



## An approach to Kidney parameter identification by digital computer curve fitting of radiorenograms

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<p>Title and author(s)</p> <p>An Approach to Kidney Parameter Identification by Digital Computer Curve Fitting of Radiorenograms</p> <p>by K. Søb Højberg</p>	<p>Date June 1972</p> <p>Department or group Electronics Department</p> <p>Group's own registration number(s) R - 4 - 72</p>
<p>7 pages + 2 tables + illustrations</p>	
<p><b>Abstract</b></p> <p>The renogram test involved includes detection over the heart region, the kidneys, and the bladder after injection of radiohippuran.</p> <p>The external system as well as the kidneys are simulated by means of two-compartment models. The bladder is represented by one compartment.</p> <p>The compartment parameters are determined by curve fitting of the model and patient renograms. The background contribution from the blood circulation system, to the bladder and the kidney detectors, is subtracted.</p>	<p>Copies to</p>
<p>Available on request from the Library of the Danish Atomic Energy Commission (Atomenergikommisjonen Bibliotek), Rissø, Roskilde, Denmark. Telephone: (03) 35 51 01, ext. 334, telex: 5072.</p>	<p>Abstract to</p>

In the renogram test considered, the course of radioactivity is measured over each kidney and the heart as well as the bladder region. The test is initiated with an intravenous injection of radiohippuran. The blood background contribution to the kidney and bladder detectors is evaluated by a separate test with radioactively labelled albumin.

## METHOD

### Model

The model (fig. 1) is a simplified version of a model developed in 1967 (1, 2, 4). As the measuring method is not selective, only a simple model could be justified in connection with the curve fitting to be described here. A special feature of the model is the compartment  $Q_{10}(Q_{20})$  which acts like a tube with pure transport delay, and in a rough way represents the kidney nephrons.  $Q_{11}(Q_{21})$  gives a rough representation of the pelvis region. Together  $Q_3$  and  $Q_4$  are a simplified model of the total blood circulation and the extravascular system.

### Analytical solution

The solution of the compartment equations can be expressed as a sum of exponential terms of the form

$$R = a + \sum_{i=1}^n b_i e^{k_i T}$$

$R$  is one detector output (radioactivity in cps).

$a$  and  $b_i$  represent positive or negative constants which are a function of dose, detector gain, and rate constants.

$k_i$  represents rate constants.  $T$  is the time as measured from the beginning of the renogram phase considered (fig. 2). If the background contribution is subtracted, the blood curve and the first renogram phase contain 2 exponential terms each ( $n=2$ ,  $T=t$ ). The second renogram phase includes 3 exponential terms ( $n=3$ ,  $T=t-\tau_1$ ), and the bladder curve 6 terms ( $n=6$ ,  $T_1=t-\tau_1$ ,  $T_2=t-\tau_2$ ). On the basis of the equations for the first renogram phases the dose  $D$  (fig. 2) can be calculated for comparison with the measured initial dose.

### Digital computer curve fitting

The curve fitting is carried out by means of Wood's Pattern-search Method (3). Limitation of the parameter range has been included. Arbitrary compartment parameters are selected for the model. The digital computer then solves the compartment equations described and adds the blood background contribution. Then the computer compares the calculated curves with the patient renograms and calculates the difference between the curves according to the least-squares method. On the basis of this error the digital computer adjusts the parameters and calculates a new error, etc., until the result is satisfactory.

The blood curve is analysed separately. The error criterion is

$$E = \sum_{t=0}^{t_{\max}} (R_{mt} - R_{pt})^2$$

where  $R_{mt}$  is the radioactivity of the model curve, and  $R_{pt}$  the radioactivity of the patient curve at the time  $t$ .

The kidney and the bladder curves are analysed together. The error criterion is

$$E = E_1 + E_2 + E_3 + V(D_m - D_p)^2$$

$E_1$ ,  $E_2$ , and  $E_3$  are the curve fitting errors for the 3 sets of curves as explained above.  $D_m$  is the calculated model dose (fig. 2).  $D_p$  is the patient dose as measured by a separate measurement.  $V$  is a weight factor (0.01 selected here).

Input data used for the kidney and the bladder curve fitting are:

- 1) 4 constants determined by the previous blood curve analysis;
- 2) initial conditions  $Q_{10}(0)$  and  $Q_{20}(0)$  read from the renograms (fig. 2) at  $t=0$  by drawing a tangent to the initial phase of each renogram.  $t=0$  is defined to be one minute after the first renogram radioactivity increase;
- 3) kidney and bladder detector output  $R_{pt}$  read for each minute;
- 4) dose  $D_p$  measured separately;
- 5) arbitrary parameter values (here  $k_1=0.5$ ,  $\tau_1=3$ ).

## RESULTS

Parameters determined for four patients with normal kidneys are given in table I. Input factors  $k_{13D}$  and  $k_{23D}$  include detector constants.  $k_{13}$  and  $k_{23}$  (fig. 1) can be derived by means of the total rate  $k_3$  obtained from the blood curve analysis. In table II the measured dose is compared with the kidney model dose as explained. The model is tested by comparison of the urine activity of the patient with the model urine activity obtained by

extending the simulated renogram test time. In fig. 3 the patient and model renograms are compared.

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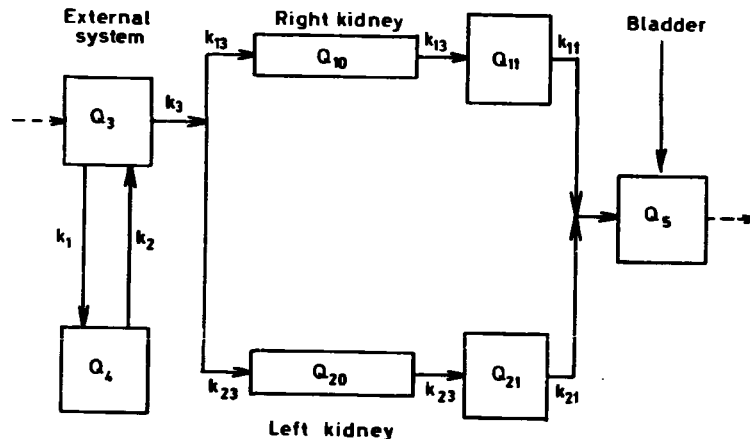


Fig.1. Compartmental model for renogram test simulation.

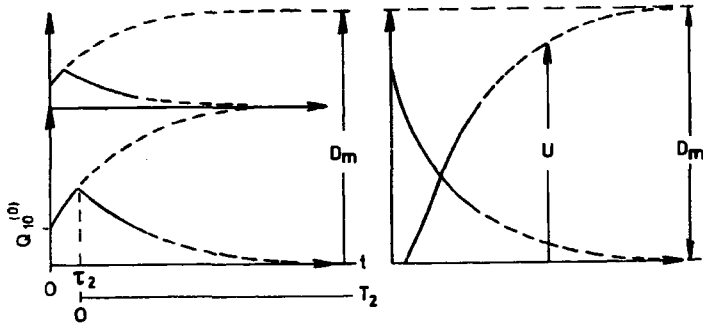


Fig. 2. Schematic renogram test curves (background subtracted).

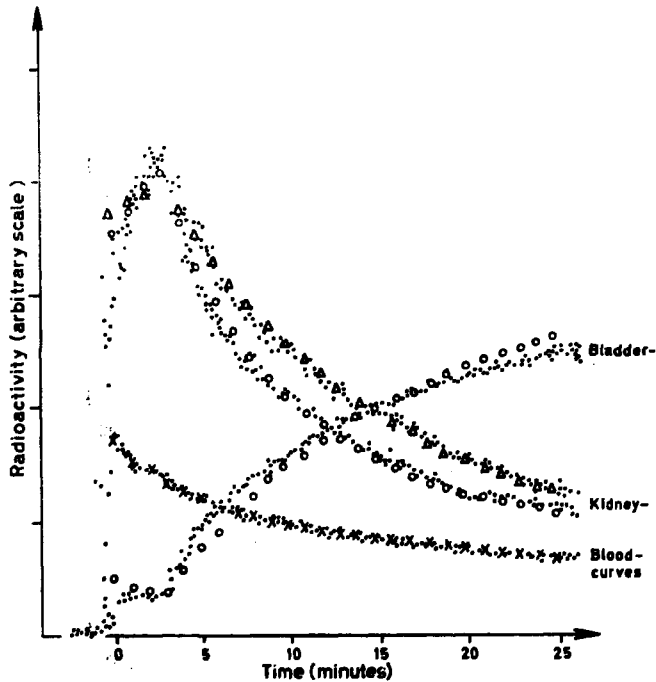


Fig. 3. Curve fitting results for the kidney and bladder renograms and blood curves.

Table I  
Kidney parameters obtained by curve fitting.

NO.	$k_{13D}$ $m^{-1}$	$k_{23D}$ $m^{-1}$	$k_{11}$ $m^{-1}$	$k_{21}$ $m^{-1}$	$c_5$	$\tau_1$ m	$\tau_2$ m	$E_{AV}$ %
1	0,098	0,113	0,286	0,276	0,268	4,18	2,89	0,89
2	0,077	0,146	0,108	0,211	0,254	3,24	2,95	0,79
3	0,036	0,145	0,081	0,436	0,298	7,17	10,69	1,38
4	0,024	0,039	0,067	0,077	0,269	5,40	4,48	1,28
5	0,059	0,111	0,136	0,250	0,272	5,00	5,25	1,09

NO., patient no;  
 k, rate constant;  
 c, bladder detector constant;  
 $\tau$ , delay;  
 E, average error % of dose;  
 M, all patients, mean value;  
 m, minutes.

**Table II**

**Dose obtained by curve fitting.**

**Comparison of model and patient urine activity.**

<b>NO.</b>	<b>D<sub>p</sub> cps</b>	<b>D<sub>m</sub> cps</b>	<b>U<sub>p</sub> %</b>	<b>U<sub>m</sub> %</b>
<b>1</b>	<b>1890</b>	<b>1990</b>	<b>87.1</b>	<b>74.9</b>
<b>2</b>	<b>1770</b>	<b>2070</b>	<b>76.7</b>	<b>51.6</b>
<b>3</b>	<b>1790</b>	<b>1600</b>	<b>66.1</b>	<b>49.7</b>
<b>4</b>	<b>2080</b>	<b>1430</b>	<b>60.7</b>	<b>60.3</b>

**D, dose;**

**p, patient;**

**m, model;**

**NO. , patient no. ;**

**U, urine activity % of dose (t = 30 to 35 minutes).**