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Modelling Zonal Pricing Design under Uncertainty in Electricity Markets

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Abstract

In deregulated electricity markets with zonal pricing the market is partitioned into a number of zones, each of which is assigned a market price to which market participants react at any given point in time . We discuss the problem of designing such zones for a market subject to uncertainty. A two-stage stochastic program is presented and its complexity is discussed. In particular, we show that, when the stochastic parameters are independently distributed, the problem is #P-hard. Furthermore, the stochastic program contains integer variables. Hence, the problem is potentially difficult to solve. This motivates a Dantzig-Wolfe reformulation of the problem based on scenario decomposition, as we conjecture that for large instances decomposing the problem will lead to more efficient solution procedures. Finally, we present a formulation ensuring spatially contiguous zones.

Keywords: Zonal pricing, Electricity market design, Transmission networks, Dantzig-Wolfe reformulation, #P-hardness

JEL Classification: C61, L11, L94

1. Introduction

Deregulated electricity markets may employ different transmission pricing mechanisms. *Nodal pricing* refers to a system with individual market prices for each physical node in the network, whereas in *zonal pricing* the network is partitioned into zones and a market price is assigned to each zone. The partitioning of the network may be based on physical characteristics of the network (e.g. capacity constraints) as well as political (national borders) and organisational divisions. In this paper we shall not delve into the discussion on nodal versus zonal pricing (see e.g. [1] for a discussion), but rather assume that a zonal pricing regime is chosen exogenously. However, we may note that

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while nodal pricing may be optimal in a perfect market, zonal pricing may offer greater transparency to market participants and a greater sense of *fairness*.

We refer to the problem of determining optimal price zones as the zonal design problem. This involves allocating each node in the tranmsission network to a particular zone. We will here assume that the number of zones is fixed. The zonal design problem for a single period with linear marginal generation cost and demand curves has been treated in [2].

In general, the resulting zonal design must be static in the short to medium term, but may be changed in the medium to long term. Johnsen et. al. [3] report in 1999 that the Norwegian zonal system may be changed on a weekly basis. The nordic market pool operator, NordPool, announced that Sweden will change from a single zone to four zones (to better reflect bottlenecks in the transmission network) in 2011 following a 17 months notice [4]. Figure 1 illustrates the current (September 2011) zonal design of the Nord Pool Spot electricity market [5]. The stochastic nature of electricity systems means that a particular zonal design must accommodate a variety of supply and load conditions in the network as well as potential line failures. This leads us to propose a stochastic version of the zonal design problem, that maximises the expected social welfare of the system. That is, the total generation cost (and potential transmission cost) except total consumer benefits is minimised.

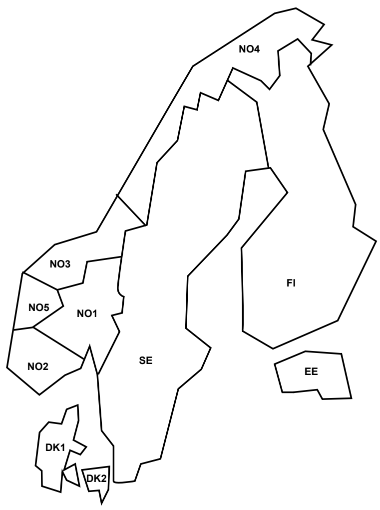


Figure 1: Schematic outline of the 10 price zones in the Nord Pool Spot market area comprising Norway (NO), Sweden (SE), Finland (FI), Estonia (EE), and Denmark (DK) as of September 2011.

Another attribute of price zones is contiguity. Often price zones are required to be spatially contiguous. That is, two nodes in the same zone must be connected by at least one path, that does not go through any other zone. When requiring that zones are contiguous with respect to the transmission network, the resulting problem is a graph clustering problem with an underlying equilibrium dispatch of electricity generation and network flow. Graph clustering problems have been studied within various applications for many years. For example, Augustson and Minker [6] explores clustering techniques for information retrieval systems.

The contribution of this paper is three-fold. Firstly, we present a linear mixed integer formulation of the deterministic zonal design problem. Secondly, a two-stage stochastic formulation based on scenario decomposition using a split variable approach (see e.g. [7]) is presented and we show that the stochastic problem is #P-hard when the stochastic parameters are independent. Thirdly, we provide a formulation that ensures spatially contiguous zones based on a minimum spanning forest formulation suggested by Martin [8] and show that this may lead to higher total generation cost.

We begin the paper by introducing the deterministic zonal design problem in section 2 and motivate the need for considering uncertainty. Subsequently, we state the stochastic version of the problem in section 3 and discuss its complexity. A Dantzig-Wolfe reformulation [9] and column generation framework for solving the stochastic problem more efficiently is suggested. In section 4 we provide a formulation ensuring spatially contiguous zones. Finally, some concluding remarks are given in section 5.

2. Model Formulation

We assume a linear direct current approximation of the optimal alternating current power flow (see e.g. [10, 11]) with linear generation costs and no line losses.

Consider the directed graph $G = (\mathcal{N}, \mathcal{A})$ with a source/sink node s. For each arc $a \in \mathcal{A}$, the cost, lower-, and upper bound on power flows, as well as reactance coefficients are given by c_a, l_a, u_a, and r_a, respectively. The flow on each arc $a \in \mathcal{A}$ is denoted by x_a, while w_i denotes the voltage phase angle for each node $i \in \mathcal{N}$. Let the set of arcs $\mathcal{F}(i)$, respectively, $\mathcal{T}(i)$ denote the set of arcs with tail, resp. , head i. Let the set of supply and demand arcs $\mathcal{S} = \mathcal{F}(s) \cup \mathcal{T}(s) \subseteq \mathcal{A}$ be defined by having s as the tail, respectively, head. Let the set of transmission arcs be denoted by $\mathcal{R} = \mathcal{A} \setminus \mathcal{S}$.

The economic dispatch of generation, consumption, and flows in the network, at any given time, may be found by solving the following linear program,

$$\min \sum_{a \in \mathcal{A}} c_a x_a \tag{1}$$

subject to

$$-x_a \geq -u_a \qquad (\lambda_a) \qquad \forall a \in \mathcal{A} \tag{2}$$

$$x_a \geq l_a \qquad (\mu_a) \qquad \forall a \in \mathcal{A} \tag{3}$$

$$\sum_{a \in \mathcal{F}(i)} x_a - \sum_{a \in \mathcal{T}(i)} x_a = 0 \qquad (\pi_i) \qquad \forall i \in \mathcal{N} \tag{4}$$

$$r_a x_a + w_j - w_i = 0 \qquad (\gamma_a) \qquad \forall a = (i,j) \in \mathcal{A} \setminus \mathcal{S} \tag{5}$$

where symbols in parenthesis denotes dual prices and in particular π is a vector of nodal prices. The objective (2) maximises total social welfare. Constraints (2) and (3) provides upper, respectively, lower bounds on the arc flows, constraints (4) ensures conservation of energy, and constraints (5) is Kirchhoff's voltage constraints.

We now introduce a set of zones $\mathcal{K}$ and we wish to restrict the market prices so that the price in two nodes belonging to the same zone is equal. Let $\rho \in \mathbb{R}^{\mathcal{N}}$ denote the vector of market prices. In a nodal pricing regime we have $\rho = \pi$.

A vector $z \in \{0,1\}^{(|\mathcal{N}|-1)|\mathcal{K}|}$ of binary variables denotes the allocation of nodes to zones, such that $z_{ik} = 1$ if and only if node i belongs to zone k. That is,

$$z_{ik} = z_{jk} = 1 \Rightarrow \rho_i - \rho_j = 0 \qquad \forall i \neq j \in \mathcal{N} \setminus \{s\}, k \in \mathcal{K} \tag{6}$$

$$\sum_{k \in \mathcal{K}} z_{ik} = 1 \qquad \forall i \in \mathcal{N} \setminus \{s\} \tag{7}$$

$$z_{ik} \in \{0,1\} \qquad \forall i \in \mathcal{N} \setminus \{s\}, k \in \mathcal{K} \tag{8}$$

Introducing some sufficiently large number M, we can rewrite (6) in a linear form as,

$$-M(2 - z_{ik} - z_{jk}) \leq \rho_i - \rho_j \leq M(2 - z_{ik} - z_{jk}) \quad \forall i \neq j \in \mathcal{N} \setminus \{s\}, k \in \mathcal{K} \tag{9}$$

We may without loss of generality allocate the first transmission node to the first zone.

For the market dispatch of generation (and consumption) to be feasible, we must ensure that if generation (consumption) is at the lower bound, then the cost is at least the price difference beween the end nodes (and profit is non-positive). Similarly, if generation (consumption) is at capacity, then the cost is at most the price difference and the corresponding profit is non-negative. Also, we must ensure that for a generator (demand segment) producing (consuming) strictly in the interval $]l_a, u_a[$ the price difference must equal the cost. Otherwise the generator (demand segment) would either increase or decrease generation (consumption). That is,

$$
\begin{aligned}
x_a = l_a \quad &\Rightarrow \quad c_a \geq \rho_j - \rho_i \quad && \forall a = (i,j) \in \mathcal{S} \\
l_a < x_a < u_a \quad &\Rightarrow \quad c_a = \rho_j - \rho_i \quad && \forall a = (i,j) \in \mathcal{S} \\
x_a = u_a \quad &\Rightarrow \quad c_a \leq \rho_j - \rho_i \quad && \forall a = (i,j) \in \mathcal{S}
\end{aligned}
$$

We can write this using the shadow prices λ_a and μ_a as,

$$
c_a - \lambda_a + \mu_a = \rho_j - \rho_i \qquad \forall a = (i,j) \in \mathcal{S} \tag{10}
$$

and complementarity constraints,

$$
0 \leq \lambda_a \perp u_a - x_a \geq 0 \qquad \forall a \in \mathcal{S} \tag{11}
$$

$$
0 \leq \mu_a \perp x_a - l_a \geq 0 \qquad \forall a \in \mathcal{S} \tag{12}
$$

In a nodal pricing scheme, constraint set (10) corresponds to the set of dual constraints associated with the flow variables x on supply and demand arcs with $\rho = \pi$. For simplicity, we may without loss of generality assume that $\rho_s = 0$.

The complementarity conditions (11) - (12) may be linearised (due to Fortuny-Amat [12]) by introducing new auxillary binary variables v_a^+ and v_a^- for each a in $\mathcal{S}$ and a sufficiently large constant M. That is, we can replace (11) - (12) by

$$
u_a - x_a \leq M v_a^+ \qquad \forall a \in \mathcal{S} \tag{13}
$$

$$
\lambda_a \leq M(1 - v_a^+) \qquad \forall a \in \mathcal{S} \tag{14}
$$

$$
x_a - l_a \leq M v_a^- \qquad \forall a \in \mathcal{S} \tag{15}
$$

$$
\mu_a \leq M(1 - v_a^-) \qquad \forall a \in \mathcal{S} \tag{16}
$$

$$
\lambda_a, \mu_a \geq 0 \qquad \forall a \in \mathcal{S} \tag{17}
$$

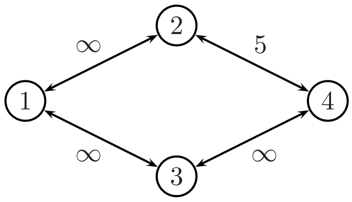
$$
v_a^+, v_a^- \in \{0, 1\} \qquad \forall a \in \mathcal{S} \tag{18}
$$

Now we can formulate the problem of finding optimal zones by minimising $\sum_{a \in \mathcal{A}} c_a x_a$ subject to the constraints $(2) - (5), (6) - (8), (10) - (12)$ or as the equivalent mixed integer linear program

$$
\min \sum_{a \in \mathcal{A}} c_a x_a \text{ s.t. } (1) - (5), (7) - (9), (10), (13) - (18)
$$

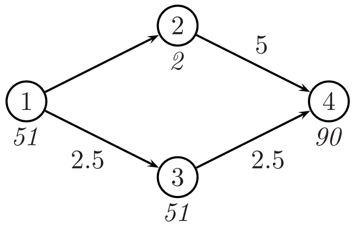
For notational convenience define the vector of binary variables $v = \begin{pmatrix} v^- \\ v^+ \end{pmatrix}$.

We will now look at a tiny instance with four transmission nodes, two generators, and two demands. The parameters are shown in Figure 2. All transmission arcs have reactance coefficient 1, zero cost and infinite capacities except for the arc from node 2 to 4, that has capacity 5. Figure 3 shows the optimal flow and prices in a nodal pricing scheme, while Figure 4 shows the flows and price for



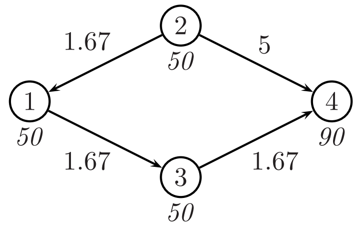
	supply		demand	
node	1	2	2	4
c	51	2	-50	-90
u	20	10	5	15

Figure 2: Small instance with four transmission nodes, two supply arcs, and two demand arcs. Arc labels $u = -l$ indicate capacities of the transmission arcs. All transmission cost and reactance coefficients are 0 respectively 1.



	supply		demand	
node	1	2	2	4
x	2.5	10	5	7.5

Figure 3: Optimal flow with nodal pricing (four zones). Arc labels indicate flows, while node labels indicate prices. Solution value is -777.50.



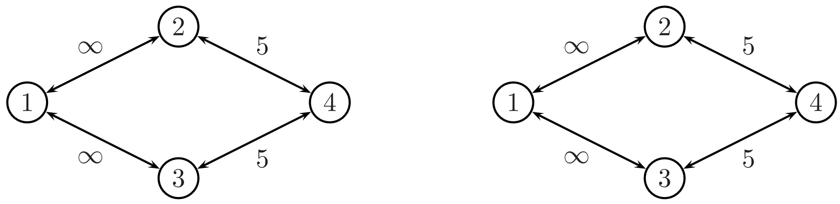
	supply		demand	
node	1	2	2	4
x	0	10	3.33	6.67

Figure 4: Optimal flow with two price zones consisting of the nodes $1, 2, 3$ respectively node 4. Arc labels indicate flows, while node labels indicate prices. Solution value is -746.67.

zonal pricing scheme with only one zone. Having a single price zone reduces the total social surplus of the system as well as the consumption in node 2.

Usually, the zonal design is static in the short to medium term, while costs and capacities may vary over time. For instance, the capacity of a wind power generator varies from hour to hour with the wind velocity etc., while the cost of generation from a natural gas turbine varies with the market price on natural gas. Also, thermal transmission line capacities may vary over the year due to temperature differences. Hence, a good zonal design must be robust to such changes. The following two-period example illustrates the problem. The parameters of the example are shown in Figure 5. The transmission network consists of four nodes and four lines.

An optimal solution with two zones for each of the two periods is depicted in Figure 6. However, these solutions dictates a dynamic zonal allocation, since the low price zone consists of node 2 and 3 in period 1, while in period 2 it



(a)

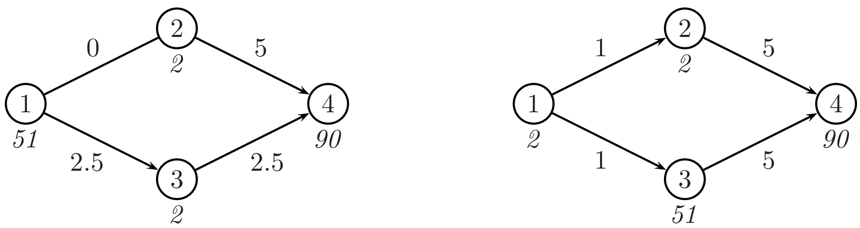
	supply		demand	
node	1	2	2	4
c	51	2	-50	-90
u	20	10	5	15

(b)

	supply			demand		
node	1	2	3	1	2	4
c	2	2	51	-50	-50	-90
u	5	10	5	3	5	15

Figure 5: Small instance with two scenarios (a) and (b) and four transmission nodes. Arc labels show transmission capacities $u = -l$. All transmission reactance coefficients are 1. The tables show supply and demand arc coefficients. Lower bound on supply and demand is 0.

consists of node 1 and 2. Also, imposing the optimal zonal design obtained for period 1 will yield a suboptimal flow for period 2.



(a)

	supply		demand	
node	1	2	2	4
x	2.5	10	5	7.5

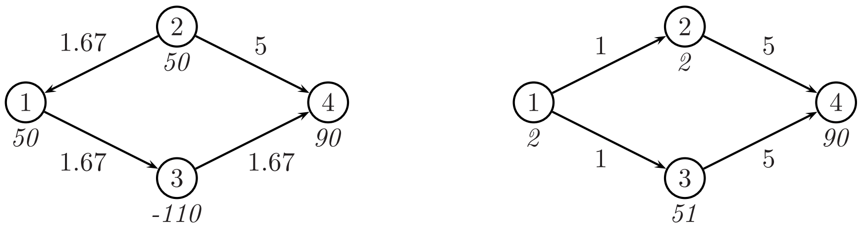
(b)

	supply			demand		
node	1	2	3	1	2	4
x	5	9	4	3	5	10

Figure 6: Resulting flows and prices when the scenarios are optimised separately with 3 price zones. Arc labels indicate transmission flows, while node labels indicate prices. Scenario (a) has an optimal cost of -777.50, while scenario (b) has optimal cost -1068.00.

When we require the zonal allocation to be identical in the two scenarios, the total surplus decreases in scenario (a), while it remains the same in scenario (b). This is due to a reduction of consumption in node 2 and 4 and a reduction of generation in node 1. The result is shown in Figure 7

Based on these observations the optimal design of zones is not obvious. In the following section, we propose a two-stage stochastic programming formulation for the zonal design problem minimising the total expected cost over a number of scenarios.



	supply		demand	
node	1	2	2	4
x	0	10	3.33	6.67

(a)

	supply			demand		
node	1	2	3	1	2	4
x	5	9	4	3	5	10

(b)

Figure 7: Resulting flows and prices when the scenarios are co-optimised with 3 price zones. Arc labels indicate transmission flows, while node labels indicate prices. Scenario (a) has an optimal cost of -746.67, while scenario (b) has optimal cost -1068.00.

3. A Two-Stage Stochastic Model

Now, we extend the problem of identifying optimal zones to a stochastic setting in which costs and capacities are not constant.

Let $z_{ik}(\omega)$ denote a *request* for node i to belong to zone k in scenario ω.

Consider a two-stage stochastic model, where the first stage decisions determine the zonal design, while the second stage models operational decisions (x, w, z, v) of dispatch and zonal allocation requests in a number of scenarios $\omega \in \Omega$, each occurring with probability $p(\omega)$. For each scenario $\omega \in \Omega$ let

$$\mathcal{Q}(\omega) = \{(x(\omega), w(\omega), z(\omega), v(\omega)) \mid (2) - (5), (7) - (9), (10), (13) - (18)\} \tag{19}$$

be the set of feasible dispatch (and zonal design) solutions for a particular scenario ω.

The problem of identifying an optimal zonal design may now be formulated as

$$\text{SZDP:} \quad \min \quad \sum_{\omega \in \Omega} p(\omega) c(\omega)^\top x(\omega) \tag{20}$$

$$\text{s.t.} \quad z(\omega) = y \qquad \forall \omega \in \Omega \tag{21}$$

$$(x(\omega), w(\omega), z(\omega), v(\omega)) \in \mathcal{Q}(\omega) \qquad \forall \omega \in \Omega \tag{22}$$

The objective (20) minimises the expected operational costs, while (21) ensures that zone allocation is static (over all scenarios).

3.1. Complexity

Two-stage stochastic programs are in general #P-hard even when efficient algorithms exist for solving the single scenario problem. This is shown by Dyer and Stougie by reduction from the graph reliability problem for discrete probability distributions and by reduction from the *volume of a knapsack polytope problem* for continuous probability distributions [13].

In the following we show that the stochastic zonal design problem is also #P-hard, which motivates the decomposition of the problem presented in the succeeding section. The proof relies on the stochastic parameters being independently distributed, which will lead to an exponential number of scenarios.

In many practical cases this may not hold. E.g. if the stochastic parameters represent capacity of wind turbines or level of water in hydro reservoirs across the network, these are likely to be highly correlated.

Define the graph $G' = (\mathcal{N}', \mathcal{R})$, where $\mathcal{N}' = \mathcal{N} \setminus \{s\}$. Consider the problem of finding a path from i to j in G' following a random event which renders each arc in $\mathcal{R}$ unusable with probability 1/2 corresponding to the failure of a transmission arc. Furthermore, assume that arc failures are independently and identically distributed.

This corresponds to the following two-stage stochastic program SP with $2^{|\mathcal{R}|}$ scenarios, where each scenario ω in Ω corresponds to an outcome of the random event occuring with equal probabilities $p(\omega) = (1/2)^{|\mathcal{R}|}$.

$$\text{SP:} \quad \max \quad Z = \sum_{\omega \in \Omega} p(\omega) x_{a'}(\omega) \tag{23}$$

$$\text{s.t.} \quad 0 \leq x_a(\omega) \leq u_a(\omega) \quad \forall \omega \in \Omega, a \in \mathcal{A} \tag{24}$$

$$\sum_{a \in \mathcal{T}(i)} x_a(\omega) - \sum_{a \in \mathcal{F}(i)} x_a(\omega) = 0 \quad \forall i \in \mathcal{N}, \omega \in \Omega \tag{25}$$

SP is obtained from SZDP by setting the number of zones to the number of transmission nodes $|\mathcal{K}| = |\mathcal{N}'|$ so that each transmission node constitutes its own price zone. This makes the zonal pricing constraints (7) - (9), and the equilibrium constraints (10), (13) - (18) redundant. Furthermore, we let the reactance coefficients $r_a = 0$ for all transmission arcs a in $\mathcal{R}$. This eliminates the constraints (5), as the flows are decoupled from the voltage phase angles. For each scenario $\omega \in \Omega$, we set the lower bound on arc flows to $l_a(\omega) = 0$. Finally, the objective function is defined by a negative unit cost on supply $c_{a'}(\omega) = -1$, and $c_a(\omega) = 0$ for all $a \neq a' \in \mathcal{A}$ and $\omega \in \Omega$.

The graph reliability problem is defined as follows [13],

Definition 3.1. *Given a directed graph G and a pair of vertices (i, j). $R_{ij}(G)$ is an instance of the graph reliability problem defined by the problem of finding the probability that i and j are connected, if each arc fails independently with probability 1/2.*

Proposition 3.1. *SP is equivalent to the graph reliability problem.*

Proof. Take any instance $R_{ij}(G')$ of the graph reliability problem on the graph $G' = (\mathcal{N}', \mathcal{R})$. Add to G' the node s and the arcs $a' = (s, i)$ and $a'' = (j, s)$

and assign to them the fixed capacities $u_{a'} = u_{a''} = 1$. For all arcs a in $\mathcal{R}$ assign random capacities u_a, that are independent and identically distributed with discrete probability distribution $p(u_a = 0) = p(u_a = 1) = 1/2$. Define a set of scenarios Ω, such that each scenario ω in Ω corresponds to an outcome of the random vector u occurring with probability $p(\omega) = (1/2)^{|\mathcal{R}|}$. Let $u(\omega)$ denote the realisation of arc capacities u in scenario ω.

Suppose, that for a realisation of arc failures in the graph reliability instance corresponding to the scenario ω, there exist a path $\mathcal{P}$ from i to j. The corresponding partial solution $x(\omega)$ to SP is constructed by letting $x_{a'}(\omega) = x_{a''}(\omega) = 1$ and $x_a(\omega) = 1$ for all a in the path $\mathcal{P}$ and $x_a(\omega) = 0$ for all remaining arcs a in $\mathcal{R} \setminus \mathcal{P}$. Similarly, if for a realisation of arc failures in the graph reliability instance corresponding to the scenario ω, there does not exist a path $\mathcal{P}$ from i to j, the corresponding partial solution $x(\omega)$ is constructed by letting $x_a(\omega) = 0$ for all a in $\mathcal{R} \cup \{a', a''\}$. The combined solution for all realisations of arc failures yields the optimal solution x^* with value Z^* being the realibility of the graph reliability instance $R_{ij}(G')$.

Conversely, an optimal solution x^* to SP will have for each scenario ω in Ω, corresponding to some realisation of arc failures in the graph reliability problem, $x_{a'}(\omega) = 1$ if and only if the graph G' contains a path from i to j and $x_{a'}(\omega) = 0$, otherwise. Hence, the optimal value Z^* is the realiability of the graph G'. ■

It follows from Proposition 3.1 and the fact that the graph reliability problem is #P-hard [14], that SP is also #P-hard. Hence, SZDP is #P-hard.

3.2. Dantzig-Wolfe reformulation

We have shown in section 3.1 that the stochastic zonal design problem is #P-hard, and hence potentially hard to solve. In the following, we provide a Dantzig-Wolfe reformulation of the problem, that allows us to decompose the problem based on scenarios and solve it using column generation and branch-and-price. We conjecture that for large instances a decomposition of the problem will lead to more efficient solution procedures.

The Dantzig-Wolfe reformulation follows in the line of [15]. Let the binary vector $z(\omega)$ define a feasible zonal design (FZD) for scenario ω if there exists $x(\omega), w(\omega), v(\omega)$ such that $(x(\omega), w(\omega), z(\omega), v(\omega)) \in \mathcal{Q}(\omega)$. Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of all FZD's for scenario ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can write any element in $Z(\omega)$ as

$$\begin{aligned} z(\omega) &= \sum_{j \in J(\omega)} \varphi^j(\omega) \hat{z}^j(\omega) \\ \sum_{j \in J(\omega)} \varphi^j(\omega) &= 1, \quad \varphi^j(\omega) \in \{0,1\}, \ \forall j \in J(\omega). \end{aligned}$$

Assume that for each feasible zonal design $\hat{z}^j(\omega)$ the corresponding optimal dispatch is given by $\hat{x}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{x}$ as

$$\text{MP: } \min \sum_{\omega\in\Omega} p(\omega) \sum_{j\in J_\omega} c(\omega)^\top \hat{x}^j(\omega)\varphi^j(\omega) \tag{26}$$

$$\text{s.t.} \quad \sum_{j\in J(\omega)} \varphi^j(\omega) = 1 \qquad [\nu(\omega)]\,,\ \forall\omega\in\Omega \tag{27}$$

$$y - \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) = 0 \qquad [\rho(\omega)]\,,\ \forall\omega\in\Omega \tag{28}$$

$$\varphi^j(\omega) \in \{0,1\}, \qquad j\in J(\omega) \tag{29}$$

$$y \in \{0,1\}^{|\mathcal{A}||\mathcal{K}|} \tag{30}$$

where $\nu(\omega)$ and $\rho(\omega)$ denote the dual prices associated with the respective constraints.

The master problem MP is a two-stage stochastic integer program with integer variables in both stages.

Proposition 3.2. *If y is chosen to be a fixed vector of binary integers, then the linear programming relaxation of MP has integer extreme points.*

For a proof we refer the reader to [15].

It is convenient to consider only a subset $Z(\omega)' \subseteq Z(\omega)$ of feasible zonal designs for each scenario ω in the master problem. We define this restricted master problem (RMP) by (26) - (30) with $J(\omega)$ replaced by $J(\omega)'$ the index set of $Z(\omega)'$. A column generation algorithm is applied to dynamically add FZD's to the linear relaxation of the master problem.

In each iteration of the algorithm, the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices ν and ρ. A new column $(p(\omega)c(\omega)^\top\hat{x}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = p(\omega)c(\omega)^\top\hat{x}^j(\omega) + \rho(\omega)^\top\hat{z}^j(\omega) - \nu(\omega)$ is negative.

A column for scenario ω may therefore be constructed by solving the subproblem:

$$\begin{array}{ll} \min & p(\omega)c(\omega)^\top x + \rho(\omega)^\top z - \nu(\omega) \\ \text{s.t.} & (x,w,z,v) \in \mathcal{Q}(\omega), \end{array}$$

where $\nu(\omega)$ and $\rho(\omega)$ are the dual prices returned from RMP-LP.

Any feasible solution $(x,w,z,v) \in \mathcal{Q}(\omega)$ with negative objective function gives rise to a potential candidate column for RMP-LP. If no columns with negative reduced cost exist then we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral then (φ^*, y^*) is an optimal solution to the master problem (26) - (30) and y^* is the optimal zonal design. Otherwise, we may resort to a branch-and-price framework for finding optimal integral solutions. Note that a fractional solution will always have at least one fractional y-value (see Proposition 3.2). Hence, we branch on one of the fractional y-variables and hope that this will resolve the fractionality. If not, one may continue branching on y-variables until the fractionality is resolved.

4. Contiguous Zones

The formulation presented so far does not restrict zones to be spatially contiguous. This means that a feasible zone may consist of nodes that are separated by nodes from another zone. In this section we provide a spanning forest formulation that requires zones to be contiguous. The formulation is based on the minimum spanning tree formulation by Martin [8].

Let $\mathcal{H}_{\mathcal{K}}$ be a spanning forest of $|\mathcal{K}|$ trees on the graph $(\mathcal{N} \setminus \{s\}, \mathcal{A} \setminus \mathcal{S})$. We can now replace (6)-(8) by

$$a \in \mathcal{H}_{\mathcal{K}} \Rightarrow \rho_i - \rho_j = 0 \qquad \forall a = (i, j) \in \mathcal{A} \setminus \mathcal{S} \tag{31}$$

Let χ be binary vector defining a spanning forest on the transmission network. The following is due to Martin [8]. Arc a belongs to $\mathcal{H}_{\mathcal{K}}$ if and only if $\chi_a = 1$ and (χ, q) is a feasible solution to,

$$\sum_{a \in \mathcal{R}} \chi_a = |\mathcal{N}| - |\mathcal{K}| - 1 \tag{32}$$

$$\chi_a = q_{hij} + q_{hji} \qquad \forall h \in \mathcal{N}, a = (i, j) \in \mathcal{R} \tag{33}$$

$$\sum_{j \neq h} q_{hhj} \leq 0 \qquad \forall h \in \mathcal{N} \tag{34}$$

$$\sum_{j \neq i} q_{hij} \leq 1 \qquad \forall h \neq i \in \mathcal{N} \tag{35}$$

$$\chi_a, q_{hij}, q_{hji} \; \{0, 1\} \qquad \forall h \in \mathcal{N}, a = (i, j) \in \mathcal{R} \tag{36}$$

where q is vector of binary auxillary variables.

We can now write (31) as,

$$-M(1 - \chi_a) \leq \rho_i - \rho_j \leq M(1 - \chi_a) \qquad \forall a = (i, j) \in \mathcal{A} \setminus \mathcal{S} \tag{37}$$

Example

For the purpose of illustration we consider in the following a single scenario instance of the zonal design problem on a network with 13 transmission nodes. The network is described in [16], however the generation data is modified to give interesting zonal designs. The topology of the transmission network is shown in Figure 8. All transmission line capacities are set to 55, that is $u_a = -l_a = 55$ for all arcs a in $\mathcal{R}$. Reactance coefficients are given in Table 1, while demand and generation is summarised in Table 2. Lower bound on generation for all generators is 0, that is $l_a = 0$ for all a in $\mathcal{S}$. We wish to find a partition of the nodes into three zones, that minimises the total generation cost of the system.

An optimal zonal design for the 13 node instance, when zones are not required to be contiguous is shown in Figure 9 with a total generation cost of 3926.77, while Figure 10 shows an optimal design when contiguity is enforced

transmission arc		reactance
from	to	r_a
1	2	0.1515
1	5	0.1515
2	5	0.1887
2	4	0.1563
2	3	0.1020
3	4	0.1333
4	5	0.1515
4	7	0.1961
4	8	0.5263
7	8	0.1695
8	9	0.1099
9	10	0.1667
6	10	0.1887
5	6	0.2326
6	12	0.2703
12	13	0.3448
6	13	0.3125
11	12	0.4545
8	11	0.2632

Table 1: Reactance coefficients for 13 node transmission network.

node	demand	supply	
i	d_i	capacity	marg. cost
1	0.0	65	10
2	77.6		
3	7.8		
4	94.7		
5	7.6	200	20
6	11.2		
7	0.0		
8	29.5	200	40
9	9.0		
10	3.5		
11	6.1		
12	13.5	200	10
13	14.9		

Table 2: Supply and demand coefficients for the 13 node network.

yielding a total cost of 4150.24. When not requiring contiguous zones the optimal solution involves generation strictly within bounds for all generators (that is, $l_a < x_a < u_a$ for all a in $\mathcal{S}$), which requires that the corresponding zonal price equals the marginal generation cost. If zones must be contiguous, this is no longer possible (with only three zones). Hence, the generation pattern is changed shifting generation to nodes with higher cost generation, yielding a solution at a considerably higher cost.

5. Conclusion

In this paper, we have presented a linearised version of the stochastic zonal design problem and we have shown that when the stochastic parameters are independently distributed the problem is #P-hard. The complexity of the problem motivated a Dantzig-Wolfe reformulation based on a split variable approach. Finally, a formulation ensuring spatially contiguous zones based on a spanning forest is provided.

The Dantzig-Wolfe reformulation is prone to the symmetry of the zonal requests and we do not expect a column generation algorithm based on this formulation to be efficient unless this symmetry is broken. A similar Dantzig-Wolfe reformulation for the stochastic model with contiguous zones may be deduced based on using tree variables as master problem variables. However, this construction exhibits similar symmetry problems, as many trees may represent the same zone. One approach may be to define linking constraints between scenarios based on the arcs belonging to the cuts between zones as these will be uniquely determined.

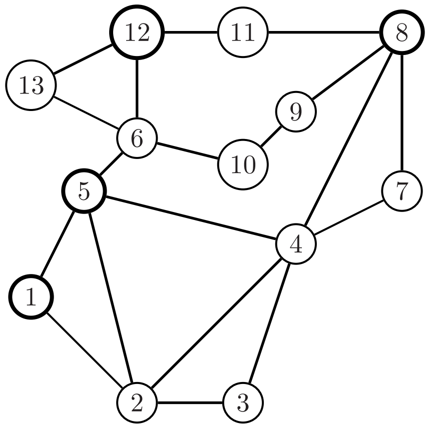


Figure 8: 13 node network with four generators located in the four emphasised (bold) nodes (1,5,8,12). All transmission line capacities are $u_a = -l_a = 55$. Supply arc capacities are all 200 except for the arc into node 1 which have $u_a = 65$.

Two-stage stochastic programs with integer variables are in general hard to solve due to both the non-convexities and potential explosion in the number of scenarios (as shown). However, it remains to be shown whether the integrality constraints yields the problem NP-hard. In practice, scenarios may be correlated and the resulting zonal design problem may not be #P-hard. Hence, further research should be dedicated to computational experiments to verify the efficiency of an algorithm based on the Dantzig-Wolfe reformulation to practical instances.

If, indeed, the stochastic problem is difficult to solve for large instances with many scenarios, further research should be directed towards stronger formulations of the stochastic zonal design problem.

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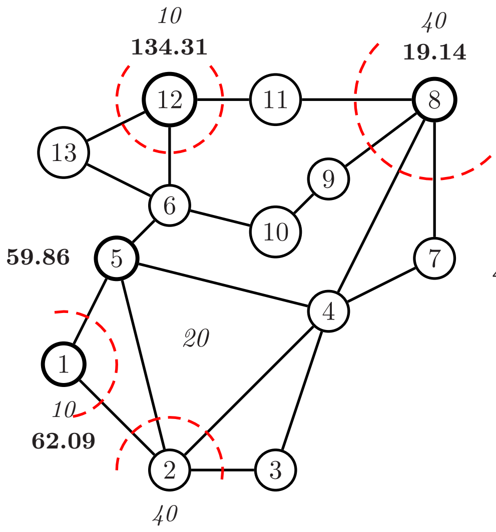

Figure 9: Resulting zonal design with three zones when zones are not required to be contiguous. Total generation cost is 3926.77. Dashed lines indicate zonal borders and italic font zonal prices.

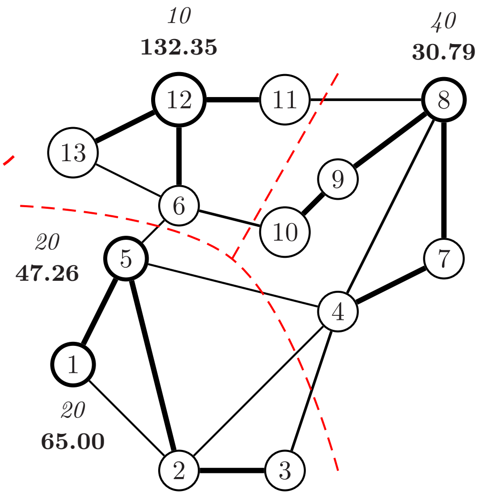

Figure 10: Resulting zonal design with 3 zones when zones must be contiguous. Bold lines indicate the forest defining the zonal design, while dashed lines indicate the corresponding zonal borders. Total generation cost is 4150.24.

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In deregulated electricity markets with zonal pricing the market is partitioned into a number of zones, each of which is assigned a market price to which market participants react at any given point in time. We discuss the problem of designing such zones for a market subject to uncertainty.
A two-stage stochastic program is presented and discussed. In particular, we show that when the stochastic parameters are independently distributed, the problem is #P-hard. Furthermore, the stochastic program contains integer variables.
Hence, the problem is potentially difficult to solve. This motivates a Dantzig-Wolfe reformulation of the problem based on scenario decomposition, as we conjecture that for large instances decomposing the problem will lead to more efficient solution procedures. Finally, we present a formulation ensuring spatially contiguous zones.

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