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Evaluation of Response Prediction Procedures using Full Scale Measurements for a Container Ship

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Abstract

This paper deals with the analysis of recent full-scale strain measurements in the hull of a large container carrier covering several months of operation. The focus is on the real-time prediction accuracy of responses 5-15 seconds ahead of the measurements. Such results are less applicable in the operation of container carriers but are important in e.g. loading/unloading operations at sea or helicopter landings.

Three different procedures are discussed: Conditional processes with analytical estimates of the mean values and standard deviations, the autoregressive predictor method and a method based on superposition of sinusoidal components. The conditional processes do not need offline training and will be applied to measured time series in order to evaluate the accuracy of response predictions within the next 1-30 seconds. The number of measured points and the time distances between them are varied to determine the best solutions. A procedure based on 11 measured points spaced 1 sec, covering the last 10 sec of the instantaneous measured signal seems generally able to give fair predictions up to 5-10 sec ahead of the current time.

The full-scale data is provided through the EU FP7 project Tools for Ultra Large Container Ships (TULCS) project no. 234146.

Keywords

Conditional processes; auto regression, ship response; wave loads.

Introduction

Estimation of ship responses for the time $t > 0$ using the measured responses for $t \leq 0$ is important for various offshore operations as e.g. crane operations for shifting cargo between ships or mobile platforms and helicopter landings offshore. Today, numerous ship responses are often measured continuously during operation and the measurements are available real-time.

The memory effect in wave-induced hydrodynamic responses is usually quite small and of the order 30 seconds. Therefore, measurements older than this do not provide useful information for prediction of future responses; neither can the predictions more than 30 sec ahead be made better than what can be obtained just using statistical estimations under stationary conditions. The aim of the present study is evaluate conditional processes based on the autocorrelation function for the current stationary stochastic condition together with real-time measured responses taken just prior to current time. Hence, procedures needing offline training like neural networks, principal or minor component analysis and autoregressive procedures are not dealt with here except for some comments regarding similarities with the conditional processes. An interesting discussion of the offline training procedures can be found in Zhao, Xu and Kwan (2004).

The focus is thus on conditional processes, with some discussion on the auto-regressive predictor and sinusoidal decomposition. In all cases the response processes are assumed to be normal distributed stationary, stochastic processes.

Conditional Processes

Two different schemes are considered. In the first only the last measurement at $t = 0$ is used together with corresponding time derivatives of the response whereas in the second scheme a number of measured response values at different times prior to $t = 0$ are applied without using time derivatives.

Conditional Process Based on Known Current Value

Consider a normal distributed process $X(t)$ with the corresponding first and second derivate $\dot{X}(t)$ and $\ddot{X}(t)$. It is assumed that all three functions are known (measured or derived from measurements) at $t = 0$. Thereby, the associated conditional processes become normal distributed with the probability density functions:

$$\begin{aligned}
p(x(t)|x(0) = x_0) &= \varphi(x(t); \mu_0(t), \sigma_0(t)) \\
p(x(t)|x(0) = x_0, \dot{x}(0) = \dot{x}_0) &= \\
\varphi(x(t); \mu_{00}(t), \sigma_{00}(t)) & \\
p(x(t)|x(0) = x_0, \dot{x}(0) = \dot{x}_0, \ddot{x}(0) = \ddot{x}_0) &= \\
\varphi(x(t); \mu_{000}(t), \sigma_{000}(t)) &
\end{aligned} \tag{1.1}$$

Here, $\varphi(x; \mu, \sigma)$ denotes the probability density function of a normal distributed variable:

$$\varphi(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \tag{1.2}$$

The conditional mean processes $\mu_0(t), \mu_{00}(t), \mu_{000}(t)$ in Eq. (1.1) become

$$\begin{aligned}
\mu_0(t) &= E[X(t)|X(0) = x_0] = x_0 r(t) \\
\mu_{00}(t) &= E[X(t)|X(0) = x_0, \dot{X}(0) = \dot{x}_0] = \\
&x_0 r(t) + \dot{x}_0 \sqrt{\frac{m_0}{m_2}} s(t) \\
\mu_{000}(t) &= \\
&E[X(t)|X(0) = x_0, \dot{X}(0) = \dot{x}_0, \ddot{X}(0) = \ddot{x}_0] = \\
&x_0 \frac{\alpha}{\alpha-1} \left(r(t) + \frac{1}{\alpha} u(t)\right) + \\
&\dot{x}_0 \sqrt{\frac{m_0}{m_2}} s(t) + \ddot{x}_0 \frac{m_0}{m_2} \frac{1}{\alpha-1} (r(t) + u(t))
\end{aligned} \tag{1.3}$$

depending on whether the measured response x_0 at $t=0$ is used, only or if the information of the time derivatives \dot{x}_0, \ddot{x}_0 at $t=0$ are included. Due to the normal distribution the most probably response for $t > 0$ becomes the same as the conditional mean process.

The variances of the three conditional processes, Eq. (1.1), become

$$\begin{aligned}
\sigma_0^2(t) &= m_0 \{1 - r^2(t)\} \\
\sigma_{00}^2(t) &= m_0 \{1 - r^2(t) - s^2(t)\} \\
\sigma_{000}^2(t) &= m_0 \left\{1 - r^2(t) - s^2(t) - \frac{(r(t) + u(t))^2}{\alpha - 1}\right\}
\end{aligned} \tag{1.4}$$

The derivations of Eqs. (1.3)-(1.4) follow directly from the definition of the joint probability density function of dependent normal distributed variables. The first derivation of these results was given by Lindgren (1970), but several alternative derivations have since been published. Note, that the actual measurements at $t=0$ do

not appear in the expressions in Eqs. (1.4), for the standard deviations $\sigma_0(t), \sigma_{00}(t), \sigma_{000}(t)$.

In Eqs. (1.3)-(1.4) $r(t), s(t), u(t)$ are the normalized time-dependent autocorrelation function $r(t)$ and its first, $s(t)$, and second, $u(t)$, time derivative:

$$\begin{aligned}
r(t) &= \frac{1}{m_0} E[X(0)X(t)] \\
s(t) &= \frac{1}{\sqrt{m_0 m_2}} E[\dot{X}(0)X(t)] = -\sqrt{\frac{m_0}{m_2}} \dot{r}(t) \\
u(t) &= \frac{1}{m_2} E[\ddot{X}(0)X(t)] = \frac{m_0}{m_2} \ddot{r}(t)
\end{aligned} \tag{1.5}$$

Here, these quantities are expressed in terms of their definition in the time domain, but the corresponding frequency domain definitions using the spectral density $S(\omega)$ are straightforward.

The normalizations in Eqs. (1.3)-(1.5) make use of the spectral moments m_n of the motion response $X(t)$:

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega \tag{1.6}$$

Furthermore, the bandwidth parameter

$$\alpha = \frac{m_0 m_4}{m_2^2} \tag{1.7}$$

is needed. Note, that for broad-banded processes $m_4 \rightarrow \infty \Rightarrow \alpha \rightarrow \infty$. For such cases the inclusion of the second derivative, $\ddot{X}(0)$, does not add anything to the prediction. For the wave process itself this might happen, but ship responses and their first and second time derivatives will usually be more narrow-banded due to the filtering effect of the motion transfer function. However, as $r(t)$ and $u(t)$ are nearly equal but with opposite sign, the inclusion of the second derivative is only important if α is close to, but not equal to 1.

For an extremely narrow-banded process, $\alpha = 1$, with center frequency ϖ , the variance tends to zero and the response to the deterministic value

$$X(t) = x_0 \cos(\varpi t) + \frac{\dot{x}_0}{\varpi} \sin(\varpi t) \tag{1.8}$$

Numerical results for the standard deviations $\sigma_0(t), \sigma_{00}(t), \sigma_{000}(t)$ for the vertical wave-induced bending stresses in the deck amidships will be shown in the section on numerical results. Response predictions based on $\mu_0(t), \mu_{00}(t), \mu_{000}(t)$ will also be compared to measured time traces.

Conditional Process Based on a Set of Known Values

An alternative to conditioning on only the measured response value and its derivatives at $t=0$ is to use measured values for a set of time values $0 > t_1 > t_2 > \dots > t_n$ prior to $t=0$. The corresponding conditional probability density function is normal distributed:

$$p(x(t)|x(0) = x_0, x(t_1) = x_1, \dots, x(t_n) = x_n) = \varphi(x(t); \mu_n(t), \sigma_n(t)) \quad (1.9)$$

As the influence of the knowledge of x diminishes with time t measured from $t=0$, the results for $n=1$ will be considered first:

$$\mu_1(t) = E[X(t)|X(0) = x_0, X(t_1) = x_1] = \frac{\{r(t) - r(t_1)r(t-t_1)\}x_0 + \{r(t-t_1) - r(t)r(t_1)\}x_1}{1 - r^2(t_1)} \quad (1.10)$$

$$\sigma_1^2(t) = m_0 \left\{ 1 - \frac{r^2(t-t_1) + r^2(t) - 2r(t)r(t_1)r(t-t_1)}{1 - r^2(t_1)} \right\} \quad (1.11)$$

The autocorrelation function $r(t)$ is given by Eq. (1.5). It can be shown by a Taylor series expansion of both $r(t)$ and $x(t)$ that if $t_1 \rightarrow 0$, then Eqs (1.10) - (1.11) reduce to $\mu_{00}(t)$, $\sigma_{00}(t)$ given in Eqs (1.3) - (1.4).

If more terms are included, $n>1$, the result simply becomes:

$$\mu_n(t) = E[X(t)|X(0) = x_0, X(t_1) = x_1, \dots, X(t_n) = x_n] = \frac{\underline{r}^T(t)\underline{R}^{-1}\underline{x}}{\sigma_n^2(t) = m_0(1 - \underline{r}^T(t)\underline{R}^{-1}\underline{r}(t))} \quad (1.12)$$

in matrix notation with

$$\underline{r}(t) = [r(t), r(t-t_1), r(t-t_2), \dots, r(t-t_n)]^T$$

$$\underline{x} = [x_0, x_1, x_2, \dots, x_n]^T \quad (1.13)$$

$$\underline{R} = \begin{bmatrix} 1 & r(t_1) & r(t_2) & \dots & r(t_n) \\ & 1 & r(t_2-t_1) & \dots & r(t_n-t_1) \\ & & 1 & \dots & r(t_n-t_2) \\ & & & \dots & \\ & & & & 1 \end{bmatrix}$$

As previously, the standard deviation is independent of

the actual measured values x_i . For $n=1$ it is easy to show that Eqs. (1.12) - (1.13) yield Eqs. (1.10) - (1.11). For measurements taken under stationary stochastic conditions the autocorrelation matrix \underline{R} does not change and needs therefore to be calculated and inverted only once. Furthermore, the autocorrelation vector $\underline{r}(t)$ does not depend on the measured values and can therefore be re-used for each time step. Thus, the vector $\underline{y}(t)$

$$\underline{y}(t)^T \equiv \underline{r}^T(t)\underline{R}^{-1} \quad (1.14)$$

can be pre-calculated and used in each time step to determine the most probable future response:

$$\mu_n(t) = E[X(t)|X(0) = x_0, X(t_1) = x_1, \dots, X(t_n) = x_n] = \underline{y}(t)^T \underline{x} \quad (1.15)$$

Autoregressive Predictor

The discussion of the autoregressive (AR) predictor method will just focus on its relation to the conditional processes considered above. The Yule-Walker equations for the AR can be written in the present notation:

$$\begin{aligned} \text{AR: } x(t) &= \underline{a}(t)^T \underline{x}; t > 0 \\ \text{Yule-Walker: } \underline{a}(t) &= \underline{R}^{-1} \underline{r}(t) \\ \Rightarrow x(t) &= \underline{r}(t)^T \underline{R}^{-1} \underline{x} = \mu_n(t) \end{aligned} \quad (2.1)$$

Thus, the two formulations are conceptually the same. However, in the AR the time step (i.e. t) is assumed to be just the next time step and not a continuous variable as in Eq. (1.12).

From the measurements the autocorrelation $r(t)$ can be estimated. An alternative is to base the prediction on offline training. This is done by collecting N time series $\underline{x}_i = [x_0, x_1, x_2, \dots, x_n]^T; i = 1, 2, \dots, N$ from the measurements in the beginning of a stationary stochastic period. For each of these series the response $x_i(t); t > 0$ is determined with time t measured from the time where $(x_0)_i$ was measured. The error ε_i in each AR estimation is:

$$\varepsilon_i = x_i(t) - \underline{a}^T \underline{x}_i; \quad i = 1, 2, \dots, N \quad (2.2)$$

or for all series

$$\begin{aligned} \underline{\varepsilon} &= [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]^T = \underline{X}(t) - (\underline{a}^T \underline{X})^T \\ \underline{X}(t) &= [x_1(t), x_2(t), \dots, x_n(t)]^T \\ \underline{X} &= [\underline{x}_1, \underline{x}_2, \dots, \underline{x}_n] \end{aligned} \quad (2.3)$$

A least square fit for \underline{a} can be obtained by minimization of the error measure $\underline{\varepsilon}^T \underline{\varepsilon}$ with respect to \underline{a} :

$$\begin{aligned} \frac{d}{da} \{ \underline{\varepsilon}^T \underline{\varepsilon} \} = \\ \frac{d}{da} \{ \underline{X}(t)^T \underline{X}(t) - 2\underline{a}^T \underline{X}\underline{X}(t) + \underline{a}^T \underline{X}\underline{X}^T \underline{a} \} = \quad (2.4) \\ -2\underline{X}\underline{X}(t) + 2\underline{X}\underline{X}^T \underline{a} = 0 \end{aligned}$$

Thus, the best least square choice of the AR coefficients \underline{a} becomes

$$\underline{a} = \underline{a}(t) = \left(\underline{X}\underline{X}^T \right)^{-1} \underline{X}\underline{X}(t) \quad (2.5)$$

stressing again that \underline{a} is a function of the time step used.

Sinusoidal Decomposition

In this method, Chung, Bien and Kim (1990), the response $x(t); t > 0$ is estimated assuming the following form of the response:

$$\mu_s(t) \equiv x(t) = \underline{b}(t)^T \underline{x}; t > 0 \quad (3.1)$$

where

$$\underline{b}(t) = [b_{1s} \sin(\omega_1 t), b_{1c} \cos(\omega_1 t), \dots, b_{nc} \cos(\omega_n t)]^T \quad (3.2)$$

The frequency range and increments chosen $[\omega_1, \omega_2, \dots, \omega_n]$ must obviously reflect the spectral content of the response.

The coefficients b_{is}, b_{ic} can be estimated by offline training in the same way as for AR by collecting N time series $\underline{x}_i = [x_{i0}, x_{i1}, x_{i2}, \dots, x_{in}]^T; i = 1, 2, \dots, N$ from the measurements in the beginning of a stationary stochastic period. For each of these series the response $x_i(t); t > 0$ is determined with time t measured from the time where $(x_{i0})_i$ was measured. The error ε_i in each estimation is:

$$\varepsilon_i = x_i(t) - \underline{b}(t)^T \underline{x}_i; \quad i = 1, 2, \dots, N \quad (3.3)$$

or for all series

$$\begin{aligned} \underline{\varepsilon} &= [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]^T = \underline{X}(t) - \left(\underline{b}(t)^T \underline{X} \right)^T \\ \underline{X}(t) &= [x_1(t), x_2(t), \dots, x_N(t)]^T \\ \underline{X} &= [\underline{x}_1, \underline{x}_2, \dots, \underline{x}_N] \end{aligned} \quad (3.4)$$

A least square fit for $\underline{b}(t)$ can be obtained by minimization of the error measure $\underline{\varepsilon}^T \underline{\varepsilon}$ with respect to $\underline{b}(t)$ in the same way as for the AR, Eq. (2.4), and the result becomes

$$\underline{b}(t) = \left(\underline{X}\underline{X}^T \right)^{-1} \underline{X}\underline{X}(t) \quad (3.5)$$

Finally, the coefficients b_{is}, b_{ic} are obtained by dividing

the components in $\underline{b}(t)$ with the corresponding sine or cosine term. As the least square estimation depends on t it is as in the AR most convenient to choose a fixed value of t . The similarities between the AR and sinusoidal decompositions have previously been shown by e.g. From et al. (2011).

The variance follows directly from the error measure:

$$\sigma_s^2(t) = \underline{\varepsilon}^T \underline{\varepsilon} = m_0 - \left(\underline{b}(t)^T \underline{x} \right)^2 \quad (3.6)$$

with $\underline{b}(t)$ given by Eq. (3.5).

Comparisons with Measurements

The measured data collected during the TULCS project on board a 9,400 TEU container vessel is used in the present study. The main dimensions of the ship are given in Table 1.

Table 1: Main dimensions of ship.

L _{OA}	349.0 m
Beam	42.8 m
Draught	15.0 m
DWT	113,000 ton

On 2 October 2011 a severe sea state was encountered off Hong Kong. The ship was going North West with wave heights between 3 and 10 m according to the WaMoS II ® wave radar system installed on the bridge. Strain measurements covering one hour from this period are used here. Two long base strain gauges of the displacement measuring type are installed amidships in port and starboard side, respectively, as illustrated in Fig. 1.

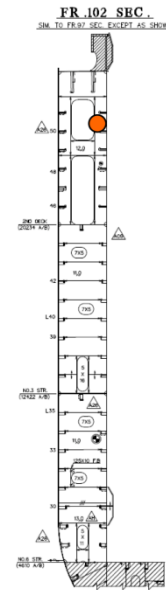


Fig. 1: Location of long base strain gauge close to the deck amidships.

The strain measurements are sampled at 20 Hz and converted into stress signals. The mean of the port and starboard signal is used to exclude the possible contributions from horizontal and torsional stress components. Further details of the measurements can be found in Andersen and Jensen (2013).

Variance estimations

The normalized autocorrelation function $r(t)$ and its time derivatives $s(t)$ and $u(t)$, Eqs. (1.5), are shown in Fig. 2 as derived from the measurements. The signal has been filtered to remove noise and high frequency vibration components as well as components with a period higher than 50 sec, as these components are not important for the present study. Please refer to Andersen and Jensen (2013) for details.

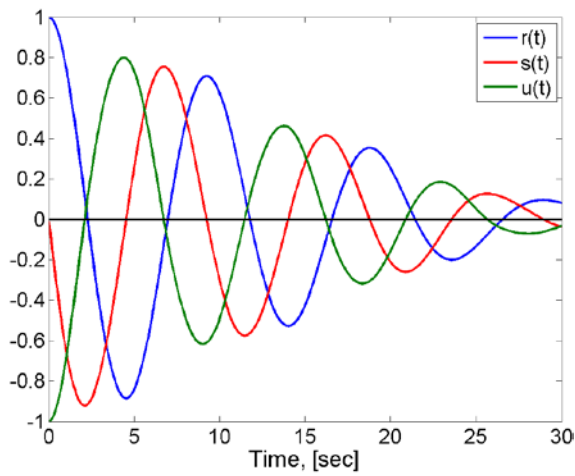


Fig. 2: Normalized autocorrelation function $r(t)$ and its time derivative $s(t)$ and $u(t)$ for the measured stress signal.

It is clearly seen that the memory time is about 30 sec implying that measurements older than 30 sec are not useful for response predictions. Consequently, response predictions more than 30 seconds ahead cannot be improved by conditioning the response on measured data.

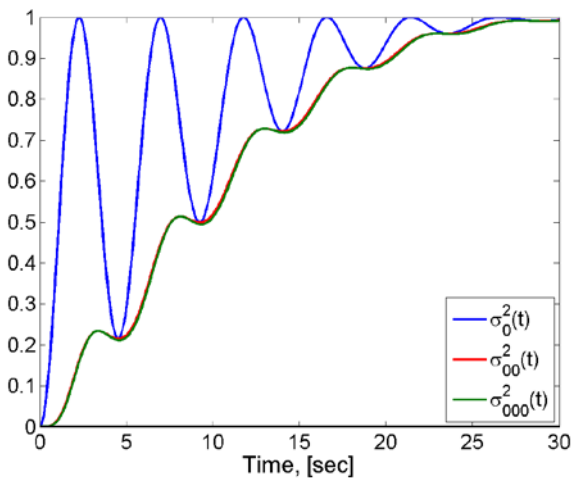


Fig. 3: Normalized variances $\sigma_0^2(t)$, $\sigma_{00}^2(t)$, $\sigma_{000}^2(t)$ of the conditional processes for the measured stresses, Eq. (1.4).

The normalized variances, Eqs. (1.4), are shown in Fig. 3 and obviously, the inclusion of information on the derivative $s(t)$ reduce the variance quite significantly as compared to the reduction using information of the response alone. However, additional information of the second derivative, $u(t)$, does not add much reduction to the variances except for narrow-banded processes, where α , Eq. (1.7), is close to 1. Here, α is found to be 3.0. Generally, the inclusion of information about $r(t)$ and $s(t)$ yields a rather smooth reduction of the unconditional variance in the range from 0 to 30 seconds. The zero-upcrossing period of the response is 8.8 sec, corresponding well with the variations shown in Fig. 3. Thus, any influence of past values has vanished from the response after approximately three times the zero-upcrossing period.

Fig. 4 shows the variance $\sigma_1^2(t)$ for four different values of the time lag t_1 . If the time lag is larger than 10 sec, it does not reduce the standard deviation, whereas for smaller values its effect is closer to the effect obtained by inclusion of the first derivative, Eq. (1.4), i.e. $\sigma_1^2(t) \rightarrow \sigma_{00}^2(t)$ for $t_1 \rightarrow 0$. It can also be noted from Fig. 4 that the reduction in variance is not monotonic decreasing with increasing time lag t_1 . For instance, the reduction using $t_1 = 7$ sec is larger than for $t_1 = 5$ sec.

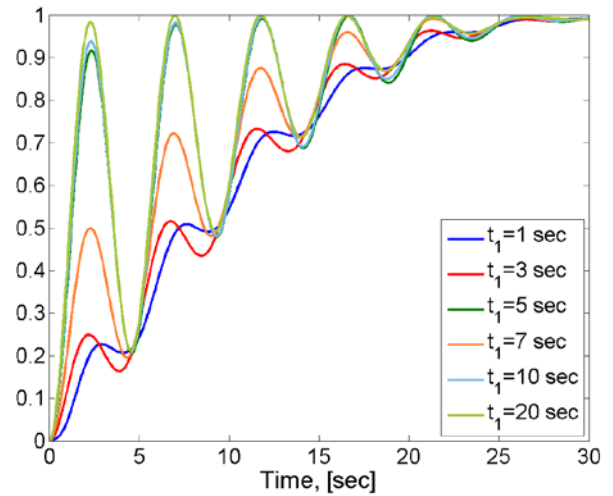


Fig. 4: Normalized variances $\sigma_1^2(t)$; $t_1 = 1, 3, 5, 7, 10$ and 20 sec determined for the conditional process for the measured stresses, Eq. (1.11).

If the response prediction is made conditional on more than one measured value the variance can be reduced as shown in Fig. 5. Here, a constant time step of 1 sec between the measured values x_0, x_1, \dots, x_n applied in the conditioning is used, and the figure shows that going from two terms (at $t = 0$ and $t_l = -1$ sec, i.e. $n = 1$) to eleven terms (at $t = 0$ and $t_l = -1$ sec, $t_2 = -2$ sec, ..., $t_{10} = -10$ sec, i.e. $n = 10$) reduces the variance to some extent.

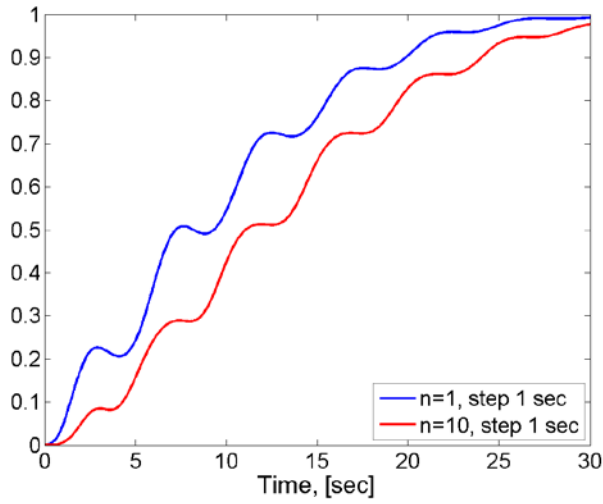


Fig. 5: Normalized variances $\sigma_n^2(t)$ for $n = 1$ and $n = 10$ with constant time step $t_i - t_{i+1} = 1$ sec, determined for the conditional process for the measured stresses, Eqs. (1.12)-(1.13).

The same result will be obtained by the autoregressive predictor (AR) method based on the Yule-Walker formulations as discussed in the previous section.

For the sinusoidal decomposition method the variance can be estimated by Eq. (3.6). However, as it needs offline training, Eq. (3.5), it will not be considered further in the present paper.

Comparison with measured stress responses

The conditional mean processes, Eqs. (1.3), (1.10) and (1.15) are shown in Fig. 6a-i together with the measured response. The time difference in the measurements between each figure is 5 sec and the time axis shown is from 10 sec before $t = 0$ to 30 sec after. Some observations are:

- Estimations based on $\mu_0(t)$, Eqs. (1.3), are not useful.
- Estimations based on $\mu_{00}(t)$ and $\mu_{000}(t)$, Eqs. (1.3), give in general good results for the first 3-5 sec. The inclusion of the second derivative, $\mu_{000}(t)$, only marginally improve the predictions.
- Estimations based on $\mu_1(t)$, Eqs. (1.10), give predictions not as good as those from $\mu_{00}(t)$.
- Estimations based on $\mu_n(t)$, Eqs. (1.12), (1.15), i.e. using measured points spaced 1 sec and taken from the last 10 sec of the measured signal are generally far better than the other predictions and give fair predictions typically up to 5-10 sec ahead of the current time.

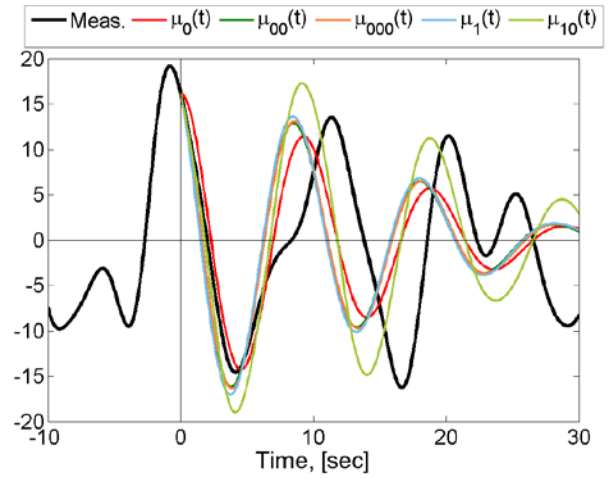


Fig. 6a: Measurements and conditional mean responses.

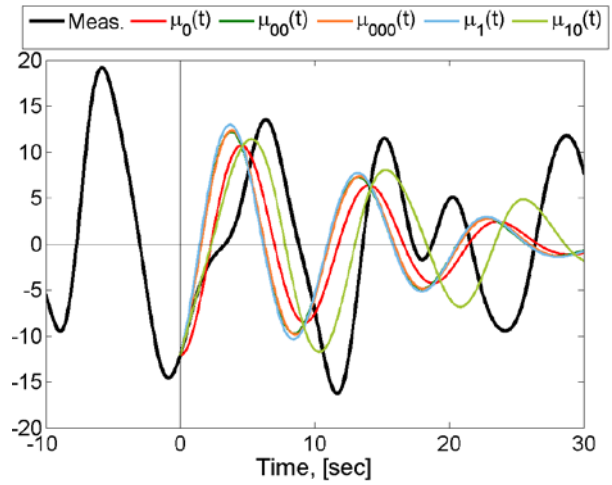


Fig. 6b: Measurements and conditional mean responses.

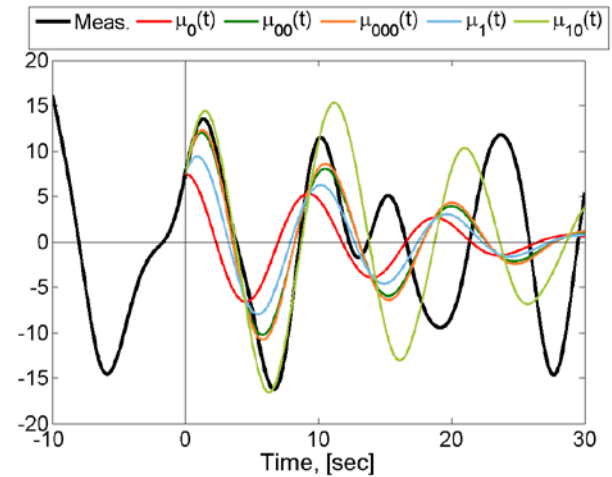


Fig. 6c: Measurements and conditional mean responses

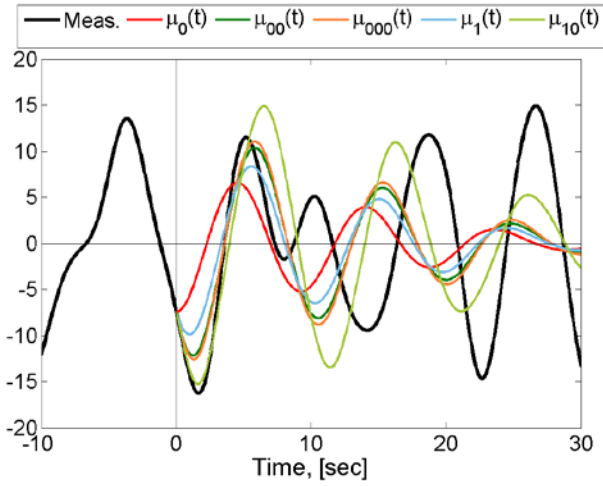


Fig. 6d: Measurements and conditional mean responses.

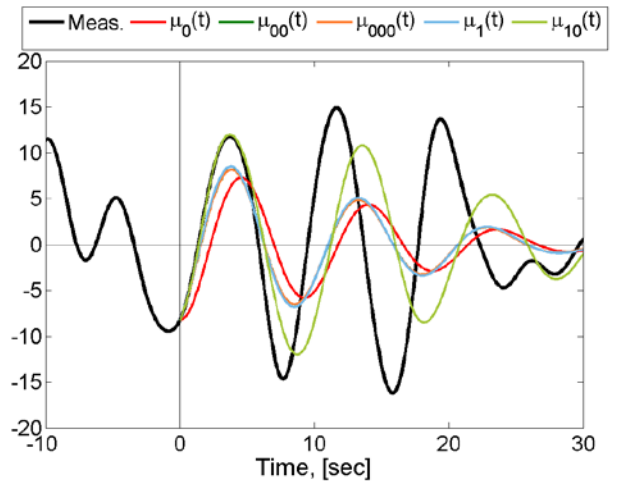


Fig. 6g: Measurements and conditional mean responses.

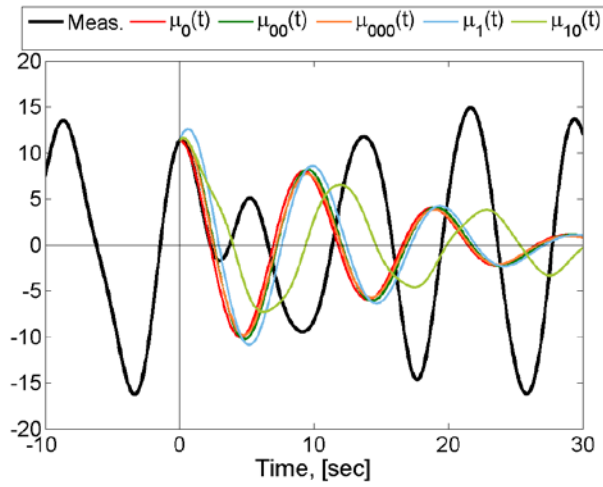


Fig. 6e: Measurements and conditional mean responses.

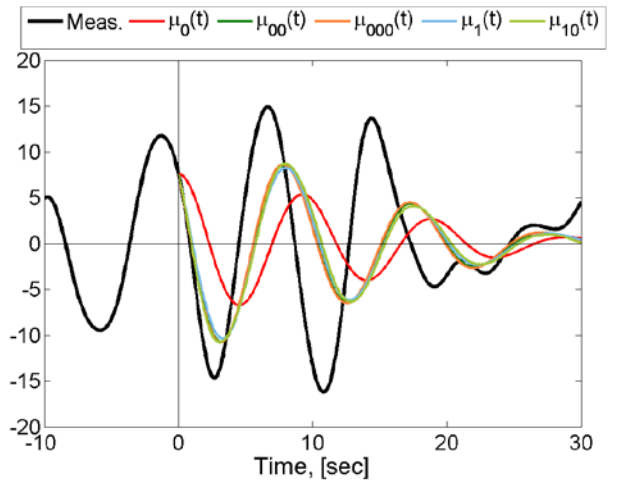


Fig. 6h: Measurements and conditional mean responses.

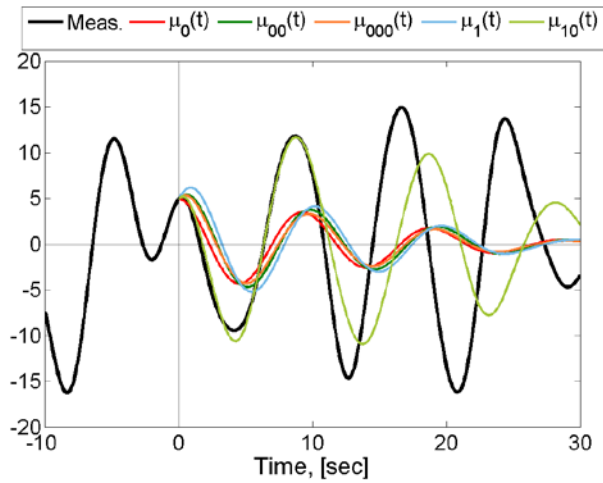


Fig. 6f: Measurements and conditional mean responses.

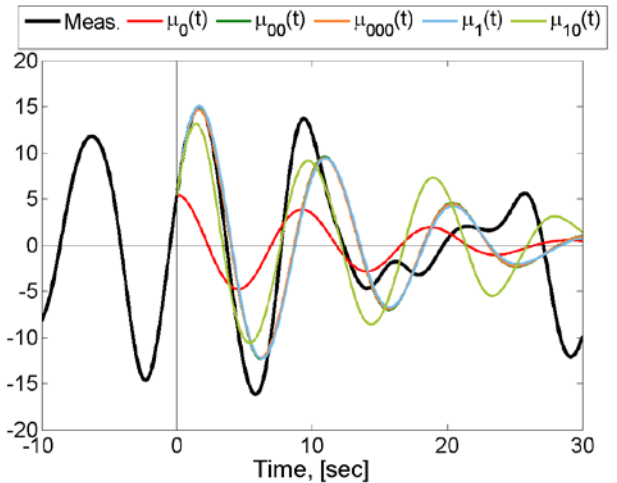


Fig. 6i: Measurements and conditional mean responses.

Conclusions

Various procedures for estimations of ship responses in waves covering the next 1-30 seconds ahead have been described and evaluated by comparison with full scale measurements of hull girder bending stresses. A procedure based on 11 measured points spaced 1 sec, covering the last 10 sec of the instantaneous measured signal seems generally able to give fair predictions up to 5-10 sec ahead of the current time.

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