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A Path Based Model for a Green Liner Shipping Network Design Problem

Mads K. Jepsen, Berit Løfstedt, Christian E. M. Plum , David Pisinger and Mikkel M. Sigurd

Abstract—**Liner shipping networks are the backbone of international trade providing low transportation cost, which is a major driver of globalization. These networks are under constant pressure to deliver capacity, cost effectiveness and environmentally conscious transport solutions. This article proposes a new path based MIP model for the Liner shipping Network Design Problem minimizing the cost of vessels and their fuel consumption facilitating a green network. The proposed model reduces problem size using a novel aggregation of demands. A decomposition method enabling delayed column generation is presented. The subproblems have similar structure to Vehicle Routing Problems, which can be solved using dynamic programming.**

Index Terms—**liner shipping, network design, mathematical programming, column generation, green logistics**

I. Introduction

GLOBAL liner shipping companies provide port to port transport of containers, on a network which represents a billion dollar investment in assets and operational costs.

The liner shipping network can be viewed as a transportation system for general cargo not unlike an urban mass transit system for commuters, where each route (service) provides transportation links between ports and the ports allow for transhipment in between routes (services). The liner shipping industry is distinct from other maritime transportation modes primarily due to a fixed public schedule with weekly frequency of port calls as an industry standard (Stopford 1997). The network consists of a set of *services*. A *service* connects a sequence of ports in a cycle at a given frequency, usually weekly. In Figure 1 a service connecting Montreal-Halifax and Europe is illustrated. The weekly frequency means that several vessels are committed to the service as illustrated by Figure 1, where four vessels cover a round trip of 28 days placed with one week in between vessels. This roundtrip for the vessel is referred to as a *rotation*. Note that the Montreal service carries cargo to the Mediterranean and Asia. This illustrates that transhipments to other connecting services is at the core of liner shipping. Therefore, the design of a service is complex, as the set of rotations and their interaction through transhipment is a transportation system extending the supply chains of a multiplum of businesses. Figure 2 illustrates two services interacting in transporting goods between Montreal-Halifax and the Mediterranean, while individually securing transport between Montreal-Halifax and Northern Europe, and Northern Europe and the Mediterranean respectively. The Montreal service additionally interacts with a service between Europe and Asia, which is partly illustrated.

A. Modelling the Liner Shipping Network Design Problem (LSNDP)

The Liner Shipping Network Design Problem (LSNDP) aims to optimize the design of the networks to minimize cost, while satisfying customer service requirements and operational constraints. The mathematical formulation of the LSNDP may be very rich as seen in (Løfstedt et al. 2010), where a compact formulation along with an extensive set of service requirements and network restrictions is presented. A rich formulation like (Løfstedt et al. 2010) serves as a description of the LSNDP domain, but is not computationally tractable as the number of feasible services is exponential in the number of ports. Therefore, a formulation of the LSNDP is typically restricted to an interpretation of the domain along with the core costs and constraint structures of the problem. The LSNDP has been modelled as a rich Vehicle Routing Problem (VRP) (Baldacci et al. 2008) for instances, where transhipments are not allowed and vessels can be assumed to return empty to a single main port of a voyage in (Fagerholt 2004), (Karlaftis et al. 2009). The structure is applicable for regional liner shippers referred to as *feeder* services as opposed to global liner shipping in focus in the present paper. Models where the LSNDP is considered as a specialized capacitated network design problem with multiple commodities are found in (Reinhardt and Kallehauge 2007), (Agarwal and Ergun 2008), (Alvarez 2009), (Plum 2010). The network design problem is complicated by the network consisting of disjoint cycles representing container vessel routes as opposed to individual links. The models handle transhipments although transhipment cost is not included in (Agarwal and Ergun 2008). The vessels are not required to be empty at any time. The works of (Agarwal and Ergun 2008, Alvarez 2009) identify a two tier structure of constraint blocks: the first deciding the rotations of a single or a collection of vessels resulting in a capacitated network and the second regarding a standard multicommodity flow problem with a dense commodity matrix. The cost structure of LSNDP places vessel related costs in the first tier and cargo handling cost and revenue in the second tier. The work of (Plum 2010) has identified two main issues with solving the LSNDP as a specialized capacitated network design problem:

1) Economy of scale on vessels and the division of cost and revenue on the two tiers results in highly fractional LP solutions.
2) The degeneracy of the multicommodity flow problem results in weak LP bounds.

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Mads K. Jepsen {makj@man.dtu.dk}, Berit Løfstedt {blof@man.dtu.dk}, David Pisinger {pisinger@man.dtu.dk} and Christian E. M. Plum are with the Technical University of Denmark, Operations research group at DTU Management, Produktionstorvet, bygning 426, DK-2800 Kgs. Lyngby

Presenting and corresponding author Christian E. M. Plum {christian.edinger.munk.plum@maersk.com} and Mikkel M. Sigurd {mikkel.muhldorff.sigurd@maersk.com} are with Maersk line, Esplanaden 50, DK-1098 København K.

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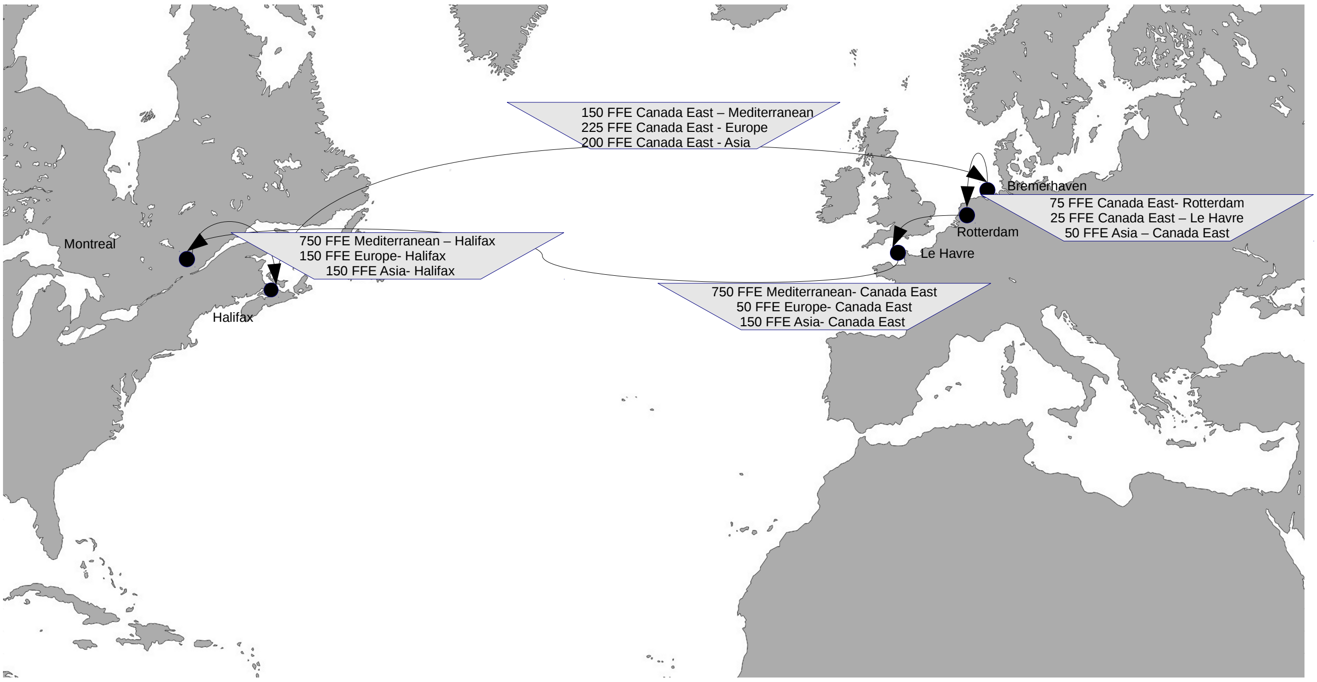

Fig. 1. A Canada-Northern Europe service. Total roundtrip time is 28 days and to provide weekly frequency, four vessels are sailing one week apart. The cargo headed for Europe is both destined for Europe, the Mediterranean and Asia. The cargo for the Mediterranean and Asia will tranship in Bremerhaven to connect to appropriate services. Vice versa cargo headed for Canada has multiple origins. FFE is Forty Foot Equivalent unit container used to express the amount of containers in each cargo category. Demands are expressed as interregional demands on the vessels in deep sea between regions and as region to distinct ports once the vessel has entered the region and is performing port calls.

Furthermore, it is well known that the linear multicommodity flow problem and hence capacitated network design problems do not scale well with the number of distinct commodities. Computational results for existing models confirm the hardness of this problem and the scalability issues, struggling to solve instances with 10-15 ports and 50-100 commodities.

The model presented in this paper has a single tier and combines revenue with total cost in the service generation problem. The motivation is to ensure efficient capacity utilization of vessels and avoid highly fractional LP solutions. Service generation is based on pick-up-and-delivery of cargoes transported entirely or partly on the service. The cost of a service reflects asset, operational and port call costs of the vessels on the service along with the the cargo handling cost and revenue of collected cargo on the service. The cargo handling cost includes load, unload and transhipment costs. The model is inspired by the Pick-up-and-Delivery VRP problem, but is considerably more complex as we allow transhipments on non-simple cyclic routes, where the vessel is not required to be empty at any point in time.

The degeneracy of the multicommodity flow problem is mitigated both by modelling the flow as assignments to services as opposed to the traditional multicommodity flow formulation, but also by exploiting the liner shipping concept of *trade* lanes to aggregate the number of distinct commodities to a minimum. Trade lanes are based on the geographic distances within a set of ports and their potential to import/export to another region.

Maritime shipping produces an estimated 2.7% of the worlds CO_2 emission, whereof 25% is accounted to container vessels according to the (World Shipping Council 2010). The value proposition of liner shipping companies has focus on the environmental impact of their operation and the concept of *slow steaming* has become a standard for some liner shipping companies (Lloyd's List 2010). (Cariou 2010) estimate that the emissions have decreased by 11 % since 2008 by slow steaming alone. Breaking down the cost of a service to each vessel (Stopford 1997) state that 35-50% of the cost is for fuel (bunker) whereas capital cost accounts for 30-45%, OPEX (crew, maintenance and insurance) accounts for 6-17% and port cost for 9-14%. Slow steaming minimizes the fuel cost, but comes at an asset cost of additional vessels deployed to maintain weekly frequency (Notteboom and Vernimmen 2009). Slow steaming is not always an option as some cargo may have crucial transit times. Current models of LSNDP assumes fixed speed on a service. The model of

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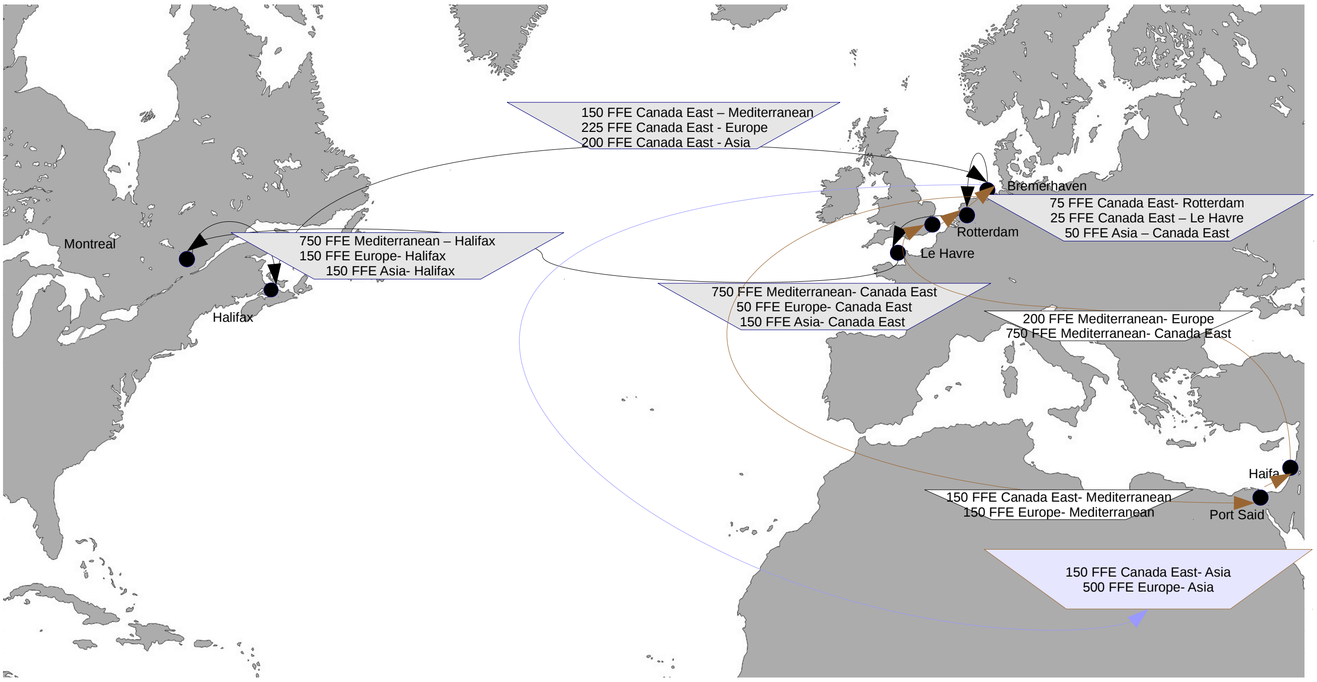

Fig. 2. Two connecting services. The Montreal service from Figure 1 and a Europe-Mediterranean service with a roundtrip time of 2 weeks illustrated by two white vessels. The cargo composition onboard vessels illustrate transhipments at the core of the liner shipping network design. The light blue incomplete service illustrates a larger service transporting cargo between Europe and Asia. FFE is Forty Foot Equivalent unit container used to express the amount of containers in each cargo category. Demands are expressed as interregional demands on the vessels in deep sea between regions and as region to distinct ports once the vessel has entered the region and is performing port calls.

(Alvarez 2009) explicitly aims at minimizing the fuel cost and consumption in the network by varying the speed of services in the model. (Løfstedt et al. 2010), (Notteboom and Vernimmen 2009) and (Fagerholt et al. 2009) state that the speed on a service is variable on each individual voyage between two ports and as the fuel consumption is a cubic function of speed (Stopford 1997) the cost calculated on an average fixed speed on a roundtrip is an approximation. As a result the actual fuel consumption of a service cannot be estimated untill the schedule is fixed. Tramp shipping often model their routing and scheduling as rich Pick-up-and-Delivery VRP problems with Time Windows (Fagerholt 2007, Korsvik 2010). (Fagerholt et al. 2009) is the first article within tramp shipping with variable speed between each port pair in the routing. The optimization of speed and hence minimizing the fuel consumption and environmental impact is driven by the time windows and the optional revenue of spot cargoes. (Fagerholt et al. 2009, Norstad 2010) report significant improvements in solutions using variable speed. Minimizing the fuel consumption of the network can be a post optimization regarding speed of the liner shipping network, when deciding on the schedule in terms of berthing windows or the transit time of individual cargo routings. The path based model assumes a fixed speed for each vessel class and in the dynamic programming algorithm the number of vessels deployed to a service is ceiled in order to ensure that a weekly frequency can be maintained on each service.

The path based model is inspired by operations research techniques within the airline industry, where the optimization is divided into faces. Therefore, a solution to the path based model is a generic capacitated network of cyclic services based on a weekly frequency of port calls. The generic network is transformed into an actual network by making an actual schedule, deploying vessels and deciding on the speed of the individual voyages and actual flow of all distinct commodities. The slow steaming speed of a vessel is 12 knots and depending on size and age a vessel has a maximal speed of 18 to 25 knots. If the fixed speed is chosen 30-40% above slow steaming speed for each vessel the ceiling of the number of vessels will allow post optimization of the schedule to achieve an energy efficient network with focus on slow steaming, while ensuring the transit time of products. The generic network allows for a green liner shipping network, while at the same time enabling scalability due to a more general description of the network.

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B. Demand Aggregation

In models of the LSNDP using a specialized capacitated network design formulation the second tier is a standard multicommodity flow problem. The work of (Alvarez 2009) identifies solving the multicommodity flow problem as prohibitive for larger problem instances due to the large number of commodities considered. The model of (Alvarez 2009) is aggregating the flow combining it by destination, giving a smaller model to solve. This could result in worse LP bounds as identified in (Croxton 2007), as the LSNDP will have a concave cost function, due to the economies of scales of deploying larger vessels, and high startup costs, as at least one vessel must be deployed.

A contribution of this paper is to formulate a model that considers aggregated aspects of the demand instead of specific origin-destination (*o-d*) pairs. This is motivated by the *trade*-centric view of liner-shipping present in the liner shipping industry instead of the *o-d*-centric view considered in the literature. As seen in Figure 1, the o-d demand from Halifax to Rotterdam could be considered, but in practice it will be hard to estimate such a specific demand. More realistically one could estimate the volume of exports from Halifax to Northern Europe and reversely the volume of imports from East Coast Canada to Rotterdam (or exports from Mediterranean to Halifax as in Figure 2). Each commodity $k \in K$ will then be characterized by a volume d^{XY} from a region X to a region Y i.e. East Coast Canada or Northern Europe as seen in Figure 1 on the vessels in deep sea. Each set of X,Y will symbolize a *trade*. Each port $p \in X$ will also have an export and import in the trade: d^{pY}, d^{Xp}, where $\sum_{p\in X} d^{pY} = \sum_{p\in Y} d^{Xp}$ as seen in Figure 1 on the vessels in a region. In effect a port as Halifax will be ensured a volume of export to Mediterranean ports and each of these will be insured a volume of imports from East Coast Canadian ports, without specifying the concrete origin-destination pairs. Note the difference in aggregation approach, compared with the models of (Croxton 2007), as we are now aggregating by *trade* origin-region to destination-region, instead of aggregation by destination port. This should give the benefit of fewer variables due to the aggregation, while we still have quite tight LP-relaxations.

The aggregation of demand may be more or less fine grained giving the ability to "zoom" according to the definition of ports, regions and trade lanes, enabling both detailed networks for a smaller region and coarse network designs for a larger set of ports that may be refined by subsequent optimization methods. We foresee a computational tractability trade-off between the number of ports and the number of distinct commodities when defining regions for ports.

In the following we will present a path-based formulation of the LNSDP and a column generation approach generating capacitated cyclic rotations with assigned flow. We will outline a dynamic programming algorithm to solve the pricing problem.

II. Service Based Model

In the following we introduce a model based on a combination of feasible services for each vessel class, into a generic liner shipping network solution. The service based model is based on a Dantzig-Wolfe decomposition of the model in (Løfstedt et al. 2010). Let S_v denote the set of feasible services for a vesselclass $v \in V$ and let $S = \cup_{v\in V} S_v$. Let α^{XY}_{kps} and β^{XY}_{kps} be the amount of respectively load and unload of containers from region X to region Y on the k'th visit to port p on service $s \in S$. We assume that $\alpha^{XY}_{kps} = \beta^{XY}_{kps} = 0, \forall p \notin X \cup Y \cup G^{XY}$, where G^{XY} is the set of ports where transhipments is allowed for *trade* XY. Let M_p be the maximal number of port visits to port p for each service. Furthermore, let γ_{pq} equal the number of times the service sails between ports $p \in P$ and $q \in P$. The move cost in a port p for a *trade* $XY \in K$ consist of the unload cost u^{XY}_p and load cost l^{XY}_p. For ports $p \in X$ the transhipment cost is included in the unload cost and the revenue is r^{XY}_p. For ports $p \in P \setminus X$ the transhipment cost is included in the load cost. Each vessel of vesseltype $v \in V$ has costs c_v for fuel-, crew- and depreciation of vessel value or timecharter-costs per week. The cost of vesseltype v calling a port q is c^v_q. The number of vessels used by the service is the roundtrip distance of the service divided by W^v_d, the weekly distance covered by vesseltype v at the predefined speed. This value is ceiled to ensure the vessels can complete the roundtrip at the predefined speed. The number of vessels used by the service is given as n_s. The cost of a service $s \in S$ is given as:

$$
\begin{aligned}
c_s = & \sum_{XY\in K} \sum_{p\in X} \sum_{k\in M_p} r^{XY}_p \alpha^{XY}_{kps} \\
& - \sum_{XY\in K} \sum_{p\in P} \sum_{k\in M_p} (l^{XY}_p \alpha^{XY}_{kps} + u^{XY}_p \beta^{XY}_{kps}) \\
& - c_v \left[\sum_{p\in P} \sum_{q\in P} \frac{d_{pq}\gamma_{pq}}{W^v_d} \right] - \sum_{p\in P} \sum_{q\in P} c^v_q \gamma_{pq}
\end{aligned}
$$

The model based on services then becomes:

$$\max \sum_{s\in S} c_s \lambda_s \tag{1}$$

$$\texttt{s.t}\ 0 \le \sum_{s\in S} \sum_{k\in M_p} (\alpha^{XY}_{pks} - \beta^{XY}_{pks}) \lambda_s \le d^{pY} \qquad \forall XY \in K, \forall p \in X \tag{2}$$

$$0 \ge \sum_{s\in S} \sum_{k\in M_p} (\alpha^{XY}_{pks} - \beta^{XY}_{pks}) \lambda_s \ge -d^{Xp} \qquad \forall XY \in K, \forall p \in Y \tag{3}$$

$$\sum_{s\in S} \sum_{k\in M_p} (\alpha^{XY}_{pks} - \beta^{XY}_{pks}) \lambda_s = 0 \qquad \forall p \in G^{XY}, \forall XY \in K \tag{4}$$

$$\sum_{s\in S} \sum_{p\in X\cup Y} \sum_{k\in M_p} (\alpha^{XY}_{pks} - \beta^{XY}_{pks}) \lambda_s = 0 \qquad \forall XY \in K \tag{5}$$

$$\sum_{s\in S_v} n_s \lambda_s \le |v| \qquad \forall v \in V \tag{6}$$

$$\lambda_s \in \{0, 1\} \qquad \forall s \in S \tag{7}$$

The objective (1) maximizes the profit, constraints (2) and (3) ensure that the difference between what is loaded and unloaded (unloaded and loaded) by all services in a port is positive and less than the export capacity (import capacity) of the port for the given *trade*. Constraints (4) ensure

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that the amount of containers loaded equals the amount of containers unloaded in a transhipment port and constraints (5) ensure that all containers loaded are unloaded for each *trade*. Constraints (6) ensure that the number of available vessels for each vessel class is not exceeded and the binary domain on the variables is defined by (7).

The key issue with the service based model is that the set of feasible services S can be exponential in the number of ports. Therefore it cannot be expected to solve instances of significant size. To overcome this issue we propose to write up the model gradually using delayed column generation and then solve the problem through Branch-and-Cut-and-Price. Branching is done by imposing a limit on the number of times an arc can be used by a given vessel class. When possible an enumeration technique similar to the one used within CVRP (Baldacci et al. 2008) will be used. The upper bound needed will be obtained using heuristics adapted from (Løfstedt et al. 2010).

A. *Pricing Problem*

To use delayed column generation we start by considering the reduced cost of a column (service). For each $XY \in K$ a port $p \in P$ is present in at most one of the constraints (2) to (4). Let $\omega_p^{XY}, \forall XY \in K, \forall p \in X \cup X \cup G^{XY}$ denote the duals from (2) to (4). Let δ^{XY} be the dual variables of constraints (5) and π^v are the duals of constraints (6).

For each vessel class $v \in V$ the reduced cost of a service(column) $s \in S_v$ then becomes:

$$\begin{aligned}
\hat{c}_s =& c_s + \sum_{XY \in K} \sum_{p \in X \cup Y \cup G^{XY}} \sum_{k \in M_p} \omega_p^{XY} (\alpha_{kp}^{XY} - \beta_{kp}^{XY}) + \\
& \sum_{XY \in K} \sum_{p \in G^{XY}} \sum_{k \in M} \delta^{XY} (\alpha_{kp}^{XY} - \beta_{kp}^{XY}) + \pi_v n_s \\
=& \sum_{XY \in K} \sum_{p \in X} \sum_{k \in M_p} (r_p^{XY} + \omega_p^{XY} - l_p^{XY}) \alpha_{kp}^{XY} + \\
& \sum_{XY \in K} \sum_{p \in X} \sum_{k \in M_p} (\omega_p^{XY} - u_p^{XY}) \beta_{kp}^{XY} + \\
& \sum_{XY \in K} \sum_{p \in Y} \sum_{k \in M_p} (\omega_p^{XY} - l_p^{XY}) \alpha_{kp}^{XY} + \\
& \sum_{XY \in K} \sum_{p \in Y} \sum_{k \in M_p} (\omega_p^{XY} - u_p^{XY}) \beta_{kp}^{XY} + \\
& \sum_{XY \in K} \sum_{p \in G^{XY}} \sum_{k \in M_p} (\omega_p^{XY} + \delta^{XY} - l_p^{XY}) \alpha_{kp}^{XY} + \\
& \sum_{XY \in K} \sum_{p \in G^{XY}} \sum_{k \in M_p} (\omega_p^{XY} - \delta^{XY} - u_p^{XY}) \beta_{kp}^{XY} + \\
& \pi_v n_s - c_v \left[\sum_{p \in P} \sum_{q \in P} \frac{d_{pq} \gamma_{pq}}{W_d^v} \right] - \sum_{p \in P} \sum_{q \in P} c_q^v \gamma_{pq}
\end{aligned}$$

The reduced cost can be rewritten as a cost connected to loading, unloading, sailing and a number of vessels used on a service as follows:

$$\hat{l}_p^{XY} = \begin{cases} r_p^{XY} + \omega_p^{XY} - l_p^{XY} & \forall p \in X \\ \omega_p^{XY} - l_p^{XY} & \forall p \in Y \\ \omega_p^{XY} + \delta^{XY} - l_p^{XY} & \forall p \in G^{XY} \end{cases}$$

$$\hat{u}_p^{XY} = \begin{cases} \omega_p^{XY} - u_p^{XY} & \forall p \in X \cup Y \\ \omega_p^{XY} - \delta^{XY} - u_p^{XY} & \forall p \in G^{XY} \end{cases}$$

Finally there the cost $\hat{c_v} = \pi_v - c_v$ is inferred each time the distance of W_d^v is traveled.

The $|V|$ pricing problems can then formally be formulated as the following graph problem. Given a directed graph $G^v(P^v, A^v)$, where P^v are the ports compatible with the vessel class. Each arch in A^v has a distance d_{pq} for sailing between ports p and q. Our task is to find a rotation, a load and unload pattern and the number of vessels to use, such that the cost of the service is minimal and the distance of the rotation at the fixed speed is feasible for the number of vessels chosen. Furthermore, the capacity of the vessel class (C) is not exceeded at any port visit.

The above problem has a similar structure to the pricing problems that arise in the context of Vehicle Routing (a Resource Constraint Shortest Path). Dynamic programming is the preferred solution approach for these problems due to the limited resources imposed on the solution space. The resources are: the capacity of the vessels, which is limiting the amount of containers carried out of a port, the distance of the schedules and the number of ports called can limit the length of the route and the limited number of ports where transhipment is possible can also be used to bound the length of the route. We will therefore use the dynamic programming principle and use the basic ideas surveyed in (Irnich and Desaulniers 2005).

To ensure that the number of vessels and the capacity is not exceeded we define the accumulated distance d and the vector of flows F. $F_c = \sum_{XY \in K} F^{XY}$ is the current amount of containers carried and F^{XY} is the amount of containers carried of *trade* XY. The revenue/cost t is a function of the distance d, the current flow F, the load L and the unload U, the current port p_c and the start port p_s. A state in the dynamic programming can be reached in the following three ways: We sail from another port q, we make a load or an unload in the current port. Let I_p^{XY} denote a matrix with dimension $|K| \times |P|$ and a 1 at position (XY,p). To add scalar value r to the position L_p^{XY} the notation $L + rI_p^{XY}$ is used. Similar we define I^{XY} as the unit vector with the same dimension as F. When maximizing the revenue the cost is a function of the other parameters and we obtain the following dynamic programming recursion:

$$\begin{aligned}
& t(F, L, U, d, p_c, p_s) = \\
& \max \begin{cases} \max_{q \in P \setminus \{p_c\}} t(F, L, U, d - d_{qp_c}, q, p_s) - \\ c_q^v - \left(\left\lceil \frac{d}{W_d^v} \right\rceil - \left\lceil \frac{d - d_{qp_c}}{W_d^v} \right\rceil \right) \hat{c}_v \\ \max_{XY \in K} \max_{0 \le h \le \min\{F_c, d^{pY} - L_p^{XY}\}} h\hat{l}_p^{XY} + \\ t(F^{XY} + hI^{XY}, L_p^{XY} + hI_p^{XY}, U, d, p_c, p_s) \\ \max_{XY \in K} \max_{0 \le h \le \min\{F^{XY}, d^{Xp} - U_p^{XY}\}} h\hat{u}_p^{XY} + \\ t(F^{XY} - hI^{XY}, L, U_p^{XY} +_p^{XY}, d, p_c, p_s) \end{cases}
\end{aligned}$$

A state is feasible when the start node is reached ($p_c = p_s$) and there is balance between the containers loaded and unloaded for all trades ($\sum_{p \in P}(L_p^{XY} - U_p^{XY}) = 0 \ \forall XY \in K$). The generated service is added to the service model iff the cost is greater than 0. To obtain the solution to a service the auxiliary data of what has actually been loaded and unloaded has to be stored and a mapping from L to α and from U to β creates the column entries for (un)load in the master problem.

Let T denote an upper bound on the distance.

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The running time of the algorithm can be shown to be $O((T|P|C^{|K|} \prod_{p \in X} d^{pY} \prod_{p \in G^{XY}} C)^2)$. Increasing the number of *trades* and the number of transhipment ports will increase the number of states in the Dynamic Programming algorithm. To solve practical problem instances it is therefore important to make a careful choice of the *trades* and the ports, where transhipment is allowed.

In CVRP a pseudo polynomial relaxation is used when solving the strongly NP-hard pricing problem (Baldacci et al. 2008) to reduce the practical running time of the algorithm. The method has proven to be very powerful and we therefore suggest a pseudo polynomial relaxation of our pricing problem. This relaxation can be obtained as follows: Each port is assigned the minimal load and unload cost and the bounds on the load is removed. In each port the number of different states will then be limited to $T|P||C|$ and a running time of $O((T|P|^2|C|))$ can be obtained.

As in other column generation algorithms we will not solve the pricing problem to optimality in each iteration but will stop once a sufficient amount of columns with positive reduced cost is found. An easy way to do this is to run the dynamic programming algorithm using a greedy variant adding any reduced cost column instead of the best reduced cost column.

III. Conclusion

We have presented a new model for LSNDP and presented a solution approach using column generation. Among the benefits of the proposed model is a novel view of demands in liner-shipping, which are considered on a *trade* basis. This has the advantage of both being intrinsic to the liner shipping business, giving a natural understanding, and requiring fewer variables. Additionally the proposed subproblem is related to the pricing problems in VRP where Branch-and-Cut-and-Price has been used with great success. We have shown that a pseudo polynomial relaxation can be used as bounding to solve the pricing problem in combination with heuristics and other techniques that have been effective in solving VRP problems. This encourages us to believe that the method scales well to larger instances. In the VRP context resource limitations have proven to be effective for the dynamic programming algorithms in reducing the state space. Therefore, further work with richer formulations of LSNDP, considering aspects as transit time limits on paths, draught limits in ports and other operational constraints from liner shipping might tighten the pricing problems. This will help us scale to larger instances while adding real life complexity to the model. At the time of the conference we aim to present preliminary computational results for the dynamic programming algorithm based on data from the benchmark paper of (Løfstedt et al. 2010).

References

[1] R. Agarwal and O. Ergun,"Ship Scheduling and Network Design for Cargo routing in Liner Shipping",*Transportation Science* 42, 175-196 (2008).

[2] J.F. Alvarez, " Joint Routing and Deployment of a Fleet of Container Vessels", *Maritime Economics and Logistics* 11, 186-208 (2009).

[3] R. Baldacci, N. Christofides, and A. Mingozzi. An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts. *Mathematical Programming*, 115(2):351–385, 10 2008.

[4] Pierre Cariou "Is slow steaming a sustainable means of reducing CO2 emissions from container shipping?" Euromed-Management, http://www.c4tx.org/ctx/pub/liner_co2.pdf, 2010

[5] Maersk Line Chief Operating Officer, Morten Engelstoft: "slow steaming is here to stay", Lloyd's List, 2010

[6] Christiansen, M., Ronen, D., Nygreen, B. & Fagerholt, K., "Maritime Transportation",*Handbooks in Operations Research and Management Sciences* 14, 189-284, 2007.

[7] Keely L. Croxton, Bernard Gendron and Thomas L. Magnanti, Variable Disaggregation in Network Flow Problems with Piecewise Linear Costs, *Operations research* 55, 1, 146-157, 2007.

[8] K. Fagerholt, "Designing optimal routes in a liner shipping problem", *Maritime Policy & Management*, 31, 259-268, 2004.

[9] K. Fagerholt and H. Lindstad, "TurboRouter: An interactive optimisation-based decision support system for ship routing and scheduling". *Maritime Economics and Logistics* 9, 214-233,2007.

[10] K. Fagerholt, G. Laporte & I. Norstad, "Reducing fuel emissions by optimizing speed on shipping routes", *The Journal of the Operational Research Society* 61, 523-529, 2009.

[11] M. G. Karlaftis and K. Kepaptsoglou and E. Sambracos, "Containership routing with time deadlines and simultaneous deliveries and pick-ups", *Transportation Research Part E*,45, 210-221, 2009.

[12] S. Irnich and G. Desaulniers, Shortest Path Problems with Resource Constraints, *Column Generation*, Springer, G. Desaulniers and Jacques Desrosiers and M.M. Solomon, ,2,33-65, 2005.

[13] J. E. Korsvik, K. Fagerholt and G. Laporte "A large neighbourhood search heuristic for ship routing and scheduling with split loads". Accepted for publication in Computers & Operations Research, 2010.

[14] B. Løfstedt, J. F. Alvarez, C.E.M. Plum, D. Pisinger & M.M. Sigurd, "An integer programming model and benchmark suite for liner shipping network design", *Technical University of Denmark*, Technical Report 19, 2010.

[15] I. Norstad, K. Fagerholt and G. Laporte, "Tramp ship routing and scheduling with speed optimization", *Transportation Research Part C: Emerging Technologies*, In Press, Corrected Proof, 2010.

[16] Notteboom, T.E. and Vernimmen, B., "The effect of high fuel costs on liner service configuration in container shipping", *Journal of Transport Geography* 17, 325-337, 2009.

[17] C.E.M. Plum, "The Linearized Simultaneous String-Design and Cargo-Routing Problem", *EURO 2010 conference*, Lisbon, July 2010.

[18] L. B. Reinhardt and B. Kallehauge, "Network Design for Container Shipping Using Cutting Planes",Conference on Maritime & Intermodal Logistics, Singapore, December 2007.

[19] M. Stopford, "Maritime Economics",1997

[20] World Shipping Council, "Liner Shipping and CO2 Emissions Policy", http://www.shippingandco2.org/ LinerShippingandCO2EmissionsPolicySeptember.pdf, 2010

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