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Heuristic Approaches to a Pickup and Delivery Problem with Sequencing Constraints

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Heuristic Approaches to a Pickup and Delivery Problem with Sequencing Constraints

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Problem Definition

- Pickup and delivery problem from real life
  - pickups and deliveries are performed in separated areas/graphs, ie. all pickups lie before all deliveries
  - each order consists of one item, which has a pickup address and a delivery address
  - all orders served by the same container, no repacking
  - sequencing constraints on loading, the available loading positions form a grid on the floor of the container
  - all items uniform, no stacking

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- With only one available loading row the pickup and delivery routes must strictly obey the LIFO principle.
- With one loading row available per item the two routes are completely independent.

The Problem II

- The problem has not previously been described in literature
- It is an extension of the regular travelling salesman problem (TSP), with
  - pickups and deliveries
  - multiple loading rows (individually accessible)
- Given: A set of orders, each with a pickup and a delivery address.
- Produce:
  - pickup route
  - delivery route
  - loading plan
- Objective: Minimise travelled distance

A Mathematical Model

First variable: $x_{ij}^{\tau} = 1$ if edge (i,j) is used in graph τ, 0 otherwise.

Objective:

$$\min \sum_{i,j \in N_0, \tau \in \{\mathrm{P},\mathrm{D}\}} c_{ij}^{\tau} \cdot x_{ij}^{\tau}$$

Flow balance:

$$\sum_{i} x_{ij}^{\tau} = 1 \qquad \forall j \in N_0$$

$$\sum_{j} x_{ij}^{\tau} = 1 \qquad \forall i \in N_0$$

Precedence constraints I

Second variable: $y_{ij}^{\tau} = 1$ if the address of item i is visited prior to that of item j in graph τ, 0 otherwise.

$$
\begin{aligned}
y_{ij}^{\tau} + y_{ji}^{\tau} &= 1 && \forall i, j, \tau, i \neq j \\
y_{ik}^{\tau} + y_{kj}^{\tau} &\leq y_{ij}^{\tau} + 1 && \forall i, j, k, \tau \\
x_{ij}^{\tau} &\leq y_{ij}^{\tau} && \forall i, j, \tau
\end{aligned}
$$

Precedence constraints II

Precedences are only relevant when two items in the same row.

Third variable: $z_{ir} = 1$ if item i is placed in row r, 0 otherwise.

$$y_{ij}^{\mathrm{P}} + z_{ir} + z_{jr} \leq 3 - y_{ij}^{\mathrm{D}} \qquad \forall i, j, r$$

Finally keep track of the row assignments:

$$\sum_r z_{ir} = 1 \qquad \forall i$$

$$\sum_i z_{ir} \leq L \qquad \forall r$$

Heuristics

The mathematical model has been implemented in GAMS, which can only solve instances with up to 12 orders (typical size in real life is 33 orders for one container).

Heuristics!

- Steepest Descent
- Tabu search
- Simulated Annealing
  - Modified to take running time as parameter instead of the temperature reduction coefficient:

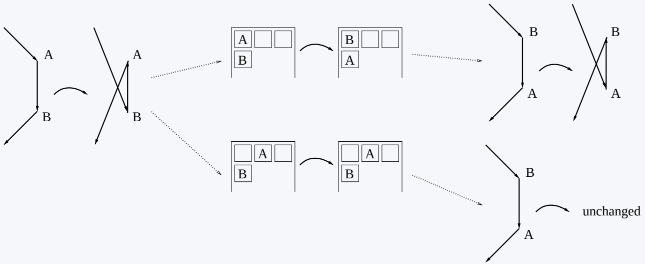
$$T(t) = T_{\text{start}} \cdot \frac{T_{\text{end}}}{T_{\text{start}}}^{\frac{t}{t_{\text{total}}}}$$

Complication: The LIFO Condition

- strict LIFO (one row problem) gives a tighter and simpler problem - the two trips must exactly be eachother's opposite:
  - add the two distance matrices
  - solve a regular TSP
- this gives an initial solution

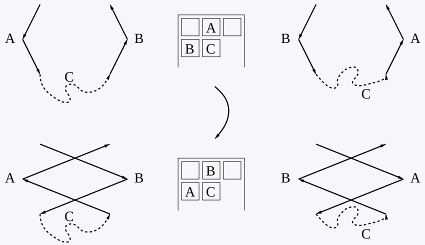
Two Different Neighbourhood Structures

- Change routing, keep row assignment
  - swap any pair of customers that are adjacent in either route
  - if they are in the same row, also swap in opposite route (loading positions will be swapped)
  - if they are in separate rows, nothing else needs to be changed (loading positions will be unchanged)



Two Different Move Structures II

- Swap rows
  - swap the loading positions of any two items that are in separate rows
  - also swap their positions in both routes



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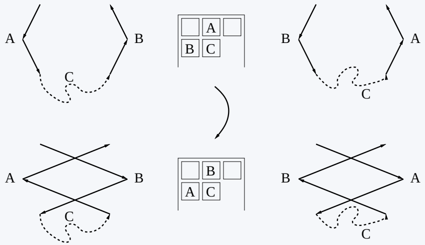
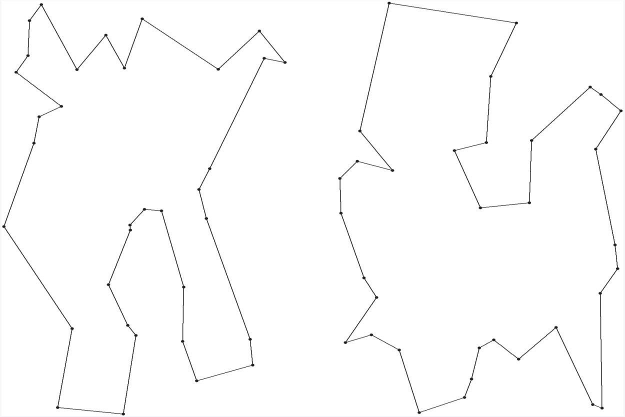
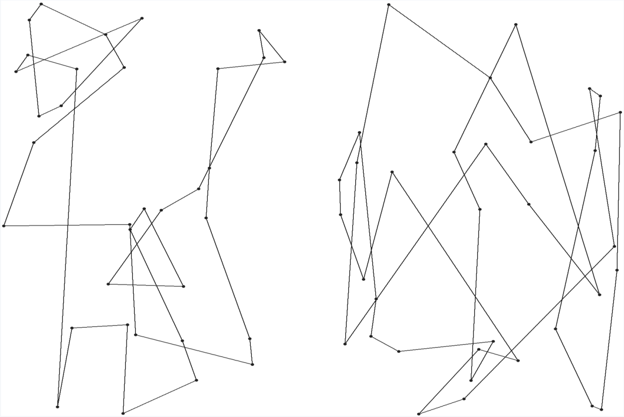


- In the end both moves are necessary to cover the solution space

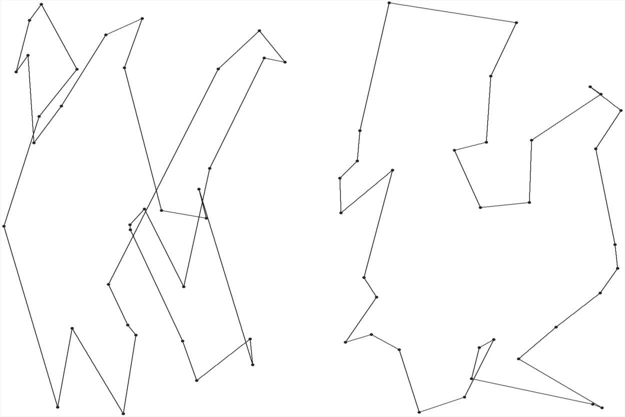
TSP Solution



Initial Solution (Savings Algorithm)



Best Heuristic Solution



Summary

- The real-life version of the problem has 3 rows and $n = 33$ orders.
- GAMS can solve to optimality for up to 12 orders (in 30-45 minutes). (15 orders: 945 v., 8400 c.; 33 orders: 4,500 v., 80,000 c.)
- A feasible solution can be obtained by adding the distance matrices of the graphs and solving TSP (ie. strict LIFO).
- A (weak) lower bound can be obtained by solving a TSP in each graph (ie. relaxing LIFO conditions).

Results of Heuristics

- Testing has been done on 10 randomly generated instances with 33 orders.
- Simple steepest descent does not give acceptable results (75% above best known solution).
- Simulated Annealing outperforms Tabu Search.
- Solution not very stable - with running times around 10 minutes solutions vary within 3-20% above best known solution.
- Best results from SA (ie. best known) are 10-20% above the "independent TSPs" lower bound

Results

Best feas.	LB	LB-ratio	Init.	Init.-ratio
1092	914	1.19	1813	1.66
1066	875	1.22	1747	1.64
1106	935	1.18	1645	1.49
1128	961	1.17	1818	1.61
1097	933	1.18	1700	1.55
1038	898	1.16	1606	1.55
1127	998	1.13	1757	1.56
1162	962	1.21	1816	1.56
1117	976	1.15	1746	1.56
1100	982	1.12	1669	1.52

Further work

- Examine the effects of:
  - the number of loading rows (one row and n rows are trivial)
  - having several items in one order, ie. sharing pickup and delivery adresses
- Expand the problem:
  - multiple vehicles
  - multiple depots
- Solve to optimality

Thank you for your attention.

Questions?