simulations performed with threshold values equal to zero indicate that the initial stress intensity range for the shape-optimised cutout is larger than for the conventional cutout (due to the larger load carried by the weld seam between the longitudinal and the web frame) but that it drops to zero at a given distance. The stress intensity range for the conventional structure starts at a
lower range but it increases continuously as the crack propagates. The same trend is to be expected at a stress level above the threshold limit. Thus the optimised slot design seems to be superior to the conventional design. Finally, crack propagation initiated at the toe of the bracket used in the conventional design should enter a complete evaluation of the relative fatigue strength of the two designs.
Chapter 9

Conclusion and Recommendations for Further Work

9.1 Conclusion

The large number of longitudinal to web frame connections present an essential part of the production costs of a very large crude oil carrier (VLCC). The slot structures are complicated welded details which are exposed to dynamic loads during the service of the vessels and if not adequately designed, significant fatigue cracking may take place and cause major repair costs as in the case of the early cracking of the second-generation VLCCs. The objective of the present work has been to develop and investigate a new type of slot design suitable for application of welding robots. This included establishing a probabilistic model for non-linear estimation of fatigue lives and a general-purpose mixed mode crack simulation procedure for prediction of curved crack propagation in welded structures. The models have been applied in a comparison of the fatigue strength of the new slot structure relative to the strength of a conventional design.

In conventional slot structures, cracks can often be found at the toe and the heel of the bracket used to transfer load from the pressure on the side shell into the web frame. In the new design, the stiffener is removed in order to avoid these cracks and to ease the application of welding robots (no manual fitting of the bracket). The load to be carried by the weld seam between the web frame and the longitudinal is thus increased and the cutout becomes a hard-spot area. A finite element based shape optimisation was applied to reduce the stresses at the cutout. Before the shape optimisation, the main stress concentrations were at the welds which are known to be crack initiators. The shape optimisation reduced the stress level by a factor of 1.6 in this region and in the new design the main stress concentrations were found in the bends away from the welds.

Before the introduction of significant structural changes at locations known to be prone to fatigue cracking, it is good design practice to estimate the fatigue life of the new design.
A fatigue damage model with a probabilistic load generation based on the sailing route of the vessel was established for that purpose. By applying the quasi-stationary narrow-band model it was possible to include the non-linear effect of intermittent submergence of the side shell found in the zone close to the load line. Fatigue lives of the new and the conventional designs were predicted by the probabilistic model for details located approximately 2 metres below the still water load line in the region where the longitudinal to web frame connections are most prone to fatigue. The estimated fatigue lives of the weakest points of the shape-optimised cutout were found to be approximately of the same order of magnitude close to 30 years. For the conventional structure it was predicted that cracks are most likely to initiate at the toe of the bracket with an estimated lifetime of 5 years i.e. 6 times less than the predicted lifetime of the new design.

Cracks can be initiated at imperfections caused by welding and thermal cutting of edges. Conventional fatigue damage approaches are normally based on the assumption that if subsequent crack propagation occurs it is in a self-similar manner. However, branched crack propagation can be found in structures with residual stresses or in details exposed to bi-axial loading where curved crack propagation may have a significant influence on the crack growth rate. A two-dimensional multipurpose crack propagation procedure has been established in order to investigate mixed mode crack propagation in welded structures subjected to high-cycle loads. The method was based on a step-by-step finite element procedure with continuous remeshing of the domain surrounding the propagating crack. A two-dimensional free mesh generator was established for the meshing of the crack domain. The mesh generator based on the advancing-front method was found to be stable and robust and thus well suited for support of discrete crack propagation. Special emphasis was put on the numerical evaluation of the mixed mode stress intensity factors which are the controlling parameters in a linear elastic fracture mechanical approach. The formulated model allows for determination by the displacement correlation technique as well as the J-integral in the equivalent domain integral formulation with decomposition applied. Three common methods for estimation of crack propagation angles were employed. The simulation procedure was validated by comparison with experimental and numerical data found in the literature covering both the influence of holes and biaxial loading. A good correlation was obtained. The simulations also revealed that the choice of procedures for determination of crack propagation angle and stress intensity factors does not influence significantly the prediction of the crack paths. It is therefore concluded that the presented crack simulation scheme is able to handle crack propagation in general two-dimensional domains.

Crack closure is when the two crack surfaces come into contact. The phenomenon affects the crack growth and it becomes of major importance when cracks are located in compressive residual stress fields. The occurrence of closure is influenced by a large number of factors such as plasticity, oxide on the crack surface and roughness of the crack faces. The complexity of closure mechanisms does not allow for direct modelling of the phenomenon by the finite element method. The effective stress intensity range is defined as the part of the load cycle which causes damage to the material. Crack closure was modelled by empirical formulas which related the effective stress intensity range to the stress state at the crack tip. An evaluation of methods for determination of effective mixed mode stress intensity ranges left
some doubt concerning the determination of the effective mode II range. Two models were used: 1) An effective range assumed to follow the effective mode I range and 2) the whole mode II range regarded as effective.

Two simple procedures for qualitative estimations of residual stresses caused by flame-cutting and welding have been outlined with the latter approach based on the inherent strain method. Obtained strain distributions can be used as an input for a linear elastic determination of residual stresses in the structures. For a known residual stress distribution can be given as an input. To account for the residual stresses in the determination of the stress intensity factors, the superposition method was employed. The capabilities of the procedure were tested by several benchmark examples which revealed that good estimations were provided for high-cycle fatigue crack growth as well as brittle fracture close to welds. Two different determinations of the mode II range in the presence of compressive residual stresses were tested against experimental results for cracks propagating in a three-dimensional plate structure. The best result was obtained for the “full-range” philosophy. However, more simulations and experiments are needed for a general conclusion on the effective mode II range.

The crack simulation procedure was applied to the conventional and the shape-optimised slot structures. Residual stresses were estimated on the basis of relatively simple inherent strain distributions. From the simulations, it was observed that compressive residual stresses at the location of the weld reduce the risk of crack propagation in this area, which is an important observation because imperfections from welds are often found in this region. The assumptions made in the qualitative prediction of the inherent strain fields should be kept in mind in evaluations of the influence of residual stresses.

An equivalent wave height was determined by the probabilistic model described in Chapter 3. However, the stress range obtained by applying the equivalent wave resulted in prediction of crack arrest at the initial stage, which makes it difficult to conclude that one of the designs is superior to the other with respect to fatigue. Nevertheless, the crack simulations performed with threshold values equal to zero indicate that the initial stress intensity range for the optimised cutout is larger than for the conventional cutout but it drops to zero at a given distance. The stress intensity range for the conventional structure starts at a lower range but it increases continuously as the crack propagates. The same trend is to be expected at a stress level above the threshold limit. Thus, the optimised slot design seems to be superior to the conventional design.
Chapter 9. Conclusion and Recommendations for Further Work

9.2 Recommendations for Further Work

The outlined probabilistic model and the crack simulation procedure are believed to cover main issues of fatigue assessment. However, improvements of the models could be made and additional effects could be included. In the work on the probabilistic model further research might deal with

- taking into account the effect of corrosion which is believed to be the single most important cause of failure of ship structures
- investigation of different longitudinal profile types such as L- and bulb profiles for other vertical positions of the T-profile
- taking into account effects of the load sequence and the mean stress level
- use of three-dimensional finite element models and strip programs for the determination of loads.

Further work on the simulation procedure can be divided into subjects which involve minor efforts or changes of the established program

- control of the crack increment size based on the changes in stress intensity at the crack tip
- multocrack propagation
- a comparison of residual stresses estimated by the inherent strain procedure with distributions predicted by state-of-the-art programs which include plasticity and thermally caused material changes
- investigation of crack propagation initiated at the toe of the bracket

and issues which require a major effort such as

- variable amplitude loading (related to sequence-effects)
- establishing a model for determination of residual stresses from plasma cutting
- better understanding of crack closure in mixed mode stress fields
- taking into account material changes due to welding and thermal cutting
- including the residual stresses from the rolling of the plates
- redistribution of residual stresses during loading of the structures.
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Appendix A

Singular Triangular Quarter-Point Elements

This appendix outlines shortly how the singular stress state is obtained along the crack face of a singular triangular quarter-point element. For a general proof of singularity within the element see Barsoum [15].

A normal isoparametric 8-noded quadratic element is used for the formulation. The element stiffness is well documented for the isoparametric elements and in accordance with the notation of Zienkiewicz [154] an 8-noded plane isoparametric element is mapped into the normalised space $(\xi, \eta), \xi \leq 1, \eta \leq 1$ through the transformation

\begin{align*}
    x_1 &= \sum_{i=1}^{8} N_i(\xi, \eta)x_{1,i} \\
    x_2 &= \sum_{i=1}^{8} N_i(\xi, \eta)x_{2,i} \\
    N_i &= [(1 + \xi \xi_i)(1 + \eta \eta_i) - (1 - \xi^2)(1 + \eta \eta_i) - (1 - \eta^2)(1 + \xi \xi_i)]\xi_i^2 \eta_i^2/4 + \\
    &\quad (1 - \xi^2)(1 + \eta \eta_i)(1 - \xi_i^2)\eta_i^2/2 + (1 - \eta^2)(1 + \xi \xi_i)(1 - \eta_i^2)\xi_i^2/2
\end{align*}

(A.1)

where $N_i$ is the shape function corresponding to the node $i$ whose coordinates are $(x_{1,i}, x_{2,i})$ in the global $x_1-x_2$ system and $(\xi_i, \eta_i)$ in the normalised system. Figure A.1 shows the collapsed element in which

- $x_{1,1} = x_{1,4} = x_{1,8} = 0$, $x_{1,5} = x_{1,7} = \frac{h}{4}$, $x_{1,2} = x_{1,3} = x_{1,6} = h$

and

- $x_{2,1} = x_{2,4} = x_{2,6} = x_{2,8} = 0$, $x_{2,5} = -x_{2,7} = -\frac{l}{4}$, $x_{2,2} = -x_{2,3} = -l$

(A.2)
By substitution Eq. (A.2) into Eq. (A.1) and collection of the terms of the following expressions are derived:

\[ x_1 = \frac{h}{4}(1 + \xi)^2 \]

and

\[ x_2 = \frac{l}{4}\eta(1 + \xi)^2 \] (A.3)

For simplicity only the displacement of the edge with the points 12Γ and 5 is evaluated. Here the displacement \( u_1 \) are given by

\[ u_1 = -\frac{1}{2}\xi(1 - \xi)u_{1,1} + \frac{1}{2}\xi(1 + \xi)u_{1,2} + (1 - \xi^2)u_{1,5} \] (A.4)

If the displacement is written in terms of \( x_1 \):

\[ u_1 = -\frac{1}{2}\left(-1 + 2\sqrt{\frac{x_1}{h}}\right) \left[2 - 2\sqrt{\frac{x_1}{h}}\right] u_{1,1} + \frac{1}{2}\left(-1 + 2\sqrt{\frac{x_1}{h}}\right) \left[2\sqrt{\frac{x_1}{h}}\right] u_{1,2} + \left(4\frac{x_1}{h} - 4\frac{x_1}{h}\right) u_{1,5} \] (A.5)

the strain in the \( x_1 \)-direction can be found by use of the chain rule:

\[ \varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = \frac{\partial u_1}{\partial x_1} \frac{\partial x_1}{\partial \xi} \]

\[ = -\frac{1}{2}\left[3\frac{3}{\sqrt{x_1}h} - \frac{4}{h}\right] u_{1,1} + \frac{1}{2}\left[-\frac{1}{\sqrt{x_1}h} + \frac{4}{h}\right] u_{1,2} + \left[\frac{2}{\sqrt{x_1}h} - \frac{4}{h}\right] u_{1,5} \] (A.6)

It is seen that strain singularity along the side 1-2 is of the type \( 1/\sqrt{r} \), which is equal to the theoretical strain field for linear elastic materials.
Appendix B

Numerical Examples

Stress intensity factors calculated by the procedures described in Section 5.3 are in this section compared to different benchmark examples. For the simple mode I case, stress intensity factors collected in compendiums are used for the evaluation. The collected stress intensity factors are typically of the form:

\[ K_\alpha = \sigma_0 \sqrt{\pi a} f(a, w, h) \quad \alpha = I \text{ or } II \tag{B.1} \]

where \( \sigma_0 \) is the far field stress, \( a \) is the crack length. The width and the height of the specimen are denoted by \( w \) and \( h \) respectively, while the effect of the boundaries is taken into account by the parameter \( f \) which is equal to one for a crack in an infinite plate.

Stress intensity factors obtained by the numerical procedure are in the mixed mode case compared to a theoretical result, data obtained by Aliabadi et al. [1] using boundary element analysis, and an experiment where the path of a propagating crack has been measured.

B.1 Single- and Double-Edge Cracks, Mode I

The first state considered is a single-edge crack in a plate subjected to uniform tensile stresses acting perpendicularly to the direction of the crack. The second case is similar to the first state just with a double-edge crack configuration. The two states are illustrated in Figure B.1 and Figure B.2 respectively.
Plane stress conditions are assumed for the two cases and the following data is used for the calculations (arbitrarily chosen):

- Field stress $\sigma_0 = 960$ Pa.
- Height $= 2h = 20$ m.
- Width $= 2w = 20$ m.
- No body forces.

The stress intensity factors determined for different crack sizes are presented in Figure B.3 and Figure B.4. A good correlation is obtained between the stress intensity factors calculated by the three methods presented in Section 5.3 and the stress intensity factors based on Eq. (B.1). The best results are produced for the single-edge specimen where all the numerical results are within a 3% error in a comparison to Eq. (B.1). The numerical results from the specimen with the double-edge crack configuration show a fine agreement for relatively small crack lengths. But a small divergence starts as the crack length increases and the maximum error is about 4% for the $J$-contour integral method when $a/w = 0.4$. 

Figure B.1: Single-edge crack, Mode I.  
Figure B.2: Double-edge crack, Mode I.

Figure B.3: Single-edge crack, Mode I.  
Figure B.4: Double-edge crack, Mode II.
B.2 Single-Edge Crack in Mixed Mode Condition

By application of a boundary element method together with Bueckner weight functions, Aliabadi et al [1] obtained very accurate results for the mixed mode example described in Section 7.6.3. Thus, their results for a specimen with the single-edge inclined crack configuration illustrated in Figure B.6 are here used as a benchmark example. In the present case, the comparison is carried out for the following configuration (arbitrarily chosen):

- Crack propagation angle $\beta = 67.5^\circ$.
- Field stress $\sigma_0 = 960$ Pa.
- Width = $w = 10$ m.

The outcome of the calculations is depicted in Figure B.6. From the figure it is observed that all the presented stress intensity factors are almost identical with the results of Aliabadi et al [1] for both mode I and mode II.
B.3 Comparison with Experimental Results

With the theory outlined in Chapter 4 it should be possible to carry out a crack path simulations using one of the described methods to calculate the stress intensity factors and the relation given by Eq. (4.11) to determine the crack propagation direction.

The starting point for simulations is a plate which is to be welded to another plate by a double fillet weld. Full penetration is not obtained during the welding and a crack is formed as illustrated in Figure B.7. In this primitive simulation some assumptions are made: 1) there are no residual stresses in the structure; 2) the weld and the base material can be considered as a homogeneous body; and 3) the structure is only exposed to tensile forces. The last assumption makes it possible to disregard the effect of crack closure. The simulation was carried out for a structure:

- made of high-tensile steel st 52
- subjected to a sinusoidal tensile force \( F \) ensuring that crack closure would not occur.

Lyngbye and Nickelsen [75] performed some experiments for the configuration described above. A crack path was simulated using the displacement correlation technique for the determination of stress intensity factors and the maximum tangential stress criterion for the determination of the crack propagation angle. A comparison between the simulated and experimentally obtained paths is given in Figure B.8. It is seen that the simulation "underestimates" the propagation direction but the difference is relatively small.
B.3. Comparison with Experimental Results

Figure B.8: Experimental and simulated crack path.
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