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Reflection and absorption coefficients for use in room acoustic simulations

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Two ideas to improve the boundary conditions for room acoustic simulations are presented. First, all rooms have finite boundary surfaces, thereby a reflection coefficient for finite surfaces should be physically more suitable than that for infinitely large surfaces. Second, absorption coefficients measured by the chamber method, so-called the Sabine absorption coefficients, have certain problems to be used in geometrical acoustics simulations: one serious problem is that they often exceed unity for porous absorbers due to the finite sample size and non-uniform intensity in the test reverberation chamber. Therefore the Sabine absorption coefficients should be converted into the random incidence absorption coefficients, which never exceed unity, thus are more proper for room acoustic simulations.

1. Introduction

For room acoustic simulations, the absorption/reflection characteristics of the boundary surfaces should be characterized as precisely as possible. In practice, measured absorption coefficients in reverberation chambers and theoretically calculated reflection coefficients based on the infinite panel theory have been widely used. However, the absorption coefficients measured in reverberation chambers, so-called the Sabine absorption coefficients, are problematic to be used in computer simulations since they are often higher than unity. The reflection coefficient based on the infinity panel theory is inaccurate particularly at low frequencies, since the actual room boundary surfaces are not infinitely extended. Thus, this study aims to suggest a better reflection/absorption coefficient for more accurate room acoustic simulations.

2. Finite-sized reflection coefficient

The plane-wave reflection coefficient based on the infinite panel theory is expressed as [1]

\[
\frac{r_{\text{inf}}}{\zeta} = \frac{\zeta - \zeta_{\text{rad,inf}}}{\zeta + \zeta_{\text{rad,inf}}} = \frac{\zeta - 1/\cos \theta}{\zeta + 1/\cos \theta}. \tag{1}
\]

Here, \( \zeta \) is the normalized surface impedance and \( \theta \) is the incidence angle. This is known to be inaccurate at low frequencies and for small surfaces. Thomasson theoretically derived a radiation impedance for a finite panel backed by a rigid wall as [2],

\[
\zeta_{\text{rad,buffer}} = -\frac{jk}{S} \iiint_{S} G e^{-jkR} \left[ \mu_x (x-x_0) + \mu_y (y-y_0) \right] dx dy dz. \tag{2}
\]

where \( k \) is the wavenumber, \( S \) is the surface area, \( \mu_x = \sin \theta \cos \phi \), \( \mu_y = \sin \phi \), \( G = -\exp(-jkR)/2\pi R \) and \( R = \sqrt{(x-x_0)^2 + (y-y_0)^2} \). A new radiation impedance for the finite-sized reflection coefficient is defined as the
mean value of $\zeta_{\text{rad,baffle}}$ and $1/\cos \theta$ as [3].

$$
\eta_{\text{inc}} = \frac{\zeta - \zeta_{\text{rad,baffle}}}{\zeta + \zeta_{\text{rad,baffle}}} = \frac{\zeta - \left(\zeta_{\text{rad,baffle}} + 1/\cos \theta\right)}{\zeta + \left(\zeta_{\text{rad,baffle}} + 1/\cos \theta\right)}
$$  \hspace{1cm} (3)

3. A simulation example

A rectangular room of dimensions of $5 \times 1 \times 1$ m is simulated. A point source is positioned at $(0.1, 0.1, 0.4)$ representing a talker. A total of 36 receivers are positioned with $x$ changing from 0.5 to 4.5 with steps of 0.5, and $y$ changing from 0.2 to 0.8 with steps of 0.2, and a fixed $z$ of 0.3. Transfer functions are calculated using a phased beam tracing method (PBTM) and boundary element method (BEM) from 20 Hz to 710 Hz at 2 Hz intervals. From the calculated transfer functions, 1/3 octave band levels re. 1 Pa are computed, and named as $\text{SPL}_{\text{PBTM,oct}}$ and $\text{SPL}_{\text{BEM,oct}}$. Then the simulation error is defined as

$$
e(f_c) = \frac{\text{SPL}_{\text{PBTM,oct}}(f_c) - \text{SPL}_{\text{BEM,oct}}(f_c)}{(\text{dB})},
$$  \hspace{1cm} (4)

where $f_c$ means the center frequencies of the 1/3 octave bands from 31.5 Hz to 630 Hz. When the normalized surface impedance of the boundary surfaces is set to 3.9, which is equivalent to an absorption coefficient of 0.8, the error of PBTM using the infinite panel theory in Eq. (1) and using the newly suggested finite-sized reflection coefficient in Eq. (3) is shown in Fig. 1 [3]. Particularly at low frequencies, improvements are noticeable, which is natural since the ratio of the surface dimension to the wavelength is small, thus it is difficult to assume that the surface is infinitely large.

4. Converting Sabine to random incidence absorption

The Sabine absorption coefficient has been widely used in room acoustic simulations, but it is well known that this quantity is likely to be overestimated [2,4]. Therefore the random incidence absorption coefficient calculated by Paris’s law should be used in geometrical acoustics simulation. There are several ways of converting the Sabine absorption coefficient into the random incidence absorption [5]. First, one can convert the Sabine absorption to the surface impedance, then calculate the random incidence absorption coefficient based on local reaction. Second, the flow resistivity value can be inversely estimated from the measured Sabine absorption coefficient, and then the random incidence absorption coefficient can be finally calculated. In
this study, the matlab function 'fminsearch' is used for optimizing the surface impedance and the flow resistivity value. The cost function is defined as

\[
e_{\text{cost}} = \sum_{f} \left| \alpha_{\text{measured}}(f) - \alpha_{\text{opt}}(f) \right|, \tag{5}
\]

where \( \alpha_{\text{measured}} \) is the Sabine absorption coefficient and \( \alpha_{\text{opt}} \) is the size-corrected absorption coefficient.

First for estimating the surface impedance, \( Z = \mathbf{R} + j \chi \), the resistance term (\( R \)) can be assumed to be a linear function of \( \log(f) \), as \( R = A + B \log(f) \). For a fairly constant resistance over the frequency range, the slope (\( B \)) is resultantly negligible. The reactance term is generally written as \( \chi = -C \rho \theta \cot(k d) \), where \( \omega \) is the angular frequency, \( m \) is the mass, and \( d \) is the cavity depth. In most cases, but not necessarily, the mass term can be negligible if a nearly massless thin film or membrane is loaded on the porous material. Therefore the reactance term becomes \( -C \rho \theta \cot(k d) \). The parameters to be optimized are \( A \) and \( B \) in the resistance term, and \( C \) and \( d \) in the reactance term. Often, if the air cavity depth (\( d \)) of the construction is known, it can be excluded from the optimization set.

\[
\alpha_{\text{opt}} = 2 \int_{0}^{\pi} \frac{4[A + B \log(f)]}{\left|[A + B \log(f) - jC \rho \theta \cot(k d)] + Z \right|^2} \sin(\theta) d\theta. \tag{6}
\]

Second, the flow resistivity value can be estimated from the measured Sabine absorption coefficient based on theoretical/empirical formulae. In this study, Miki's empirical model was used [6]. Estimating the flow resistivity based on Miki's model has two advantages. First, it uses only one optimization parameter, the flow resistivity, provided that one already knows the absorber thickness and the cavity depth. If the information about the thickness or cavity depth is unknown, the optimization parameters are set to \([ \sigma, d, d] \) instead. Second, knowledge of the flow resistivity enables surfaces of extended reaction as well as locally reacting absorbers to be modeled. Therefore, the flow resistivity optimization is suitable for absorbers behaving as extendedly reacting, e.g., having a backing cavity between the specimen and the hard wall of the test chamber. The size-corrected absorption coefficient is given by

\[
\alpha_{\text{opt}} = 2 \int_{0}^{\pi} \frac{4Z_{\alpha} \sin(\theta)}{|Z_{\alpha} + Z_{\chi}|^2} d\theta. \tag{7}
\]

5. A conversion example

A mineral wool absorber from [7] was 5 cm thick and the density was 50 kg/m\(^3\). A square specimen with an edge length of 3.6 m was measured in two reverberation chambers of volumes 190 and 200 m\(^3\). Note that the Sabine absorption coefficient is overestimated compared to the random incidence absorption coefficient, on average by 0.12 as shown in Fig. 3. The random incidence absorption coefficients estimated by the suggested methods are also shown in Fig. 3. The surface impedance estimation method produce an absorption difference of 0.02, whereas the differences between the flow resistivity estimation by Eq. (7) and the true random incidence absorption data is 0.04. The average optimized parameters \([A, B, C, d] \) are found to be \([-556, 121, -0.04, 0.002] \) for Eq. (6). The optimized flow resistivity value is 40161 Ns/m\(^4\).

![Figure 3. Estimated random incidence absorption coefficients for a mineral wool of 5 cm thickness. Measured data from [7].](image-url)
6. Conclusion
This study is concerned with proper reflection and absorption coefficients for use in geometrical room acoustic simulations. The finite-sized reflection coefficient can improve simulation results mainly at low frequencies, and the conversion from the Sabine to random incidence absorption is beneficial, particularly for porous materials that yield overestimated absorption coefficients measured by the chamber method.

References