Investigation of Turbulence and Flow Structures in Electrostatic Precipitator

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Investigation of Turbulence and Flow Structures in Electrostatic Precipitator

by

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Preface

This dissertation\textsuperscript{1} is submitted as partial fulfillment of the requirements for the Ph.D. degree. The dissertation is based on research work carried out during the period August 2000 to September 2003 at the Department of Mechanical Engineering, Fluid Mechanics Section, at the Technical University of Denmark. The study was financed by a stipend from the Technical University of Denmark and supervised by Professor Poul Sheel Larsen to whom I would like to offer my special thanks for giving me the opportunity to undertake this Ph.D. I extend my sincere thanks to Professor Larsen for his genuine interest and excellent guidance. Also thanks to my co-supervisor Dr Knud Erik Meyer for his good advice, particularly on optical methods, and to Dr.s Niels Finderup Nielsen and Leif Lind of FLS Miljo A/S for stimulating input of fundamental applied aspects of industrial electrostatic precipitators.

Professional and personal thanks also to my colleagues at the Fluid Mechanics Section for the stimulating and friendly atmosphere. A special thanks to Oktay Özcan at Yildiz Technical University in Istanbul for his inspiration and advice. In particular I express my gratitude to Professor Pierre Sagué for letting me visit his group and work at ONERA in Paris during fall of 2001.

Lyngby, September 2003

Thorvald Uhrsköv Ullum

\textsuperscript{1}Part of the present research work has been published, see [54] and [55], and accepted for publication, see [53].
Abstract

The optical measurement technique, stereo Particle Image Velocimetry (PIV), was applied to a laboratory scale model of an Electrostatic Precipitator (ESP). The test section consisted of 2 smooth collector plates and 7 barbed discharge electrodes. Charged particles, moving relative to the gas, were used as light scatters for the PIV. The PIV results were supplemented with and compared to results obtained by Laser Doppler Velocimetry (LDV) and Large Eddy Simulation (LES) of the flow.

The electrostatic equations were solved numerically to get particle drift velocities and the Coulomb body force acting on the charge carrying gas. The drift velocities were used to correct measured particle velocities, and the Coulomb body force was used as input for the numerical solution of the gas flow by LES.

Detailed information on instantaneous spatial structures were obtained. The main feature consisted of axial counter-rotating rolls superposed the bulk flow. The rolls were very unsteady with secondary velocities as high as 0.6 m/s at reference conditions (bulk flow velocity of 1.0 m/s and mean current density of 0.40 mA/m²). The spatial symmetry and roll strength increased with increasing electrostatic field strength and decreasing bulk flow velocity. However, the swirl number was found to level off at a maximum value of about 0.3.

In addition to secondary motion the electrostatics induced high levels of anisotropic and inhomogeneous turbulence. Turbulence intensities up to 18% were measured with PIV. Turbulence intensities increased almost linearly with the inverse Froude modulus with no sign of leveling off. The Froude modulus is defined as the ratio of axial flow inertia to transverse electric force.

The turbulence was presumably produced by shear stresses only, i.e. no electrically induced turbulence. High production of turbulence was found at all axial positions close to the collector plate where the counter-rotating rolls met. However, the highest production rates were found just upstream the discharge pins where the discharge current caused a local reverse axial flow. This was found both experimentally by PIV and numerically by LES.

The Kolmogorov length scale and the Taylor micro scale were estimated for reference conditions to 0.5 mm and 10-15 mm, respectively. Similarly, the temporal Taylor micro scale was estimated from LDV and LES time series to 0.01-0.015 s. The integral time scale decreased with the electrostatic field strength to about 0.05 s at reference conditions.

Finally, precipitation efficiencies were estimated based on particle counting at one point and grey scale levels of PIV images. The one point measurements gave unrealistically high efficiencies, whereas the levels obtained by the PIV images seemed more realistic. The efficiencies increased with electric field strength to about 25% at reference conditions for 0.75 m of the active test section.
Synopsis (in Danish)

Particle Image Velocimetry (PIV), der er en optisk målemetode, blev anvendt i en skala-model af et elektrostatisk filter (ESP). Testsekptionen bestod af 2 glatte udfaldningsplader og 7 udladningselektroder. Elektrisk ladende partikler, som bevægede sig relativt til gassen, blev benyttet som lysspredere for PIV. PIV-resultaterne blev suppleret og sammenlignet med resultater opnået med Laser Doppler Velocimetry (LDV) og Large Eddy Simulation (LES).

De elektrostatiske ligninger blev løst numerisk for at få dels drifthastigheder for partiklerne og dels Coulomb volumenkraven virkende på den ladiningsbærende gas. Drifthastighederne blev benyttet til at korrigerre de målte partikelhastigheder, og Coulomb volumenkraven blev benyttet som input til den numeriske løsning af gasflowet med LES.

Strømningsundersøgelserne har givet detaljeret information om instantane rumlige strukturer. Den dominerende struktur bestod af aksiale roller overlejet den aksielle hovedstrømning. Rullerne var meget instationære med sekundære hastigheder på op til 0,6 m/s for reference betingelser (hovedstrømningshastighed på 1,0 m/s og middel elektrisk strømtæthed på 0,40 mA/m²). Den rumlige symmetri og rullestyrken forågedes med forøget elektrostatisk feltstyrke og øget hovedstrømningshastighed. Imidlertid blev det fundet, at swirltallet fladede ud ved en maksimal værdi på omkring 0,3.

Udover sekundære strømnings inducerede det elektrostatiske felt højere niveauer af anisotropisk og inhomogen turbulens. Turbulensintensiteter på op til 18 % blev målt med PIV. Turbulensintensiteter voksende omtrent lineært med det inverse Froude tal, uden tegn på at flade ud. Froude tallet er defineret som forholdet mellem axial flow inerti og tværgående elektrisk kraft.

Turbulens blev antagelig genereret alene ved forskydningspænderinger, d.v.s. ingen elektrisk genererer turbulens. En stor produktion af turbulens blev fundet ved alle aksielle positioner tæt på udfaldningspladen, hvor de to modsat roterende roller mødtes. Den største produktion blev dog fundet opstrøms for udladningspindene, hvor udladningsstrømmen foråage mellem de aksialstrømning. Dette blev observeret eksperimentelt med PIV og numerisk med LES.

Kolmogorov længdeskalalen og Taylor mikro-skalaen blev estimert for reference betingelser til henholdsvis 0,5 mm og 10-15 mm. Ligeledes blev den tidlige Taylor mikro-skala estimert v.h.a. LDV og LES tidserier til 0,01-0,015 s. Den integrerte tidsskala af gav den elektrostatiske feltstyrke til omkring 0,05 s ved reference betingelser.

Endeligt blev udfaldningsvirkningsgrader estimert ud fra partikel-tælling i et punkt og grå-skala værdier fra PIV billeder. Et-punktsmålingerne gav urealistiske høje virkningsgrader, hvorimod værdier opnået med PIV billeder virkede mere realistiske. Virkningsgraderne voksende med elektrisk feltstyrke til omkring 25 % ved reference betingelser for 0,75 m af den aktive filter model.
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Nomenclature

Roman letters

$A$ area

$b$ mobility

$B$ magnetic field strength

$C$ particle concentration

$C'$ proportionality vector

$D$ diffusion coefficient

$d_e$ particle image diameter

$d_p$ particle diameter

$d_{p'}$ mean particle diameter

$\sigma^2_p$ particle diameter variance

$d_s$ diffraction image diameter

$d_r$ CCD pixel diameter

$E$ electric field strength

$E_0$ characteristic field strength

$E$ expectation value

$f$ distribution function

$f_0$ frequency

$f_e$ effective distribution function

$f_#$ numerical aperture

$f_s$ selection function

$F$ scattering function

$F_p$ body force on gas

$F_p$ coulomb force on particle

$G_x$ axial momentum flow

$G_y$ angular momentum flow

$i$ \( \sqrt{-1} \)

$i$ unit vector

$I$ light intensity

$I_0$ incident light intensity

$I_p$ ionic current at particle surface

$J$ electrical current density

$J_m$ mean current density

$J_p$ electrical current density at particle surface

$k$ turbulent kinetic energy

$k$ wave number

$K$ number of particles

$l_s$ electrode to plate distance

$L_H$ hydraulic radius

$m$ relative refractive index

$M$ geometric magnification

$n$ refractive index

$\bar{n}$ mean sampling rate

$n_p$ particle surface normal

$N$ number of samples

$N_w$ filter width

$p$ pressure

$P$ scattering power

$P$ point in $yz$-plane

$P_{C}$ point in $yz$-plane of max($\Gamma$)

$P$ production of $k$

$q_c$ kinetic energy at test scale

$q_p$ charge on particle

$q_{p\infty}$ saturation charge on particle

$r$ radius

$R_{xx}$ temporal velocity correlation function

$\text{Re}$ Reynolds number

$s_{ij}$ strain rate tensor

$S$ uncertainty

$S$ swirl number

$\hat{S}_{xx}$ power spectral density function

$t$ time

$T$ temperature

$T_\eta$ Kolmogorov time scale

$T_\lambda$ temporal Taylor micro scale

$T_I$ integral time scale

$T_u$ turbulence intensity

$U_0$ bulk flow velocity

$u$ fluid velocity, $u = (u, v, w)$

$u_e$ particle drift velocity
\( u_p \) \hspace{1em} \text{particle velocity} \hspace{1em} W \hspace{1em} \text{energy} \\
\( V \) \hspace{1em} \text{volume} \hspace{1em} x \hspace{1em} \text{position, } x = (x, y, z) \\
\( w \) \hspace{1em} \text{Hanning filter function}

**Greek letters**

\( \alpha \) \hspace{1em} \text{dimensionless particle diameter} \hspace{1em} \Pi \hspace{1em} \text{pseudo pressure} \\
\( \delta t \) \hspace{1em} \text{laser pulse duration} \hspace{1em} \phi \hspace{1em} \text{azimuth angle} \\
\( \delta_{ij} \) \hspace{1em} \text{Kronecker delta function} \hspace{1em} \phi \hspace{1em} \text{electric potential} \\
\( \Delta \) \hspace{1em} \text{cut-off length scale} \hspace{1em} \phi_0 \hspace{1em} \text{electrode potential} \\
\( \epsilon \) \hspace{1em} \text{exposure} \hspace{1em} \rho \hspace{1em} \text{gas density} \\
\( \epsilon \) \hspace{1em} \text{viscous dissipation of } k \hspace{1em} \rho_C \hspace{1em} \text{total space charge density} \\
\( \epsilon_0 \) \hspace{1em} \text{permittivity of vacuum} \hspace{1em} \rho_E \hspace{1em} \text{electron space charge density} \\
\( \epsilon_r \) \hspace{1em} \text{relative permittivity } (\epsilon_r \equiv \epsilon / \epsilon_0) \hspace{1em} \rho_i \hspace{1em} \text{ion space charge density} \\
\( \eta \) \hspace{1em} \text{precipitation efficiency} \hspace{1em} \rho_p \hspace{1em} \text{particle space charge density} \\
\( \eta \) \hspace{1em} \text{Kolmogorov length scale} \hspace{1em} \rho_{xx} \hspace{1em} \text{autocorrelation coefficient} \\
\( \Gamma \) \hspace{1em} \text{scalar function} \hspace{1em} \sigma^2 \hspace{1em} \text{variance} \\
\( \lambda \) \hspace{1em} \text{mean free path of molecules} \hspace{1em} \sigma_{PIV}^2 \hspace{1em} \text{variance due to PIV} \\
\( \lambda \) \hspace{1em} \text{wave length} \hspace{1em} \sigma_{PIV/IP}^2 \hspace{1em} \text{variance due to PIV in image plane} \\
\( \lambda_{fx} \) \hspace{1em} \text{longitudinal spatial micro scale of } x \hspace{1em} \tau \hspace{1em} \text{time} \\
\( \mu \) \hspace{1em} \text{gas absolute viscosity} \hspace{1em} \tau \hspace{1em} \text{transmissivity} \\
\( \nu \) \hspace{1em} \text{gas kinematic viscosity} \hspace{1em} \tau \hspace{1em} \text{subgrid stresses} \\
\( \nu_{sgs} \) \hspace{1em} \text{subgrid scale viscosity} \hspace{1em} \tau_p \hspace{1em} \text{particle relaxation time} \\
\( \Omega \) \hspace{1em} \text{solid angle} \hspace{1em} \theta \hspace{1em} \text{angle} \\
\( \Omega \) \hspace{1em} \text{vorticity source} \hspace{1em} \theta_0 \hspace{1em} \text{threshold angle} \\
\( \omega \) \hspace{1em} \text{vorticity}

**Subscripts**

\( \text{cond} \) \hspace{1em} \text{conduction} \\
\( \text{conv} \) \hspace{1em} \text{convection} \\
\( d \text{iff} \) \hspace{1em} \text{diffusion} \\
\( E \) \hspace{1em} \text{electron} \\
\( i \) \hspace{1em} \text{ion} \\
\( m \) \hspace{1em} \text{mass} \\
\( p \) \hspace{1em} \text{particle}

**Superscripts**

\( w \) \hspace{1em} \text{window filtered function} \\
\( t \) \hspace{1em} \text{temporal or spatial fluctuation} \\
\( D \) \hspace{1em} \text{deviatoric} \\
\( MS \) \hspace{1em} \text{Mixed Scale} \\
\( + \) \hspace{1em} \text{length in wall units}
**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ACF</td>
<td>Auto Correlation Function</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge Coupled Device</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>DFT</td>
<td>Direct Fourier Transform</td>
</tr>
<tr>
<td>DPIV</td>
<td>Digital Particle Image Velocimetry</td>
</tr>
<tr>
<td>EHD</td>
<td>ElectroHydroDynamics</td>
</tr>
<tr>
<td>ESP</td>
<td>ElectroStatic Precipitator</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>HVDC</td>
<td>High-Voltage Direct-Current</td>
</tr>
<tr>
<td>IA</td>
<td>Interrogation Area</td>
</tr>
<tr>
<td>LDV</td>
<td>Laser Doppler Velocimetry</td>
</tr>
<tr>
<td>LES</td>
<td>Large Eddy Simulation</td>
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<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds Averaged Navier Stokes</td>
</tr>
<tr>
<td>SGS</td>
<td>SubGrid Scale</td>
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</table>
Chapter 1

Introduction

Electrostatic precipitators (ESPs) are devices used for separation of suspended aerosols from a carrier gas. The process was first patented by Cottrell in 1914 focused on the removal of a sulphuric acid mist. Since then, the process has been introduced in many different areas ranging in scale from applications in clean-room industries to filtering of exhaust air from cement processing or coal-fired power plants.

The ESP compete with other filter technologies such as cyclones and fabric filters. The advantages of the ESP include low maintenance and operation costs and the only apparent disadvantage is the high installation price. The popularity is further increased by the extremely high separation efficiencies in excess of 99.9%. The efficiency increases with gas residence time in the filter. The residence time can be increased in two ways by either increasing the filter length or the cross sectional area (i.e. lower gas velocity). Thus arbitrarily high collection efficiencies may be achieved at the expense of increased size of the ESP.

The physics of the electrostatic aerosol precipitation process is described in more detail in Chapter 2.

1.1 Motivation and objectives

As will be shown in Chapter 2, the precipitation velocity is approximately proportional to the particle diameter. Therefore, larger particles are precipitated very fast whereas the small (sub-micron) particles are hard to filter out. Further, the small particles are more sensitive to imperfections such as secondary gas motion and turbulence, which has been shown to decrease the collection efficiency. The secondary gas motion and increased turbulence are caused by the carrier gas being ionized and moving in a nonuniform electric field.

Most commonly used in the industry is the negative corona wire-plate ESP. The wires are kept at a high negative potential and the plates are grounded, hereby generating the interelectrode electrical field. Most experimental studies of electrohydrodynamic (EHD) flows in negative corona wire-plate ESPs have used smooth collector plates and smooth discharge wires. This configuration makes the corona tuft points be instationary and randomly distributed on the wires yielding a two-dimensional mean flow with spanwise vortical structures. The structures degenerate to undulations with increasing through-flow. Such flows were studied with hot-film by Davidson & Shaugh-
nessy [18], with hot-wire by Yamamoto & Velkoff [62] and with LDV by Kallio & Stock [25]. Leonard et al. [32] employed LDV, hot-wire and Schlieren photography to study the case of positive corona. General well documented features of the EHD flow embrace an increase of secondary velocity and turbulence level with increasing electric field level and decreasing bulk velocity.

Industrial ESP flows are usually three-dimensional due to artificially fixed corona points and details concerning these type of flow structures are still unknown. However, Zamány [63] performed both simulations and LDV experiments on different kinds of electrodes with fixed corona points. It was the purpose of this work to analyse flow structures appearing in a negative corona wire-plate ESP with smooth collector plates and barbed discharge wires. Hence, a gas flow that was three-dimensional in average. The objectives were:

- to demonstrate the application of the stereo PIV technique to a complex flow where the light scattering particles do not follow the gas motion due to a drift velocity
- to provide detailed instantaneous data of the flow field for different parametric situations
- to analyse strength of secondary motions, levels of turbulence and scales of motion
- to find out how and where turbulence is produced

The primary source of data was the stereo PIV technique supplemented by and compared with data obtained by LDV and LES.

1.2 Outline of the dissertation

The dissertation begins with a presentation of the theory that is necessary for understanding the experimental and numerical results. The results of [53] are based on the same measurements as presented here, whereas the results in [54, 55] are based on different experiments using the same test facility. Part of the text and results in [53, 54, 55] will be reproduced here. The dissertation is organized as follows:

Chapter 2: Theory of electrostatic precipitation including governing equations in reduced form. The chapter deals with particle free electrostatics, the interaction with the gas flow and the particle charging and motion.

Chapter 3: Description of the test facility, apparatus and test programs, and numerical and experimental electrostatic results from the test section geometry.

Chapter 4: Selected topics within the PIV theory that are considered relevant for the ESP application. The chapter treats the particle size dependency in PIV and measurement uncertainties.
Chapter 5: Presentation of the LES procedure used to obtain the numerical gas flow results.

Chapter 6: The data reduction used to generate the results presented in Chapter 7-9.

Chapter 7: Results of flow structures obtained primarily with PIV and to a lesser extent with LES.

Chapter 8: Results from time resolved data obtained by LES and LDV.

Chapter 9: Results from PIV and particle counting giving estimates of precipitation efficiencies.

Chapter 10: Conclusions.
Chapter 2

Theory of Electrostatic Precipitation

The process of electrostatic precipitation is quite straightforward: electrodes of different potential generate an electric field and through corona discharge create a ionized interelectrode gas in which suspended particles become charged and drift towards a collecting electrode. In general, airborne particles may carry a natural positive or negative charge attained by flame ionization, friction or the like. However, these charges are often too small for effective precipitation. Several artificial ways of charging the particles have been proposed through history, but both technically and economically the gaseous discharge called high-voltage direct-current (HVDC) corona has proved to be the best solution [60]. In particular, the negative corona process is utilized for industrial electrostatic precipitation and will therefore be the only technique described here.

The different steps of particle charging are shown in Figure 2.1. A free electron is generated by the corona process at the emitting electrode (1), the electron takes part of the electron avalanche (2), a drifting electron attaches to a neutral gas molecule forming an ion (3), and a suspended particle gets charged by ion impingement (4). The different steps are explained in more details in the following.

2.1 Electrostatics

2.1.1 Negative corona discharge and ionization

The process of particle charging by means of negative HVDC corona is sketched in Figure 2.1. In order to have a corona process a free electron must be strongly accelerated by the electric field (1) allowing the migrating electron to collide at high speed with a neutral gas molecule (2). The electron is knocked out of the molecule, and the net result is an additional free electron and a positively charged ion moving in opposite directions due to the electric field hereby constituting an electric current. The positive ion regains its neutral condition at the negative electrode and the electrons take part in the corona chain reaction (also known as electron avalanche). If the kinetic energy of the electron colliding with the molecule is too low the electron may get captured forming a negatively charged ion. In this way the interelectrode space is divided into
two subregions determined by the magnitude of the electric field governing the electron speed: (a) a small (<1 %) active region of high-level electric field, which constitutes the region of corona glow and (b) a large (>99 %) passive region where the kinetic energy of electrons is below the ionization energy making corona impossible:

(a) Active region: \( \text{electron} + \text{molecule} \rightarrow 2 \text{electrons} + \text{positive ion} \),

(b) Passive region: \( \text{electron} + \text{molecule} \rightarrow \text{negative ion} \).

The ionization energy is a characteristic quantity of molecules, however, the lowest level present in a gas mixture determines the necessary field strength [41, 60].

For smooth wire-plate negative corona, the corona points will form tufts on the wire and be very unsteady and randomly located on the wire, making both a hissing noise and an ultraviolet light. Increasing the high voltage results in closer spaced and more steadily positioned tuft points. The same phenomenon is seen with small irregularities on the wire fixing the corona points in space, which is often utilized in an industrial electrostatic precipitator (ESP) where electrodes may be barbed [17].

In the present study the negative corona process is fixed spatially at electrode discharge pins. However, the process is inherently unsteady in time generating pulses at 10-1000 kHz known as Trichel pulses [27]. This frequency is much higher than characteristic frequencies of the gas flow, thus the electric system may be treated as static when the gas kinetics is considered.
2.1.2 Electric field and current distribution

The electric field strength between electrodes in an ESP, \( \mathbf{E} \), is determined from the Maxwell equations. The Maxwell first and third law become

\[
\nabla \cdot (\varepsilon \mathbf{E}) = \rho_c, \\
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \approx 0,
\]

where \( \varepsilon \), \( \rho_c \), \( \mathbf{B} \) and \( t \) are gas permittivity, space charge density, magnetic field strength and time, respectively. For gases with negligible polarization, \( \varepsilon \) can be assumed constant, and the magnetic field of Eq. (2.2) is negligible due to the very small currents. Hence it is clear that the electric field is fixed from the electric surface charge on the electrodes and the interelectrode space charge, \( \rho_c \). In general the space charge consists in the sum of charge from electrons (\( \rho_E \)), ions (\( \rho_I \)) and charged particles (\( \rho_p \)), where the contribution from electrons usually is negligible [41]

\[
\rho_c = \rho_E + \rho_I + \rho_p \approx \rho_I + \rho_p.
\]

The irrotational nature of \( \mathbf{E} \) expressed in Eq. (2.2) allows it to be represented by the gradient of a scalar, electric potential, \( \phi \), as

\[
\mathbf{E} = -\nabla \phi.
\]

The constitutive Ohm law for the electrical current density \( \mathbf{J} \) may be written as

\[
\mathbf{J} = \mathbf{J}_{\text{cond}} + \mathbf{J}_{\text{diff}} + \mathbf{J}_{\text{conv}}
\]

where the conduction (\( \mathbf{J}_{\text{cond}} \)), diffusion (\( \mathbf{J}_{\text{diff}} \)) and convection (\( \mathbf{J}_{\text{conv}} \)) terms are given as

\[
\mathbf{J}_{\text{cond}} \equiv (\rho_E b_E + \rho_I b_I + \rho_p b_p) \mathbf{E} \\
\mathbf{J}_{\text{diff}} \equiv -(\mathbf{D}_E \nabla \rho_E + \mathbf{D}_I \nabla \rho_I + \mathbf{D}_p \nabla \rho_p) \\
\mathbf{J}_{\text{conv}} \equiv \rho_c \mathbf{u}
\]

where index \( E, I \) and \( p \) refers to electrons, ions and particles, respectively, \( b \) is mobility, \( \mathbf{D} \) is charge diffusion coefficient and \( \mathbf{u} \) is fluid velocity. Oglesby and Nichols [41] estimated the mobilities to: \( b_E \approx 6.6 \times 10^{-2} \text{ m}^2/\text{V}s \), \( b_I \approx 2.2 \times 10^{-1} \text{ m}^2/\text{V}s \) and \( b_p \approx 5 \times 10^{-7} \text{ m}^2/\text{V}s \). The conduction caused by electrons, \( \rho_E b_E \mathbf{E} \), and charged particles, \( \rho_p b_p \mathbf{E} \), may be neglected due to a small charge density of electrons, \( \rho_E \), and a small particle mobility, \( b_p \), respectively [6, 14, 16, 24]. The charge diffusion may consist of both thermal and turbulent diffusion, where the turbulent contribution is expected to be dominant [36], however, the term is usually negligible [14]. Finally, as \( \mathbf{u} \) is order of magnitudes smaller than the ion conduction velocity (\( \sim 60 \text{ m/s} \)), the convective current is usually ignored, which decouples the electrostatics from the fluid dynamics. In this way Eq. (2.5) reduces to

\[
\mathbf{J} \approx \rho_I b_I \mathbf{E}.
\]
The charge conservation reads
\[ \frac{\partial \rho_C}{\partial t} + \nabla \cdot \mathbf{J} = 0, \quad (2.10) \]
where the \( \frac{\partial \rho_C}{\partial t} \) term mainly covers small scale fluctuations leading to a charge diffusion, and the term is difficult to estimate. However, Zamany modeled it with a diffusion term \( \frac{\partial \rho_C}{\partial t} \equiv -D' \nabla^2 \rho_C \) with \( D' \gtrsim D_f \) [65]. Introducing this definition together with the corresponding steady current density, \( J' \equiv J - D' \nabla \rho_C \), into Eq. (2.9) and (2.10) and assuming sparse particle flow (\( \rho_C \approx \rho_l \)) the final set of equations reads
\[ \nabla \cdot \mathbf{E} = \frac{\rho_l}{\varepsilon}, \quad (2.11) \]
\[ \nabla \cdot \mathbf{J}' = 0, \quad (2.12) \]
\[ \mathbf{J}' = \rho_l b_I \mathbf{E} - D' \nabla \rho_C. \quad (2.13) \]
Solution method and results from the above equations applied to the laboratory scale model precipitator are presented in Section 3.3.

2.2 Electrohydrodynamics

The interaction of electrodynamics and hydrodynamics is often named electrohydrodynamics (EHD) and appears whenever a dielectric fluid is moving in an electric field. The dielectric fluid (e.g. air) is characterized by a very small electrical conductivity, which gives rise to only small currents even when an intense electric field is present. As a consequence, magnetic effects are negligible as discussed in the preceding section.

The flight of ions in an ESP is interrupted by collisions with neutral gas molecules, which are in Brownian equilibrium. Since the frequency of these collisions is very high, complete transfer of momentum from ions to the bulk gas is achieved. Thus the Coulomb force on the ions becomes an electric body force, \( \mathbf{F} \), acting on the gas as a continuum
\[ \mathbf{F} = \rho_l \mathbf{E} = \frac{1}{b_I} \mathbf{J}, \quad (2.14) \]
where \( b_I \) is treated as constant making \( \mathbf{F} \) dependent on only one variable, \( \mathbf{J} \). The Navier Stokes equations for an incompressible fluid yields
\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1}{\rho} F_i, \quad (2.15) \]
\[ \frac{\partial u_i}{\partial x_i} = 0, \quad (2.16) \]
where \( p \) is pressure, \( \rho \) is gas density and \( \nu \) is gas kinematic viscosity. The mean value of \( \mathbf{F} \) merely causes a pressure stratification, but the departure from the mean drives secondary flows. Further, it is noteworthy that unlike \( \mathbf{E} \), both \( \mathbf{F} \) and \( \mathbf{J} \) are generally rotational, and thus may be looked upon as a vorticity source. This becomes clear from inspection of the vorticity equation [52]
\[ \frac{\partial \omega_i}{\partial t} + u_j \frac{\partial \omega_i}{\partial x_j} = \omega_j \frac{\partial u_i}{\partial x_j} + \nu \frac{\partial^2 \omega_i}{\partial x_j \partial x_j} + \frac{1}{\rho} (\nabla \times \mathbf{F})_i, \quad (2.17) \]
where $\omega \equiv \nabla \times \mathbf{u}$ is the vorticity of the flow.

It can also be shown that the applied body force may influence the budget of turbulent kinetic energy, $k \equiv \frac{1}{2}u''_i' u''_j'$, where the Reynolds decomposition, $\mathbf{u}(\mathbf{x}, t) = \mathbf{U}(\mathbf{x}) + \mathbf{u}'(\mathbf{x}, t)$, has been introduced. The transport equation for $k$ reads

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \mathcal{D} + \mathcal{P} - \epsilon \tag{2.18}$$

where $\mathcal{D}$ is diffusion of $k$ which simply redistributes $k$ in the fluid, $\mathcal{P}$ is production of $k$ by deformation work, and $\epsilon$ is viscous dissipation of $k$. In the ESP flow case only $\mathcal{P}$ is affected

$$\mathcal{D} = -\frac{\partial}{\partial x_j} \left( \frac{1}{\rho} \frac{\partial p}{\partial x_j} + \frac{1}{2} \overline{u_i' u_i'} - 2\nu \overline{s_{ij}'} \right), \tag{2.19}$$

$$\mathcal{P} = -\overline{u_i' u_j'} S_{ij} + \frac{1}{\rho \dot{\theta}} \overline{u_i' J_i'}, \tag{2.20}$$

$$\epsilon = 2\nu \overline{s_{ij}' s_{ij}'} \tag{2.21}$$

where $S_{ij}$ and $s_{ij}'$ are the mean and fluctuating rate of strain respectively.

$\mathbf{F}$ may cause turbulence production in two ways, directly and indirectly. The indirect way is by the extra strain produced by the generated secondary flows. The direct way is through the second term on the right of Eq. (2.20). However, this term is only non-zero if a correlation exists between $\mathbf{u}'$ and $\mathbf{J}'$. In the case of fixed corona points the only apparent frequency of $\mathbf{J}'$ is the Trichel frequency of 10-1000 kHz, which is order of magnitudes higher than typical frequencies of $\mathbf{u}'$, making $\overline{u_i' J_i'} \approx 0$. If the corona points were not fixed (i.e. smooth wires) it is possible that the varying positions would give frequencies comparable to the energy containing frequencies of $\mathbf{u}'$ hence contributing directly to the turbulence production. Some numerical results of $\mathbf{F}$ are shown in Section 3.3, where the theory is applied to the test section geometry.

### 2.3 Particle charging and motion

The basic concept of electrostatic precipitation is to precipitate particles by use of the Coulomb force acting on the individual particles, $\mathbf{F}_p = q_p \mathbf{E}$, with $q_p$ being the individual particle charge. Thus the precipitation efficiency is strongly dependent on the particle charge level and it becomes desirable to charge the particles to a high degree.
2.3.1 Charging regimes

The corona charging process, step (4) in Figure 2.1, consists primarily of an ionic current to the particle surface,\(^1\) \(I_p\), found by integrating the current density, \(J_p\), normal to the particle surface, over the particle area, \(A_p\)

\[
I_p = \int_{A_p} -J_p \cdot n_p \, dA, \tag{2.24}
\]

where \(n_p\) is the particle surface normal. The constitutive equation describing the current density at the particle surface is very similar to the one describing current densities in the interelectrode domain, Eq. (2.5), when only the ionic currents are considered

\[
J_p = -D_T \nabla \rho_T + \frac{b_T \rho_T E}{\varepsilon}. \tag{2.25}
\]

The two terms on the right hand side are named \textit{diffusion charging} and \textit{field charging} respectively. As stated in the equation, the two processes are in play simultaneously, but one may dominate the other in different situations. Because of the generally high field levels in ESPs the relative importance of the two terms is primarily particle size dependent dividing the continuum regime described by Eq. (2.5) in the presence of moderate fields (50 kV/m < |\(E|\) < 300 kV/m) into a) a field charging region for particles greater than 0.5 \(\mu\)m, and b) a diffusion charging region for particles smaller than 0.1 \(\mu\)m [20]. For intermediate particles of 0.1-0.5 \(\mu\)m none of the processes may be neglected.

\textbf{A) Field charging:} Particles get charged by the field charging process when a gas ion attach to the particle under the influence of the electric field – the process is also referred to as impact charging. If a particle, submerged into an electric field, is either conducting or dielectric its presence will distort the electric field by intersecting field lines, and ions transported along these field lines will attach to the particle surface. This ionic current will continue, though with decreasing charging rate, until the charge induced electric field around the particle is strong enough to prevent any further ions to reach the particle, i.e. no intersection of field lines.

\textbf{B) Diffusion charging:} By diffusion charging the ion attachment is due to the ion density gradient at the particle surface, and caused by the Brownian (thermal) motion of the gas ions. The charging thus increases with temperature and is independent of the electric field. The ions diffuse through the gas and may collide with particles. The frequency of collision is increased by the attractive electrical image-force caused by the image-charge in the particle. The image-force is usually quite small and short range (\(\sim\) particle diameter), which is also the case for charged particles approaching a conducting surface [12, 33].

\(^1\)Some authors claim that also free electron charging is occurring at elevated temperatures in negative corona. However, this process has at present not been described theoretically, and no experiments have been able to isolate this effect [35].
The simplicity of the widely used two-regime models leads in some cases to non-
acceptable results, especially in the intermediate particle size range. This is dealt with
by the continuum models, among which the one proposed by Fjeld et al. [20] presently
is the standard against which other models are judged. For a review on the subject
and current models in use the reader is referred to Lawless [29] and Fjeld et al. [20].

2.3.2 Saturation charge

The particle field charging is usually a very fast process and saturation is for typical
ESP situations achieved within approximately 10 ms [42, 60]. The saturation charge,
$q_p^\infty$, determines the particle mobility and is mainly dependent on particle size, electrical
field strength, temperature and particle material. Different models have been proposed
where the one by Cochet [15] generally provides adequate accuracy

$$q_p^\infty = \left(1 + 2\lambda/d_p\right)^2 + \frac{2}{1 + 2\lambda/d_p} \left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right) \pi \epsilon_0 |E|d_p^2,$$

(2.26)

where $\lambda$ is the mean free path of molecules, $\epsilon_0 = 8.85 \times 10^{-12}$ C$^2$/(m$^2$N) is the vacuum
permittivity, $\epsilon_r \equiv \epsilon/\epsilon_0$ is the relative permittivity of the particle material, and $d_p$ is the
particle diameter. Since $E$ is a function of position the saturation charge is a function
of the individual particle paths. However, particles retain their highest charge level,
thus a representative value of $E$ is usually used, which could be based on the mean value
of $|E|$. In air $\lambda$ decreases with temperature from about 0.07 $\mu$m at room temperature
to about 0.1 $\mu$m at 150$^\circ$C, however, the influence is only important for submicron
particles. This is obvious from Figure (2.2), where the predicted saturation charge is
shown for particles in atmospheric air at room temperature with an average electric
field corresponding to the one used in the laboratory experiments of this work. The
three different models shown differ in complexity - the simplest one is by Zamany [63],
$q_p^\infty = 1.2 \times 10^{-6} q_p^{1.75}$, and the next one by White [60] is $q_p^\infty = 3\pi \epsilon_0 |E|d_p^2$. The Cochet
model is the only one presented, which takes into account both field and diffusion
charging.

In the following the model of Cochet has been used assuming fully charged particles
and neglecting the influence of the mean free path of gas molecules giving

$$q_p = q_p^\infty = \frac{3\epsilon_r}{\epsilon_r + 2} \pi \epsilon_0 E_0 d_p^2,$$

(2.27)

where $E_0$ is a characteristic field level approximated as $E_0 = \phi_0/l_y$ with $l_y = 0.1$ m
being the distance between pin electrode and collector plate, and $\phi_0$ is the applied high
voltage.

2.3.3 Particle motion and Coulomb drift

The motion of a particle suspended in a gas is governed by Newton’s laws of classical
mechanics. In 1888 Basset derived a general equation of motion for a single spherical
particle taking into account the different forces acting on the particle. Since that several
Figure 2.2: Particle saturation charge vs particle diameter using three different models. 
$T = 25 \, ^\circ$C, $p = 1$ atm, $|E| = 280$ kV/m and $\varepsilon_r = 8.6$.

papers have appeared correcting or modifying terms in the equation [34]. The simple
Basset equation [37] is shown here with the additional Coulomb body force term

$$
\frac{\pi d_p^3 \rho_p}{6} \frac{d\mathbf{u}_p}{dt} = -3 \pi \mu d_p (\mathbf{u}_p - \mathbf{u}) + \frac{\pi d_p^3 \rho}{6} \frac{d\mathbf{u}}{dt} - \frac{3}{2} \frac{\pi d_p^3 \rho}{6} \frac{d(\mathbf{u}_p - \mathbf{u})}{dt} - \frac{3}{2} \sqrt{\frac{\pi \rho \mu}{t}} \int_{t_0}^{t} \frac{d(\mathbf{u}_p - \mathbf{u})}{dt'} \frac{1}{\sqrt{t - t'}} \, dt' + q_p \mathbf{E},
$$

(2.28)

where the $p$-index refers to the particle and no index refers to the fluid. The term on
the left is the acceleration force and the first term on the right is the viscous resistance
according to Stokes’ law, which is applicable when the Reynolds number based on the
particle diameter and the slip velocity, $Re_p \equiv d_p |\mathbf{u}_p - \mathbf{u}| / \nu \approx 0.01$, is smaller than 0.25
(Stokes range) [47]. The second term on the right is due to a pressure gradient in the
flow and the third term represents the resistance of an inviscid fluid to the acceleration
of the sphere. The fourth term on the right is the ‘Basset history integral’, which
expresses viscous drag forces arising from unsteadiness in the flow. The last term is the
Coulomb body force term. The gravitational and centrifugal forces have been neglected
since a rough estimate showed them to be negligible. Neglecting the ‘Basset history
integral’ and having a large density ratio, $\rho_p / \rho \gg 1$, the equation of motion (2.28) may
be greatly simplified to only involve the acceleration force, drag force and Coulomb
force [37]

$$
\frac{d\mathbf{u}_p}{dt} = - \frac{1}{\tau_p} (\mathbf{u}_p - \mathbf{u}) + \frac{q_p}{m_p} \mathbf{E},
$$

(2.29)
2.3 Particle charging and motion

where $\tau_p$ is the relaxation time

$$\tau_p \equiv \frac{\rho_p d_p^2}{18 \mu},$$

and $m_p$ is the mass of the particle. For the mean diameter Rolovit seeding particle of Table 3.5 page 22 the relaxation time is $\tau_p \approx 25 \mu s$.

Treating the Coulomb force as static, Eq. (2.29) may be analyzed by the method proposed by Hjelmfelt and Mockros in 1966 to study the particle response to turbulent frequencies, $f$, of the flow [37]

$$\frac{\overline{u_p^2}}{\overline{u^2}} = (1 + 2\pi \tau_p f)^{-1},$$

where the velocity variance ratio is chosen as a parameter of the ability of the particle to follow the gas flow frequencies. The particle response of Eq. (2.31) is plotted in Figure 2.3 for the poly-disperse particles used in this study (see Figure 3.3 on page 23). Values of mean aerodynamic diameter with interval of standard deviation diameter are shown. It appears that the response is accurate up to $\sim 100$ Hz and acceptable up to $\sim 1$ kHz, which is fully adequate since numerical simulations showed that practically no turbulence frequencies above 80 Hz are present in the flow (see Chapter 8). Since the solid particles used were not spherical the whole theory described in this section becomes much more complicated. However, the instrument used to measure particle size (see Section 3.2.3) measured the aerodynamic diameter based on the Stokesian drag assumption (sometimes denoted as 'Stokes diameter'). Thus, Eq. (2.29) and Eq. (2.31) may be used with an acceptable accuracy.

Since turbulent frequencies of the flow generally are smaller than 80 Hz the inertia term of Eq. (2.29) is several orders of magnitude smaller than the Coulomb force. If the term is neglected it is possible to calculate a steady three-dimensional drift velocity, $u_e$, defined as the velocity slip

$$u_e \equiv u_p - u = \frac{q_p}{3 \pi \mu d_p} E = \frac{1}{\mu \varepsilon_r + 2 d_p E_0 E},$$

where Eq. (2.27) has been applied. Note that $u_e$ is proportional to $d_p$ and has the direction of the electrostatic field.

2.3.4 Particle collection

In the preceding section the kinetics of a single particle was treated. But with regard to particle collection and overall efficiency, the industrial precipitation conditions are so complicated that the single particle theory becomes too troublesome. The gas itself is moving in a complex turbulent flow pattern and myriads of poly-disperse particles are present in cloud form making single particle tracking practically impossible. In addition, the high particle density (i.e. upstream in an ESP) and momentum may
well be sufficient to influence gas momentum. As a result the entire problem must be handled by statistical methods in terms of e.g. a concentration of particles. One of the first models was developed by Deutsch in 1922 [41] giving a collection efficiency as a function of axial position, \( x \), electrical drift velocity, \( V_e \), bulk velocity, \( U_0 \) and electrode to plate distance, \( l_y \)

\[
\eta(x) \equiv 1 - \frac{<C(x, y, z)>_{yz}}{<C(0, y, z)>_{yz}} \\
= 1 - \exp \left(-\frac{V_e}{U_0 l_y} x \right)
\]  (2.33)

where \( <C(x, y, z)>_{yz} \) is the average particle concentration in the \( yz \)-plane at the axial position, \( x \). The one-dimensional Deutsch equation is based on several assumptions [41] among which is the uniform distribution of particles at any axial position, hereby underestimating \( \eta \). In fact, a finite diffusion of particles makes the distribution non-uniform. A general transport equation for the concentration is

\[
\nabla (C \mathbf{u}_p) - \mathcal{D}_p \nabla^2 C = 0,
\]  (2.35)

where \( \mathcal{D}_p \) is the turbulent particle diffusivity due to turbulent gas fluctuations. The convective diffusion equation (2.35) is among others treated by Leonard et al. [31] who estimates \( \mathcal{D}_p \). However, since particles follow the gas motion it is common practice to assume that \( \mathcal{D}_p \) is of the same order of magnitude as the turbulent diffusivity of gas.

Figure 2.3: Particle tracking response for a \( d_p = 1.75 \) \( \mu m \) spherical particle (solid line) and \( d_p = 1.75 \pm 0.71 \) \( \mu m \) particle (dashed lines). Levels of \( d_p \) corresponds to mean value and interval of standard deviation for the Rolovit seeding particles used.
momentum. The influence of secondary gas velocities is represented in \( u_p \), and White [60] shows that for smaller particles \( (d_p < 10-20 \ \mu \text{m}) \) the motion is mainly determined by the gas flow and to a lesser extend by electrical effects. Similar results were found numerically by Larsen & Sørensen [28].

The efficiency determined from Eq. (2.35) is useful for ideal laboratory precipitators. But the fact that industrial precipitator efficiencies often are far below indicates that nonideal effects are dominating. These could be in the form of re-entrainment, sneakage, rapping losses and back corona [31, 60], which will not be treated in this work.

### 2.4 Summary

In order to understand gas and particle behavior during the process of electrostatic precipitation, this chapter outlined the general principles. Initially, the equations governing the electrostatics were stated and then reduced as much as possible. The electrostatics was shown to be independent of the fluid dynamics, hereby decoupling the numerical problems of electrostatics and fluid dynamics. However, the one-way interaction was shown to influence gas flow behavior in several ways. The extra terms appearing in gas flow transport equations of both momentum, vorticity an turbulent kinetic energy were outlined and discussed.

The second half of the chapter treated the suspended particles affected by both the gas flow turbulence and electric effects such as ion current and electrostatic field. First, the charging process, which would last less than \( \sim 10 \ \text{ms} \) was described. Second, an estimation of the saturation charge level was given. The motion of a single charged particle was developed through the reduced Basset equation, and it was shown that the seeding particles of this study would follow the turbulent gas frequencies up to approximately 1 kHz. Neglecting the particle inertia force the steady three-dimensional drift velocity became proportional to the particle diameter. Finally, statistical particle collection was described based on the convective diffusion equation, and the Deutsch equation was given.

The suspended particles may be used as transmitters of scattered light when optical measuring techniques are applied. The primary measuring technique of this work was the Particle Image Velocimetry, which will be treated in Chapter 4.
Chapter 3

Experiments

3.1 Experimental facility

A laboratory-scale ESP facility has been designed with focus on maximum optical accessibility for LDV and PIV measurements. Figure 3.1 presents a schematic drawing of the facility, which is an open-circuit, variable-speed wind tunnel with a 1 m long test section (5) of cross section 0.2×0.2 m. The flow is driven by a fan (9) placed downstream of the facility exhausting into the atmosphere. Air from the laboratory is drawn through the inlet nozzle (1) and an 0.8×0.8 by 1.0 m long settling chamber (2) with two screens (orifice diameters of 12 and 3 mm) and a wire-mesh for eliminating large scale structures. A 16:1 contraction (3) forms the inlet to a 1.0 m straight extension section of 0.2×0.2 m cross section (4) for further improvement of flow uniformity. The test section (5) supports up to 7 discharge electrodes (denoted with dots in Figure 3.1) and is described in more details in the following paragraph. A downstream extension section of 0.3 m (6) serves as a buffer between the test section and the filter box (7). The filter box has a cross section of 0.5×0.5 m and a length of 0.75 m and contains fabric filter bags for collecting non-precipitated particles. Bulk flow rate is measured by an orifice-plate flow meter located 1.2 m upstream of the fan and 4.5 m downstream of the inlet to the 121 mm diameter straight pipe (8). The convergent-divergent inlet nozzle (1) serves the purpose of ensuring a uniform distribution of seedling particles added to the air intake. Liquid seeding (water-glycerol mixture, olive oil) was generated by pressurized air, and solid particles (limestone) were supplied from a vertical cylinder-piston arrangement. The solid particles were brushed off the top of the cylinder and sucked into the flow, hereby minimizing agglomeration effects.

Figure 3.1: Experimental facility. 1: inlet nozzle, 2: settling chamber with screens, 3: contraction, 4: extension, 5: test section with electrodes, 6: extension, 7: filter box, 8: straight pipe with orifice plate, 9: exhaust fan.
Three sectional views of the test section are shown in Figure 3.2 together with the coordinate system to which all experimental data (PIV and LDV) are referred. The origin is located at the midpoint of the bottom plate at the inlet to the test section. The test section consists of grounded aluminum top and bottom plates (collector plates) and 8 mm thick glass sidewalls for optical access. The glass walls support seven 3 mm brass rod electrodes placed mid between the two collector plates and spaced 100 mm apart. Each pin electrode is pierced by two, axially oriented, 10 mm long and 1 mm diameter pins placed 50 mm from the glass walls. The pin electrodes are fixed by teflon holders at the glass walls and kept at an adjustable high negative DC-potential of $\phi_0 \in [18, 28]$ kV. The two pins act as fix points for tuft coronas (i.e. 4 corona points on each electrode). In this way the mean electric field is well defined, though still unsteady in nature due to the unsteadiness of the corona process.

The high-voltage is applied by a low ripple, DC power supply (Hypotronics 800PL) interfaced to the electrodes outside the channel by a brass bus. The bus is displaced 250 mm in order to reduce electric field distortions causing lack of symmetry inside the channel. Also the PIV setup (CCD cameras and laser sheet) are shown.

**Figure 3.2:** Three sectional views of the $0.2 \times 0.2 \times 1.0$ m test section from Figure 3.1 showing the PIV test volumes (hatched areas), coordinate system and PIV setup. Upper: HVDC bus outside the test section supports seven electrodes. Middle: dots indicate electrode positions along center plane. Lower: electrodes with axially oriented corona pins and PIV setup.
The geometry of the test section with pin electrodes suggests a spatially periodic arrangement of \( l_x \times l_y \times l_z = 100 \times 100 \times 50 \) mm 'unit cells' with boundaries of electric symmetry. However, the no-slip boundary condition at the side-walls influences the flow in these cells. Therefore measurements were focused on the central unit cells \((-50 \leq z \leq 50 \) mm). As indicated in Figure 3.2 measurements were also confined to the lower half of the section mid between electrodes for \( x \in [150, 550] \) mm and to the volume extending from \( x = 650 \) mm to \( x = 750 \) mm through flow-mapping.

The mean bulk flow velocity \( U_0 \) was measured by an ISO 5167-1980 international standard orifice-plate flow meter, the pressure drop being read from a precision micro-manometer. The rate of solid particle addition was controlled by a step motor interfaced to a PC. The mean current density \( J_m \), which determines the electric field strength, was measured from an isolated 60×160 mm section of the bottom plate underneath the mapped volume to avoid boundary effects from the glass walls.

### 3.2 Measurements

Measurements were primarily performed with stereoscopic digital PIV, but supplementary LDV measurements were carried out. This section describes the apparatus used and the different test series including the seeding material. Finally, the spatial and temporal resolutions are compared and discussed.

#### 3.2.1 PIV setup and test program

The PIV system of Figure 3.2 was mounted on a three-axes traversing bench with the vertically diverging laser sheet horizontally directed and perpendicular to the bulk flow. The cameras were placed in angular displacement on each side of the laser sheet mounted in Scheimpflug condition [44] with a viewing angle of \( \theta \approx \pm 45^\circ \) to the laser sheet. The cameras were 1k×1k CCD cameras (Kodak Megaplus ES 1.0) with 60 mm lenses (Nikon) using an F-number of 5.6. The laser sheet was supplied by a double cavity Nd-YAG laser (Continuum Surelite I-10) delivering 170 mJ light pulses, and the sheet thickness was about 3 mm, which was a compromise between spatial accuracy and capturing particles having a large out-of-plane velocity \( (U_0 \geq 1.0 \) m/s). The area covered by both cameras was \( z \in [-75, 75] \) mm by \( y \in [1, 94] \) mm. A Dantec FlowMap PIV2100 processor was employed to handle data acquisition and velocity calibration, and data were taken in dual frame, single exposure mode at 2 Hz with a pulse separation of 400 \( \mu \)s and a minimum sample size of 400. Processing was performed in 32×32 pixel interrogation areas using adaptive correlation with 50% overlap between interrogation areas. The final vector maps contained 53×52 three-component velocity vectors giving a spatial resolution of \((\Delta y, \Delta z) = (1.87, 2.89) \) mm. Table 3.1 summarizes the essential PIV parameters.

Table 3.2 shows the 4 PIV test series based on variation of axial position, \( x \), bulk flow velocity, \( U_0 \), and mean current density, \( J_m \). A) Reference flow covers the reference situation concerning \( U_0 \) and \( J_m \) and is measured as far downstream as possible to have a developed flow. In B) Flow development the axial position of the PIV plane is varied to study the flow development. For the C) Parametric effects case both \( U_0 \) and \( J_m \) have
Flow geometry | axial velocity normal to light sheet
Observation distance | $\sim 0.60$ m
Illumination | Nd:YAG laser, 170 mJ/pulse
Laser sheet thickness | $\sim 3$ mm
Pulse delay | $\Delta t = 400 \mu$s
Seeding material | $CaCO_3$ (limestone)
Recording medium | 2 frame CCD (1008×1018 px)
Recording method | dual frame, single exposure
Recording lens | $f = 60$ mm, $f_\# = 5.6$
Field of view | $\Delta Y \times \Delta Z \approx 93 \times 150$ mm
Interrogation area | 32×32 px
Correlation method | adaptive
Overlap | 50%
Subpixel interpolator | Gaussian
Validation method | local median
Data rate | 2 Hz
Number of vector maps | 400-1000

<table>
<thead>
<tr>
<th>x [mm]</th>
<th>$U_0$ [m/s]</th>
<th>$J_m$ [mA/m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Reference flow</td>
<td>750</td>
<td>1.0</td>
</tr>
<tr>
<td>B) Flow development$^\dagger$</td>
<td>150,250,…,750</td>
<td>1.0</td>
</tr>
<tr>
<td>C) Parametric effects</td>
<td>750</td>
<td>1.0</td>
</tr>
<tr>
<td>D) Unit cell flow</td>
<td>650,660,…,750</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 3.1: PIV acquisition and processing parameters.

been varied, and in D) Unit cell flow a volume of unit cell length has been flow-mapped by 10 mm separation in x to study unit cell variations.

<table>
<thead>
<tr>
<th>x [mm]</th>
<th>$U_0$ [m/s]</th>
<th>$J_m$ [mA/m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Reference flow</td>
<td>750</td>
<td>1.0</td>
</tr>
<tr>
<td>B) Flow development$^\dagger$</td>
<td>150,250,…,750</td>
<td>1.0</td>
</tr>
<tr>
<td>C) Parametric effects</td>
<td>750</td>
<td>1.0</td>
</tr>
<tr>
<td>D) Unit cell flow</td>
<td>650,660,…,750</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 3.2: PIV test series. $^\dagger$ The measurements at $x = 150$ mm is performed at $x = 250$ by removing the first electrode.

3.2.2 LDV setup and test program

LDV measurements were performed to compare with PIV velocity moments and to get time resolved experimental data. The LDV system was a two-component, four-beam, fiber optics system with back scatter collection optics and two Dantec BSA (57N21/57N35) signal processors. The light was supplied by a 4 W Coherent Innova 90 Argon-Ion laser using the blue and the green line. The LDV probe was mounted on a three-axes traversing bench and the same seeding as in the PIV experiments was used. Table 3.3 summarizes the essential LDV parameters.
### Optics

<table>
<thead>
<tr>
<th></th>
<th>Green (u)</th>
<th>Blue (v)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>514.5</td>
<td>488</td>
<td>nm</td>
</tr>
<tr>
<td>Beam diameter</td>
<td>1.35</td>
<td>1.35</td>
<td>mm</td>
</tr>
<tr>
<td>Beam expander ratio</td>
<td>1.98</td>
<td>1.98</td>
<td>-</td>
</tr>
<tr>
<td>Beam spacing</td>
<td>35.1</td>
<td>35.1</td>
<td>mm</td>
</tr>
<tr>
<td>Lens focal length</td>
<td>310</td>
<td>310</td>
<td>mm</td>
</tr>
</tbody>
</table>

### Measurement volume

<table>
<thead>
<tr>
<th></th>
<th>Value 1</th>
<th>Value 2</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>76</td>
<td>73</td>
<td>μm</td>
</tr>
<tr>
<td>Length</td>
<td>682</td>
<td>638</td>
<td>μm</td>
</tr>
<tr>
<td>Fringe spacing</td>
<td>2.3</td>
<td>2.2</td>
<td>μm</td>
</tr>
<tr>
<td>Number of fringes</td>
<td>33</td>
<td>33</td>
<td>-</td>
</tr>
</tbody>
</table>

### BSA setup

<table>
<thead>
<tr>
<th></th>
<th>Value 1</th>
<th>Value 2</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSA filter center</td>
<td>1.08</td>
<td>0</td>
<td>m/s</td>
</tr>
<tr>
<td>BSA filter span</td>
<td>2.16</td>
<td>4.62</td>
<td>m/s</td>
</tr>
<tr>
<td>PM high voltage</td>
<td>1000</td>
<td>1400</td>
<td>V</td>
</tr>
<tr>
<td>Signal gain</td>
<td>35</td>
<td>35</td>
<td>dB</td>
</tr>
</tbody>
</table>

### Data collection

<table>
<thead>
<tr>
<th></th>
<th>Value 1</th>
<th>Value 2</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of samples</td>
<td>500</td>
<td>500</td>
<td>-</td>
</tr>
<tr>
<td>Dead time</td>
<td>200</td>
<td>200</td>
<td>ms</td>
</tr>
</tbody>
</table>

**Table 3.3:** LDV setup parameters. †For time series, no dead time is applied and the number of samples is 70,000-120,000 sampled in 100 s.
Table 3.4 shows the 2 LDV test series. A) Reference flow is a line traverse in $z$ for reference settings of $U_0$ and $J_m$. The purpose is to compare velocity moments. B) Time series are one-point measurements with no dead-time to study temporal scales and frequencies.

<table>
<thead>
<tr>
<th>$x$ [mm]</th>
<th>$y$ [mm]</th>
<th>$z$ [mm]</th>
<th>$U_0$ [m/s]</th>
<th>$J_m$ [mA/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Reference flow</td>
<td>750</td>
<td>50</td>
<td>$[-95,95]$</td>
<td>1.0</td>
</tr>
<tr>
<td>B) Time series</td>
<td>750</td>
<td>50</td>
<td>50</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 3.4: LDV test series.

3.2.3 Seeding particles

Measurements were initially made with seeding droplets of a water-glycerol mixture, but the electric conductivity of this seeding introduced electrical asymmetries as droplets deposited on the walls. Thus olive oil was used subsequently and formed the basis for the measurements presented in [54, 55]. Finally, the liquid seeding was substituted with solid particles to simulate real ESP behavior more correctly in regard to agglomeration, deposition, attachment, re-entrainment etc.

The physical properties of the olive oil and solid particles (Rolovit) appear in Table 3.5. The size measurements of olive oil particles were carried out by a Lasair 1002 instrument from Particle Measuring Systems. The aerodynamic diameter distribution of the Rolovit seeding particles is shown in Figure 3.3 and was measured 250 mm downstream of the last electrode by a TSI-3320 Aerodynamic Particle Sizer [7] by the time-of-flight procedure for reference conditions ($U_0 = 1.0$ m/s and $J_m = 0.40$ mA/m²).

Upstream compositions were slightly different with $\overline{d_p} = 2.00$ µm and $\overline{d_p^2} = 0.56$ µm² (cf. Figure 9.1, page 92), but for simplicity $\overline{d_p} = 1.75$ µm and $\overline{d_p^2} = 0.50$ µm² are used in all data reduction. Particles smaller than $d_p = 0.5$ µm are added to one interval in Figure 3.3 and shows a relatively high concentration.

<table>
<thead>
<tr>
<th>Type</th>
<th>Olive oil</th>
<th>Rolovit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composition</td>
<td>spherical droplets</td>
<td>solid particles</td>
</tr>
<tr>
<td>Size measurements</td>
<td>light scatter</td>
<td>time-of-flight</td>
</tr>
<tr>
<td>Mean diameter, $\overline{d_p}$</td>
<td>0.41 µm</td>
<td>1.75 µm</td>
</tr>
<tr>
<td>Variance, $\overline{d_p^2}$</td>
<td>0.063 µm²</td>
<td>0.50 µm²</td>
</tr>
<tr>
<td>Density, $\rho_p$</td>
<td>900 kg/m³</td>
<td>2,700 kg/m³</td>
</tr>
<tr>
<td>Refractive index, $n$</td>
<td>1.47</td>
<td>1.68</td>
</tr>
<tr>
<td>Relative permittivity, $\epsilon_r$</td>
<td>3.2</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Table 3.5: Physical properties of seeding materials.
Figure 3.3: Aerodynamic diameter distribution (by number) of Rolovit particles, \( f(d_p) \), measured by TSI-3320 Aerodynamic Particle Sizer for \( d_p \in [0, 20] \) \( \mu m \). Abscissa is truncated at \( d_p = 8 \) \( \mu m \).

### 3.2.4 Spatial and temporal resolutions

The experimental resolutions of PIV and LDV from Table 3.2 and 3.4 are repeated in Table 3.6 together with computational resolutions obtained by LES (cf. Chapter 5). The idea is to compare both spatial and temporal resolutions, which were very different for the three techniques. The spatial resolution of PIV is subdivided into 4 cases depending on the cross correlation window size, i.e. interrogation area (IA). It is clear that LDV provided excellent spatial and temporal resolutions. LES was performed with an even finer temporal resolution, but the spatial resolution was not as fine. PIV, on the other hand, had a very low temporal resolution, and the spatial resolution was also coarse.

<table>
<thead>
<tr>
<th></th>
<th>( \Delta x ) [mm]</th>
<th>( \Delta y ) [mm]</th>
<th>( \Delta z ) [mm]</th>
<th>( \Delta t ) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDV</td>
<td>0.07</td>
<td>0.07</td>
<td>0.7</td>
<td>( \sim 10^{-3} )</td>
</tr>
<tr>
<td>PIV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IA = 8\times8 px</td>
<td>( \sim 3 )</td>
<td>0.47</td>
<td>0.72</td>
<td>0.5</td>
</tr>
<tr>
<td>IA = 16\times16 px</td>
<td>( \sim 3 )</td>
<td>0.94</td>
<td>1.4</td>
<td>0.5</td>
</tr>
<tr>
<td>IA = 32\times32 px</td>
<td>( \sim 3 )</td>
<td>1.9</td>
<td>2.9</td>
<td>0.5</td>
</tr>
<tr>
<td>IA = 64\times64 px</td>
<td>( \sim 3 )</td>
<td>3.7</td>
<td>5.8</td>
<td>0.5</td>
</tr>
<tr>
<td>LES</td>
<td>3.3</td>
<td>[0.5, 4.8]</td>
<td>3.3</td>
<td>( 2 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

Table 3.6: Approximate spatial and temporal resolution of LDV, PIV and LES. Default PIV interrogation area (IA) is 32\times32 px.

One problem with the presented PIV setup is the inferior spatial resolution defined by the area of the IA and the thickness of the light sheet. In fact, the measured displacement is the spatial 'mean' displacement in the interrogation volume. The effect of
changing the IA size is shown in Figure 3.4, where 1000 snapshots have been processed with the 4 different IA sizes. Note that the thickness of the interrogation volume, \( \Delta r \), is kept constant. Spatial mean values of \( V \) and \( V_{\text{rms}} \) along a line in the PIV image are shown, normalized by the value at 32×32 px IA. Clearly, the registered turbulence increases with smaller IA. This phenomenon may be explained by the spatial integration allowing turbulent fluctuations in different directions within the IA to cancel out. However, smaller IA does also increase the measurement uncertainty and may cause a fictitious increase in turbulence, but this effect is estimated to be small. The variation in the mean value, \( V \), is small and non-monotonous. One may conclude that mainly higher order moments are influenced by the relatively poor spatial resolution. 1st and 2nd order moments are compared to Laser Doppler Velocimetry measurements in Section 7.1 (cf. Figure 7.3). All subsequent image processing is done with 32×32 px IA since the number of erroneous vectors would increase considerably in some areas if smaller IA’s were used.

![Figure 3.4: Mean values of \(|V|\) and \(V_{\text{rms}}\) at \((x, y) = (750, 50)\) mm, \(z \in [-50, 50]\) mm vs. size of interrogation area (IA). Values normalized by levels at 32×32 pixel. \(U_0 = 1.0\) m/s, \(J_m = 0.40\) mA/m².](image)

### 3.3 Electrostatic results

To correct measured particle velocities for electric drift and to solve the Navier Stokes equations with the Coulomb body force it is necessary to solve the electrostatic equations of Section 2.1 applied to the geometry of the test section. Prior to this, measurements of the voltage-current behaviour was performed in the test section, with the results of Figure 3.5. The mean current density was measured in three 60×160 mm ports in the lower collector plate, and each point in the figure represents the mean of 1000 samples measured at 35 Hz. The standard deviation at each point is also shown. It appears that the corona onset voltage is \(\phi_0 \approx 15\) kV with both \(J_m\) and its standard deviation increasing with \(\phi_0\).

The nonlinear numerical problem consists of solving the reduced electrostatic equations, Eq. (2.11)-(2.13), which has been done by the code developed by Zamany [63]. The charge diffusivity and ion mobility was estimated to \(\mathcal{D}^+ = 10^{-3}\) m²/s and
3.3 Electrostatic results

![Graph with data points and error bars](image)

**Figure 3.5:** Measured mean current density at collector plate vs. applied high voltage. Error bars denote standard deviation.

\[ b_I = 2.1 \times 10^{-4} \text{ m}^2/\text{s}, \text{ respectively} \ [64]. \] The imposed boundary conditions were: \( \phi = 0 \) at the collector plate, \( \phi = \phi_0 \) at the electrode, and \( \rho_I = \rho_{\text{in}} \) at corona points. \( \rho_{\text{in}} \) was adjusted iteratively to get the experimentally determined current density \( J_m = J_m(\phi_0) \) at the collector plate (cf. Figure 3.5).

The result in form of the Coulomb body force, \( \mathbf{F} = \rho_I \mathbf{E} \), is shown in Figure 3.6 in one unit cell. Field lines are also representative for the electric drift velocity since \( \mathbf{u}_e \propto \mathbf{E} \). It appears that \( \mathbf{F} \) is practically constant and perpendicular to the collector plate except in the vicinity of the electrode. The size of \( \mathbf{F} \) is very large at the edges of the discharge pins and almost constant and small throughout the rest of the domain.

As argued in Section 2.2 only the departure from the mean of \( \mathbf{F} \) drives secondary flows, i.e. \( \mathbf{F} - \langle \mathbf{F} \rangle_{yz} \), where \( \langle \ldots \rangle \) denotes spatial averaging in the directions given by the indices. Of particular interest are the \( yz \)-components perpendicular to the direction of bulk flow, because this flow is fairly free to develop and be sustained. Due to the large inertia of the bulk fluid, secondary flows in the axial direction will hardly appear. Figure 3.7 shows axial mean body force (left) and the net axial mean body force (middle), averaged over the length of the unit cell (\( l_x = 100 \text{ mm} \)), the latter being responsible for generating secondary flows in the form of axial rolls superposed the axial bulk flow. This may also be viewed as a result of the axial vorticity source distribution, \( \Omega_x = (\nabla \times \mathbf{F})_x \), also appearing in Figure 3.7 (right) averaged in axial direction.

Based on Eq. (2.32) and numerical solution of \( \mathbf{E} \), the drift velocity, \( \mathbf{u}_e = \mathbf{u}_e(d_p, \mathbf{x}, J_m) \), may be calculated. The predominant drift velocity component is \( v_e \) directed towards the bottom plate, thus responsible for the precipitation. \( v_e \) is practically constant in space except close to the electrode where electric field lines are far from parallel. Figure 3.8 shows spatially averaged values of \( v_e \) for a mean diameter Rolovit particle, with Error bars denoting the levels for \( d_p = \overline{d_p} \pm \left( \overline{d_p^2} \right)^{0.5} \).
Figure 3.6: Field lines and contours in [N/m$^3$] of calculated Coulomb body force, $F$, in $xy$-
and $yz$-planes of symmetry in a unit cell. Electrode at $(x, y) = (100, 100)$ mm, discharge pin
at $(x, y, z) = (100, 100, -50)$ mm and collector plate at $y = 0$. 
3.3 Electrostatic results

Figure 3.7: Calculated Coulomb body force, $\mathbf{F}$, of Figure 3.6. Left: axial mean, $<\mathbf{F}_x>$. Middle: net axial mean, $<\mathbf{F}_x> - <\mathbf{F}_{xyz}>$. Right: contours of axial mean of axial curl component, $(\nabla \times \mathbf{F})_x$ in $[N/m^4]$.

Figure 3.8: The calculated vertical drift velocity, $|<V_{c}>_{xyz}|$, averaged spatially in one unit cell. Values correspond to a mean diameter Rolovit particle, $d_p = 1.75 \mu m$. Error bars denote standard deviation for $d_p$ according to values of Table 3.5.
3.4 Summary

The experimental setup used for PIV and LDV measurements was presented. The instrumentation (including seeding material) was described along with the test series and relevant acquisition and processing parameters. Also, the spatial and temporal resolutions were presented showing a bias problem with higher order moments measured by PIV.

Finally, some electrostatic results were shown. An experimental current-voltage characteristic as well as numerical results of the Coulomb body force and the particle drift velocity were presented.
Chapter 4

Application of Particle Image Velocimetry

Particle Image Velocimetry (PIV) is a non-intrusive optical measurement technique that provides instantaneous velocities in a plane of measurement. Standard PIV makes use of one camera to get in-plane velocities whereas stereo PIV employs two cameras to provide all 3 velocity components. The fundamentals of PIV will not be treated here, but the reader might consult some of the numerous texts in the area [1, 44, 58]. Especially the handbook of Raffel et al. [45] serves as an excellent guide in practical PIV. Instead this section will focus on the particle scattering and recording since the polydisperse seeding particles used in this study tend to bias the PIV results. However, it is possible to overcome these problems from the theory treated in the following. This is done in the subsequent data reduction in Section 6.1.

As the particles are expected to trace the motion of the fluid and at the same time act as transmitters of scattered light, the choice of optimal size of seeding particles in PIV is a compromise between an adequate tracer response, requiring small particles, and a clear image i.e. high signal-to-noise ratio requiring large particles. Also the accuracy of the calculated velocity, which is proportional to the ratio of image diameter to image separation, has to be considered. Hence the optimal particle size is the smallest one capable of producing enough light to be photographed [3]. Section 2.3.3 served to outline the suspended particle kinetics, and the following section treats the scattering of spherical particles.

4.1 Particle scattering

A spherical particle illuminated by a light source scatters light with an intensity, $I$, very dependent on angle of observation. The intensity of the scattered light is proportional to the incident intensity, $I_0$, and $1/r^2$ (conservation of energy) with the spherical coordinate system $(r, \theta, \phi)$ having its origin in the particle center

$$I = I_0 \frac{F(\theta, \phi)}{k^2 r^2}, \quad [\text{W/m}^2]$$

(4.1)

where $F(\theta, \phi)$ is a dimensionless scattering function depending on the direction of observation $(\theta, \phi)$, and $k \equiv 2\pi/\lambda$ is the wave number based on the wave length of the
incident light, $\lambda$, $\theta$ is the angle with the direction of propagation of the incident light, and $\phi$ is the azimuth angle. When the incident light is linearly polarized $\phi$ could be the angle with the polarization of the incident light. Examples of scattering diagrams, $F(\theta)$, are shown in Figure 4.1 for the mean diameter seeding particle used in this study.\footnote{Calculated by the Mie scattering toolbox developed by Barnett \cite{Barnett}.}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.1.png}
\caption{Scattering diagrams, $F(\theta)$, for a 1.75 $\mu$m spherical Rolovit particle (see Table 3.5 page 22) for $\theta \in [0, 2\pi]$. Particle located at origin and incoming light from the left. Left: $\phi = 0^\circ$. Right: $\phi = 90^\circ$. Identical log\textsubscript{10}-scaling.}
\end{figure}

Usually $F(\theta, \phi)$ is dominant in forward direction (cf. Figure 4.1), but for very small particles the forward and backward scatter intensities take the same values. The directional intensity may be integrated over a sphere of area $A = 4\pi r^2$ to get the total radiation power scattered from the particle

$$P \equiv \int_A I \, dA = \int_{\Omega} r^2 I \, d\Omega = \frac{I_0}{k^2} \int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin(\theta) \, d\theta \, d\phi, \quad [W] \quad (4.2)$$

where $\Omega = A/r^2$ is a solid angle. In general $F(\theta, \phi)$ may be determined from the Mie theory, which corresponds to solving the Maxwell equations in spherical coordinates with results given as infinite series \cite{Born, Wolf}.

In some cases the Mie theory approaches a simplified theory, which is the Rayleigh theory for small particles and the geometric theory for large particles, depending on the dimensionless particle diameter, $\alpha$, and relative refractive index, $m$,

$$\alpha \equiv \pi d_p/\lambda, \quad (4.3)$$
$$m \equiv n_p/n, \quad (4.4)$$

with $n_p$ and $n$ being the refractive index of the particle and the surrounding fluid respectively. Table 4.1 shows how $P$ scales with particle diameter, $d_p$, and light wave length, $\lambda$, dependent on $\alpha$ and $m$. In the geometrical optics regime the radiant energy is treated as independent rays of light by the Fresnel equations. Physically each ray may undergo the processes of diffraction, reflection and refraction and the scattered power scales with the incident power, i.e. the particle cross section ($P \sim d_p^2$). On the contrary, the Rayleigh theory is based upon the assumption of the particle being isotropic, homogeneous and dielectric and placed in a uniform field. In this case the solution to the Maxwell equations is well known. The result is an electric field, $\mathbf{E}$, generated in the entire volume, $V$, which explains the $d_p^2$ dependence on $P$ ($P \sim |\mathbf{E}|^2 \sim V^2 \sim d_p^6$). For intermediate-sized particles one has to rely on the general (and more complicated) Mie theory.
\[
\begin{array}{ll}
\alpha \ll 1, \ \omega m \ll 1 & \text{Rayleigh scattering} & P \sim I_0 \rho^6 / \lambda^4 \\
\alpha \approx 1 & \text{Intermediate (Mie theory)} \\
\alpha \gg 1, \ \alpha (m - 1) > 1 & \text{Geometrical optics} & P \sim I_0 \rho^2 / \lambda^2 \\
\end{array}
\]

Table 4.1: Radiant power behaviour in different scattering regimes.

Figure 4.2 shows values of \(P/I_0\) from Eq. (4.2) for both olive oil seeding and Rolovit (cf. Table 3.5 page 22).\(^2\) The trends of both the Rayleigh and geometrical optics regimes are clear when plots are compared to the 6 and 2 m\(^2\) slopes.

![Scattering power per incoming intensity](image)

Figure 4.2: Scattering power per incoming intensity, \(P/I_0\) for olive oil (\(m = 1.47\)) and Rolovit (\(m = 1.68\)). Slopes of 6 m\(^2\) and 2 m\(^2\) are also shown.

Small liquid particles are always spherical due to the surface tension, but solid particles are rarely spherical making the scattering theory much more complicated. In addition, the intensity from a cloud of scattering particles equals the sum of the individual intensities. For in-depth information on the scattering of particles the reader is referred to van de Hulst [57] and Kerker [26].

### 4.2 Illumination and recording

The pulsed double-cavity laser is widely used in PIV in order to provide a monochromatic (i.e. constant frequency) light pulse with a high energy content, \(W\). The energy

\(^2\)Calculated by the Mie scattering toolbox developed by Barnett [8].
intensity, \( I_0 \), in the measuring volume (measuring plane of finite thickness) is

\[
I_0 = \frac{W}{\Delta y \Delta z \delta t}, \quad \text{[W/m}^2\text{]} 
\]

where \( \Delta y \) and \( \Delta z \) are the dimensions of the measuring volume normal to the direction of propagation, and \( \delta t \) is the pulse duration (see Figure 4.3). In the present study a frequency doubled Nd:YAG has been employed with a maximum output of \( W = 170 \) mJ and \( \delta t = 4-6 \) ns giving \( I_0 \approx 10^{11} \) W/m\(^2\) in the \( \Delta y \Delta z \approx 3 \) cm\(^2\) measuring volume. The recorded amount of light scattered from a particle is limited by the solid angle of observation, \( \Omega \), determined from the lens aperture as sketched in Figure 4.3.

![Figure 4.3: Particle image recording.](image)

The recording of the particle scatter can be done in several ways, but recent advances in electronic recording has made digitally recorded PIV (DPIV) more attractive than the traditional photo-chemical recording [61]. In DPIV the recording medium is a CCD camera containing an array of isolated pixels, which are electronic sensors capable of converting light into electric charge. The spatial resolution is still small compared to photo-chemical recording, but progress is made towards high-resolution CCD cameras. The electric charge in each pixel continues to accumulate during the time of exposure, thus each pixel charge is integrated over the pixel area and the exposure time, which for pulsed lasers equals the pulse duration, \( \delta t \). Hence, the mean exposure, \( \tau \), over an image area of diameter \( d_e \) yields

\[
\tau = \frac{\tau}{4\pi d_e^2} \int_0^\Omega \int_{\delta t} r^2 I \, d\Omega \, dt \quad \text{[J/m}^2\text{]} 
\]

where \( \tau \) is the transmissivity of the receiving optics (< 1), \( \Omega \) is the solid angle subtended by the aperture of the camera lens and \( I \) is the intensity at the distance \( r \) from the object plane (see Figure 4.3). In Eq. (4.6) it is assumed that the image is larger than one pixel, i.e. \( d_e > d_r \), where \( d_r \) is the equivalent diameter of one pixel. If \( d_e < d_r \) one may substitute \( d_e \) with \( d_r \) in Eq. (4.6), which does not take into account the image
location on the CCD array. If, for instance, one image is placed mid between two pixels, both of these will accumulate charge. Since $I$ is highly dependent on the scattering direction and $\Omega$ usually is quite small, $\tau$ will depend strongly on the angle of observation (cf. Figure 4.1).

The image of the particle is given as the convolution of a point spread function with the geometric image. Figure 4.4 shows an example of the intensity distribution along the diameter of the image. The image of the point spread function forms concentric rings of zero and non-zero intensity due to the distance between the aperture and the image plane, known as Fraunhofer diffraction. Usually the exposure is so low that only the inner core of the Fraunhofer diffraction is visible, thus the image diameter yields \cite{2}

$$d_s = 2.44(M + 1)f_\# \lambda,$$

where $M$ is the geometric magnification and $f_\#$ is the numerical aperture. The geometric image is a simple top hat function with the diameter $Md_p$. Assuming that both the diffraction image and the geometric image are Gaussian the convolution yields the effective image diameter \cite{2}

$$d_e = \sqrt{M^2d_p^2 + d_s^2}.$$

Again it is assumed that the photographic resolution is capable of resolving the image ($d_e > d_s$).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{convolution.png}
\caption{Schematic of the convolution ($\otimes$) of Fraunhofer diffraction image (left) and geometrical image (right).}
\end{figure}

Assuming that the viewing angle, $\Omega$, of Eq. (4.6) is large enough to capture the general trends of the scattered power, i.e. $7 \sim P$ and combining the results from Table 4.1 with Eq.'s (4.6)-(4.8) gives the results of Table 4.2. The table gives the asymptotic dependency of different parameters on the mean exposure of the particle image, $\tau$. The entries are two-fold: a) the scattering regime, which may be one of the two extremes of Rayleigh scattering or geometrical scattering and b) the recorded image diameter, which may be smaller than the CCD resolution (One-pixel image) or either limited by one of the two extremes of diffraction or the geometric image. One should note the very different dependencies of $d_p$ and $\lambda$ on $\tau$, which will be used in the subsequent data reduction in Section 6.1. Experimentally, Adrian & Yao \cite{3} investigated the dependency of $d_p$ on $\tau$ for diffraction limited images and found similar results but with considerable variations due to a small viewing angle, $\Omega$. No simple laws of proportionality exist in the intermediate size range where only the general Mie theory applies.
<table>
<thead>
<tr>
<th>One-pixel image</th>
<th>Diffraction image $d_c &lt; d_r$</th>
<th>Geometrical scattering $d_c = d_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha \ll 1, \alpha m \ll 1$</td>
<td>$d_p^2 / (d_c^2 \lambda^2)$</td>
<td>Geometrical scattering $\alpha \gg 1, \alpha (m - 1) &gt; 1$</td>
</tr>
<tr>
<td>$d_c &lt; d_r$</td>
<td>$d_p^2 / (d_c^2 \lambda^2)$</td>
<td>1 / $(M^2 \lambda^2)$</td>
</tr>
<tr>
<td>$d_c &gt; d_r$</td>
<td>$d_p^2 / (d_c^2 \lambda^2)$</td>
<td>1 / $(M^2 \lambda^2)$</td>
</tr>
</tbody>
</table>

Table 4.2: Asymptotic dependency of selected parameters on mean exposure in digital PIV, $\tau = \tau (d_r, d_p, \lambda, M, f_\#)$.

4.3 Measurement uncertainty

The variance, $\sigma_u^2$, of the measured velocity, $u$, has contributions from both turbulent fluctuations, $\sigma_T^2$, and measurement uncertainties, $\sigma_{PIV}^2$

$$\sigma_u^2 = \sigma_T^2 + \sigma_{PIV}^2.$$ (4.9)

Here the measurement error, $\sigma_{PIV}^2$, is defined as the difference between the true mean displacement and the measured displacement within the interrogation area (IA). Thus, the bias error due to the poor spatial resolution affecting higher order moments is not included (cf. Section 3.2.4). The main contribution to the measurement error, $\sigma_{PIV}^2$, is a result of two factors. First, the estimation of the correlation peak introduces an error known as peak locking [56]. Second, velocity gradients within the IA and system noise (e.g. thermal noise of the CCD sensor) cause errors observed as a broadened correlation peak [61]. In the early years of DPIV, the measurement precision was given by the pixel dimension, i.e. 0.5 px, but introduction of the sub-pixel interpolation improved the measurement precision substantially to about 0.1 px [59, 61]. Further improvements were obtained by the adaptive correlation (offset interrogation windows) reducing the error of correlation peak estimation to about 0.04 px [59].

The above described measurement error is observed on the CCD sensor in the image plane and is referred to as $\sigma_{PIV,IP}^2$. In order to relate this error to the error of the displacement in the object plane geometric parameters are used. In this study a calibration-based reconstruction was used, which minimizes registration errors compared to geometric reconstruction [44]. Thus, assuming perfect registration the measurement error for the in-plane and out-of-plane motions for stereo PIV become [44]

$$\sigma_{PIV,IP}^2 \approx \frac{1}{2 M^2} \sigma_{PIV,IP}^2,$$ (4.10)

$$\sigma_{PIV,IP}^2 \approx \frac{1}{2 (M \tan(\theta))^2} \sigma_{PIV,IP}^2,$$ (4.11)

where $\theta$ is the angle between the cameras and the laser sheet. The factor of $1/2$ is due to the enhancement obtained by using two cameras instead of one. Of course, the stereo arrangement also removes the perspective error.
In the present experiment \( d_p \approx 1.75 \ \mu \text{m}, \ M \approx 0.10, \ f_\# = 5.6, \) and \( \lambda = 532 \ \text{nm}, \) which gives by Eq. (4.8) an image diameter of \( d_s \approx 8.0 \ \mu \text{m} \) or 0.9 px.\(^3\) With this image size the Gaussian estimator gives a standard deviation due to peaking of \( \sim 0.005 \) px [45, 56]. In steady flows this error causes biasing, but for highly turbulent flows it may be treated as a random error. The error due to broadening of the correlation peak is also random and mainly dependent on relative displacement in the image plane and size of IA. For this study the error of 2-6 px displacement in a 32\times32 px IA gives a standard deviation of \( \sim 0.04 \) px [58]. The combined PIV measurement variance in the image plane then becomes \((0.005 \text{ px})^2 + (0.04 \text{ px})^2 \approx 0.0016 \text{ px}^2, \) which equals the approximate value given by Westerweel [59]. The value corresponds to \( \sigma^2_{PIV,IP} \approx 8.1\times10^{-7} \text{ m}^2/\text{s}^2 \) when the time separation of \( \Delta t = 400 \ \mu\text{s} \) is used.

With \( \theta = 45^\circ \) Eq. (4.10)-(4.11) gives \( \sigma_{PIV}^2 \approx 4\times10^{-5} \text{ m}^2/\text{s}^2 \) for both in-plane and out-of-plane motion. The variance due to turbulence is \( \sigma_T^2 \approx 0.04 \text{ m}^2/\text{s}^2, \) i.e. three orders of magnitudes larger than the uncertainties due to measurement errors, \( \sigma_{PIV}^2, \) giving

\[
\sigma_u \approx \sigma_T. \tag{4.12}
\]

The uncertainties in the velocity moments may then be estimated using analytical expressions given by Benedict & Gould [10]. When Gaussian distributions are assumed, the uncertainties in \( U \) and \( U_{rms} \) with a 95 % confidence interval are given by

\[
S(U) = \pm 1.96 \frac{\sigma_u}{\sqrt{N}}, \tag{4.13}
\]

\[
S(U_{rms}) = \pm 1.96 \frac{\sigma_u}{\sqrt{2N}}, \tag{4.14}
\]

where \( N \) is the number of snapshots. With \( \sigma_u = 0.2 \) \text{ m/s} \) and \( N = 400 \) the uncertainties are \( S(U) = \pm 0.020 \) \text{ m/s} \) and \( S(U_{rms}) = \pm 0.014 \) \text{ m/s}. With a mean velocity of 0.5 \text{ m/s} \) and RMS velocity of 0.2 \text{ m/s} \) the relative uncertainties become 4 % and 7 %, respectively.

The above PIV error analysis treated the difference between the true mean displacement and the measured displacement and showed that practically no measurement error was present. However, the relatively low spatial resolution of PIV was shown in Section 3.2.4 to introduce substantial errors with regard to higher order moments.

### 4.4 Summary

This chapter provided a discussion on the application of digital Particle Image Velocimetry especially with regard to particle scattering, its recording and the introduced measurement errors. The general scattering behavior of a spherical particle was treated and asymptotic dependencies of selected parameters on mean exposure were deduced.

The measurement error was found to consist mainly of uncertainties due to the gas turbulence, whereas the PIV measuring error was found to be negligible.

\(^3\) The magnification is only approximately uniform over the field of view due to the rotational PIV setup, see Figure 3.2.
Chapter 5

Application of Large Eddy Simulation

Up to today, most simulations in Computational Fluid Dynamics (CFD) are carried out with traditional RANS (Reynolds-Averaged Navier-Stokes). In RANS, the flow variables are split into a time-averaged (mean) part and a turbulent (fluctuating) part, where the latter is modelled with a turbulence model such as $k-\varepsilon$ or Reynolds Stress Model. However, it may not always be appropriate to use RANS, since the turbulent part might be dominating the flow. The more appropriate solution would then be to use Large Eddy Simulation (LES) or alternatively unsteady RANS.

In LES the governing equations are filtered to separate the large-scale and small-scale motion. The large-scale motion is solved for explicitly by the discretized equations whereas the small-scale motion is modelled. Since the small scales tend to be more homogeneous and universal, and less affected by the boundary conditions than the large ones, their models may usually be simpler and require fewer adjustments than for the RANS equations [43, 46]. Thus, LES is preferable over RANS simulations especially when large-scale eddy structures are dominating. Because turbulence is three-dimensional and unsteady of nature, LES must be carried out as 3D, unsteady simulations.

LES of the laboratory scale ESP flow has been performed to supplement the PIV and LDV measurements. LES was chosen, since the ESP flow is very unsteady and dominated by large turbulent structures. The calculated 3D steady body force, $F$, of Figure 3.6 page 26 has been used as input to the finite difference solver PEGASE developed at ONERA [30, 38]. This Navier-Stokes solver was developed for the plane channel flow, thus directly applicable to the ESP flow. However, several modifications were needed for the body force implementation. Modifications included 3D interpolation of $F$ to the LES mesh and adjustments of momentum equations, pressure equation and boundary equations as described in the following. This chapter does not include general text on LES, but only serves to outline the techniques used in the presented simulations. Details on LES may be found in e.g. Sagaut [48].
5.1 Governing equations

To separate the resolved scales from the small scales, LES applies a spatial filtering operation. In this study the top-hat filter has been applied with a filter length equal to the grid size \([48]\). Filtering the incompressible Navier-Stokes equations, Eq. (2.15) and (2.16) with a filtered (resolved) variable, denoted with an over-bar, yields

\[
\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial (\overline{u}_i \overline{u}_j)}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 \overline{u}_i}{\partial x_i \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \overline{F}_i, \tag{5.1}
\]

\[
\frac{\partial \overline{u}_i}{\partial x_i} = 0, \tag{5.2}
\]

where the equations have been made dimensionless and the non-linear term in the momentum equation (5.1) has been rewritten by use of a subgrid scale (SGS) stress term

\[
\tau_{ij} \equiv \overline{u}_i \overline{u}_j - \overline{u}_i \overline{u}_j. \tag{5.3}
\]

The filtered equations (5.1) and (5.2) govern the evolution of the large, energy-containing scales of motion. The effect of the small scales appears through the SGS term, which is the large scale momentum flux caused by the action of the non-resolved scales. The term remains to be modeled. The fundamental hypothesis is that the energy transfer from the resolved to the non-resolved scales is analogous to the viscous dissipation of energy, dependent on the filtered strain-rate tensor (i.e. the Boussinesq approximation)

\[
\overline{S}_{ij} \equiv \frac{1}{2} \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right). \tag{5.4}
\]

Due to continuity \(\overline{S}\) has zero trace, thus it can only model the deviatoric part of \(\tau\), which is

\[
\tau_{ij}^D \equiv \tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij}, \tag{5.5}
\]

where \(\delta_{ij}\) is the Kronecker delta function.\(^1\) The isotropic part of the SGS stress term is added to the pressure term hereby introducing a pseudo-pressure, \(\Pi\)

\[
\Pi \equiv \overline{p} + \frac{1}{3} \tau_{kk}, \tag{5.6}
\]

which needs no modelling. A Poisson equation for the pressure is obtained by taking the divergence of the momentum equation, and the final set of equations becomes

Momentum: \[
\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial (\overline{u}_i \overline{u}_j)}{\partial x_j} = -\frac{\partial \Pi}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 \overline{u}_i}{\partial x_i \partial x_j} - \frac{\partial \tau_{ij}^D}{\partial x_j} + \overline{F}_i, \tag{5.7}
\]

Pressure: \[
\frac{\partial \Pi}{\partial x_i \partial x_i} = -\frac{\partial^2}{\partial x_i \partial x_j} (\overline{u}_i \overline{u}_j + \tau_{ij}^D) + \frac{\partial \overline{F}_i}{\partial x_i}, \tag{5.8}
\]

SGS viscosity: \[
\tau^D = -2\nu_{sgs} \overline{S}, \tag{5.9}
\]

\(^1\delta_{ij} = 1 \text{ if } i = j \text{ and } \delta_{ij} = 0 \text{ otherwise.}\)
where \( \nu_{sgs} \) is the subgrid viscosity. In Eq. (5.8) the viscous and inertia terms have been discarded by virtue of the continuity equation (5.2). It should be noted that by this technique the continuity equation is replaced by the pressure equation, and the closure now consists in determining the subgrid viscosity of Eq. (5.9) based on the resolved scales, \( \nu_{sgs} = \nu_{sgs}(\overline{\sigma}) \).

### 5.2 Subgrid scale model

The employed SGS model developed by Sagaut [48] is referred to as the Mixed-Scale model, \( \nu_{sgs}^{MS} \), depending on both the large and the small scales. Actually it exhibits a triple dependency on 1) the vorticity of the resolved scales, \( \overline{\sigma} = \nabla \times \sigma \), 2) the kinetic energy of the highest resolved frequencies, \( q_{c} \equiv \frac{1}{2}u_{i}^{2}u_{i} \), and 3) the cut-off length scale, \( \Delta = (\Delta x \Delta y \Delta z)^{1/3} \), based on the local mesh dimensions. The relation is

\[
\nu_{sgs}^{MS} = 0.1 \ |\overline{\sigma}|^{1/2} q_{c}^{1/4} \Delta^{3/2}.
\]  

(5.10)

The \( q_{c} \) term is assumed to approximately equal the kinetic energy at the subgrid scale. In practice \( q_{c} \) is found by applying a test filter of \( 2 \Delta x \times 2 \Delta y \times 2 \Delta z \), which gives

\[
q_{c} = \frac{1}{2} \left( (\overline{\sigma} - \tilde{\sigma})^{2} + (\overline{\sigma} - \tilde{\sigma})^{2} + (\overline{\sigma} - \tilde{\sigma})^{2} \right),
\]  

(5.11)

where a tilde denotes that the test filter has been applied. In practice the discrete test filter velocities on a cartesian mesh of index \((i, j, k)\) are found as

\[
\overline{u}(i, j, k) = \frac{1}{4} \left[ u(i - 1, j, k) + 2u(i, j, k) + u(i + 1, j, k) \right],
\]  

(5.12)

\[

\overline{v}(i, j, k) = \frac{1}{4} \left[ v(i, j - 1, k) + 2v(i, j, k) + v(i, j + 1, k) \right],
\]  

(5.13)

\[

\overline{w}(i, j, k) = \frac{1}{4} \left[ w(i, j, k - 1) + 2w(i, j, k) + w(i, j, k + 1) \right].
\]  

(5.14)

In order to improve the prediction of flow intermittency it is possible to design a self-adaptive SGS model by combining the Mixed-Scale model, \( \nu_{sgs}^{MS} \), with a selection function, \( f_{s} \). The idea is that the selection function must turn off the SGS model when all scales are resolved. Sagaut [48] stated that a flow may be considered as locally under-resolved when the local angular fluctuations of the vorticity vector of the highest resolved frequencies are greater than a certain threshold level. Thus, the selection function becomes a function of the angle, \( \theta \), between the filtered vorticity vector, \( \overline{\sigma} \), and the one of the test filter, \( \overline{\omega} = \nabla \times \overline{u} \),

\[
\nu_{sgs} = \nu_{sgs}^{MS} f_{s}(\theta),
\]  

(5.15)

where the selection function is

\[
f_{s}(\theta) = \begin{cases} 
1, & \text{if } |\theta| \geq \theta_{0} \\
\frac{r}{\theta_{0}}, & \text{if } |\theta| < \theta_{0},
\end{cases}
\]  

(5.16)

\[
r = \frac{\tan^{2}(\theta_{0}/2)}{\tan^{2}(\theta/2)}.
\]  

(5.17)

In the present simulations a threshold angle of \( \theta_{0} = 20^\circ \) was used [48].
5.3 Solution method

In the PEGASE code, the presented equations are discretized by the finite difference method and time integration is performed using a second-order accurate semi-implicit method. A time-adaptive homogeneous forcing term is added to the momentum equation in order to keep a constant mass flow rate through the channel. Details on the numerical methods may be found in Sagaut et al. [49].

The computational domain of Figure 5.1 was \((L_x, L_y, L_z) = (200, 200, 200)\) mm corresponding to two unit cell lengths of the full cross section of the experimental test section (see Figure 3.2 on page 18), i.e, a total of 16 unit cells. The spatial resolution was \(61 \times 61 \times 81\) nodes and was equidistant in \(x\) and \(z\) and stretched in \(y\) with a distance to the first node of \(\Delta y^+ \approx 2\) in wall units. The boundary conditions were periodic in axial \((x)\) and spanwise \((z)\) directions, and the no-slip condition was employed in the wall-normal \((y)\) direction corresponding to solving for the fully developed flow with no side-walls. The volume of the 3-mm electrodes was not resolved, since the wake effects were assumed negligible when the electrostatic field was applied [51]. The initial condition was a perturbed Poiseuille flow and a developed flow was achieved after 1-2 seconds. The time step was kept as low as \(\Delta t = 2 \times 10^{-5}\) s in order to stabilize the computations, and the presented results are based on averages of 8-10 seconds of developed flow. Essential LES simulation parameters are summarized in Table 5.1.

![Figure 5.1: Computational domain. 2 electrodes at \(x = 50, 150\) mm and \(y = 100\) mm with discharge pins at \(z = 50, 150\) mm, 2 collector plates at \(y = 0, 200\) mm.](image-url)
<table>
<thead>
<tr>
<th>Method</th>
<th>3D, transient, incompressible LES</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGS model</td>
<td>Mixed Scale with selection function</td>
</tr>
<tr>
<td>Fluid</td>
<td>air, $\nu = 1.5 \times 10^{-5}$ m$^2$/s</td>
</tr>
<tr>
<td>Domain</td>
<td>$(L_x, L_y, L_z) = (200, 200, 200)$ mm</td>
</tr>
<tr>
<td>2 collector plates</td>
<td>$y = 0, 200$ mm</td>
</tr>
<tr>
<td>2 electrodes</td>
<td>$y = 100$ mm, $x = 50, 150$ mm</td>
</tr>
<tr>
<td>4 emission pins</td>
<td>$y = 100$ mm, $x = 50, 150$ mm, $z = -50, 50$ mm</td>
</tr>
<tr>
<td>Time step</td>
<td>$\Delta t = 2 \times 10^{-5}$ s</td>
</tr>
<tr>
<td>Spatial resolution</td>
<td>$N_x \times N_y \times N_z = 61 \times 81 \times 61$ nodes</td>
</tr>
<tr>
<td>BC’s</td>
<td>periodic ($x$), no-slip ($y$), periodic ($z$)</td>
</tr>
<tr>
<td>Bulk velocity</td>
<td>$U_0 = 1.0$ m/s</td>
</tr>
<tr>
<td>Mean current density</td>
<td>$J_m = 0, 0.05, 0.10, 0.20, 0.30, 0.40$ mA/m$^2$</td>
</tr>
</tbody>
</table>

Table 5.1: LES simulation parameters.

### 5.4 Summary

This chapter provided information on the LES application to the ESP flow. The governing equations were shown as well as the employed SGS model (Mixed Scale with a selection function). Finally, simulation parameters such as computational domain, boundary conditions etc. were presented.
Chapter 6

Data reduction

6.1 Correction for drift of polydisperse particles

Using an optical measuring technique such as LDV or PIV the velocity of the seeding particle, \( u_p \), is measured. The corresponding local gas velocity, \( u \), may be found by subtracting the drift velocity, \( u_c \), of the seeding particle, \( u = u_p - u_c \). This is easily done when the particles have identical drift velocities, which, however, is not the case when polydisperse particles are used. It is the purpose of this section to derive corrections to measured particle velocities in order to get unbiased gas velocities. It will be assumed that particles of all sizes perfectly follow the carrier gas, thus the slip velocity is simply the electric drift velocity. Eq. (2.32) on page 13 relating particle diameter, \( d_p \), to \( u_c \) may be rewritten as

\[
u_c = Cd_p, \tag{6.1}
\]

where \( C \) is a proportionality vector whose direction is given by the local field strength, \( E \). The Reynolds decomposition is applied to gas, particle and drift velocities as

\[
u = U + u', \quad u_p = U_p + u'_p, \quad u_c = U_c + u'_c, \tag{6.2}
\]

with \( U_c = C\bar{d}_p \), where \( \bar{d}_p \) is the particle mean diameter. The mean diameter and its variance, \( d_p^2 \), are calculated by the particle size distribution, \( f(d_p) \), as

\[
\bar{d}_p \equiv \int d_p f(d_p) \dd d_p, \tag{6.3}
\]

\[
\bar{d}_p^2 \equiv \int (d_p - \bar{d}_p)^2 f(d_p) \dd d_p. \tag{6.4}
\]

In LDV the measured instantaneous particle velocity, \( u_p \), is based on the motion of a single particle, but in PIV the particle velocity is the average motion of all the particles in the interrogation area (IA). The instantaneous mean diameter, \( \bar{d}_p \), of these particles is

\[
\bar{d}_p \equiv \int d_p \bar{f}(d_p) \dd d_p, \tag{6.5}
\]
where \( \bar{f} \) is the size distribution of the particles that \( u_p \) is based on (e.g. particles in the IA). The instantaneous gas velocity becomes

\[
\mathbf{u} = \mathbf{u}_p - \mathbf{u}_e(\bar{d}_p), \tag{6.6}
\]

If the number of recorded particles, \( K \), is large enough to represent the overall size distribution, \( f \), it follows that \( \bar{d}_p = \bar{d}_p \) and

\[
\mathbf{u} = \mathbf{u}_p - \mathbf{U}_e, \quad \text{if } K \gg 1. \tag{6.7}
\]

If the number of particles is small \( \mathbf{u} \) can only be estimated if the individual particle diameters are measured. The mean gas velocity is

\[
\mathbf{U} = \mathbf{U}_p - \mathbf{u}_e \left( \bar{d}_p \right), \tag{6.8}
\]

where

\[
\bar{d}_p = \frac{1}{N} \sum_{n=1}^{N} \left[ \int \bar{d}_p \bar{f}(\bar{d}_p) \, \mathrm{d} \bar{d}_p \right]_n. \tag{6.9}
\]

and \( N \) is the total number of samples. It is seen that if either \( N \) or \( K \) is large it follows that \( \bar{d}_p = \bar{d}_p \) and

\[
\mathbf{U} = \mathbf{U}_p - \mathbf{U}_e, \quad \text{if } K \gg 1 \text{ or } N \gg 1. \tag{6.10}
\]

The Reynolds stresses are calculated as

\[
\begin{align*}
\bar{u}'_i u'_j &= u'_p,i u'_p,j \frac{\bar{d}_p - \bar{d}_e}{\bar{d}_p - \bar{d}_p} - u'_e,i u'_e,j - u'_j u'_e,i \left( \bar{d}_p - \bar{d}_e \right) \bar{d}_p - \bar{d}_p \\
&= u'_p,i u'_p,j - u'_e,i u'_e,j, \tag{6.11}
\end{align*}
\]

where the last step assumes no correlation between gas and drift velocities. Using Eq. (6.1) the co-variance of the drift velocities becomes

\[
\begin{align*}
\bar{u}'_{e,i} u'_{e,j} &= \frac{1}{N} \sum_{n=1}^{N} \left[ C_i \left( \bar{d}_p - \bar{d}_p \right) C_j \left( \bar{d}_p - \bar{d}_p \right) \right]_n \\
&= \frac{C_i C_j}{N} \sum_{n=1}^{N} \left[ \left( \bar{d}_p - \bar{d}_p \right)^2 \right]_n. \tag{6.12}
\end{align*}
\]

If \( N \gg 1 \) it follows that \( \bar{d}_p = \bar{d}_p \), and if \( K \gg 1 \) then \( \bar{d}_p = \bar{d}_p \) and \( u'_{e,i} u'_{e,j} = 0 \). If on the other hand \( K = 1 \) the correction due to drift velocity variance becomes

\[
\lim_{N \to \infty} \frac{C_i C_j}{N} \sum_{n=1}^{N} \left[ \left( \bar{d}_p - \bar{d}_p \right)^2 \right]_n = C_i C_j \int \left( \bar{d}_p - \bar{d}_p \right)^2 \bar{f}(\bar{d}_p) \, \mathrm{d} \bar{d}_p, \tag{6.13}
\]

\[
= U_{e,i} U_{e,j} \frac{\overline{d^2}}{\overline{d_p}}. \tag{6.13}
\]
Basically, Eq. (6.11) states that the Reynolds stresses may require a modification by the variance of the particle drift due to the particle size distribution, \( f(d_p) \). However, when \( K \) is sufficiently large to be representative of the complete particle size distribution the influence of the particle size variance vanishes.

The time velocity correlation for \( u \) is calculated as

\[
R_{uu}(\tau) = \frac{\overline{u'(t)u'(t+\tau)}}{\overline{u'_p(t)u'_p(t+\tau)}} \approx \frac{\overline{u'_p(t)u'_p(t+\tau)}}{\overline{u'_p(t)u'_p(t+\tau)}},
\]

(6.14)

where it has been used that there is no correlation between gas and drift velocities. Also, the \( \overline{u'_p(t)u'_p(t+\tau)} \) term is approximately zero since the sampling frequency used for time averaging is at least one order of magnitude smaller than the frequency of drift velocity fluctuations.

The results are summarized in Table 6.1, where \( N \) is assumed large enough to represent the size distribution, \( f \), i.e. \( d_p = \overline{d_p} \). \( K = 1 \) is represented by the LDV technique, whereas the PIV technique is somewhere in between the two entries of \( K \) depending on the seeding density and the homogeneity of \( d_p \). It is noteworthy that it is impossible to calculate instantaneous gas velocities since \( u_e \) is not recorded and \( K \) is rarely large enough to represent \( f \). An estimate of the instantaneous velocity field may however be calculated as \( u \approx u_p - U_e \) as long as the drift correction is small. However, first and second order velocity moments may be estimated with corrections based on the particle size distribution provided that the number of samples is large.

<table>
<thead>
<tr>
<th>( u_i )</th>
<th>( K \gg 1 )</th>
<th>( K = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_p,i - U_{e,i} )</td>
<td>( u_p,i - u_{e,i} )</td>
<td></td>
</tr>
<tr>
<td>( U_{i} )</td>
<td>( U_{p,i} - U_{e,i} )</td>
<td>( U_{p,i} - U_{e,i} )</td>
</tr>
<tr>
<td>( u_p,i u_p,j )</td>
<td>( u_p,i u_p,j - U_{e,i} U_{e,j} \overline{d_p^2} / \overline{d_p}^2 )</td>
<td></td>
</tr>
<tr>
<td>( R_{ii} )</td>
<td>( u_p,i(t)u_p,i(t+\tau) )</td>
<td>( u'_p,i(t)u'_p,i(t+\tau) )</td>
</tr>
</tbody>
</table>

Table 6.1: Drift velocity corrections. The number of samples, \( N \), is assumed large.

### 6.1.1 Application to PIV data

The PIV images of Rolovit particles were recorded on a CCD chip with an 8 bit resolution, i.e. 256 pixel grey levels, and in accordance with the principles of digital PIV the pixel values were subsequently cross correlated to get the velocity in each interrogation area. Due to the dependency of particle size on both drift velocity and the recorded light intensity (exposure) this procedure introduced biasing. To overcome this problem the measured size distribution \( f(d_p) \) of Figure 3.3 page 23 used to estimate \( \overline{d_p} \) (Eq. (6.3)) and \( \overline{d_p^2} \) (Eq. (6.4)), was replaced with an effective size distribution \( f_e(d_p) \) based on the results of Table 4.2 page 34.

The mean dimensionless particle diameter was \( \alpha \approx 10 \) and the relative refractive index was \( m = 1.68 \) giving scattering in the lower end of the geometrical regime (cf.
Figure 4.2 page 31). Further, the magnification factor was $M \approx 0.10$, giving a diffraction image diameter of $d_s \approx 8 \, \mu m$, which was slightly smaller than the pixel size, $d_p = 9 \, \mu m$ and considerably higher than the geometric image, $Md_p \approx 0.2 \, \mu m$. Thus the mean exposure would give an approximate $d_p^2$ dependency.

Due to the subsequent cross correlation procedure involving products of intensities, the relative contribution of each particle in the interrogation area scaled with $d_p^4$, provided that none of the CCD pixels became saturated. However, inspection of the images showed that 3-5\% of the particles gave rise to saturation, corresponding to particles of size $d_p > d_p' \approx 4 \mu m$, where $d_p'$ is the smallest diameter causing saturation. The effective size distribution $f_e(d_p)$ thus yielded

$$f_e(d_p) = \begin{cases} (d_p/[m])^4 f(d_p), & \text{for } d_p < d_p' \\ (d_p'/[m])^4 f(d_p), & \text{for } d_p \geq d_p' \end{cases}$$ (6.15)

Using $f_e(d_p)$ instead of $f(d_p)$, the mean value, $\langle d_p \rangle$, increased from 1.75 $\mu m$ to 2.7 $\mu m$ and the variance, $\sigma_p^2$, from 0.50 $\mu m^2$ to 1.1 $\mu m^2$, yielding a spatial mean drift velocity of $\langle U_\epsilon \rangle_{xy} \approx 0.053 \, m/s$ for $J_m = 0.40 \, mA/m^2$. In the same case, the maximum difference between the two expressions of $\langle \ddot{u}_j' \rangle$ of Table 6.1 was then $\sim 4 \times 10^{-1} \, m^2/s^2$ or about 1 \%. This small difference was well within the range of uncertainty, thus the PIV data reduction was based on the expression valid for $K \gg 1$.

### 6.2 Velocity statistics

In order to display instantaneous measured velocities the assumption of $u_i = u_{p,i} - U_e,i$ was used for PIV measurements, and 1st and 2nd order gas velocity moments were calculated by use of Table 6.1 as

**LES:** \[ U_i = \frac{1}{N} \sum_{n=1}^{N} u_{i,n}, \] (6.16)

**PIV:** \[ U_i = \frac{1}{N} \sum_{n=1}^{N} u_{p,i,n} - U_e,i, \] (6.17)

**LDV:** \[ U_i = \frac{\sum_{n=1}^{N} u_{p,i,n} u_n}{\sum_{n=1}^{N} u_n} - U_e,i, \] (6.18)

and for the variances

**LES:** \[ \overline{u_i' u_j'} = \frac{1}{N} \sum_{n=1}^{N} u_{i,n}' u_{j,n}', \] (6.19)

**PIV:** \[ \overline{u_i' u_j'} = \frac{1}{N} \sum_{n=1}^{N} u_{p,i,n}' u_{p,j,n}', \] (6.20)

**LDV:** \[ \overline{u_i' u_j'} = \frac{\sum_{n=1}^{N} u_{p,i,n}' u_{p,j,n}' u_n}{\sum_{n=1}^{N} u_n} - U_e,i U_e,j \overline{d_p^2/d_p^2}, \] (6.21)
where the LDV particle transit time, \( \nu \), (sometimes referred to as residence time) was used as a weighting factor to overcome the problem of particle-rate/velocity correlation [11, 13].

The turbulence intensity was calculated for all three velocity components as

\[
Tu_u \equiv \frac{\sqrt{\langle u'^2 \rangle}}{U_0}, \quad Tu_v \equiv \frac{\sqrt{\langle v'^2 \rangle}}{U_0}, \quad Tu_w \equiv \frac{\sqrt{\langle w'^2 \rangle}}{U_0},
\]

(6.22)

based on the bulk flow velocity, \( U_0 \). The 3 dimensional turbulence intensity was

\[
Tu \equiv \frac{\sqrt{\nu \langle u'^2 \rangle}}{U_0}.
\]

(6.23)

6.3 Turbulence production and dissipation

Turbulence production, \( \mathcal{P} \), and dissipation, \( \epsilon \), were estimated from PIV and LES by use of results from Section 2.2

\[
\mathcal{P} = -\overline{u'_i u'_j s_{ij}} + \frac{1}{\rho \rho_i} \frac{\partial}{\partial x} \overline{u'_i u'_j} \approx -\overline{u'_i u'_j s_{ij}},
\]

(6.24)

\[
\epsilon = 2\nu \overline{s_{ij} s_{ij}}.
\]

(6.25)

All of the above terms could be estimated from LES by use of central difference approximation to the differential operator. But since PIV is a planar measuring method gradients perpendicular to the plane of investigation may only be estimated with respect to mean velocities. This means that if PIV axial planes are spaced close enough to resolve axial gradients in the mean flow, then \( \partial U_i / \partial x \) may be estimated in the same manner as \( \partial / \partial y \) and \( \partial / \partial z \). However, \( \partial u'_i / \partial x \) terms are impossible to get from the PIV measurements. These terms were thus estimated from an assumption of

\[
\frac{\partial u'_i}{\partial x} = \frac{1}{2} \left( \frac{\partial u'_i}{\partial y} + \frac{\partial u'_i}{\partial z} \right),
\]

(6.26)

even though the turbulence was anisotropic.

6.4 Characteristics of swirling motion

In the presence of an axial bulk flow, the body forces created secondary flows whose dominating feature appeared as axial rolls. The roll positions were highly unsteady and the flow possessed high levels of turbulence and axial vorticity with opposing sign.

6.4.1 Axial vorticity

The vorticity field is defined as the curl of the velocity field, \( \omega \equiv \nabla \times \mathbf{u} \). But since planar PIV only provides velocities in a 2D domain it is impossible to calculate instantaneous derivatives with respect to the out-of-plane direction. Thus, only the component of \( \omega \) in the direction normal to the PIV plane (i.e. \( \omega_x \)) is possible to calculate. Again,
finite difference may be employed to calculate the spatial derivatives, but Raffel et al. [45] argued that a two-dimensional vorticity estimation based on the Stokes theorem yields the most accurate results

\[ \omega_x = \lim_{s \to 0} \frac{1}{S} \int \mathbf{u} \cdot d\mathbf{l}, \tag{6.27} \]

where \( l \) describes the closed contour path around the surface of area \( S \) containing the point of evaluation. This means that \( \omega_x \) is calculated from the surrounding 8 velocities in cartesian mesh of the \( yz \)-plane. Eq. (6.27) was used for both PIV and LES data reduction.

### 6.4.2 Swirl center

The need for analyzing the instantaneous large-scale coherent structures in a \( yz \)-plane was met by the vortex identification algorithm proposed by Graftieaux et al. [22]. A global scalar function \( \Gamma \) was introduced, its maximum defining the center of an unsteady swirling flow superposed on a small-scale turbulent velocity field by only considering the topology of the flow and not the absolute velocity levels

\[ \Gamma(P) = \frac{1}{A} \int_A \frac{\mathbf{r} \times \mathbf{u} \cdot \mathbf{i}_x}{\mathbf{r} \cdot |\mathbf{u} \times \mathbf{i}_x|} \, dA, \tag{6.28} \]

where \( P \) is a point in the \( yz \)-plane, \( A \) the two-dimensional area surrounding \( P \), \( \mathbf{r} \) the vector from \( P \) to a fixed point in \( A \), \( \mathbf{u} \) the point velocity, and \( \mathbf{i}_x \) a unit vector in the \( x \)-direction. The position, \( P_C \), of the maximum absolute value of the variable \( \Gamma \) then specifies the instantaneous swirl center location (i.e. one center for each roll). For increased accuracy, the swirl center locations were found by local maximum detection on a refined mesh of \( \Delta = 0.5 \) mm by using bicubic interpolation on a 5×5 stencil surrounding the initial swirl center. The same procedure was used to determine the mean swirl center, \( \overline{P_C} \), of the mean velocity field.

### 6.4.3 Swirl number

Based on \( P_C \) it is possible to determine a swirl number, \( S \), characterizing the swirling motion around \( P_C \). \( S \) is the ratio of angular momentum flow, \( G_{\theta} \), to axial momentum flow, \( G_x \), over the area of the unit cell plane with a hydraulic radius \( L_H \equiv 2 \times \text{Area/Perimeter} \)

\[ S = \frac{G_{\theta}}{L_H G_x}, \tag{6.29} \]

\[ G_{\theta} = \int_A \rho u (\mathbf{r} \times \mathbf{u}) \cdot \mathbf{i}_x \, dA, \tag{6.30} \]

\[ G_x = \int_A \rho u^2 \, dA, \tag{6.31} \]

where contributions from pressure and turbulent velocity fluctuations have been ignored. Similarly, the mean swirl number \( \overline{S} \) was found with respect to \( \overline{P_C} \) for the mean velocity field.
6.5 Power spectrum and auto correlation function

Auto correlation functions and 1D power spectra may be calculated from time resolved data, such as LDV and LES. As shown in Section 6.1, the LDV measured particle velocities may be used to calculate the auto correlation function, and instead of treating the LDV data directly by a Direct Fourier Transform (DFT) the data was reconstructed and resampled equidistantly hereby allowing the use of Fast Fourier Transform (FFT) [11]. The method used to estimate Power Spectral Density (PSD) and Auto Correlation Function (ACF) is sketched in Figure 6.1. The individual steps will be described in the following.

![Diagram of the process](image)

**Figure 6.1:** Method of estimating PSD and ACF for LDV and LES velocity time series.

A) **Removal of mean velocity:** The fluctuating velocity \( u' \) was calculated from the time series, \( u \), by use of Eq. (6.16) and (6.18) for the \( N \) samples.
B) Reconstruction and resampling: The purpose of reconstruction and resampling of the LDV data is to get equidistant sampled data allowing a FFT processing. Several ways of reconstructing a continuous signal are explained by Schmidt [50], where the zeroth-order sample-and-hold procedure is the simplest and recommended by several authors [4, 39]. For the sample-and-hold interpolation the most recent sample is retained until a new one is available. Subsequently the continuous velocity is resampled with a prescribed constant frequency, which generally is higher than the mean frequency of the original time series. This is possible as the probability of a particle arrival decreases with time separation, thus much high-frequency information is contained in the LDV data [5]. The LDV time series were resampled at 5 kHz giving ~500,000 samples.

One advantage of the sample-and-hold technique is a vanishing velocity bias caused by the correlation of particle-rate with velocity. This is due to the fact that large interarrival times are re-sampled more often than are values with small interarrival times [11]. This principle is similar to the transit time weighting used in the moment estimations in Eq. (6.18) and (6.21). However, the sample-and-hold technique introduces some errors in the PSD, which were first discovered by Adrian & Yao [4]. They derived an expression for the expectation of the PSD \( E\{S_{xx}(f)\} \) based on the true PSD \( \hat{S}_{xx} \)

\[
E\{\hat{S}_{xx}(f)\} = \frac{1}{1 + f^2/\hat{n}^2} \left( \hat{S}_{xx}(f) + \frac{2\sigma_0^2}{\hat{n}^2 T_\lambda^2} \right), \tag{6.32}
\]

where \( f \) is frequency, \( \hat{n} \) is LDV mean sampling rate, \( \sigma_0^2 \) is velocity variance, and \( T_\lambda \) is temporal Taylor microscale. The step noise is due to the step-like jumps in a sample-and-hold signal, but the term was negligible for the present LDV data. The low-pass filter has a cut-off frequency of \( \hat{n}/2\pi \) and was accounted for by an inverse low-pass filter invoked at the ACF (see Figure 6.1).

C) Zero padding: The FFT method is based on the assumption that the input signal is cyclic with a period corresponding to the sampling period. If this is not true an error, known as circular correlation, is introduced. This error can be avoided completely by use of the zero padding technique, by which the sampling period is artificially doubled. This is done by adding to the signal a zero signal of the same length \( (N) \) as the original signal

\[
x_i = \begin{cases} 
\nu_i, & \text{for } i = 1, \ldots, N \\
0, & \text{for } i = N + 1, \ldots, 2N.
\end{cases} \tag{6.33}
\]
D) PSD by FFT of velocity: A first estimate of the PSD \( \hat{S}_{xx} \) may now be found by applying the FFT procedure to \( x \)

\[
\hat{S}_{xx,j} = \frac{\Delta \tau}{2N} \left| \sum_{n=0}^{2N} x_n \exp(-2\pi i j n/2N) \right|^2, \quad j = 1, \ldots, 2N
\]

where \( \Delta \tau \) is the time between the elements in \( x \), which was 200 \( \mu \)s and 20 \( \mu \)s for the LDV and LES series, respectively.

E) ACF by inverse FFT of PSD: The ACF \( R_{xx} \) was determined by invoking the inverse FFT to \( \hat{S}_{xx} \)

\[
R_{xx,k} = \frac{1}{2N \Delta \tau} \sum_{j=1}^{2N} \hat{S}_{xx,j} \exp(2\pi i j k/2N), \quad k = 1, \ldots, 2N
\]

In fact, the ACF consists of the first \( N \) elements only, since \( R_{xx} \) is symmetric.

F) ACF window filter: Due to limited number of samples the PSD appeared noisy. To overcome this problem frequency smoothing was applied. In practice a window filter was used in the time-domain to improve calculation time. \( R_{xx} \) was multiplied by a Hanning filter function \( w \) to get the window filtered ACF \( R_{xx}^w \)

\[
R_{xx,k}^w = w_k R_{xx,k}, \quad k = 1, \ldots, 2N
\]

\[
w_k = \begin{cases} 
\frac{1}{2} \left( \cos(\pi k/N_w) + 1 \right), & \text{for } k \leq N_w \\
0, & \text{for } k > N_w 
\end{cases}
\]

where \( N_w \) is the filter width, which was fixed to give a temporal filter width of 2 s.

G) ACF inverse low-pass filter: As noted under the Reconstruction and re-sampling section the sample-and-hold procedure introduces a low-pass filter for the PSD of the LDV data. This was accounted for by an inverse low-pass filter of the FIR type [9] introduced by Nobach & Tropea [39, 40]. The inverse filter was applied to the LDV window filtered ACF

\[
R_{xx,k}^{w, FIR} = \begin{cases} 
R_{xx,1}^w, & \text{for } k = 1 \\
(2c+1)R_{xx,k}^w - c \left( R_{xx,k-1}^w R_{xx,k+1}^w \right), & \text{for } k = 2, \ldots, 2N - 1 
\end{cases}
\]

with

\[
c \equiv \frac{\exp(-\hat{n} \Delta \tau)}{(1 - \exp(-\hat{n} \Delta \tau))^2}.
\]
H) PSD by FFT of ACF: The final estimation of the PSD was found by invoking
the FFT once again

\[ \hat{S}_{xx,j} = \Delta \tau \sum_{n=0}^{2N} R_{xx,n}^{IR} \exp(-2\pi ijn/2N), \quad j = 1, \ldots, 2N \]  

(6.40)

\[ \hat{S}_{xx,j} = \Delta \tau \sum_{n=0}^{2N} R_{xx,n}^{IR} \exp(-2\pi ijn/2N), \quad j = 1, \ldots, 2N \]  

(6.41)

Again only the first \( N \) elements of \( \hat{S}_{xx} \) were considered for the PSD, as for the ACF
in step E).

6.6 Length and time scales

The integral time scale, \( T_I \), was estimated from LDV and LES time series based on the
autocorrelation coefficient, \( \rho_{xx}(\tau) \equiv R_{xx}(\tau)/R_{xx}(0) \)

\[ T_I = \int_0^{T_w} \rho_{xx}(\tau) \, d\tau. \]  

(6.42)

In the definition of \( T_I \) the integration time is \( T_w = \infty \), but due to large scattering
of \( R_{xx} \), which increased with \( \tau \), a maximum time lag of \( T_w = 2 \) s was chosen. This
Corresponds to prefiltering with a top-hat filter of filter width 2 s.

The temporal Taylor micro-scale, \( T_\lambda \), could also be estimated from the ACFs pro-
vided that the temporal resolution is fine enough

\[ \frac{2}{T_\lambda} = - \frac{\partial^2 \rho_{xx}}{\partial \tau^2} \bigg|_{\tau=0}. \]  

(6.43)

Employing central difference discretization and using that \( \rho_{xx} \) is symmetric in \( \tau \),
Eq. (6.43) may be approximated by

\[ T_\lambda \approx \sqrt{-2 \frac{\Delta \tau^2}{\rho_{xx}(2\Delta \tau) - 1}}, \]  

(6.44)

where \( \rho_{xx}(2\Delta \tau) \) is the value of \( \rho_{xx} \) at \( \tau = 2\Delta \tau \).

Spatial scales could be estimated from PIV and LES by assuming isotropic hom-
ogeneous turbulence. In this way the longitudinal Taylor micro scales became [23]

\[ \lambda^2_{fu} = 2 \frac{\overline{u'u'}}{\left( \frac{\partial u'}{\partial x} \right)^2}, \quad \lambda^2_{fv} = 2 \frac{\overline{v'v'}}{\left( \frac{\partial v'}{\partial y} \right)^2}, \quad \lambda^2_{fw} = 2 \frac{\overline{w'w'}}{\left( \frac{\partial w'}{\partial z} \right)^2}, \]  

(6.45)

where \( \lambda_{fu} \) is the Taylor scale of \( u \) in \( x \)-direction, \( \lambda_{fv} \) of \( v \) in \( y \) and \( \lambda_{fw} \) of \( w \) in \( z \). Again
spatial derivatives were estimated from central difference discretization, and obviously
\( \lambda f_n \) could not be calculated from PIV, since axial gradients were lacking by the planar technique.

Assuming that the dissipation of turbulent kinetic energy, \( \varepsilon \), from Eq. (6.25) was properly estimated, one could calculate the Kolmogorov length scale, \( \eta \). Based on the Kolmogorov’s *Universal equilibrium Theory* [52] the length scale is defined as

\[
\eta \equiv \left( \frac{\nu^3}{\varepsilon} \right)^{1/4}.
\] (6.46)  

An order of magnitude estimate of the corresponding Kolmogorov time scale, \( T_\eta \), based on scale relations yields [52]

\[
T_\eta \sim \left( \frac{\nu^3}{\varepsilon} \right)^{1/2}.
\] (6.47)

### 6.7 Summary

This chapter showed the different post-processing techniques applied to both experimental and numerical data. Initially, a way of dealing with polydisperse seeding particles was presented. Corrections to measured particle velocities were introduced in order to get unbiased gas velocities.

A way of estimating turbulence production and dissipation was shown. After that, swirling motion quantities such as vorticity, swirl center detection and swirl number were introduced.

The last part dealt with estimation of power spectra, auto correlation function and length and time scales from time resolved data.
Chapter 7

Results: Flow structures

7.1 Reference flow

\((U_0 = 1.0 \text{ m/s}, J_m = 0.40 \text{ mA/m}^2, x = 750 \text{ mm})\)

In this section results from PIV are presented supplemented with LDV and LES results. Reference settings of bulk velocity \(U_0 = 1.0 \text{ m/s}\) and mean current density \(J_m = 0.40 \text{ mA/m}^2\) were used, and experimentally measurements were performed as far downstream in the test section as possible to be as close as possible to fully developed condition, i.e. \(x = 750 \text{ mm}\). Presented results are limited to the two center unit cells, \(z \in [-50, 50] \text{ mm}\). Instantaneous data as well as ensemble averages of 1000 PIV snapshots, 500 LDV samples, and 9 seconds of fully developed flow from LES were analyzed for steady and unsteady flow structures and other turbulent quantities. The test series are shown in Table 3.2 (page 20), Table 3.4 (page 22) and Table 5.1 (page 41) for PIV, LDV and LES, respectively.

Figure 7.1 shows secondary velocity \((v, w)\), axial velocity \(u\) and axial vorticity \(\omega_z\) measured by PIV. Both instantaneous and mean (ensemble average) values are shown, including locations of swirlcenters, \(P_C\). The mean values are based on 1000 snapshots. On average, the flow is directed towards the collector plate \((y = 0)\) in the \(xy\)-planes of electrode pins \((z = \pm 50 \text{ mm})\) and returns in between \((z = 0)\). The two axial rolls of opposite sign and approximately same strength have a higher axial velocity near centers of rolls. Regarding the instantaneous values, on the other hand, the velocity field is much more chaotic due to turbulence giving spots of high velocity and vorticity.

Generally, the secondary velocity is less than \(\sim 0.6 \text{ m/s}\), axial velocity is between 0.8 and 1.2 m/s and the absolute axial vorticity is less than \(\sim 50 \text{ s}^{-1}\). It is clear that swirlcenters \(P_C\) coincide with points of maximum absolute vorticity for the average velocity field, but this is not true for the instantaneous velocity field. However, it seems that the vortex identification algorithm (see Section 6.4.2) is able to locate the swirl center, \(P_C\), even though the flow is superposed the small scale turbulence. The flow is almost similar to the one measured by olive oil seeding presented by Ullam et al. [55], but with a higher degree of symmetry between the two unit cells. The lack of symmetry could be caused by current irregularities such as different currents from the discharge pins. A larger current in one side will cause a larger roll with higher secondary velocity. Due to the large axial inertia the observed asymmetries might be
due to irregularities occurring far upstream of the measuring plane.

LES results corresponding to the PIV results of Figure 7.1 are shown in Figure 7.2. The mean values are based on \( \sim 9 \) seconds of fully developed flow corresponding to 450,000 instantaneous realizations. Even though the flow is more regular in the mean than the measured flow, asymmetries are still present suggesting a very unstable flow. The main difference is, however, the smaller secondary velocities, which will be discussed in the following. The figure also shows that, although fully developed, the wall boundary layer is thin yielding an essentially flat axial profile. This is a result of the secondary flows.

Velocity moments in form of mean velocities, \( U \) and \( V \), and corresponding RMS velocities, \( U_{rms} \) and \( V_{rms} \), are presented in Figure 7.3 for PIV, LDV and LES. The values are taken from a line crossing the channel mid between the last 2 electrodes, \( x = 750 \) mm, halfway between bottom plate and electrode, \( y = 50 \) mm. The corresponding axial location in LES is \( x = 0 \) (cf. Figure 5.1). The different boundary conditions in experiments and simulations at \( z = \pm 100 \) mm appears through a no-slip velocity at the glass walls, which is seen in the LDV data as the wall is approached, although the wall boundary layer is very thin.

The difference in \( U \) and \( V \) between PIV and LDV may only be explained in different electrical conditions at the two times of measuring, e.g. discharge current variations. The difference between experimental and LES values in the two central unit cells, \( z \in [-50, 50] \) mm, is likely to be caused by the presence of the glass walls. The no-slip velocity at the side walls reduces the recirculation, hence the strength and size of the rolls in these cells, allowing the central rolls to enlarge. Inspection of the PIV velocity fields in Figure 7.1 shows center rolls of approximate sizes \( z \in [-60, 0] \) mm and \( z \in [0, 60] \) mm for the left and right roll, respectively. Referring back to Figure 3.7 on page 27 it is obvious that the center rolls benefit from a greater part of the large downward directed body force underneath electrode pins, which in turn increases the roll strength of the center rolls. A schematic drawing of the roll structures is presented in Figure 7.4.

Regarding turbulence, the level of \( U_{rms} \) is practically constant across the channel, whereas \( V_{rms} \) reflects the large gradients in \( V \). The levels found by the three methods are not identical. LDV gives the highest levels and PIV the lowest. This difference can not be explained, as with \( U \) and \( V \), by different electrical conditions, but is more likely caused by the different methods used. As explained in Section 3.2.4 and shown in Figure 3.4 page 24 the poor spatial resolution of PIV performs spatial filtering hereby lowering the RMS level. Thus the PIV resolved turbulence level is lower than the actual level. The length of the LDV measuring volume has the opposite effect, due to velocity gradient broadening within the measuring volume. Velocity gradient broadening causes a fictive turbulence caused by gradients in the mean velocity [19]. However, the PIV error is estimated to be about 10 % and the LDV error only about 1 %, which then may not explain the large differences in measured RMS values.

The 6 different components of the Reynolds stress tensor, \( \bar{u}_i \bar{u}_j \), are shown in Figure 7.5 for the 2 center rolls measured by PIV. The PIV area has been truncated for \( y > 90 \) mm due to erroneous data recorded in this area. The data reveal a significant anisotropy and non-homogeneity with \( \bar{u}_i \bar{u}_j \) being the most homogeneous of the three normal Reynolds stresses. The deviatoric Reynolds stresses, taking values between
Figure 7.1: Central unit cell PIV results of instantaneous (left) and mean values (right) of secondary velocity (upper), axial velocity with contour line spacing $\Delta = 0.1$ m/s and gray regions $>1.0$ m/s (middle), and axial vorticity with $\Delta = 20$ s$^{-1}$ and gray regions $>0$ (lower). Swirl center locations, $P_C$, are denoted with a plus.
Figure 7.2: Numerical LES results corresponding to experimental PIV data of Figure 7.1. (See caption of Figure 7.1).
7.1 Reference flow

Figure 7.3: Velocity moments at \((x, y) = (750, 50)\) mm from LDV, PIV and LES respectively. \(U_0 = 1.0\) m/s, \(J_m = 0.40\) mA/m².

Figure 7.4: Schematic of the roll structures in the channel cross section. Pin electrode at \(y = 100\) mm with two discharge pins at \(z = \pm 50\) mm, denoted with crosses. Upper half: idealized case with no effects from side-walls (LES). Lower half: wall effects are present (PIV and LDV).
$-0.01$ and $0.015 \text{ m}^2/\text{s}^2$, are generally smaller than the normal components. Generally, the spatial variation may be explained by locally generated shear stresses due to mean velocity gradients, except for the high values of $\overline{v'w'}$. When the vertical level of electrodes ($y = 100 \text{ mm}$) is approached $\overline{v'w'}$ is seen to increase. This increase could be due to axially convected turbulence generated at the electrode upstream of the measuring plane.

The unsteadiness of the swirling motion appears from Figure 7.6, where the instantaneous swirl centers, $P_c$, of the two center unit cells are shown. Both rolls show large variations between the individual realizations, and the asymmetry of the two rolls is evident through the distributions of $P_c$. The left roll is, on the mean, closer to the $z = 0$ plane and displays smaller spatial deviations than the right roll. Thus one may conclude that the right roll is more unsteady than the left one, which is supported by the spatial distributions of velocity variances of Figure 7.5.

The distribution of production, $\mathcal{P}$, and dissipation, $\epsilon$, of turbulent kinetic energy measured by PIV are shown in Figure 7.7. $\mathcal{P}$ is numerically larger than $\epsilon$, but large areas of negative values of $\mathcal{P}$ in the roll centers makes the spatial mean values comparable, of the order of $<\mathcal{P}>_{yz} \approx 2<\epsilon>_{yz}$, with $<\cdot>_{yz}$ denoting spatial averaging in $y$- and $z$-directions. The distribution of $\mathcal{P}$ corresponds to the distribution of $\overline{w'w'}$ of Figure 7.5, i.e. $\overline{w'w'}$ is presumably generated locally near the collector plate mid between discharge pins. This may be explained by the fact that this is where the two rolls with opposing $w$-velocity meet. This generates large levels of $\overline{w'w'}$ contributing to $\mathcal{P}$ by the term $\overline{w'w'} \partial W/\partial z$ (cf. Eq. (6.24)). Contrary to $\mathcal{P}$, $\epsilon$ is non-symmetric in $z = 0$, with larger dissipation rates in the right unit cell than in the left, caused by more small scale fluctuations.

While the flow is developing, the level of turbulent kinetic energy, $<k>_{yz}$, increases with $x$ as $<\mathcal{P}>_{yz}$ is greater than $<\epsilon>_{yz}$. But when the flow is fully developed, i.e. $<k>_{yz}$ is constant, one would assume $<\mathcal{P}>_{yz} \approx <\epsilon>_{yz}$. However, the ratio is close to 2, meaning that either the flow still develops at $x = 750 \text{ mm}$ or some estimations are wrong. Assuming that the dissipation, $\epsilon$, is properly estimated, the Kolmogorov length scale, $\eta$, is given by Eq. (6.46). The range of values of $\epsilon$ in Figure 7.7 gives values of $\eta$ between $\eta = 0.41 \text{ mm}$ to $\eta = 0.54 \text{ mm}$, which is below the spatial resolution of the PIV system (see Table 3.6 on page 23). Especially all of the small scale data from PIV should then be interpreted with the reservation that PIV performs spatial filtering. Thus, the large scale $\mathcal{P}$ is probably more correct than the small scale $\epsilon$.

Distributions of the Taylor micro scales from longitudinal correlations of $v$ in $y$-direction, $\lambda_{fv}$, and of $w$ in $z$-direction, $\lambda_{fw}$, are presented in Figure 7.8. $\lambda_{fw}$ is more or less constant over the area taking values around $10-15 \text{ mm}$. $\lambda_{fv}$ on the other hand is small close to the wall and large in the center of the rolls $\sim 15-20 \text{ mm}$. The Taylor scales are thus anisotropic like the turbulence.
Figure 7.5: PIV measured velocity variances and co-variances, $\overline{u_i'u_j'}$. Contour levels in $m^2/s^2$ and bold faced line indicates $\overline{u_i'u_j'} = 0$. 
Figure 7.6: 1000 PIV measured instantaneous swirl center locations, $P_C$, for the two center unit cells.

Figure 7.7: PIV based production (left) and dissipation (right) of turbulent kinetic energy. Mean values are $\langle P \rangle_{y_z} = 0.16 \text{ m}^2/\text{s}^3$ and $\langle \epsilon \rangle_{y_z} = 0.073 \text{ m}^2/\text{s}^3$. 
Figure 7.8: PIV based Taylor micro scales, $\lambda_{fv}$ (left) and $\lambda_{fw}$ (right). Contour levels in mm.
7.2 Flow development

\((U_0 = 1.0 \text{ m/s}, J_m = 0.40 \text{ mA/m}^2, x \in [150, 750] \text{ mm})\)

In this section results of the axial development in the test section from upstream of the first electrode, \(x = 150\), to upstream of the last electrode, \(x = 750\), is presented. Reference settings of bulk velocity \(U_0 = 1.0 \text{ m/s}\) and mean current density \(J_m = 0.40 \text{ mA/m}^2\) were used. The parametric settings appear in Table 3.2 page 20 as B) Flow development. The purpose of the study was to investigate how secondary velocities and turbulence developed along the test section.

The general flow patterns, at the different axial positions, appear through the sectional streamlines of Figure 7.9. The secondary velocities are already developing at \(x = 150 \text{ mm}\) upstream of the first electrode, but the roll pattern does not appear until \(x = 250 \text{ mm}\) after the first electrode. The streamlines may be compared to the vertical velocity component, \(V\), at \(y = 50\) shown in Figure 7.10 for the same PIV data.

It is interesting to note that the rolls develop with increasing symmetry and strength until \(x = 650 \text{ mm}\), where a clear asymmetry appears as a dominating left roll. As the results have proved reproducible this results suggests an electrical asymmetry at the 5th electrode \((x = 600 \text{ mm})\). One explanation could be that no or only a weak corona was present at the right discharge pin on the 5th electrode hereby reducing the right roll in favour of the left roll. At the next downstream location \((x = 750 \text{ mm})\) the symmetry was re-established, i.e. the left roll had lost some of its momentum to the right roll.

A way to describe the secondary velocity field by a single scalar is through the swirl number, \(S\), as used in Figure 7.11 for each of the two center unit cells. Apparently \(S\) does not increase from \(x = 350 \text{ mm}\) to \(x = 550 \text{ mm}\) despite the increasing secondary velocity. The reason for this is that the swirl centers (cf. Figure 7.9) move closer to the unit cell centers as axial distance increases hereby decreasing the angular momentum. The levels are identical to those found by olive oil seeding presented by Ullum et al. [55].

As the velocity develop so does the turbulence level as presented in Figure 7.12 for each of the three velocity components, \(Tu_u\), \(Tu_v\) and \(Tu_w\), as well as the three-dimensional turbulence intensity, \(Tu\). The intensities have been averaged over the entire PIV area from Figure 7.9. Reference levels at \(x = 750 \text{ mm}\) with no applied electrostatic field \((J_m = 0)\) are shown. The anisotropy is the same at all locations with \(<Tu_w>_{yz}\) being the largest and \(<Tu_u>_{yz}\) the smallest component. The turbulence level increases from about 0.07 at \(x = 150 \text{ mm}\) to about 0.18 at \(x = 650-750 \text{ mm}\). However, this is still the resolved level, which presumably is lower than the actual level. Although not conclusive, it seems that \(Tu\) is leveling off at \(x = 750 \text{ mm}\), like \(S\), at least for the left roll.
Figure 7.9: Sectional streamlines and swirl centers (denoted with a plus) for the mean velocity field. Axial position of measurement plane is indicated at upper right corner in each plot. Swirl center positions, $P_C$, for the 2 center rolls are indicated with a plus.
Figure 7.10: Vertical velocity component, $V$, at $y = 50$ mm and $x = 150, 250, \ldots, 750$ mm marked as 1, 2, \ldots, 7 respectively.

Figure 7.11: Axial development of absolute swirlnumber for the two rolls based on swirl centers of Figure 7.9.
Figure 7.12: Mean turbulence levels vs axial position. Reference levels for $J_m = 0$ at $x = 750$ mm are indicated with dashed lines.
7.3 Parametric effects

\((U_0 \in [1.0, 2.0] \text{ m/s}, J_m \in [0, 0.40] \text{ mA/m}^2, x = 750 \text{ mm})\)

In the preceding section the axial development from \(x = 150 \text{ mm}\) to \(x = 750 \text{ mm}\) was investigated. This section treats the parametric effects of varying either bulk flow velocity, \(U_0\), or mean current density, \(J_m\). The PIV measurements were performed as far downstream in the test section as possible (i.e. \(x = 750 \text{ mm}\)) to be as close as possible to fully developed flow. The experimental settings appear in Table 3.2 page 20 as C) Parametric effects. The experimental results are compared to LES results for varying mean current density as shown in Table 5.1 page 41.

The general flow patterns, at the different parametric settings, appears through the sectional streamlines of Figure 7.13 (reference case is no. 5) and the vertical velocity component, \(V\), at \(y = 50\) of Figure 7.14. Obviously the secondary velocities and degree of symmetry increase with increasing \(J_m\) and decreasing \(U_0\). The asymmetry is increasing from \(J_m = 0.05\)–\(0.30\) mA/m\(^2\) in favour of the left roll, but at \(J_m = 0.40\) mA/m\(^2\) the symmetry is established. This phenomenon could be explained by lack of corona glow at one or more discharge pins for \(J_m < 0.40\) mA/m\(^2\). However, the same effect is seen from varying \(U_0\) and keeping \(J_m\) constant. Usually it is assumed that the electrostatics is unaffected by the velocity field. This means that in this case it must be the fluid inertia that effected the secondary velocities and degree of symmetry by making upstream imperfections visible at the measuring position.

Looking at the fully developed flow from LES, the flow is much more symmetric even at low current densities. But secondary velocity increases with \(J_m\) as shown in Figure 7.15, and the levels of secondary velocities are comparable to the ones found by PIV.

The combined effects of varying bulk velocity and mean current density may be analyzed by the dimensionless Froude modulus

\[
Fr = \frac{1}{2} \rho \frac{U_0^2}{l_y J_m / b_f},
\]

being the ratio of axial flow inertia to transverse electric force. In Figure 7.16 the swirl number is used to describe the velocity field dependency on the Froude modulus. The left roll continues to increase until \(Fr^{-1} \approx 0.24\), whereas the swirl number of the right roll is almost constant until \(Fr^{-1} \approx 0.24\). The good correlation of the data by \(Fr\) confirms that the magnitude of \(S\) is a bulk flow phenomenon.

The LES results have also been analyzed for swirling motion by the swirl number as shown in Figure 7.17. The deviations between the two rolls are smaller than in the PIV results, but relatively large differences are still present. The general trend of \(S\) leveling off suggests that the angular momentum has a maximum value which is reached close to \(Fr^{-1} \approx 0.32\). As noted earlier, \(S\)-values from LES are smaller than those from PIV.

The effect of \(Fr\) on the measured mean turbulence level in the area of investigation is shown in Figure 7.18. For \(U_0 = 1.0 \text{ m/s}\) the turbulence appears to increase almost linearly with \(Fr^{-1}\) from about 0.08 to 0.17 with no sign of leveling off. The anisotropy is almost preserved and identical to the flow developing anisotropy of Figure 7.12 page 67. Comparing the two plots of Figure 7.18 shows once again the good correlation with \(Fr\).
Figure 7.13: PIV measured sectional streamlines and swirl centers (denoted with a plus). 1-5: $U_0 = 1.0$ m/s and $J_m = 0.05, 0.1, 0.2, 0.3, 0.4$ mA/m$^2$. 6-8: $J_m = 0.4$ mA/m$^2$ and $U_0 = 1.15, 1.41, 2.0$ m/s.
Figure 7.14: PIV measured vertical velocity component, $V$, at $(x, y) = (50, 750) \text{ mm}$. Upper: $U_0 = 1.0 \text{ m/s}$ and $J_m = 0.05, 0.1, 0.2, 0.3, 0.4 \text{ mA/m}^2$ denoted as 1, 2, ..., 5. Lower: $J_m = 0.4 \text{ mA/m}^2$ and $U_0 = 1.0, 1.15, 1.41, 2.0$ denoted as 6, ..., 8.
Figure 7.15: LES simulated vertical velocity component, $V$, at $(x, y) = (50, 750)$ mm. $U_0 = 1.0$ m/s and $J_m = 0.05, 0.1, 0.2, 0.3, 0.4$ mA/m$^2$ denoted as 1, 2, …, 5.

Figure 7.16: PIV measured absolute swirl number vs inverse Froude modulus based on swirl centers from Figure 7.13. Empty symbols: $U_0 = 1.0$ m/s and $J_m \in [0, 0.40]$ mA/m$^2$. Filled symbols: $J_m = 0.40$ mA/m$^2$ and $U_0 \in [1.0, 2.0]$ m/s.
Figure 7.17: Absolute swirl number from LES vs inverse Froude modulus.

The LES results have also been analyzed for turbulence levels as appearing in Figure 7.19. A good agreement with the experimental results of Figure 7.18 is seen in the 3-dimensional turbulence level, $<Tu>_y$. However, the anisotropy is smaller and different with $<Tu_z>_y$ being the largest and $<Tu_u>_y$ the smallest.
Figure 7.18: PIV measured mean turbulence levels vs inverse Froude modulus. Upper: $U_0 = 1.0$ m/s and $J_m \in [0, 0.40]$ mA/m$^2$. Lower: $J_m = 0.40$ mA/m$^2$ and $U_0 \in [1.0, 2.0]$ m/s.
**Figure 7.19:** Mean turbulence levels vs inverse Froude modulus for LES results.
7.4 Unit cell flow

\( (U_0 = 1.0 \text{ m/s, } J_m = 0.40 \text{ mA/m}^2, x \in [650, 750] \text{ mm}) \)

The results in this section has appeared in Ulhum & Larsen [53]. It serves to investigate how variations occur through a unit cell for reference conditions of \( U_0 = 1.0 \text{ m/s and } J_m = 0.40 \text{ mA/m}^2 \), as previous measurements were confined to planes mid between two electrodes. PIV was used to make a 3-dimensional flow mapping of the length of one unit cell from \( x = 650 \text{ mm to } x = 750 \text{ mm} \). The domain was mapped in 11 equidistantly spaced planes giving a spatial resolution of \( \Delta x = 10 \text{ mm} \) in axial direction. The experimental settings appears in Table 3.2 page 20 as D) Unit cell flow, and the experimental results are supplemented with LES results.

The velocity field of Figure 7.1 page 57 is characterized by large values of axial vorticity, \( \omega_z \), of opposite sign for the two unit cells. The axial vorticity field based on the measured mean velocities, \( \overline{\omega_z} \), at all axial planes is illustrated in Figure 7.20, showing iso-surfaces for \( |\overline{\omega_z}| = 25 \text{ s}^{-1} \). Asymmetry is clear for \( x < 690 \), where the contour surface of the right roll \( (z \text{ positive}) \) contracts in spanwise direction. Axial variations in the left roll \( (z \text{ negative}) \) are not as pronounced, but a tendency towards lower levels midway in the cell is noticeable. This is an effect of the body force being three-dimensional, which would be more pronounced if the bulk flow were decreased or the electrostatic field increased.

\[ \text{Figure 7.20: Contours of PIV measured absolute axial vorticity at } |\overline{\omega_z}| = 25 \text{ s}^{-1} \text{ based on mean velocity field. Electrode with pins is shown.} \]
Studying the unsteadiness of the swirling motion it appears that the right roll has two possible vertical positions for $x < 690$ mm between which it alternates in time making the mean value difficult to interpret and decreasing the mean axial vorticity as seen in Figure 7.20. This phenomenon may be explained by the left roll propagating into the right unit cell around $x = 650$ mm hereby decreasing the width of the right unit cell especially near $y = 50$ mm (cf. Figure 7.9 page 65). The largest roll allowed in the right unit cell will then be located either in the top or in the bottom of the unit cell. This unsteadiness of the right roll is supported by Figure 7.21, where the standard deviation, $\sigma$, of the measured swirl center, $\overline{R_C}$, is shown. The general level is about 12 mm, though for the right roll $\sigma$ decreases from about 32 mm at $x = 650$ before it reaches the 12 mm at $x = 690$ mm. No distinct axial variation of $\sigma$ for the left roll is noticeable.

![Figure 7.21: Standard deviation for the measured mean swirl center locations, $\overline{R_C}$, for both rolls vs axial position, $x$, of $yz$-plane. $J_m = 0.40$ mA/m$^2$ and $U_0 = 1.0$ m/s.](image)

Figure 7.22 shows the turbulence intensity, $Tu$, at the 11 PIV planes of the unit cell. $Tu$ takes values between 0.08 and 0.33 with large spatial variations. The unsteadiness of the right roll for $650$ mm $< x < 690$ mm is seen as areas of large $Tu$. Most remarkable, though, are the small areas just upstream the electrode ($x = 690$-700 mm), where $Tu$ attains very high values close to discharge pins ($z = \pm 50$ mm and $y \rightarrow 100$ mm). The spots of high turbulence were smoothed out with downstream distance by diffusion and secondary flow (primary reason).

Figure 7.7 page 62 showed the distribution of turbulent kinetic energy production, $\mathcal{P}$, at $x = 750$ mm. The same algorithm has been applied to the 11 PIV planes of the unit cell and the results appear in Figure 7.23. The area where the two rolls meet at $z \approx 0$ close to the wall is characterized by large $\mathcal{P}$ for all axial positions. But just upstream the discharge pins $\mathcal{P}$ attains much higher values. Apparently a large production of turbulent kinetic energy is found just upstream the discharge pins and not downstream. The current flux is symmetric in $x = 700$, but the bulk flow is antisymmetric. This leads to the explanation that the asymmetry in $\mathcal{P}$ must be related to the direction of the current relative to the bulk flow direction. Downstream of the electrode the current is directed in the bulk flow direction opposite upstream the electrode.
Figure 7.22: Axial variation of PIV measured turbulence intensity, $T_u$, for $x \in [650, 750]$ mm.
Figure 7.23: Axial variation of PIV measured turbulent kinetic energy production, $P$, for $x \in [650, 750]$ mm. Contours in [m$^2$/s$^3$].
7.5 Summary

It was not possible to apply PIV close to the discharge pins due to large depositions of seeding material close to the electrode holders on the glass walls making optical access impossible. But the LES flow field close to a discharge pin is shown in Figure 7.24. The figure shows both sectional streamlines and contours of $P$. It should be noted that the electrodes have not been resolved, thus the flow field is incorrect since it is only affected by the applied body force. However, it is clear that the current at the upstream discharge pin is capable of generating a region of negative axial flow and very high levels of $T_u$ and $P$. This turbulence producing region explains the spots of high $T_u$ and $P$ seen in the PIV images at $x = 690$ mm and 700 mm in Figure 7.22 and 7.23. Also the maximum measured value of $P$ of $\sim 10$ m$^2$/s$^3$ in Figure 7.23 is close to values read from Figure 7.24 10-15 mm from the electrode.

![Figure 7.24: Streamlines and estimated level of turbulent kinetic energy production, $P$, in [m$^2$/s$^3$] based on LES. Position of electrode and pin are shown, but not resolved, and coordinate system refers to experimental setup.](image)

7.5 Summary

This chapter provided results in form of flow structures obtained from PIV and LES and to a lesser extent from LDV. Focus was on the two center unit cells, $z \in [-50, 50]$ mm, containing two rolls superposed the axial bulk flow. Initially, the reference flow was presented in detail with distributions of $(v, w), (V, W), u, U, \omega, \Omega, u_{ij}$, $P_C, \lambda, \epsilon, \lambda_{fv}$ and $\lambda_{fw}$. The spatial positions of the two rolls were very unstable, thus benefiting from different amounts of the body force. As a result, the flow would be asymmetric due
to even small electrical imperfections. The Kolmogorov length scale and the Taylor micro scale were, for the reference flow, estimated to $\eta \sim 0.5$ mm and $\lambda \sim 10$-15 mm, respectively. Turbulence was anisotropic and inhomogeneous.

Second, the axial flow development through the test section was investigated. It was shown that the body force reached upstream the first electrode, where secondary velocities started to develop. The flow developed in axial direction with increasing degree of symmetry, roll strength and turbulence level. The turbulence level and roll strength, in form of swirl number, showed sign of levelling off at the downstream end, $x = 750$ mm.

Third, the parametric effects of changing either mean current density or bulk flow velocity were treated. The two parameters were combined in the Froude modulus, Fr. It was found that degree of symmetry, roll strength and turbulence level increased with $F_r^{-1}$ similar to the axial development with $x$. Again, the roll strength levelled off, but the turbulence level continued to increase almost linearly with $F_r^{-1}$ with preservation of anisotropy.

Finally, a detailed study of unit cell variations was presented. It showed that the right roll was much more unsteady than the left roll for the first $\sim 40$ mm, but the degree of symmetry increased with axial position. Apparently turbulence was mainly generated in two ways. For all axial positions turbulence was created where the two rolls met at $z \approx 0$, close to the collector plate. More locally, turbulence was also created upstream of the discharge pins due to corona induced axial recirculation. This was probably the main source of turbulence. The turbulence was distributed to the entire domain by primarily secondary flow and, to a lesser extent, by turbulent diffusion.
Chapter 8

Results: Analysis of time series

\( U_0 = 1.0 \, \text{m/s}, \, J_m \in [0, 0.40] \, \text{mA/m}^2, \, (x, y, z) = (750, 50, 50) \, \text{mm} \)

Time series of instantaneous velocity were measured by LDV and simulated by LES. The following results all refer to measurements in \( (x, y, z) = (750, 50, 50) \, \text{mm} \) by LDV and monitoring position of \( (x, y, z) = (100, 50, 50) \, \text{mm} \) by LES – i.e. the same spatial location relative to electrodes and assuming fully developed flow at \( x = 750 \, \text{mm} \). Mean current densities of \( J_m = 0, 0.05, 0.1, 0.2, 0.3, 0.4 \, \text{mA/m}^2 \) and a constant bulk flow velocity of \( U_0 = 1.0 \, \text{m/s} \) were used. The time series were analyzed for Power Spectral Density (PSD), Auto Correlation Function (ACF) and turbulent time scales.

By LDV \( u \) and \( v \) were measured during 100 s with a mean sample rate in the range of 700-1200 Hz yielding 70,000-120,000 samples. By LES \( u, v \) and \( w \) were monitored for \( \sim 10 \, \text{s} \) at 50 kHz. Truncating the first 2 s of the LES series to assure fully developed flow the series consisted of \( \sim 400,000 \) samples.

8.1 Spectral analysis

The LDV PSDs of \( u \) and \( v \) estimated from Eq. (6.40) are shown in Figure 8.1-8.2 for six levels of mean current density, \( J_m \). The figures also show the cut-off frequencies \((\hat{u}_m/2\pi)\) based on the mean sample rate, \( \hat{u}_m \). However, the FIR filter should eliminate the filtering effect from the sample-and-hold technique completely. Due to slow variations in the time series, \( \hat{S}_{xx} \) does not vanish as \( f \rightarrow 0 \). Also, it is difficult to distinguish when \( \hat{S}_{xx} \) falls off at the Kolmogorov scale as the \( \hat{S}_{xx} \) starts to decrease at a low frequency, \( \sim 1 \, \text{Hz} \). An order of magnitude estimate of the Kolmogorov time scale, \( T_\eta \), based on Eq. (6.47) and a dissipation rate of \( \epsilon \sim 0.07 \, \text{m}^2/\text{s}^3 \) (see Figure 7.7) gives \( T_\eta \sim 15 \, \text{ms} \) or a frequency of \( \sim 70 \, \text{Hz} \).

However, it is clear that \( \hat{S}_{xx} \) generally increases with \( J_m \), i.e. higher levels of turbulent kinetic energy at all frequencies. Also, peaks seem to appear at 5-6 Hz and 10 Hz corresponding to lengths of 15-20 cm and 10 cm based on a characteristic velocity of 1 m/s. These lengths are also the channel height and the unit cell height, respectively.

Figure 8.3 shows the PSDs for \( v \) from the LES simulations estimated from Eq. (6.41). PSDs for \( u \) and \( w \) have also been calculated, but are not shown, as they appear almost similar to the one for \( v \). Again \( \hat{S}_{xx} \) increases with \( J_m \), but the high sampling rate of 50 kHz makes it possible to capture higher frequencies than by the LDV. Hereby it is clear
that the frequency, at which $\tilde{S}_xx$ starts to drop off, increases with $J_m$, meaning that turbulence appears at higher frequencies when $J_m$ is increased. At $J_m = 0.40$ mA/m² turbulent energy is present up to approximately 80 Hz, which is in good agreement with the Kolmogorov frequency estimated to 70 Hz. The LDV peak frequency of 10 Hz are also present at all levels of $J_m$, but it becomes less pronounced with increasing $J_m$.

8.2 Auto correlation and time scales

The ACFs of the LDV data are estimated by Eq. (6.35) and the autocorrelation coefficients, $\rho_{xx}(\tau) = R_{xx}(\tau)/R_{xx}(0)$, are shown in Figure 8.4-8.5. The ACFs of the LES data are also given by Eq. (6.35) and shown for $v$ in Figure 8.6. ACFs from LES for $u$ and $w$ look similar to $v$ and are not shown. At $J_m = 0$ the LES ACF indicates a correlation period of 0.2 s. This is caused by the imposed periodic boundary conditions in the flow direction forcing structures to reappear every 0.2 s as the domain length is 0.2 m and the bulk velocity is 1.0 m/s. When $J_m$ is increased the periodicity vanishes as the secondary velocities and high turbulence levels do not allow structures to pass through the domain without strong deformation. The imposed periodicity of the LES makes the ACF difficult to interpret. However, at $J_m = 0.30$ and $0.40$ this phenomenon is weak and the ACFs indicate correlation times like the ones found by LDV.

Integral time scales estimated by numerical integration of Eq. (6.42) by the trapezian method are shown in Figure 8.7 (upper). The general trends are similar for $u$ and $v$ based $T_I$, but for $J_m = 0.05-0.30$ mA/m² the $u$-based $T_I$ is considerably higher than the $v$-based one. The increase in $T_I$ from $J_m = 0$ to $J_m = 0.05$ mA/m² must be explained by some large structures generated at the low current density. As $J_m$ is increased further these structures become smaller and at $J_m = 0.40$ mA/m² the integral time scale becomes $T_I = 0.05-0.06$ s. With a characteristic velocity of 1.0 m/s this corresponds to 50-60 mm, i.e. the width of a unit cell. This could be interpreted as the structures being confined to the individual unit cells.

Inspection of the LDV ACFs near $\tau = 0$ showed an approximately linear decrease in $\rho_{xx}$ indicating that the resolution is too coarse for Eq. (6.44) to be used for estimating the Taylor micro scale. But in general the Taylor micro scale is also the average distance between zero-crossings of a random variable in time, which is approximately true for turbulence as well [21]. Thus $T_\lambda$ for the LDV data is approximated by the zero-crossings of $u'$ and $v'$, whereas for the LES data Eq. (6.44) has been used. The results appear in Figure 8.7 (lower). $T_\lambda$ is, contrary to $T_I$, almost independent of $J_m$ and is about 0.015 s and 0.01 s for the LDV and LES data respectively. Further, no distinct difference between the velocity components considered is visible, pointing to homogeneous sized structures. The deviation between the levels of the LDV and LES data might be caused by the different methods used to evaluate $T_I$. Using the same characteristic velocity as for $T_I$ (1.0 m/s) the Taylor micro scale yields 10-15 mm, which corresponds perfectly to what was found by PIV (cf. Figure 7.8, page 63).
Figure 8.1: Power spectral density for $u$ measured by LDV in $(x, y, z) = (750, 50, 50)$ mm. Cut-off frequency is shown as dashed line.
Figure 8.2: Power spectral density for $v$ measured by LDV in $(x,y,z) = (750, 50, 50)$ mm. Cut-off frequency is shown as dashed line.
Figure 8.3: Power spectral density for $v$ by LES in $(x, y, z) = (100, 50, 50)$ mm.
Figure 8.4: Autocorrelation coefficient for $u$ measured by LDV in $(x, y, z) = (750, 50, 50)$ mm.
Figure 8.5: Autocorrelation coefficient for $v$ measured by LDV in $(x,y,z) = (750,50,50)$ mm.
Figure 8.6: Autocorrelation coefficient for $v$ by LES in $(x, y, z) = (100, 50, 50)$ mm.
8.2 Auto correlation and time scales

Figure 8.7: Integral time scale, $T_I$, (upper) and temporal Taylor micro scale, $T_{\lambda}$, (lower) estimated from $u$ and $v$ from LDV and LES time series. $T_{\lambda}$ is based on either zero-crossings of $u'$ and $v'$ (LDV) or $\rho_{uu}$ and $\rho_{uv}$ (LES).
8.3 Summary

This chapter provided results based on time resolved data from LDV and LES. Calculation of power spectral density (PSD) and auto correlation function (ACF) formed the basis of the data reduction. Peaks of high energy content were found at frequencies of 5-6 Hz and 10 Hz, which agreed well the physical dimensions of the test section and the unit cell, respectively. LES results showed that turbulent energy was present at higher frequencies when mean current density, \( J_m \), was increased.

The LES auto correlation functions were non-physical in the sense that the imposed periodic boundary conditions in axial direction forced structures to reappear. This effect, however, became less pronounced as \( J_m \) was increased.

Based on the auto correlation functions and zero-crossings of \( u' \) and \( v' \), the integral time scales, \( T_f \), and the temporal Taylor micro scales, \( T_\lambda \), were estimated. From \( J_m = 0.05 \text{ mA/m}^2 \), \( T_f \) decreased with \( J_m \), indicating smaller structures. The Taylor micro scales, on the other hand, were almost unaffected by \( J_m \) with values of \( T_\lambda \approx 0.015 \text{ s} \) for LDV data and \( T_\lambda \approx 0.01 \text{ s} \) for LES data.
Chapter 9

Results: Precipitation efficiency

Direct and indirect measurements of the local concentration of Rolovit seeding particles were executed in order to estimate the precipitation efficiency of the laboratory ESP. Direct measurements were performed by a particle counter giving also size estimation, and indirectly estimated concentrations were obtained from the grey scale levels of the PIV images.

9.1 Particle counter

Local values of the aerodynamic particle diameter distribution was measured by a TSI-3320 Aerodynamic Particle Sizer in the size range of \( d_p \in [0, 20] \) \( \mu m \) [7]. The instrument gives estimates of particle sizes based on an assumption of spherical particles influenced only by Stokesian drag. The instrument sucked in air with suspended particles from a 3 mm diameter probe inserted in one of two holes placed about 250 mm upstream of the test section or 250 mm downstream of the test section. The two positions are referred to as upstream and downstream, respectively. The probe was placed approximately in the center of the channel cross section and a 90° bend on the probe allowed the intake to face the bulk-flow direction. Since the particle concentrations were to high for the instrument a by-pass of atmospheric air was added. Consequently, it was not possible to calculate absolute concentrations since only the total flow rate was known. However, the flow rates were constant allowing relative concentration measurements. One sample lasted 300 s giving between 600,000 to 2,000,000 counts with size estimation. The sampling time minimized the effect of temporal variations in seeding addition.

Figure 9.1 shows the relative distribution functions of 1 upstream and 3 downstream samples truncated for \( d_p > 5 \) \( \mu m \). All measurements were performed with a bulk flow velocity of \( U_0 = 1.0 \) m/s while mean current densities were varied with \( J_m = 0, 0.20, 0.40 \) mA/m². The downstream concentration with \( J_m = 0 \) was measured to see if any natural precipitation was present. 5% of the particles precipitated in this way, which was found to be of minor importance. The following concentrations measured downstream of the test section will then relate to the upstream concentration.

Generally, concentrations decrease with increasing \( J_m \) and so does the mean diameter. Table 9.1 shows the different values of mean diameter based on number, \( \overline{d_{p,n}} \), and mass, \( \overline{d_{p,m}} \), assuming spherical particles. It shows how precipitation mainly affects larger particles due to their larger drift velocities. The table also shows precipitation
efficiencies based on both number, $\eta$, and mass, $\eta_m$, of particles (cf. Eq. (2.33), page 14). For reasons of comparison, the Deutsch efficiency, $\eta_D$, from Eq. (2.34) is also shown in Table 9.1, based on $x = 1.0$ m, $U_0 = 1.0$ m/s, $t_y = 0.1$ m and drift velocity for a mean diameter particle, $V_{dc}$, from Figure 3.8 on page 27. The filter length of $x = 1.0$ m has been used since particles precipitated over the entire length of the test section, even though only 0.8 m was covered with pin electrodes. The experimentally observed efficiencies are much larger than $\eta_D$, which assumes infinite diffusion, i.e. uniform concentration in the cross section. The deviation may be caused by the fact that the concentration at the measuring probe position was not representative for the entire cross section. The probe was placed approximately in the center of the cross section of the channel, $y \approx 100$ mm, where the particle concentration might be relatively low, despite the considerable degree of mixing due to secondary velocities and turbulence.

![Graph](image)

**Figure 9.1:** Measured aerodynamic diameter distribution function, $f(d_p)$, of Rolovit particles normalized by upstream function, $f_0$ (black curve). Black: upstream. Red: downstream, $J_m = 0$. Blue: downstream, $J_m = 0.20$ mA/m$^2$. Green: downstream, $J_m = 0.40$ mA/m$^2$.

### 9.2 Exposure of the PIV image

The relatively large precipitation of PIV seeding particles caused areas of low particle densities hereby impeding PIV measurements. Figure 9.2 shows two PIV images at $x = 750$ mm taken with the left camera (cf. Figure 3.2 on page 18). Reflections from the aluminum bottom plate are seen as well as the electrode with teflon holder placed at $x = 800$ mm. No electric field is present at the left image giving an approximately uniform seeding density. However, in the right image an electric field with $J_m = 0.40$ mA/m$^2$ is applied, giving lower seeding density and less uniformity.

As the total light captured by the PIV image depends on the number of scattering particles in the light sheet, one may argue that the grey scale levels of the PIV images
Table 9.1: Mean diameters, $d_p$ (based on diameter) and $d_{p,m}$ (based on mass), and efficiencies, $\eta$ (based on number) and $\eta_m$ (based on mass), from distribution functions $f(d_p)$ of Figure 9.1. $\eta_D$ is the Deutsch efficiency.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Upstream</th>
<th>Downstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_p$ [μm]</td>
<td>2.00</td>
<td>1.82</td>
</tr>
<tr>
<td>$d_{p,m}$ [μm]</td>
<td>2.29</td>
<td>2.11</td>
</tr>
<tr>
<td>$\eta$ [%]</td>
<td>–</td>
<td>58</td>
</tr>
<tr>
<td>$\eta_m$ [%]</td>
<td>–</td>
<td>67</td>
</tr>
<tr>
<td>$\eta_D$ [%]</td>
<td>–</td>
<td>19</td>
</tr>
</tbody>
</table>

Figure 9.2: PIV images at $x = 750$ mm from left camera with cropped image between horizontal white lines. Left: $J_m = 0$. Right: $J_m = 0.40$ mA/m².
could give an estimate of the particle concentration in the imaged area. To ensure that the captured light originated from particle scatter the images were cropped with 200 px from the top and 150 px from the bottom as shown with white lines in Figure 9.2. The analyzed area was then approximately \( y \in [-75, 75] \text{mm} \) and \( z \in [15, 80] \text{mm} \). However, particles deposited on the glass walls could still be present in the preserved area, and additionally, temporal variations in seeding addition and energy of light pulses forming the laser sheet constituted sources of error.

Assuming no imperfections, the image mean concentration of particles, \( < C > \), scales with the number of illuminated particles, \( K \), whereas the mass based concentration scales with \( K < d_p^3 > \). As discussed in Section 6.1.1 on page 45 the light scattering of the Rolovit particles would give an approximate \( K < d_p^3 > \) dependency on the image mean exposure, \( < \epsilon > \). The relative concentrations might then be estimated as

\[
\frac{< C >}{< C >_0} \sim \frac{K}{K_0} \sim \frac{< \epsilon >}{< \epsilon >_0} \cdot \frac{< d_p^3 >}{< d_p^3 >_0}
\]

\[
\frac{< C_m >}{< C_m >_0} \sim \frac{K < d_p^3 >}{K_0 < d_p^3 >_0} \sim \frac{< \epsilon >}{< \epsilon >_0} \cdot \frac{< d_p^3 >}{< d_p^3 >_0} = \frac{< d_p^3 >}{< d_p^3 >_0} \frac{< d_p^3 >_0}{< d_p^3 >_0}
\]

where index \( m \) refers to mass and 0 to a reference level. Since the particle size statistics of the PIV images are unknown, it is not possible to directly relate mean exposures to particle concentrations. The bias error introduced by neglecting the particle diameter variations may be estimated from the point measurements from Figure 9.1 to be

\[
1 < \frac{< d_p^3 >}{< d_p^2 >} < 1.27, \quad 0.94 < \frac{< d_p^3 >}{< d_p^3 >_0} \frac{< d_p^3 >_0}{< d_p^3 >_0} < 1
\]

where \( < \cdot > \) has been interpreted as ensemble average. The results show that the relative concentrations by number will be underestimated by up to 27%. The relative concentrations by mass will be overestimated by up to 6%.

The PIV images were stored with an 8 bit resolution giving pixel grey levels from 0 to 255, which may be interpreted as a measure of the exposure, \( \epsilon \), of each pixel. A grey level of 0 corresponds to no CCD charge giving the color black, whereas a grey level of 255 corresponds to CCD saturation giving the color white.

Figure 9.3 shows the axial evolution in spatially averaged mean grey levels, \( < \epsilon > \), in the cropped PIV image. Mean grey levels are based on ~650,000 image pixels in 10 snapshots, with generally higher levels in the right camera due to different sensitivity. Since the two curves follow the same trend one may conclude that depositions on windows and other disturbances are either identical for the two cameras or negligible. Biasing due to temporal variations in seeding addition and laser power are assumed overcome with by using several snapshots. The precipitation efficiencies from \( x = 150 \text{mm} \) to \( x = 750 \text{mm} \) are estimated to \( \eta \approx 31 \% \) and \( \eta \approx 35 \% \) for the right and left camera, respectively. For a \( \Delta x \) of 600 mm the corresponding Deutsch efficiency is \( \eta_D = 17 \% \). Again, the experimentally determined efficiency is higher than the Deutsch efficiency.

The same procedure as used to generate the results of Figure 9.3 is used for Figure 9.4, but this time for studying the effect of different electric field levels. Also, the mean pixel grey levels are now converted into efficiency by assuming
Figure 9.3: Axial development of mean pixel grey levels of PIV images, based on the test series B) Flow development of Table 3.2.

\( \langle \epsilon \rangle / \langle \epsilon \rangle \approx \langle C \rangle / \langle C \rangle_0 \). The value at \( J_m = 0 \) is used as reference level, which is the reason why efficiencies obtained from Figure 9.3 are not identical to the level at \( J_m = 0.40 \) mA/m\(^2\) of Figure 9.4. The Deutsch efficiency is also shown based on drift velocities from Figure 3.8 on page 27 and a precipitation length of \( x = 0.75 \) m. As one would expect, the efficiency increases with increasing field levels except from \( J_m = 0.05 \) mA/m\(^2\) to \( J_m = 0.10 \) mA/m\(^2\). In general the measured efficiency is higher than the Deutsch efficiency, which was also expected, but the results seems more realistic than the results presented in Table 9.1 measured in one point of the cross section. One may conclude that concentrations were inhomogeneous and the one point measurement was insufficient to get a good estimate of the mean concentration over a cross section.

9.3 Summary

Direct and indirect measurements of the particle concentrations were performed. Direct measurements were made with a particle sizer giving size distributions upstream and downstream of the test section. Efficiencies were as high as 71% based on counts or 78% based on mass for the 1.0 m test section. Efficiencies were highest for the largest particles. The unrealistic high efficiencies were likely to be caused by the measurement probe being placed in a downstream position of low concentration.

Indirect estimations of efficiencies were made by relating the mean exposure of the PIV images to particle concentrations. Efficiencies found by this approach were about 35% for 0.6 m of the test section \( (x \in [150, 750]) \) mm. Second, the mean current density was varied and concentrations were related to particle concentration at \( x = 750 \) mm
Figure 9.4: Efficiencies based on PIV images from left (PIV (left)) and right (PIV (right)) camera compared to the Deutsch (Deutsch) efficiency as function of the mean current density. Results are based on the test series C) Parametric effects of Table 3.2 with $U_0 = 1.0$ m/s.

with no field. Efficiency increased with electric field level and was in the range of 15-25%.
Chapter 10

Conclusions

The electrohydrodynamic flow in a laboratory scale electrostatic precipitator has been investigated experimentally and numerically. The non-intrusive optical measurement technique, Particle Image Velocimetry (PIV), has been employed to capture spatial flow structures in a measurement plane perpendicular to the bulk flow direction. These measurements have been supplemented by results from Laser Doppler Velocimetry (LDV) to compare statistics and get time resolved data. Also, numerical results obtained by Large Eddy Simulation (LES), were used to supplement and compare to the PIV results.

Optical measurement techniques, such as PIV and LDV, measure velocities of seeding particles. Usually this velocity will also represent the gas velocity, since particles are assumed to follow the gas motion. However, in the present study particles moved relative to the gas with an electrically induced drift velocity. Treating the drift velocity as steady, it was shown how to correct the measured particle velocities for drift, even when poly-disperse seeding particles were used. The drift velocity correction was obtained by solving the electrostatic Maxwell equations.

The numerical electrostatic results were also used to calculate the Coulomb body force needed for the LES. This was needed as the ionized gas also was influenced by the applied electrostatic field. The simulations were carried out with the PEGASE code invoking the Mixed Scale subgrid scale model with an selection function. The computational domain corresponded to two unit cell lengths of the full cross section of the experimental test section.

The electrostatic field generated secondary flow consisting of axial rolls superposed the axial bulk flow. The rolls were very unsteady, which made them vary in both position and strength. The techniques involved provided an enormous amount of data, which have been analyzed for different phenomena, such as secondary swirling motion and the generated scales and levels of turbulence. It turned out that the side-walls did not only affect the rolls at the walls but also the center rolls were influenced. As the rolls at the walls decreased in strength, they allowed the center rolls to enlarge and their strength to increase (cf. Figure 7.4 on page 59). A lack of symmetry appeared in some cases probably due to electrical imperfections upstream of the measuring position.

The strength of the swirling motion was characterized by a swirl number. The swirl number increased with axial position and level of mean current density and decreased with bulk flow velocity. Asymmetries were present in many cases, but symmetry was
established when the swirl number values leveled off at about 0.3.

The turbulence was both anisotropic and inhomogeneous. The level for a constant mean current density of \( J_m = 0.40 \, \text{mA/m}^2 \) increased from about 8% at the inlet and leveled off at about 18% at the outlet. The turbulence level increased almost linearly with mean current density with no sign of leveling off. Similarly, the turbulence level decreased with increasing bulk velocity.

It was argued that the turbulence production would only be caused by shear stresses of the flow, thus not affected by electrical fluctuations. Detailed analysis of the flow in a unit cell showed that for all axial positions turbulence was created where the two rolls met close to the collector plate. More locally, and to a higher extent, turbulence was also created 10-15 mm upstream of the discharge pins. The turbulence was distributed to the entire domain by secondary flow and, to a lesser extent, by turbulent diffusion.

Time resolved data from LDV and LES were used to calculate power spectral densities, auto correlation functions and time scales. Typically, peaks of high energy were found at 5-6 Hz and 10 Hz and very little energy was present at frequencies higher than 80 Hz. Temporal Taylor micro scales were practically constant of about 0.01-0.015 s and the integral time scale decreased with increasing current density to about 0.05 s at \( J_m = 0.40 \, \text{mA/m}^2 \).

Finally, estimations of precipitation efficiencies were obtained in two ways. One, by point measurements with a particle sizer and second, by the exposure of the PIV images. The point measurements gave efficiencies that in some cases exceeded 70%. This was unrealistic high and probably due to the position of the measuring probe in a non-uniform concentration. Mean exposures of the PIV images, on the other hand, gave efficiencies that were less than 30% and more realistic. In both cases efficiencies increased with the applied electrostatic field.
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