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Ion acoustic waves in the presence of high frequency oscillations

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Abstract

Spatially varying high frequency oscillations will influence the electron motion in plasmas (through ponderomotive forces) and in turn affect the characteristics of ion acoustic oscillations. In this report we derive the dielectric function for long wavelength, low frequency ion acoustic waves in the presence of high frequency, short wavelength oscillations. Apart from reproducing well known results this dielectric function shows the possibility of unstable ion acoustic waves for extreme intensities of high frequency oscillations. It is demonstrated that moderate intensities have a stabilizing effect on linearly unstable double humped ion distribution functions and it is argued that this effect may be of importance in laboratory experiments. Finally we point out some generalisations.

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CONTENTS

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H.L. Pécseli

Introduction

In this preliminary report we consider the influence of high frequency short wavelength electron oscillations on the propagation on low frequency long wavelength ion acoustic oscillations.[1,2] The high frequency oscillations are denoted by (ω, k) and the ion acoustic oscillations by (Ω, q). We describe the ions by the collisionfree Vlasov equation

$$\frac{\partial f_i}{\partial t} + v\frac{\partial f_i}{\partial x} + \frac{e}{M} E \frac{\partial f_i}{\partial v} = 0 \tag{1}$$

The electrons are considered as an isothermal fluid and we let the electron motion be determined by[1]

$$m\left(\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x}\right) = -eE - \frac{\kappa T_e}{n_e}\frac{\partial n_e}{\partial x} + F \tag{2}$$

The force F arises from the high frequency oscillations (plasmons) and is determined by the following argument[1,3]: Let E(x,t) be a high frequency field that varies slowly in amplitude with position. The equation of motion for an electron is then

$$\ddot{x} = \frac{e}{m} E(x,t) \tag{3}$$

We look for solutions in the form $x = y + \xi$ where y is a slow displacement of the particle due to spatially varying amplitude of E and ξ is a small displacement in the high frequency field. Expanding E(x,t) in powers of ξ we obtain

$$\ddot{y} + \ddot{\xi} = \frac{e}{m}\left(E(y,t) + \xi \frac{\partial}{\partial y} E(y,t)\right) \tag{4}$$

Taking the rapidly oscillating term in (4) we obtain

$$\xi = -\frac{e}{m}\sum_{\omega}\frac{E_\omega(y)}{\omega^2}\exp(-i\omega t) \tag{5}$$

Inserting (5) in (4) and averaging over at time τ where $\frac{1}{\omega} \ll \tau \ll \frac{1}{\Omega}$ we obtain

$$F = m\langle\ddot{y}\rangle = -\frac{\partial}{\partial y}\sum_{\omega}\frac{e^2E_\omega^2}{2m\omega^2} \tag{6}$$

A similar problem is considered in ref. 4 in a more general form. The high frequency oscillations obeys the dispersion relation

$$\omega = \omega_p\left(1+\frac{3}{2}\frac{k^2\kappa T_e}{m\omega_p^2}\right) = \omega_p\left(1+\frac{3}{2}(kd)^2\right) \tag{7}$$

where $\omega_p = \omega_p(y)$ is the local electron plasma-frequency. We prefer to use k as a summation index. Transforming the sum in (6) into an integral (with proper change in the dimensions of the integrand since $\sum_k \to \frac{L}{2\pi}\int dk$) we obtain

$$F = -\frac{e^2}{2m}\frac{\partial}{\partial y}\int\frac{E_k^2}{\omega_k^2}dk \tag{8}$$

Introducing $N_k = \frac{\epsilon_0 E_k^2}{\omega_k}$ equation (2) reduces to

$$m\left(\frac{\partial V_e}{\partial t}+V_e\frac{\partial V_e}{\partial x}\right) = -eE - \frac{\kappa T_e}{n_e}\frac{\partial n_e}{\partial x} - \frac{e^2}{2\epsilon_0 m}\frac{\partial}{\partial x}\int\frac{N_k}{\omega_k}dk \tag{9}$$

The variation of N_k is determined by

$$\frac{\partial N_k}{\partial t} + \frac{\partial\omega}{\partial k}\frac{\partial N_k}{\partial x} - \frac{\partial\omega}{\partial x}\frac{\partial N_k}{\partial k} = -2\gamma_k N_k \tag{10}$$

where γ_k is the appropriate decay time for the distribution of high frequency oscillations. Equation (10) corresponds to the zero order W.K.B. approximation for the evolution of a wave packet of high frequency oscillations. This approximation is valid since we assume $k \gg q$ and $\frac{\partial}{\partial x}$ refers to the spatial variation of the ion density.

Using (7) we reduce (10) to:

$$\frac{\partial N_k}{\partial t} + 3\frac{\kappa T_e k}{m\omega_p}\frac{\partial N_k}{\partial x} - \left(1-\frac{3}{2}(kd)^2\right)\frac{\omega_p}{2n_i}\frac{\partial n_i}{\partial x}\frac{\partial N_k}{\partial k} = -2\gamma_k N_k \tag{11}$$

Finally we introduce Poisson's equations

$$\frac{\partial E}{\partial x} = \frac{e}{\epsilon_0}(n_i - n_e) \tag{12}$$

Equations (1), (9), (11) and (12) form a closed set of equations. In the following we will use the terms "radiation pressure" for F and "number of quasiparticles" (or plasmons) for N_k although this terminology may be slightly misleading.

We linearize the equations (1), (9) and (11) and obtain

$$\frac{\partial f_i}{\partial t} + v\frac{\partial f_i}{\partial x} + \frac{e}{M}E f_0'(v) = 0 \tag{13}$$

$$m\frac{\partial v_e}{\partial t} = -eE - \frac{\kappa T_e}{n_0}\frac{\partial n_e}{\partial x} - \frac{\omega_p}{2n_0}\frac{\partial}{\partial x}\int N_k dk \tag{14}$$

$$\frac{\partial N_k}{\partial t} + \frac{3\kappa T_e k}{m\omega_p}\frac{\partial N_k}{\partial x} - \left(1-\frac{3}{2}(kd)^2\right)\frac{\omega_p}{2n_0}\frac{\partial n_i}{\partial x}\frac{\partial N_k}{\partial k} = 0 \tag{15}$$

In the integrand in the last term of (9) we have replaced $1/\omega_k$ with $1/\omega_p$. This is a reasonable approximation for the case of weak dispersion (see eq. (7)). The zero order distribution of quasiparticles, N_0, is a function of time $N_0(k) = N_0(k)\exp(-2\gamma_k t)$. In the following we assume $\gamma_k = 0$ corresponding to a stationary spectrum. We may for inst. assume that $N_0(k)$ qualitatively represents a fluctuation level due to the thermal fluctuations of the electrons.

In a more interesting case $N_0(k)$ is determined by a turbulent spectrum.

The following features of F and N_k may be noted : the force F tries to push the electrons to regions at low fluctuation level i.e. small N_k thus justifying the term "radiation pressure". The force acting on the quasi-particles, $-\frac{\partial\omega}{\partial x}$ tries to condense the quasi-particles in regions of low ion density, a feature obtained also by physical intuition when the dispersion relation (7) is taken into account.

In eqs. (13)-(15) ω_p and d refer to the unperturbed state.

The dielectric function

By considering the adiabatic response to a test charge we find the dielectric function for the medium. In the calculations we neglect electron inertia i.e. the term on the left side of (14).

$$\varepsilon(q,\Omega)=\frac{1}{(qd)^2}\left[1+(qd)^2-\left(C_s^2+\frac{\omega_p}{2M}\oint\frac{Q(k)N_0'(k)}{v_g(k)-\frac{\Omega}{q}}dk\right)\oint\frac{f_0'(v)}{v-\frac{\Omega}{q}}dv\right] \tag{16}$$

where

$$C_s^2=\frac{\kappa T_e}{M},$$

$$Q(k)=\left(1-\frac{1}{2}(kd)^2\right)\frac{\omega_p}{2n_0}$$

and

$$v_g(k)=\frac{\partial\omega}{\partial k}=3\frac{\kappa T_e}{m\omega_p}k$$

The sign $\oint$ indicates that the integration path runs below the pole. Since we have assumed that q is small ($q \ll 1/d$) we will neglect the term $(qd)^2$ in the paranthesis in (16).
The validity of (16) for the case where $N_o=0$ is well established experimentally.[6,7] We will now consider the effect of the high frequency oscillations.

We consider three special cases since eq. (16) is somewhat confusing for arbitrary $f_o(v)$ and $N_o(k)$.

I. Assume $f_o(v)=\delta(v)$.

Then

$$\mathrm{Re}\,\varepsilon(q,\Omega)=1-\left(\frac{q}{\Omega}\right)^2\left(C_s^2+\frac{\omega_p}{2M}P\int\frac{QN_0'}{v_g-\frac{\Omega}{q}}dk\right) \tag{17a}$$

$$\mathrm{Im}\,\varepsilon(q,\Omega)=-\frac{\omega_p^2 m}{6M\kappa T_e}\pi\,Q\left(\frac{\Omega\omega_p m}{3q\kappa T_e}\right)N_0'\left(\frac{\Omega\omega_p m}{3q\kappa T_e}\right)\left(\frac{q}{\Omega}\right)^2 \tag{17b}$$

For N_o=0 eq. (17) describes undamped oscillations. When $N_o \neq 0$, these oscillations are damped (if $N_o' < 0$ since $Q > 0$ for cases of interest) on quasiparticles with distribution $N_o(k)$ in a way much similar to the conventional Landau damping (This effect is discussed in Ref. 2). Conversely a distribution of quasiparticles for which $N_o' > 0$ for some k may lead to instability. (Note however that for $\Omega/q \approx C_s$ the quantity $(\Omega\omega_p m)/(3q\varkappa T_e)$ corresponds to a very small k-value. We have assumed q $\ll$ k. For small k we may let $Q \sim \frac{\omega_p}{2n_o}$).

The imaginary part of the frequency is determined by the well known expression

$$\mathrm{Im}\,\omega \simeq - \frac{\mathrm{Im}\,\varepsilon(K,\Omega_K)}{\frac{\partial\,\mathrm{Re}\,\varepsilon(K,\Omega_K)}{\partial\Omega_K}} \tag{18}$$

where Ω_K is the solution to Re $\varepsilon(K,\Omega_K)$=0. Eq. (18) is valid for small Imω.

II. As a second case assume $N_o(k)=N_oK\,\delta(k-K)$. This assumption corresponds to the case of a monochromatic high frequency oscillation. Then:

$$\mathrm{Re}\,\varepsilon = 1 - \left(C_e^2 + \frac{\omega_p N_o K}{24M\varkappa T_e}\left(\frac{3K^2\varkappa T_e + 2\omega_p^2 m - 2Km\omega_p\frac{\Omega}{q}}{n_o\left(K - \frac{\Omega\omega_p m}{3q\varkappa T_e}\right)^2}\right)\right)P\int\frac{f_o'}{v-\frac{\Omega}{q}}dv \tag{19a}$$

$$\mathrm{Im}\,\varepsilon = -\pi f_o'\left(\frac{\Omega}{q}\right)\left(C_e^2 + \frac{\omega_p N_o K}{24Mn_o\varkappa T_e}\left(\frac{3K^2\varkappa T_e + 2\omega_p^2 m - 2Km\omega_p\,\Omega/q}{\left(K - \frac{\Omega\omega_p m}{3q\varkappa T_e}\right)^2}\right)\right) \tag{19b}$$

It is well known that this particular $N_o(k)$ leads to instability if we also choose $f_o(v) = \delta(v)$. ("Instability of a cold plasmon gas" see ref. 2 and 3). In the case where $f_o(v) \neq \delta(v)$ (assume f_o to be a Maxwellian for definiteness) we consider:

$$3K^2\varkappa T_e + 2\omega_p^2 m - 2Km\omega_p\frac{\Omega}{q} = 0$$

$$K = \frac{m\omega_p\Omega}{3q\varkappa T_e} \pm \sqrt{\left(\frac{m\omega_p\Omega}{3\varkappa T_e q}\right)^2 - \frac{2}{3}\frac{\omega_p^2 m}{\varkappa T_e}}$$

When K has tworeal solutions the paranthesis in (19b) becomes negative for sufficiently large N_o, and K chosen appropriately. Obviously there is a threshold value for N_o. If Re $\varepsilon(q,\Omega) = 0$ has solutions also we get unstable oscillations. Since the case where $f_o = \delta(v)$ is unstable for <u>arbitrary</u> K* we notice a stabilizing effect of the thermal spread in ion velocities. This instability belongs to the class of parametric instabilities. For a review see for inst. F.F. Chen.[8]

III. Finally we consider the case where

$$N_o(k) = \begin{cases} 0 & \text{for } |k| > K \\ N_o & \text{for } |k| \leq K \end{cases}$$

Then:

$$\mathrm{Re}\,\varepsilon(q,\Omega) = 1 - \left(C_e^2 - \frac{\omega_p^2 mQ}{6M\varkappa T_e}\,\frac{2N_oK}{K^2 - \left(\frac{\Omega\omega_p m}{3q\varkappa T_e}\right)^2}\right)P\int\frac{f_o'}{v-\frac{\Omega}{q}}dv \tag{20a}$$

$$\mathrm{Im}\,\varepsilon(q,\Omega) = -\pi f_o'\left(\frac{\Omega}{q}\right)\left(C_e^2 - \frac{\omega_p^2 mQ}{6M\varkappa T_e}\,\frac{2N_oK}{K^2 - \left(\frac{\Omega\omega_p m}{3q\varkappa T_e}\right)^2}\right) \tag{20b}$$

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* and N_o large enough[2]

For physical reasons K is of the order 1/d (d the Debye-length).

Then

$$\frac{\Omega}{q}\,\frac{\omega_p m}{3\varkappa T_e}\,\frac{1}{K} \simeq \frac{\Omega}{q}\,\frac{1}{3d\omega_p} = \frac{\Omega}{q}\,\frac{1}{3v_{therm}} \ll 1$$

where v_{therm} is the electron thermal velocity. Equations (20a) and (20b) then reduces to

$$Re\,\varepsilon(q,\Omega) = 1 - \left(C_e^2 - \frac{\omega_p^2 m Q}{3M\varkappa T_e}\,\frac{N_o}{K}\right) P\int \frac{f_o'(v)}{v-\frac{\Omega}{q}}\,dv \tag{21a}$$

$$Im\,\varepsilon(q,\Omega) = -\pi f_o'\left(\frac{\Omega}{q}\right)\left(C_e^2 - \frac{\omega_p^2 m Q N_o}{3M\varkappa T_e K}\right) \tag{21b}$$

Assume for definiteness that $f_o(v)$ is a Maxwellian. We may then make use of the plasma dispersion function[9] in calculating the integral in (21a). For small Ω/q this integral is negative.

If

$$N_o\,\frac{\omega_p^3 m\left(1-\frac{3}{2}(Kd)^2\right)}{6Mn_o\varkappa T_e K} > C_e^2 \tag{22}$$

then a Nyquist contour encircles the origin and the plasma is unstable.[10,11]

If $K < 0.82\ d^{-1}$ and

$$N_o > \frac{(\varkappa T_e)^2\, 6 n_o K}{\omega_p^3 m\left(1-\frac{3}{2}(Kd)^2\right)} \tag{23}$$

the inequality (22) is satisfied.

The physical mechanism for this instability is easily understood: In a plasma where $N_o \neq 0$ an ion acoustic oscillation is driven by the E-field that builds up because of the electron pressure. This

E-field points from the high density region to the low density region of wave. When $N_o \neq 0$ the radiation pressure will hinder the electron motion from crest to trough of the wave. If the radiation pressure is strong enough it will overcome the electron pressure thus giving rise to an E-field in the opposite direction. This E-field will in turn force the ions towards regions of <u>higher</u> density and instability sets in. In the case where the two pressure terms cancel (equality in eq. (23)) an initial perturbation of the ions is damped solely due to ion free streaming.

We may introduce an <u>effective</u> electron temperature

$$\varkappa T_{eff} = \left(\varkappa T_e - N_o\frac{\omega_p^3 m\left(1-\frac{3}{2}(Kd)^2\right)}{6n_o\varkappa T_e K}\right) \tag{24}$$

although this definition refers to a particular choice for $N_o(k)$.

The influence of a radiation pressure on double humped ion distributions, which are unstable when $N_o=0$ is immediately realized by considering the stability diagram shown in for inst. ref. 12. This diagram shows that a distribution consisting of two ion groups with a sufficiently large velocity difference is unstable for large enough electron temperatures, T_e. Replacing T_e by T_{eff} from eq. (24) we see that the radiation pressure reduces the instability (reduces T_e), eventually stabilizes the plasma. This fact may be of importance for the study of ion acoustic instabilities in for inst. Q-machines. In these devices double humped ion distributions are created by charge-exchange processes and the electron temperature is increased by RF- or μ-wave heating[13,14,15]. These methods of electron heating are known to give rise to a high noise level[16] (high N_o), considerably above the level of thermal fluctuations. The effect of "radiation pressure" should therefore be taken into account in the analysis.

The condition (23) means (roughly) that the energy in the fluctuations should be larger than the thermal energy of the electrons thus rendering the instability mentioned somewhat academic. If we replace (7) by the dispersion relation for electromagnetic waves the basic physical mechanisms remains unchanged. The instability described may be observed for inst. when an intense laser beam interacts with a plasma.

Discussion

In the foregoing sections we considered the influence of high frequency short wavelength oscillations on long wavelength ion acoustic waves. The question remains whether their effect is of any importance compared with the influence of the long wavelength electron oscillations. In this connection we draw the attention to the case of a cylindrical plasma column confined by a homogeneous magnetic field, for inst. the plasma in a conventional Q-machine. Geometrical effects are negligible for ion-acoustic oscillations. The dispersion relation for electron oscillations is on the other hand drastically changed for $k \ll D^{-1}$ (D is diameter of plasma column), the dispersion curve (for the mode $m = 0$) going through $(\omega, k) = (0,0)$ instead of $(\omega, k) = (\omega_p, 0)$ as for infinite plasmas.[17] (For $D^{-1} \ll k \ll d^{-1}$ eq. 7 is still a good approximation). We may now excite electron oscillations in the plasma around the plasma frequency. (For inst. by band limited white noise around ω_p). Because of this particular dispersion relation only large k will be excited and our analysis is appropriate (with properly chosen $N_o(k)$'s). Example II of the previous section has obvious applications also for an infinite plasma. Note that the instability in example III will be stabilized as time goes since the constant input of energy in the oscillations will heat up the electrons through nonlinear processes.

It should be noted that a force similar to F in eq. (6) acts on the ions also. This force is small since $m \ll M$, but might be taken into account in a more sophisticated theory. It is worth noting, however, that this force will try to move ions from low density regions towards regions of higher density thus having a destabilizing effect (rather than stabilizing) on the instability discussed in example III of the previous section.

Finally we mention that the force on the quasi particles should properly read

$$-\frac{\partial\omega}{\partial x} = -\left(1 - \frac{3}{2}(kd)^2\right)\frac{\omega_p}{2n_0}\frac{\partial n_e}{\partial x} \tag{25}$$

rather than the expression used in eq. (15) where $\frac{\partial n_i}{\partial x}$ was inserted. This distinction is irrelevant since we later assumed $(qd)^2 \ll 1$ corresponding to the assumption of quasi-neutrality ($n_e \approx n_i$). The expression in eq. (15) facilitates the calculations. If we apply (25) we obtain:

$$\varepsilon(q,\Omega) = \frac{1}{(dq)^2}\left[1 + (dq)^2 - C_e^2 \int \frac{f_0'(v)}{v - \frac{\Omega}{q}}dv \right.$$

$$\left. - \frac{1}{2m}\int \frac{Q(k)N_0'(k)}{v_g(k) - \frac{\Omega}{q}}dk \left(\omega_p d^2 + \frac{1}{2m}\int \frac{Q(k)N_0'(k)}{v_g(k) - \frac{\Omega}{q}}dk\right)^{-1}\right] \tag{26}$$

It is easily shown that for $(dq)^2 \ll 1$ the equation $\varepsilon(q,\Omega)=0$ will have the same solutions independent of which expression for $\varepsilon(q,\Omega)$ is used : (26) or (16).

Note that we nowhere in the calculations make use of resonance conditions as $k_1 = k_2 + k_3$ and $\omega_1 = \omega_2 + \omega_3$. These conditions can be satisfied for the dispersion relation mentioned earlier in this section if we let for inst. (ω_2, k_2) represent an ion acoustic oscillation.

Generalization and Conclusion

The analysis in the preceding sections was based on the dispersion relation (7) for the high frequency oscillations. We emphasize that the physical arguments holds equally well if electromagnetic oscillations are considered. The expressions for F in eq. (6) remains unchanged[1,8] and eq. (7) should be replaced by

$$\omega = \sqrt{\omega_p^2 + c^2k^2} \tag{27}$$

Then

$$v_g = kc^2(\omega_p^2 + c^2k^2)^{-1/2}$$

and

$$\frac{\partial\omega}{\partial x} = \omega_p^2(\omega_p^2 + c^2k^2)^{-1/2}\frac{1}{n_0}\frac{\partial n}{\partial x}$$

The only important change is that now $k \perp E_k$. It is immediately realized (compare with the discussion p. 8-9) that a sufficiently strong radiation may drive an ion acoustic instability. Self-focusing of an intense laser beam is a related effect but does not require ion motion. This effect is easily understood using the dielectric function $\varepsilon = 1 - (\omega_p/\omega)^2$. A decrease in electron density (caused by the radiation pressure) is accompanied by an increase in ε thus causing the self-focusing. An intense beam of electromagnetic radiation with a large cross section (much larger than the Debye-length) and a uniform energy distribution is thus unstable when passing through a plasma ("filamentation"). The theory for these phenomena is well established (e.g. ref. 18,19).

We would like to point out that the previous analysis also applies to the influence of high frequency ion oscillations (around ω_{pi}) on the propagation on low frequency, long wavelength-ion acoustic oscillations. We make use of the following set of equations:

The ion Vlasov equation with the radiation pressure (ponderomotive force) included

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \left(\frac{e}{M}E - \frac{e^2}{2\epsilon_0 M}\frac{\partial}{\partial x}\int \frac{N_k}{\omega_k}dk\right) f_0'(v) = 0 \tag{28}$$

The variation of N_k is given by

$$\frac{\partial N_k}{\partial t} + v_g\frac{\partial N_k}{\partial x} - \frac{\partial \omega}{\partial x}\frac{dN_0}{dk} = 0 \tag{29}$$

Since the frequency of the ion oscillations is low we may assume that the electrons are in equilibrium and use the Boltzmann law

$$E = -\frac{\kappa T_e}{e}\frac{1}{n_0}\frac{\partial n}{\partial x} \tag{30}$$

In (30) we have used the assumption of quasi-neutrality i.e. $n_e \simeq n_i = n$, since our assumptions require $q \ll d^{-1}$ as in the previous analysis (q being the wavenumber of the low frequency, long wavelength oscillations). For the high frequency oscillations we use the dispersion relation

$$\omega = C_e k\left(1 + (dk)^2\right)^{-1/2} \tag{31}$$

(We assume $T_e \gg T_i$ in order to make (31) meaningful. Otherwise the waves will be damped within a period of oscillation for $k \sim d^{-1}$ thus making the concept of a wavelike motion questionable). Using (31) we get

$$\frac{\partial \omega}{\partial k} \equiv v_g = C_e\left(1 + (kd)^2\right)^{-3/2} \tag{32}$$

and

$$\frac{\partial \omega}{\partial x} = \frac{1}{2} C_e k^3 d^2\left(1 + (kd)^2\right)^{-3/2}\frac{1}{n_0}\frac{\partial n}{\partial x}$$

$$\equiv A(k)\frac{\partial n}{\partial x} \tag{33}$$

Since we assume that the wavelengths of the high frequency oscillations are small ($k \sim d^{-1}$) we may replace ω_k^{-1} in (28) by ω_{pi}^{-1} so eq. (28) reduces to

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \left(\frac{e}{M}E - \frac{\omega_{pi}}{2}\frac{\partial}{\partial x}\int N_k\, dk\right) f_0'(v) = 0 \tag{34}$$

We find that the zeroes of the dispersion relation are given by

$$1 - \left(C_e^2 + \frac{\omega_{pi}}{2}\int \frac{A(k)\,N_0'(k)}{v_g - \Omega/q}dk\right)\int \frac{f_0'(v)}{v - \Omega/q}dv = 0 \tag{35}$$

This equation is formally the same as (16) with $(qd)^2 \ll 1$. Apart from the difference in $\frac{\partial \omega}{\partial k}$ and $\frac{\partial \omega}{\partial x}$ the only change is that ω_{pe} in (16) is replaced by ω_{pi}. The effect of the ponderomotive forces is therefore rather weak in this case but otherwise it gives rise to the same effects as in the case considered before, in particular it has a stabilizing effect on unstable double humped ion-distribution functions. In spite of the similarity between the two problems in a linear treatment their nonlinear behaviour is entirely different, since electron oscillations are trapped in regions of low plasma density (wave troughs) while ion acoustic oscillations around ω_{pi} will be trapped in regions of high density (wave crests). A correct treatment of these phenomena must take into account the problem of turning points in the WKB-approximation used in eq. (10). Such a treatment is of particular interest with reference to soliton stability[20]. We shall not consider this problem here.

The influence of the turbulent spectrum of electron oscillations on low frequency long wavelength drift waves is investigated by Satya and Kaw[21]. One finds also in this case that the influence of the ponderomotive forces may lead to strong modifications of the wave behaviour. In particular the dispersion properties of the drift waves are drastically altered by even moderate amounts of short wavelength electron oscillations. We expect high frequency ion acoustic oscillations to give a similar, although weaker, effect (when $T_e \gg T_i$).

Ponderomotive forces will, since they modify the linear dielectric function, also affect wave-wave interaction between the low frequency, long wavelength waves, through a modification of the coupling coefficients. This effect can be investigated by applying the theory of ref. 22 to the dielectric function (16). The coupling coefficients calculated in ref. 23 are easily modified using (24) for the case of a rectangular distribution for $N_o(k)$.

In conclusion we draw attention to the fact that the radiation pressure due to high frequency oscillations is a <u>non</u>linear effect. Fortunately it is possible to render the equations describing this effect linear by the introduction of N_k.

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